Apollonius of Perga: Historical Background and Conic Sections

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Abstract

Apollonius of Perga greatly contributed to geometry, specifically in the area of conics. Through the study of the “Golden Age” of Greek mathematics from about 300 to 200 B.C., Apollonius, who lived from about 262 to 190 B.C., is seen as an innovative geometer whose theories were eventually proved factual. By investigating his theorems on how the different kinds of conic sections are formed, the standard equations for each conic will be better understood. The achievements that have been made as a result of conics include flashlights to whispering galleries.

Apollonius of Perga: Historical Background and Conic Sections

The field of mathematics oscillated between stagnate periods and periods of discovery. The first few centuries of the Greek period became known as the “Golden Age” of Greek mathematics due to the work of three well-known mathematicians: Euclid, Archimedes, and Apollonius. The work of Apollonius consisted of many areas ranging from astrology to geometry. Through the study of Apollonius of Perga during the “Golden Age”, his significant contribution to geometry can be seen, specifically in the area of conic sections.

History of Conics with Apollonius

A brief look at the era that led to the “Golden Age” of Greek mathematics consists of philosophers who were intellectually well-rounded such as Plato, Aristotle, and Eudoxus. Plato greatly influenced students to pursue the field of mathematics with the purpose of strengthening the mind. He established the Platonic Academy in Athens which promoted the need for mathematics with the motto above the entrance “Let no one ignorant of geometry enter here” (Boyer, 1968, p. 93). This school became known as the “mathematical center of the world”, and many mathematicians who later became teachers were educated here (Boyer, 1968, p. 98). A few of the students of this school included Aristotle and Eudoxus. Aristotle’s main area of study was philosophy and biology; however, his interest in mathematics is evident through his constant use of mathematical concepts to demonstrate concepts of science and philosophy (Mendell, 2004). His death marked the end of the infamous Greek period, the Helenic Age (Boyer, 1968). Eudoxus was yet another one of these great teachers and is known as the “apparent originator of the integral calculus” (Boyer, 1968, p. 102). It is evident that Eudoxus was a well-known mathematician through the works of his students. One of his students was Menaechmus who was accredited with the discovery of conic sections. These mathematical minds had a lot to do with the success of the Greek mathematicians to follow.

Before the “Golden Age” of Greek mathematics, there was information known about conic sections. Menaechmus is said to have learned through the Platonic influence (Boyer, 1968). Menaechmus discovered the curves: ellipse, parabola, and hyperbola which later become known as conic sections. He found that through the intersection of a perpendicular plane with a cone, the curve of intersections would form conic sections. The cone was constructed as a single-napped cone in which the plane was perpendicular to the axis of symmetry of the cone. Menaechmus came across this finding in search of the answer to a math problem called the Delian Problem. This answer was used to give “a simple solution to the problem of duplication of the cube” (Stillwell, 1989, p. 20). Menaechmus’ idea was accepted; however, the instrument that could construct conics could have possibly been made as late as 1000 A.D. (Stillwell, 1989).

The “Golden Age” of Greek mathematics was a time in which many great minds contributed to the field of mathematics. Minds like Euclid and Archimedes wrote a foundation on which the subject of conic sections was expounded. Euclid is known for his work the “Elements” and his contribution to fields like optics and geometry. He was well-known for his teaching ability, and his book was closer to a textbook style book than any other kind (Boyer, 1968). Archimedes “can be called the father of mathematical physics" and is known for his works "On Floating Bodies" (Boyer, 1968, p. 136). Archimedes contributed greatly to the field of naval architecture through his works.

Apollonius was born at Perga in Pamphilia, and the dates suggested for his life are 262 to 190 B.C. He studied in Alexandria for some time, and then he went on to teach at Pergamum in Pamphylia, which was modeled after the school in Alexandria (Motz, 1993). Apollonius' fellow mathematicians of the time consisted of Euclid and Archimedes who were his friendly rivals. Archimedes especially was thought of more as a rival, although Archimedes was anywhere from twenty-five to forty years older than Apollonius.

Due to the rivalry between mathematicians, their writings contain some of the same areas of study. Archimedes, in particular, shared a common subject of arithmetic calculations with Apollonius (Motz, 1993). Both Archimedes and Apollonius dealt with topics such as astrology, a scheme for expressing large numbers, and conics. Archimedes used names to describe sections of cones, and Apollonius built off of his idea and introduced the names: ellipse, parabola, and hyperbola are still used today (Gullberg, 1997). The comparison is made that, just as *Elements* by Euclid had brought an advanced way of thinking to its field, *Conics* by Apollonius did likewise for the study of the field of Conics (Boyer, 1968). Because Apollonius was so thorough, "W.W. Rouse Ball asserts that 'Apollonius so thoroughly investigated the properties of these curves that he left but little for his successors to add'" (Motz, 1993, p. 20).

Mathematicians like Aristaeus and Euclid had discussed the topic of conics in their writings; however, their knowledge on this subject was simplistic when compared to the detail of Apollonius' writings. When Apollonius introduced conic sections, he demonstrated that it was not necessary for the plane intersecting the cone to be perpendicular to it. He went further to show that it could be an oblique or scalene cone. Prior to Apollonius, the ellipse, parabola, and hyperbola were derived as sections of three distinctly different types of right circular cones. He illustrated that all three conic sections could be made from the same cone. Finally, Apollonius began using a double-napped cone instead of the single-napped cone noted earlier to better define conics (Boyer, 1968).

Apollonius is known as "The Great Geometer" because Pappus made a compilation of many of his works, as well as the works of Euclid (Boyer, 1968). What made the ideas of Apollonius so revolutionary was the instrument that could construct conics could have possibly been made as late as 1000 A.D. (Stillwell, 1989). It is intriguing to note the wealth of knowledge Apollonius was able to contribute to the mathematical world when many of his works have disappeared. He wrote an eight book series called *Conics*. There are seven books that have survived; four of which are in the original Greek translation with the other three being in an Arabic translation (Heath, 1896).

Conic Sections

Classic Definition of Conic Sections

Conics are the name given to the shapes that are obtained by a cone intersected by different planes. The seven different kinds of conic sections are a single point, single line, pair of lines, parabola, ellipse, circle (which is a special kind of ellipse) and hyperbola. Apollonius discovered that each of these sections can be acquired by different planes intersecting the same kind of cone. The conics that are found by the intersection of a cone through its vertex are called degenerate conic sections. The conic sections that fit under this classification are the single point, single line, and pair of lines. On the other hand, the rest of the conics (parabola, ellipse, and hyperbola) are classified as non-degenerate curves.

Non-degenerate curves have similar features that contribute to the formation of the standard form equation used to represent them. The eccentricity of a non-degenerate conic is used to distinguish the different kinds of conic sections. Each of these conic sections has one or two foci, a directrix and an eccentricity. The generic definition of these conics is the set of points P in the plane that satisfy the following condition: the distance of P from a fixed point (or focus) that is a constant multiple (or eccentricity) of the distance of P from a fixed line (or directrix) (Brannan, 1999). Figure 1 and 2 illustrate an ellipse that is formed by a plane intersecting a double-napped cone; the other conics are formed in a similar manner by different planes intersecting the cone. The first graph is at an angle to show the plane and the second graph is at another angle to show the ellipse formed by the intersection.

Figure 1. Side view of conic.

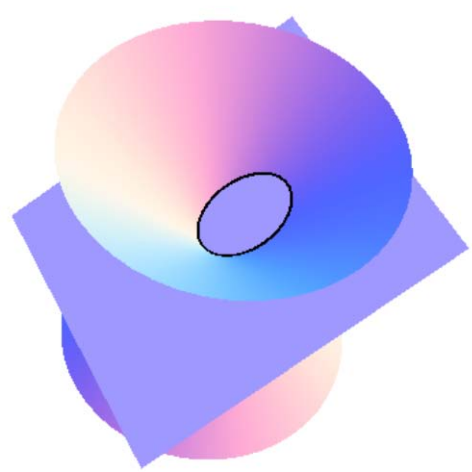

Figure 2. Top view of conic.

Affine Classification of Conics

The non-metric classification of conics in Affine Geometry can be obtained algebraically. The Affine classification of conics is solely based on shapes; there is no measure of length or angles. Through the process of completing the square and simplification of equations a conic section can be classified. These equations that describe conics have the form:

$$Ax^2 + 2Bxy + Cy^2 + 2Dx + 2Ey + F = 0 \tag{1}$$

The quadratic part of this equation is: $Ax^2 + 2Bxy + Cy^2$. The other portion is referred to as the linear part. Through the process of completing the square and simplification, the coordinate system is subsequently changed to a reduced form of itself; thus forming a conic section. There are three different kinds of polynomials to which the equation can be reduced. Assuming that A and C cannot both be zero:

$$Ax^2 = 0 \text{ or } Ax^2 + Cy^2 = 0 \tag{2}$$

$$Ax^2 + F = 0 \tag{3}$$

$$Ax^2 + Cy^2 + F = 0 \tag{4}$$

$$Ax^2 + 2y = 0 \tag{5}$$

Equations (2) and (3) represent degenerate conics: a single point, a line, or a pair of lines. In order to distinguish between the non-degenerate conics, the discriminant of the quadratic part is used. The expression for the discriminant is as follows:

$$\delta = AC - B^2 \tag{6}$$

Then, the kind is determined by the following:

1. It is an ellipse if δ< 0. The equation can be reduced to: $x^2 + y^2 - 1 = 0$.
2. It is a hyperbola if δ> 0. The equation can be reduced to: $x^2 - y^2 - 1 = 0$.
3. It is a parabola if δ= 0.The equation can be reduced to: $x^2 + 2By = 0$.

This process is illustrated by the following example problem:

$$x^2 - 2xy + y^2 + 2x - 3y + 3 = 0 \tag{7}$$

The first step is to group in order to complete the square:

$$\left(x^2 - 2x(y-1) + \_\_\right) + y^2 - 3y + 3 = 0 \tag{8}$$

The term that must fill in the blank is found according to completing the square method. It is (y-1) and is squared and then added inside and subtracted outside of the parenthesis in order to preserve the equation through additive identity.

$$\left(x^2-2x(y-1)+(y-1)^2\right)-(y-1)^2+y^2-3y+3=0 \tag{9}$$

Then, the polynomial inside the big parenthesis is factored and the terms on the right are simplified according to like terms.

$$(x-(y-1))^2-y+2=0 \tag{10}$$

Finally, to change the coordinates to better identify this equation, let $X=(x-y+1)$ and let $Y=-\frac{1}{2}y+1$. So the resulting reduced equation is:

$$X^2+2Y=0\,. \tag{11}$$

This equation can be recognized as a parabola. Because this is Affine Geometry, there are no metric properties that represent distance, only the classification can be found through this method. The discriminant can be found at any time during this process to determine the classification. In this example, it is used at the end to verify the previous work. The coefficients used to find the discriminant are: A=1, C=1, and B=1, resulting with:

$$(1)(1)-(1)^2=0\,. \tag{12}$$

Conics through Euclidean Geometry: Obtaining the Standard Form of a Conic from Geometry to Algebra

The parabola. The definition of a parabola is “the set of points P in the plane whose distance from a fixed point F is equal to their distance from a fixed line d” (Brannan, 1999). Let it be noted that unlike in Affine Geometry, in Euclidean the definition of the parabola contains a distance as a defining factor. A parabola has an eccentricity of one and has one focus. To simplify calculation, two conditions will be assumed: the focus F is on the x-axis and has the coordinates: (a, 0) and the directrix d is the line with the equation x = -a. In this case, it is evident that the vertex of the parabola is at the origin because it is halfway between the focus and directrix. In order to find the standard form equation for a parabola, let P be an arbitrary point on the parabola such that P = (x, y). Let the foot of the perpendicular from P to the directrix be M. Next, by definition of parabola FP = PM, then $|FP|^2 = |PM|^2$. Using the Pythagorean Theorem, the previous equation can be rewritten as:

$$|FP|^2 = (x-a)^2 + y^2 = (x+a)^2 = |PM|^2 \tag{13}$$

Equation 13 is illustrated in Figure 3.

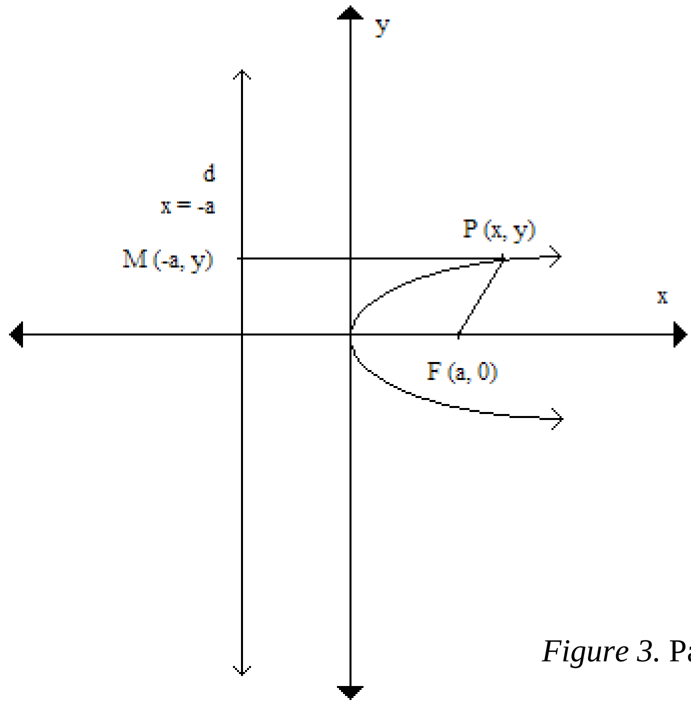

Figure 3. Parabola 1.

After expanding the square of the binomials and simplification, the resulting equation, known as the standard form, is:

$$y^2 = 4ax \tag{14}$$

It is observed that each point with coordinates (at^2, $2at$), such that $t \in \mathbf{R}$, lies on the parabola, because $(2at)^2 = 4a\,(at^2)$. The previous result can easily be affirmed using Equation (2) as follows:

$$x = \frac{y^2}{4a} \tag{15}$$

The ellipse. An ellipse is defined, with an eccentricity e between 0 and 1, as "the set of points P in the plane whose distance from a fixed point F is e times their distance from a fixed line d" (Brannon, 1999, p. 13). There are two conditions that will be assumed for this explanation: the foci F_1 and F_2 lie on the x-axis with coordinates (ae, 0) and (-ae, 0) and the directrixes d_1 and d_2 have the equations $x = \pm\frac{a}{e}$. Unlike the parabola, an ellipse has two foci and directrix. Let an arbitrary point P(x, y) be on the ellipse and the point M signify the foot of the perpendicular from P to d, illustrated in Figure 4:

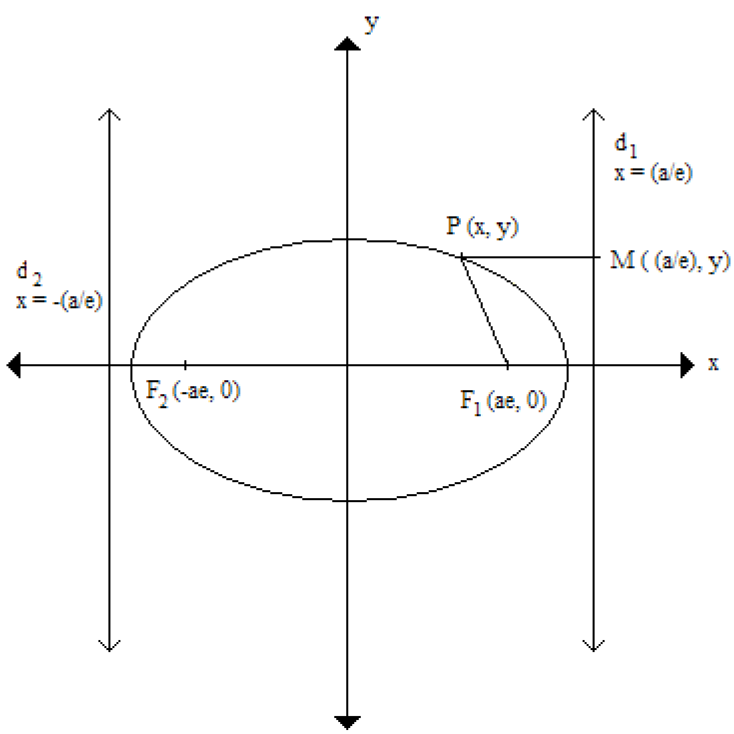

Figure 4. Ellipse 1.

Using the definition of an ellipse, it can be determined that FP = e PM; so, then $|FP|^2 = e^2 |FM|^2$. Because $|FP|^2$ is: $y^2 + (x - ae)^2$ and PM is: $\left(x - \frac{a}{e}\right)$, this equation can be rewritten in terms of coordinates:

$$(x - ae)^2 + y^2 = e^2\left(x - \frac{a}{e}\right)^2 \tag{18}$$

The square of the binomials can be expanded on each of the equal sign of Equation (18) to result in:

$$x^2 - 2aex + a^2e^2 + y^2 = e^2x^2 - 2aex + a^2 \tag{19}$$

Equation (19) is simplified to:

$$\frac{x^2}{a^2} + \frac{y^2}{a^2(1 - e^2)} = 1 \tag{20}$$

Let $b = a\sqrt{e^2 - 1}$, so then $b^2 = a^2(e^2 - 1)$, which allows for the standard form of the ellipse (where $a \geq b > 0$):

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad (21)$$

The hyperbola. The hyperbola has an eccentricity of greater than one and is "the set of all points P in the plane whose distance from a fixed point F is e times their distance from a fixed line d" (Brannan, 1999). The standard form can be found for the hyperbola with the following assumptions: the foci F_1 and F_2, lie on the x-axis and have the coordinates (-ae, 0) and (ae, 0) and the directrixes d_1 and d_2 are the lines with the equations $x = \frac{a}{e}$ and $x = -\frac{a}{e}$. This hyperbola is illustrated in Figure 5.

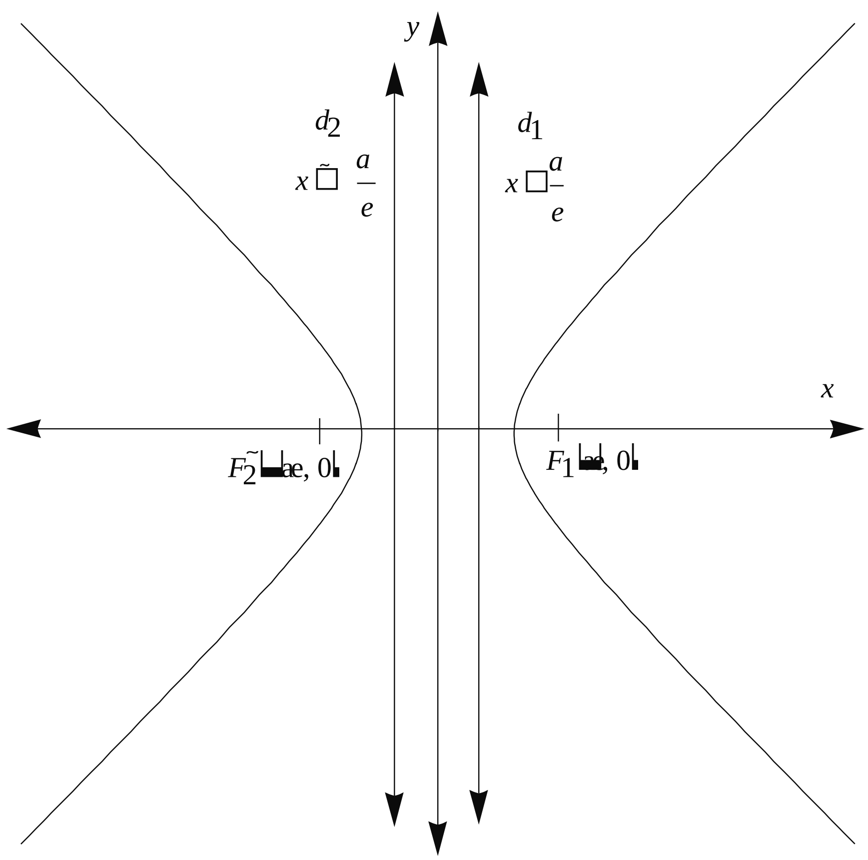

Figure 5. Hyperbola 1.

The line segment that joins the two points (a, 0) and (-a, 0) of the hyperbola that intersects the x-axis is called the major or transverse axis. The minor or conjugate axis is the line segment that joins the points (b, 0) and (-b, 0). An arbitrary point on a hyperbola P be defined as (x, y) and let the foot of the perpendicular from P to the directrix be M, shown in Figure 6.

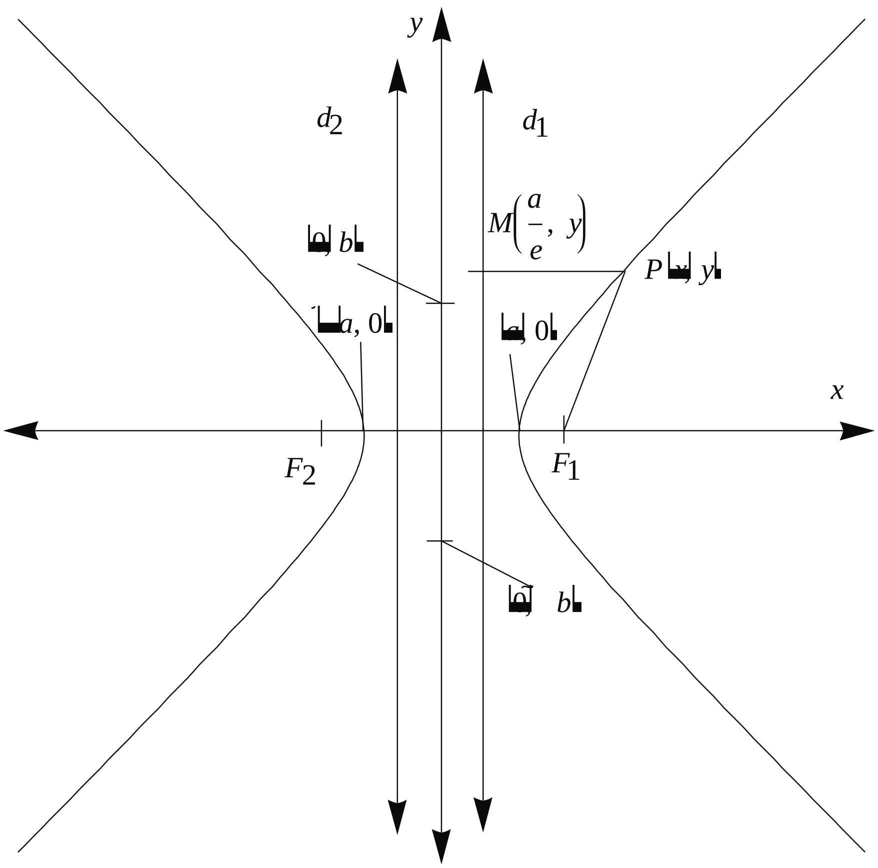

Figure 6. Hyperbola 2.

Using the definition, FP = e PM, it then can be found that $|FP|^2 = e^2 |PM|^2$. This equation can be rewritten in terms of the coordinates as:

$$(x-ae)^2 + y^2 = e^2\left(x-\frac{a}{e}\right)^2 \tag{25}$$

Through expanding the square of the binomial and the combining of like terms, the result is:

$$\frac{x^2}{a^2} - \frac{y^2}{a^2(e^2-1)} = 1 \tag{26}$$

Let $b = a\sqrt{e^2-1}$, so $b^2 = a^2(e^2-1)$. Then the standard form of a hyperbola is:

$$\frac{x^2}{a^2} - \frac{x^2}{b^2} = 1 \tag{27}$$

Each point on the hyperbola can be described using the coordinate (a sec(t), b tan(t)) where t is not an odd multiple of л/2. This can be checked as follows:

$$\frac{a^2\sec^2 t}{a^2} - \frac{b^2\tan^2 t}{b^2} = 1 \tag{28}$$

The hyperbola is made up of two branches that are separated by two asymptotes; this feature is unique to hyperbolas. As x approaches infinity, the branches of the hyperbola approach two lines with equations: $Y_1 = \frac{b}{a}x$ and $Y_2 = -\frac{b}{a}x$, this is observed in Figure 7.

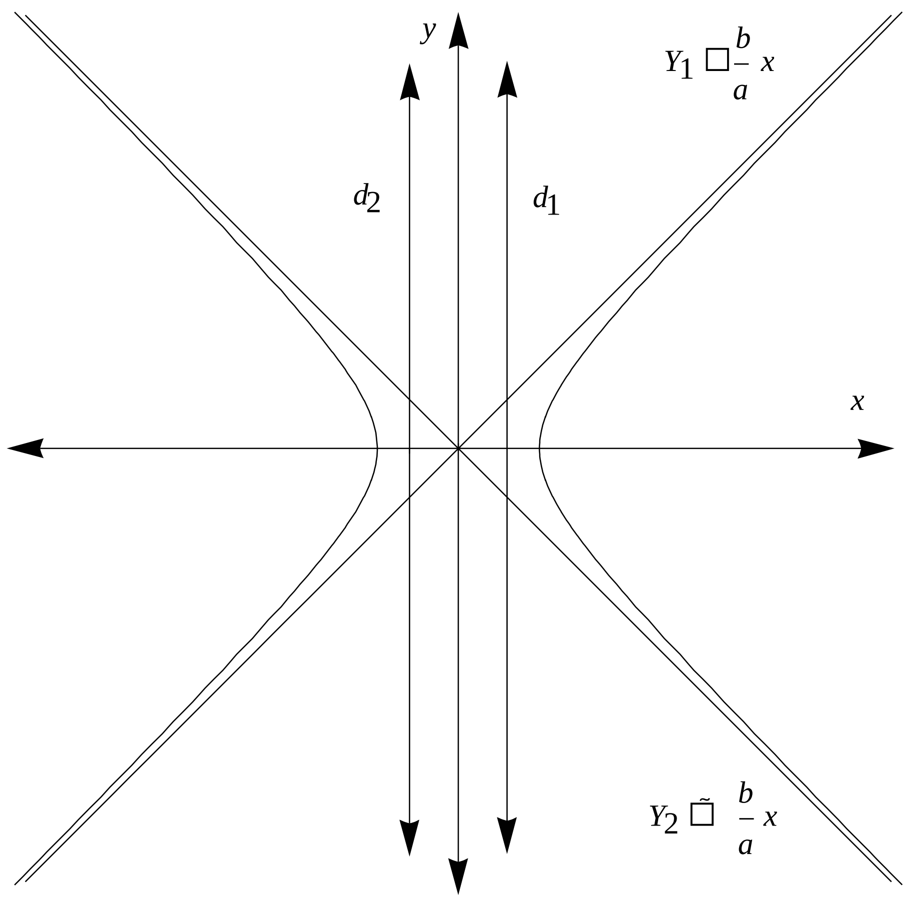

Figure 7. Hyperbola 3.

Another method for deriving the standard form for either the ellipse or hyperbola.

It is possible to derive the standard form for the ellipse and hyperbola without the knowledge of the eccentricity and directrix. The known information then is the foci and a length, l > $|F_1F_2|$, and l is equal to the sum of $|PF_1|$ and $|PF_2|$. Using the ellipse as an example, the following are the steps to solve for the standard form with this method. The known values are the foci, F_1 (a, 0) and F_2 (-a, 0), an arbitrary point P (x, y), as well as a given length of l, according to the foci and P. Using the distance formula to calculate $|PF_1|$ and $|PF_2|$.

$$\sqrt{(x+a)^2 + y^2} + \sqrt{(x-a)^2 + y^2} = l \qquad (31)$$

First, the goal is to eliminate the radicals:

$$\left(\sqrt{(x+a)^2 + y^2}\right)^2 = \left(l - \sqrt{(x-a)^2 + y^2}\right)^2 \qquad (32)$$

Then, multiply, reduce and combine like variables:

$$l^2 - 4ax = 2l\sqrt{(x-a)^2 + y^2} \qquad (33)$$

Again, the goal is to eliminate the radical:

$$(l^2 + 4ax)^2 = \left(2l\sqrt{(x-a)^2 + y^2}\right)^2 \qquad (34)$$

Next, multiply, reduce and combine common variables:

$$\left(16a^2 - 4l^2\right)x^2 - 4l^2y^2 + l^4 - 4a^2l^2 = 0 \qquad (35)$$

Then, A is the first coefficient, B the second and C is the constant. So, the equation can be written in the standard form:

$$Ax^2 + By^2 + C = 0 \qquad (36)$$

Obtaining The Standard Equation of a Conic From Algebra to Geometry

The method. In order to start with a conic representation in Algebra and work it to Geometry, the conic must go through a translation and rotation. The translation must be done to center the conic on the origin of a coordinate plane. This is done by reassigning the variables and eliminating the x and y variables, so that there are only variables squared or multiplied together. Rotation is achieved by centering the conic on the x and y axes. This action is accomplished by diagonalizing the polynomial and finding the eigenvalues. Finally, the classification of the conic can be determined.

An example using the method. The above method is used on the following equation of a conic.

$$6x^2 - 16xy + 4y^2 - 4x + 8 - 32 = 0 \tag{37}$$

Then, the first step is to work towards translating the conic; this is done by substituting new variables for x and y. Let $x = X + \alpha$ and let $y = Y + \beta$; with these new values substituted, the equation becomes:

$$6(X + \alpha)^2 - 16(X + \alpha)(Y + \beta) + 4(Y + \beta)^2 - 4(X + \alpha) + 8(Y + \beta) - 32 = 0 \tag{38}$$

After expanding the square of the binomials and gathering like variables the equation is now:

$$6X^2 - 16XY + 4Y^2 + (12\alpha - 16\beta - 4)X + (-16\alpha + 8\beta + 8)Y + 6\alpha^2 - 4\alpha - 16\alpha\beta + 8\beta + 4\beta^2 - 32 = 0 \tag{39}$$

The goal is to eliminate the X and Y variables. So, the expression of the coefficients of X and Y are set equal to zero. Then the value for α and β can be determined through a system of equations. This is demonstrated in the lines that follow:

$$12\alpha - 16\beta - 4 = 0 \quad \text{and} \quad -16\alpha + 8\beta + 8 = 0 \tag{40}$$

$$(12\alpha - 16\beta = 4) \tag{41}$$

$$\underline{+\,2(-16\alpha + 8\beta = -8)} \tag{42}$$

$$-20\alpha = -12 \tag{43}$$

$$\alpha = \frac{3}{5} \rightarrow \beta = \frac{1}{5} \tag{44}$$

Then, the equation with the values of α and β substituted in is:

$$6X^2 - 16XY + 4Y^2 + 6\left(\frac{3}{5}\right)^2 - 4\left(\frac{3}{5}\right) - 16\left(\frac{3}{5}\right)\left(\frac{1}{5}\right) + 8\left(\frac{1}{5}\right) + 4\left(\frac{1}{5}\right)^2 - 32 = 0 \quad (45)$$

$$6X^2 - 16XY + 4Y^2 - \frac{162}{5} = 0 \quad (46)$$

The equation has been translated with new variables X and Y. Now, to rotate the conic, the equation must be diagonalized. Assigning the matrix A to be:

$$A = \begin{bmatrix} 6 & -8 \\ -8 & 4 \end{bmatrix} \quad (47)$$

Then, to find the eigenvalues of A, use the formula: $\det(A - \lambda I)$.

$$\det(A - \lambda I) = \det\begin{pmatrix} 6-\lambda & -8 \\ -8 & 4-\lambda \end{pmatrix} = (6-\lambda)(4-\lambda) - 64 \quad (48)$$

$$= \lambda^2 - 10\lambda - 40 \quad (49)$$

Next, solve for the eigenvalues by using the quadratic equation.

$$\lambda = \frac{10 \pm \sqrt{(-10)^2 - 4(1)(-40)}}{2(1)} \quad (50)$$

The eigenvalues are $\lambda = 5 + \sqrt{65}$ and $\lambda = 5 - \sqrt{65}$. Using the concept from Linear Algebra of matrices, there exists a matrix P that is orthogonal to the conic. Then, applying the equation $P^T AP = \lambda I$ to the equation (Brannon, 1999):

$$(Z)^T \left(P^T AP\right)Z + \left(J^T P\right)Z + H = 0 \tag{51}$$

The new equation of the conic can be found. Set the matrix Z to be $\begin{bmatrix} X \\ Y \end{bmatrix}$ and J to be the variables of the monomials with variables X and Y which are now zero, resulting in the matrix $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$. Therefore, equation (31) would then become simply (with H being the constant:

$$(Z)^T \left(P^T AP\right)Z + H = 0 \tag{52}$$

Back to the example problem, the new equation of the conic is:

$$\begin{bmatrix} X & Y \end{bmatrix} \begin{bmatrix} 10 + \sqrt{65} & 0 \\ 0 & 10 - \sqrt{65} \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} - \frac{162}{5} = 0 \tag{53}$$

Then, multiplying the matrices yield:

$$\left(10+\sqrt{65}\right)X^2+\left(10-\sqrt{65}\right)Y^2=\frac{162}{5} \tag{54}$$

To get this equation to a more recognizable form, multiply both sides of the equal sign by $\left(\frac{5}{162}\right)$.

$$\frac{5\left(10+\sqrt{65}\right)X^2}{162}+\frac{5\left(10-\sqrt{65}\right)Y^2}{162}=1 \tag{55}$$

Now, it is evident that this conic section is an ellipse.

A line intersecting a conic section. When a line intersects a conic there are three possibilities: two points of intersection, one point of intersection, and no intersection. Whether there is an intersection can be easily determined through use of the discriminant. The equation of the line must be known and in point slope form and then solved for y: $y=m(x-x_o)+y_o$. Also, the equation for the conic must be in standard form: $ax^2+by^2+c=0$. Then, a quadratic equation is formed by substituting the equation of the line into the equation of the conic. It can be shown that the number of intersections is dependent on the slope of the line. The following steps show the substitution process and, then, how the quadratic equation of x is formed.

$$ax^2+b\left(mx-mx_o+y_o\right)^2+c=0 \tag{56}$$

$$ax^2+b\left(m^2x^2-2m^2xx_o+2mxy_o-2x_oy_o+m^2x_o^2+y^2\right)+c=0 \tag{57}$$

$$\left(a+bm^2\right)x^2+\left(-2bm^2x_o+2bmy_o\right)x+\left(-2bmx_oy_o+bm^2x_o^2+by_o^2+c\right)=0 \tag{58}$$

Then, let the coefficient of x^2 be A, the coefficient of x be B and the last term be C. Next, A,B, and C can be used in the equation to find the discriminant, in this case the equation is: $\delta = B^2 - 4AC$. The rule for the discriminant to determine how many intersection points the line will have with the conic is as follows:

1. If $\delta > 0$, then there are 2 intersection points.
2. If $\delta = 0$, then there is 1 intersection point, this line is said to be tangent to the conic.
3. If $\delta < 0$, then there are 0 intersection points.

Application of Non-Degenerate Conic Sections

Parabolas. Parabolas have a variety of useful applications. These conics are used in the reflecting mirrors of flashlights and headlights on cars. The parabola was the solution to the problem of the telescope design. In making a spherical mirror, the more curved the mirror is the more likely a blurred image will occur. A blurry image is caused by a defect in the spherical mirror known as an aberration (Giancoli, 2005). Galileo's design of the telescope had this defect in the spherical lens that was used. Then, Newton perfected the design by using concave mirrors; however, there were still aberrations. The shape of the mirror was changed to a parabolic shape which eliminated the aberration (Motz, 1993). This is because parabolas "will reflect rays to a perfect focus" (Giancoli, 2005, p. 636). Parabolas can be observed in the path of projectile objects. The actual motion of a projectile object is parabolic when air resistance is ignored and the gravitational pull is considered constant (Giancoli, 2005).

Ellipses. Ellipses can be witnessed in places that are unexpected. An interesting place that an ellipse is found is the orbit of the planets around the sun. Kepler's first law of planetary motion states that the path the planes make around the sun is elliptical with the sun at one focus (Giancoli, 2005). Another interesting place to discover an ellipse is in the phenomenon of a whispering gallery. The reason that the whispering gallery effect occurs is that a room is elliptically shaped. The shape allows all the sound waves to converge at either of the focal points, resulting in the ability to hear what is being said all over the room while standing in a focus (Egan, 2007). Places that have whispering galleries include: St. Paul's Cathedral, London, England and Union Terminal Building in Cincinnati, Ohio (Egan, 2007).

Hyperbolas. Hyperbolas have an interesting application in physics. Robert Boyle determined that there is an inverse relationship between the volume of the gas and its pressure at a constant temperature. The volume of a gas and its pressure at a constant temperature is represented on a coordinate system as an equilateral hyperbola (Motz, 1993). The path of comets is originally elliptical; however, when the comet passes a planet, the path of the comet can be affected by the gravitational pull of the planet to cause the path to be hyperbolic.

Conclusion

It is quite evident that the advancements made by Apollonius improved the world of mathematics. The rivalry between Apollonius and his fellow mathematicians allowed for the rise of mathematical discoveries. Because of Apollonius' theoretical discoveries in the field of conic sections, those that followed him were able to build from the foundations he had laid. Many discoveries are linked to conic sections whether indirectly or directly. Apollonius' addition to the subject matter of conics has contributed not only to the mathematical society, but also to a wide variety of areas. It is interesting that relationships in physics can be represented graphically to form a hyperbolic shape and that the orbit of the planets, as well as some comets, are elliptical (not to mention the parabolic path of projectile objects). Conic sections are related to many areas and fields of study.

There are many methods for classifying conics. Depending on the known information, a conic can be determined anywhere from Affine Geometry to Euclidean Geometry. The classification of a conic can also be determined without the measure of length. Conic sections are an essential element throughout math and science. Apollonius greatly affected the world through his discoveries in the field of conics.

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