AN EXAMINATION OF THE INFLUENCES OF CRITICAL THINKING TASKS ON MATHEMATICS INSTRUCTION: A QUALITATIVE STUDY

by

Glen Edmund Miller III

Liberty University

A Dissertation Presented in Partial Fulfillment

Of the Requirements for the Degree

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Abstract

The purpose of this phenomenological study will be to understand the mathematics critical thinking process for teachers in primary education in northern Virginia. The theory guiding this study is Kolb's experiential learning theory, which applies to critical thinking processes. The central research question will be: What are the experiences of primary mathematics teachers implementing critical thinking? This qualitative hermeneutic phenomenological study aims to determine the influences of critical thinking tasks on planning and instruction. Using qualified participants from northern Virginia, this study will collect data through interviews, a focus group, and document analysis. After data collection, the data will be analyzed for themes by identifying codes, creating categories, and applying the themes to the research.

Keywords: critical thinking, pedagogy, experiential learning theory, primary education, mathematics

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Dedication

To God be the glory! With Him, all things are possible. I dedicate this dissertation to the one who made it all possible. Jesus is my rock, my cornerstone, and my firm foundation. I pray that His provision and guidance forever light the path under my feet.

I would also like to thank and appreciate my wife, Breanna Miller, who has always supported me and whatever I set out to do. Thank you for your grace, support, and love through this journey.

I want to dedicate my dissertation to my beautiful little girls, Charlee Rae and Sydney Grace. Daddy loves you more than you will ever know.

I want to dedicate my dissertation to my parents. To my mom, you have always been my biggest inspiration. Dave, thank you for always being there and for providing spiritual guidance. To my dad, thank you for showing me what it looks like to work hard through the finish line. To Sharon, thank you for all the prayers and support. To my in-laws, thank you for instilling the values of compassion and grace in your daughter, as she played a massive role in this. To the rest of my family, thank you for always being there and your positive words of encouragement. Love you all!

Lastly, I would like to dedicate my dissertation to my mentors. First, Dr. Christine Remley. You have been such a blessing on this journey. I value your input, hard work, mentality, and support. Dr. Strafaccia, thank you for all the phone hours and meetings to ensure we completed this dissertation. Thank you for your support and continual blessings.

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List of Abbreviations

Annual Yearly Progress (AYP)

Assessing Math Concepts (AMC)

Beta Public County Schools (BPCS)

Common Core State Standards (CCSS)

Department of Education (DOE)

Elementary and Secondary Education Act (ESEA)

Every Student Succeeds Act (ESSA)

National Assessment of Educational Progress (NAEP)

National Council of Teachers of Mathematics (NCTM)

National Defense Education Act (NDEA)

No Child Left Behind (NCLB)

Organization for Economic Cooperation and Development (OECD)

Phonological Awareness Literacy Screener (PALS)

Project Based Learning (PBL)

Professional Learning Community (PLC)

Standards of Learning (SOL)

Trends in International Mathematics and Science Study (TIMMS)

CHAPTER ONE: INTRODUCTION

Overview

Critical thinking is essential in developing and achieving mathematics (Komariyah & Laili, 2018). Unfortunately, due to the lack of implementation of critical thinking necessary to the learning process, students are not applying the process of thinking critically in mathematical settings (Liljedahl, Clark et al., 2021; Neumann, 2016; Ridwan et al., 2022). When students are not thinking critically, mathematics achievement consistently declines across the United States, showing little signs of slowing down (U.S. Department of Education, 2022). The historical context of critical thinking highlights the significant attempts to create thinkers using problem-solving and tasks (Dahl et al., 2018). Meanwhile, the political influences of the accountability era have changed how educators teach mathematics, creating a culture of teacher-centered instruction (Barksdale-Ladd & Thomas, 2000; Gaylor, 2005; van Hover & Pierce, 2006; Vogler, 2008). This chapter explores the background of mathematics instruction, integrating critical thinking in historical, social, and theoretical contexts. Next, the problem and the purpose statements are presented, followed by the study's theoretical, empirical, and practical significance. Theorists have attempted to help bridge the knowledge gap, but educators must begin to offer opportunities for critical thinking rather than teaching to the test. Using Kolb's (1984) experiential learning theory, this study aims to understand the influences of the mathematics critical thinking process on curriculum planning and instruction for teachers in primary education.

Background

In this section, the historical, social, and theoretical background of critical thinking within mathematics instruction will be provided. First, the historical context examines the absence of critical thinking opportunities in the evolution of primary mathematics. Second, the

social context explores how society, community, and the educational system have affected the problem of underrepresentation of critical thinking skill ability in mathematics. Third, the theoretical context will examine the foundational theories that have explored this topic.

Historical Context

Critical thinking is discussed in schools as educators prepare students for the 21st century (Dahl et al., 2018). This thinking style can be traced back to the works of Socrates through the lens of cognitive skills such as analysis, evaluation, reflection, and inference (Dahl et al., 2018). From Socrates to Piaget, critical thinking has continued to evolve.

First, American philosopher John Dewey is known to be the father of the critical thinking tradition and defined it as an active, persistent, and careful consideration of a belief in a hypothetical form of knowledge (Sadeghi et al., 2020). Dewey (1916) believed there was a distinction between process and product regarding thinking and that education could help or hinder the development of critical thinking, problem-solving, and judgment (Kurfiss, 1988). Dewey's ideas supported the progressive education reform movement (Dahl et al., 2018) that promoted instructional changes designed to improve critical thinking skills at the elementary level (Kurfiss, 1988).

Following Dewey, Glaser created an assessment to test the skills of critical thinking and reasoning called the Watson-Glaser critical thinking appraisal (Watson & Glaser, 1980). The test analyzes the skills of arguments, drawing inferences, recognizing assumptions, evaluating conclusions, and assessing the strength of reasoning (Kurfiss, 1988). Glaser (1941) suggested critical thinking requires a certain attitude, openness, thoughtfulness, and persistence toward an inquiry. Glasers developed an instructional program consisting of eight lessons revealing high correlations between scores, critical thinking, and measures of intelligence (Kurfiss, 1988), further developing the process of critical thinking in educational settings.

While E. Glaser (1941) considered the elements of inquiry, Ennis (1985) built upon the process by defining critical thinking as the correct assessment of statements with reflective and reasonable thinking focused on deciding what to believe or do. Ennis (1985) believed that critical thinking belonged in the curriculum. Abilities such as clarifying questions, terms, and assumptions, assessing sources, credibility, reasoning logically, and detecting persuasive strategies were considered dispositions within Ennis & Millman's (1985) proposed curriculum. Two multiple-choice assessments testing inductive and deductive reasoning, prediction and experimentation, fallacies, definition, and identification of assumptions were created by Ennis (Ennis & Millman, 1985) to implement critical thinking in educational settings.

Social Context

The social context of critical thinking in education can often be traced through a political foundation. Democracy cannot exist without accountability. The accountability era of education has significantly influenced education **(**Dudley & Burden, 2020**;** Liljedahl, Clark et al., 2021**;** Shepard & Dougherty, 1991), creating rigorous standardized testing and national standards. Education has a long history of adapting to the world **(**Dudley & Burden, 2020**;** Liljedahl, Clark et al., 2021**;** Shepard & Dougherty, 1991). Thus, the influence of politics is critical to understanding how education got to a point where critical thinking is absent, and achievement has declined (U.S. Department of Education, 2022). In addition, the rigors of curriculum, demanding testing structure, and immense pressure tied to funding have changed how educators teach (Rottenberg & Smith, 1990). Due to the accountability era, classrooms have become dominated by a teacher-centered teach-to-the-test pedagogy (Van Harpen & Sriraman, 2013; Van Harpen & Presmeg, 2013).

Unfortunately, standardized testing negatively influences educators' time for planning, implementation, and opportunities outside the curriculum (Shepard & Dougherty, 1991).

Rottenberg and Smith (1990) found that external testing significantly affects the time available for ordinary instruction. Teachers claim they spend more time on the learning objectives of the tests than on the curriculum (Shepard, 1990). As a result, teachers have begun to spend less time on science, social studies, writing, word recognition, spelling, punctuation, and arithmetic operations (Shepard, 1990).

The immense pressure put on educators by administrators, media, and educational reform to improve test scores is causing students to lack opportunities to develop further 21st-century skills such as critical thinking (Shepard & Dougherty, 1991). This pressure leads to teaching the test content using methods presented by Darling-Hammond and Wise (1985) that give specific examples of the material to align with standardized assessments. Responsively, teachers began to change their pedagogical thinking and instructional methods to resemble testing (Shepard & Dougherty, 1991).

Also, the instructional methods affected by standardized testing are antithetical to the instruction of the National Council of Teachers of Mathematics (NCTM) (Romberg et al., 1989), further limiting critical thinking within the classroom. The NCTM (2015) provides eight mathematic teaching practices:

> (1) establish mathematics goals to focus learning, (2) implement tasks that promote reason and problem-solving, (3) use and connect mathematical representations, (4) facilitate meaningful mathematics discourse, (5) pose purposeful questions, (6) build procedural fluency from conceptual understanding, (7) support productive struggle in learning mathematics, and (8) elicit and use evidence of student thinking.

The pressures of standardized testing leave little room for most applications suggested by NCTM (Shepard & Dougherty, 1991) and the implementation of rich tasks (Liljedahl, Zager, et al., 2021). Without the opportunity for rich critical thinking tasks, lectures, and traditional methods

often leave out many of the practices set out by NCTM (Dudley & Burden, 2020). With no change in sight, educators are forced to adapt to a hybrid model of what they know to be sound teaching and preparing students for a standardized assessment.

Theoretical Context

Several theorists supply context to this study regarding the cognitive influence of critical thinking. Piaget (1964) applied structured observations, in which he wrote down everything about the participants' actions that led to his theory of cognitive development. The theory of cognitive development has four stages: sensorimotor, pre-operational, concrete operational, and formal operational (Piaget, 1962). In addition, Piaget (1962) believed that learning occurs through a balance of assimilation and accommodation. Interestingly, Vygotsky (1929) posits that a child learns through social interactions with peers and adults through scaffolding that supports and guides children to discover or understand what they could not have learned alone. Vygotsky's theory of cognitive development argues that the ability to think, feel, and communicate depends on cultural practices and social interactions with others (Zhang, 2022)**.** Vygotsky's (1929) zones of proximal development (ZPD) have profoundly influenced education (Sharkins et al., 2017).

The ZPD refers to the difference between what a child can achieve independently and what a child can achieve with scaffolding and support from another (Vygotsky, 1929). A Vygotsky (1929) education model requires reciprocal teaching, cooperative learning opportunities, and collaborative development. The work of Piaget and Vygotsky laid the groundwork for how students learn, which molded modern education (Sharkins et al., 2017). Critical thinking tasks align well with the theories as they try to explain how children learn.

Examining the learning process has also been considered a method for exploring and examining critical thinking applications in the educational setting. Experiential learning theory

(Kolb, 1984) posits that learning can be characterized by three major traditions: "(1) learning is best conceived as a process, not in terms of outcomes; (2) learning is a continuous process grounded in experience; and (3) learning is a holistic process of adaptation to the world" (Kolb, 1984, p. 28-30). In addition, Kolb (1984) created a cycle of four modes through which people experience and transform experiences into knowledge. The four modes of the cycle include active experimentation, concrete experience, abstract conceptualization, and reflective observation (Kolb, 1984). Effective learning occurs when a person progresses through the cycle (Kolb, 1984). Experiential learning theory differs from most as it sees the process of critical thinking as more valuable than the product (Kolb, 1984). Also, experiential learning removes traditional norms and immerses students in an environment where learned skills must be actively applied (Kolb, 1984). The learning cycle engages all learners, regardless of skillset and intellect, in a system of task immersion to guide success and joy in the learning process (Kolb, 1984). For this study, Kolb's (1984) experiential learning theory will explore mathematics instructors' implementation of critical thinking.

Problem Statement

The problem is that critical thinking is underrepresented in primary mathematics education. Mandated accountability testing harmed teaching as educators struggle with the rigorous demands of the curriculum and standardized assessment, leaving little time for critical thinking opportunities (Barksdale-Ladd & Thomas, 2000; Gaylor, 2005; Jones et al., 1999; Smith, 2020; van Hover & Pierce, 2006; Vogler, 2008). The accountability era of education has changed how educators think about their pedagogy (Gaylor, 2005; Liljedahl, Zager, et al., 2021). Educators have adopted methods such as teaching to the test to relieve some of the pressures of rigorous curriculum and mandated testing (Liljedahl, Zager, et al., 2021; Vogler, 2005). The

pedagogical change has negatively influenced students' opportunities to be critical thinkers (Liljedahl, Zager, et al., 2021; Neumann, 2016; Ridwan et al., 2022; Sumardi et al., 2020).

Responsively, critical thinking is underrepresented in elementary mathematics (Liljedahl, Zager, et al., 2021; Ridwan et al., 2022; Su et al., 2016). Moreover, students are not given enough opportunities to think critically (Liljedahl, Zager, et al., 2021; Ridwan et al., 2022; Su et al., 2016). In any given one-hour lesson, 75-85% of students show non-thinking behaviors (Liljedahl, Zager, et al., 2021). Examples of non-thinking behaviors are described in the literature as mimicking, faking, slacking, stalling, and lackluster attempts to try (Liljedahl, Zager, et al., 2021). In addition, students only spend approximately 20% of a given school day thinking (Su et al., 2016). Learning continues to be focused on strategies and test preparation rather than engaging learners in cognitive tasks (Ridwan et al., 2022). Today, curriculum targets and learning activities are dominated by lecture methods (Ridwan et al., 2022), limiting opportunities for educational engagement. Current mathematics pedagogy and instruction need more opportunities to engage students in the analyzing, inferential, and justification process of learning (Liljedahl, Zager, et al., 2021; Neumann, 2016; Ridwan et al., 2022; Sumardi et al., 2020). A shift needs to occur within education planning that allows opportunities for students to engage in active learning by applying 21st-century communication, collaboration, and critical thinking skills. Thus, an examination of the critical thinking process teachers utilize in primary education is necessary to add to the body of knowledge.

Purpose Statement

The purpose of this phenomenological study is to understand the mathematics critical thinking process for teachers in primary education in northern Virginia. At this stage in the research, the mathematical critical thinking process will be defined as instruction using purposeful tasks to make students think critically. The theory guiding this study will be Kolb's

(1984) experiential learning theory, which is related to the experiences associated with implementing critical thinking tasks.

Significance of the Study

Examining critical thinking skills in primary education mathematics is guided by the study's theoretical, empirical, and practical significance. Kolb's (1984) experiential learning theory ensures the participation of students during critical thinking opportunities. The empirical evidence suggests that educators need professional development in teaching using thinking tasks rather than learning about thinking (Sadeghi et al., 2020). Finally, the practical significance of mandated accountability testing has played a role in the underrepresentation of critical thinking opportunities in elementary education (Dudley & Burden, 2020).

Theoretical

The theoretical significance of examining the integration of critical thinking in mathematics lies within Kolb's (1984) experiential learning theory. Learning is an experience. Thus, learning through experiences allows children to make sense of their learning through a personalized lens (Kolb, 1984). Critical thinking opportunities offer children those experiences. Experiencing the cycle of learning ensures that students have actively participated, had a concrete experience, conceptualized an abstract idea, and had time for reflective observation (Kolb, 1984). Experiential learning theory offers unique pedagogical techniques that allow a classroom to be student-centered through critical thinking opportunities (Kolb, 1984). While many cognitive theorists have attempted to lay the framework for thinking, experiential learning theory offers a different path that focuses more on the thinking process rather than the product (Kolb, 1984). In the age of standardized testing and declining nationwide scores in the United States, a shift is necessary that allows students the opportunities to think (Camilli, 2021; (U.S. Department of Education, 2022).

Empirical

The empirical significance of examining the underrepresentation of critical thinking in primary mathematics lies within the lack of understanding of critical thinking. Dewey understood a distinction between the process and the product (Sadeghi et al., 2020). Then, Glaser created instructional programs that revealed the influences of critical thinking on measures of intelligence (Kurfiss, 1988). Today, educators believe that professional development should focus more on implementing the tasks rather than surface-level knowledge of critical thinking (Dudley & Burden, 2020). Many researchers have understood the essential underrepresentation of thinking in elementary mathematics (Liljedahl, Zager, et al., 2021; Neumann, 2016; Ridwan et al., 2022). Yet, little of the literature discusses the lived experiences of how educators felt offering critical thinking opportunities influenced their curriculum or instruction planning. Examining those influences of thinking opportunities will add to the field of research.

Practical

The practical significance of examining the influence of the accountability era in primary mathematics lies within the changes in pedagogy, planning, and curriculum. Critical thinking has been heavily researched and investigated, yet opportunities for critical thinking are still lacking within the classroom **(**Dudley & Burden, 2020**;** Liljedahl, Zager, et al., 2021**;** Shepard & Dougherty, 1991). Educators feel the pressure of mandated testing and accountability, lack the professional development to implement thinking opportunities, and have changed their pedagogy under pressure (Shepard & Dougherty, 1991). The pressure of accountability, matched with the rigorous testing, has significantly influenced the ability to offer critical thinking opportunities. This study will aid educators in planning, implementing, and sustaining a thinking classroom that allows students to showcase their thinking rather than focus on the product.

Research Questions

The proposed research questions are derived from and align with the problem and purpose statement to support this study examining the implementation of critical thinking opportunities. The central research question focuses on understanding primary mathematics teachers' lived experiences of implementing critical thinking. In addition, the sub-questions align with Kolb's (1984) theory of experiential learning as they are guided by the four elements of active experimentation, concrete experiences, abstract conceptualizations, and reflective observation.

Central Research Question

What are the experiences of primary mathematics teachers implementing critical thinking?

Sub-Question One

What are the concrete experiences of primary mathematics teachers implementing critical thinking?

Sub-Question Two

What are the reflective observation experiences of primary mathematics teachers implementing critical thinking?

Sub-Question Three

What are the abstract conceptualization experiences of primary mathematics teachers implementing critical thinking?

Sub-Question Four

What are the active experimentation experiences of primary mathematics teachers implementing critical thinking?

Definitions

1. *Active experimentation* - Trying out what you have learned (Kolb, 1984).

2. *Abstract conceptualization* - Learning from the experience (Kolb, 1984).
3. *Concrete experience* - The actual experience (Kolb, 1984).
4. *Reflective observation* - *Reflecting* on the experience (Kolb, 1984).
5. *Thinking task* - A highly engaging, clearly mathematical task that does not map well to outcomes or standards specific to the class's curriculum (Liljedahl, Zager, et al., 2021).

Summary

This phenomenological qualitative study will provide a voice to educators sharing their lived experiences implementing critical thinking in mathematics instruction. Chapter One explored the historical, social, and theoretical contextual background of critical thinking relating to mathematics instruction. In addition, the problem of critical thinking being underrepresented in primary mathematics was explored (Liljedahl, Zager, et al., 2021; Neumann, 2016; Ridwan et al., 2022). This problem has affected the ability of students to think critically and possess the necessary 21st-century skills. The purpose of this phenomenological study is to understand the mathematics critical thinking process for teachers in primary education in northern Virginia. At this stage in the research, the mathematical critical thinking process will be defined as instruction using purposeful tasks to make students think critically. The significance of this study is foundational in understanding how teachers can change, implement, or learn from using critical thinking tasks. Finally, the research questions are developed utilizing Kolb's (1984) experiential learning theory to execute critical thinking tasks.

CHAPTER TWO: LITERATURE REVIEW

Overview

A systematic review of existing literature was conducted to examine the underrepresentation of critical thinking in elementary mathematics and expand on the literature surrounding how to provide opportunities to create thinking classrooms. The first section

outlines the study's theoretical framework through the works of Kolb's (1984) experiential theory as it applies to thinking classrooms. The second section begins a synthesis of recent literature regarding building thinking classrooms, thinking tasks, teacher perceptions, and the political influence on the status of mathematics. In addition, this section outlines the current decline of mathematics education within the United States. Throughout the chapter, a gap in the literature is demonstrated, highlighting a need for this study to understand the mathematics critical thinking process for teachers in primary education in northern Virginia that will aid in expanding the body of knowledge in current research.

Theoretical Framework

The theoretical framework that will guide this study is Kolb's (1984) experiential learning theory. Kolb's (1984) experiential learning theory sees learning as a continuous process grounded in experience where knowledge is continuously derived from and assessed from the learner's experience. Learning is viewed as a process rather than an outcome. Therefore, experiential learning theory defines learning as the process whereby knowledge is created through experience transformation (Kolb, 1984). The experiential perspective emphasizes the process of adaptation and learning as opposed to content and outcomes and sees knowledge as a transformation process continuously created and recreated (Kolb, 1984).

There are three factors essential to experiential theory: "(1) learning is a continuous process, (2)-learning is a holistic process of adaptation to the world, and (3) learning involves transactions between the person and the environment" (Kolb, 1984, p. 28-30). First, Kolb views learning as a continuous process, claiming that all learning is relearning, and everyone enters every situation with articulate ideas about the topic (Kolb, 1984). Second, learning is a holistic process of adaptation to the world that describes the emergence of basic life orientations as a function of dialectic tensions between basic modes of relating to the world (Kolb, 1984). Lastly,

the theory posits that learning involves transactions between the person and the environment (Kolb, 1984). The transactional relationship makes learning an active, self-directed process that can be applied in everyday life (Kolb, 1984).

The theory of experiential learning can be seen on two levels: (1) the four-stage cycle of learning and (2) the reflective cycle (Kolb, 1984). The four-stage cycle of learning views learning as an integrated process, with each stage mutually supporting and feeding into the next (Kolb, 1984). Effective learning occurs when learners progress through the cycle of the four stages (Kolb, 1984). A learner can enter the cycle at any stage and follow it logically (Kolb, 1984). The stages within the cycle include:

> (1) having a concrete experience followed by (2) observation of and reflection on that experience, which leads to (3) the formation of abstract concepts and generalizations, which are then (4) used to test a hypothesis in future situations, resulting in new experiences. (Kolb, 1984, p. 27)

The reflective cycle works together with the four-stage cycle of learning. The cycle posits that different people naturally prefer different learning styles based on their social environment, educational experiences, or the basic cognitive structure of the individual (Kolb, 1984). The reflective cycle claims four learning styles: diverging, assimilating, converging, and accommodating (Kolb, 1984). First, the divergent learner looks at things from varying perspectives, prefers to watch rather than do, and tends to gain information and use their imagination to solve problems (Kolb, 1984). Second, the assimilating learner has a concise, logical approach and values ideas and concepts over people (Kolb, 1984). Third, the converging learner can solve problems and use learning to find solutions to practical issues (Kolb, 1984). Lastly, the accommodating learner is hands-on and relies on intuition rather than logic (Kolb, 1984).

Throughout this study, the four-stage cycle of learning and the reflective cycle will be used to examine the experiences of mathematics educators and critical thinking skill opportunities. For this study, mathematical thinking tasks encompass the process of mathematical thinking over the product. According to Kolb (1984), the educator focuses more on the mathematical processes, reasoning, and justification behind student work. Integrating Kolb's theory (1984) will guide the literature review, data collection, and data synthesis.

Related Literature

In addition to Kolb's (1984) experiential learning theory, significant literature and research regarding the influences of constructivist views on critical thinking, thinking tasks, thinking classrooms, teacher perceptions, and the status of mathematics in the United States were reviewed. Literature related to the historical perspective of critical thinking and theory, the importance of critical thinking in education, and approaches to teaching critical thinking will be discussed. The literature review will continue with a discussion regarding thinking tasks and perceptions of the tasks. In addition, research on the influences and effects of experiential learning theory (Kolb, 1984) will be shared. Finally, the literature review will outline the status of mathematics education with evidence and current research on elementary education development.

Constructivist Influences on Critical Thinking

Jean Piaget (1964) is often remembered in education for his initial work describing developmental stages (Lourenco & Machado, 1996). Piaget (1964) was a Swiss psychologist fascinated with developing cognition from a constructivist lens, spending five decades determining children's cognitive development (Butler-Bowdon, 2007). His work began when he tried to answer questions regarding why children talk, whom they speak to, and why they ask so many questions (Butler-Bowdon, 2007). To create the theory, Piaget (1964) applied structured

observations where he wrote down everything about the participant's actions. These structured observations led to Piaget's (1964) theory of cognitive development. The cognitive needs of children correspond to how educators can adapt and change curriculum planning and instruction using critical thinking tasks. Although much research has focused on how children learn, a shift towards focusing on how educators can best shift pedagogical thinking is needed.

Piaget (1964) believed there are four stages of cognitive development: sensorimotor, pre-operational, concrete operational, and formal operational. Piaget's (1972) beliefs come from the basis that development is tied to embryogenesis and that development precedes learning. The sensorimotor stage refers to children between the ages of birth to two years old and is characterized by object permanence and deferred imitation (Babakr et al., 2019; Piaget, 1962). The pre-operational stage spans ages two to seven (Babakr et al., 2019). During this stage, symbolic ability develops, and children struggle to distinguish between animate and inanimate objects (Piaget, 1962). Next, the concrete operational stage spans from age seven to eleven, where children become less egocentric and begin to understand things and solve problems (Babakr et al., 2019). Finally, the formal operational stage spans from 11 to 16 years old, when children's ability to think and understand increases significantly (Babakr et al., 2019). Piaget believed that through assimilation, children could understand new content in terms of knowledge the child already has, accommodation, modifying actions to new or similar stimuli, and equilibration, a balance between the two (Joubert & Harrison, 2021). Opportunities for critical thinking at the elementary level must fall within the stages of Piaget's (1972) stages of development to be effective. Without proper planning and implementation, opportunities can be wasted if learning is not aligned with the student's cognitive level.

Vygotsky's (1929) theory of cognitive development argues that learning precedes development as a cultural phenomenon of acquiring knowledge, beliefs, and problem strategies.

Vygotsky (1962) posited that a child learns through social interactions with peers and adults through scaffolding and support, where children are guided to discover or learn what they could not have learned alone. This process allows children to socialize and interact with others in a cultural context (Topçiu, 2015; Zhang, 2022). In addition, Vygotsky's (1929) theory concludes that parents, teachers, peers, caregivers, and society influence an individual's cognitive development through education.

Vygotsky's (1929) zone of proximal development (ZPD) has profoundly influenced education. The ZPD relates to the difference between what a child can achieve independently and what a child can achieve with scaffolding and support from another (Vygotsky, 1962). In addition, the gap between actual and potential learning is identified and argues that the gap will be bridged only through collaboration with adults and other learners (Zhang, 2022). Vygotsky (1962) often referred to this support as more knowledgeable than others (MKOs). MKO is central to the ZPD learning process (Vygotsky, 1962). An MKO can be anyone with a higher level of knowledge or skill than the earner (Vygotsky, 1962). Reciprocal teaching, cooperative learning opportunities, and collaborative development are critical in a Vygotsky (1962) education model.

Piaget's (1964) theory of cognitive development and Vygotsky's (1929) theory of cognitive development are quite similar, yet they were built upon each other. Piaget (1964) sees interaction as a source of information, not as the developmental process's essence. At the same time, Vygotsky (1929) argues that the ability to act, think, feel, and communicate depends on cultural practices and social interactions. However, Vygotsky often criticized Piaget's work for ignoring the social context in which development occurs (Zhang, 2022). Both theories provide insight into children's growth, development, and learning while directly influencing teaching

(Sharkins et al., 2017). In addition, both development theories consider that children construct their knowledge, an integral part of building critical thinkers (Sharkins et al., 2017).

The constructivist theory is conducive to working with critical thinking tasks. In a constructivist classroom, students are empowered to find their solution among many acceptable options (Sharkins et al., 2017). The students are given time to work at their own pace, self-correction is encouraged, and answers are not right or wrong (Sharkins et al., 2017). Critical thinking tasks offer the opportunity to work socially with peers and adults to solve problems. Constructivist classrooms allow students to hone auditory processing skills and express themselves orally in a space where active participation and exploration of ideas are encouraged (Sharkins et al., 2017).

Thinking tasks can be used in classrooms to get students to think critically (Liljedahl, Zager, et al., 2021; Sharkins et al., 2017). These types of tasks have foundations in cognitive development theory. Vygotsky (1962) claimed that the cognitive foundation for learners to develop begins with facing difficult situations to increase their egocentric speech. When done appropriately, thinking tasks offer a development of moral and intellectual autonomy that provides a solid foundation for a constructivist classroom (Sharkins et al., 2017). In addition, thinking tasks can offer direct interaction with objects and the types of experiences as described by Piaget (1964) in which logical-mathematical learning results from the actions taken upon those objects.

Thinking opportunities often have roots in constructivist ideas and principles. When choosing a task, it is imperative that all students can succeed. The foundation for learning begins by observing each child, determining what they already know, recognizing their strengths, and meeting them right where they are (Allington, 2005; Clay, 2014; Vygotsky, 1962). Most importantly, the constructivist classroom offers children a way to develop the ability to become

self-governed and cooperative, which in turn helps children develop a love of learning and self-discipline (Kamii, 1982).

Constructivist View of Mathematical Teaching and Learning

The constructivist ideas of Piaget's theory of cognitive development leave much unsaid about the subject areas. Therefore, cognitive models such as Steffe et al. (1988) extended the work by offering an explanation tied directly to elementary school mathematics. The constructivist view of mathematical teaching provides a different and unique perspective that differs from traditional approaches.

Constructivists such as Piaget & von Glaserfeld argue that children actively create knowledge and not passively receive it from their environment (Wood et al., 1991). Children create new mathematical knowledge by reflecting upon their actions, leading to understanding (Piaget, 1962). A constructivist believes that substantive learning occurs over periods of conflict, confusion, and surprise (Inhelder et al., 1974). Constructivists such as Bruner (1990) argue that learning is a social process in which children grow in a community rather than in isolation. Within that community, ideas are established through cooperation and collaboration by the members of the culture present (Bruner, 1990). Overall, the constructivist believes that learning opportunities occur during social interactions involving collaborative dialogue, explanation, justification, and negotiation of meaning (Voigt, 1985).

Critical Thinking

Critical thinking continues to be discussed within our schools as educators attempt to prepare students for the 21st Century. Critical thinking can be traced back to the works of Socrates through the lens of cognitive skills such as analysis, evaluation, reflection, and inference (Dahl et al., 2018). American philosopher John Dewey is the father of the critical thinking tradition and defined it as an active, persistent, and careful consideration of a belief in a

hypothetical form of knowledge (Sadeghi et al., 2020). Cash (2017) believed critical thinking is making decisions based on evidence and facts.

Many researchers believe that the upper three levels of Bloom's taxonomy define critical thinking (Sadeghi et al., 2020). Bloom's taxonomy is a hierarchical classification of different levels of thinking (Bloom et al., 1956). The bottom three levels of the pyramid contain the cognitive processes of remembering, understanding, and applying, while the top three levels contain the cognitive processes of analyzing, evaluating, and creating (Bloom et al., 1956). Students must be able to organize their thoughts, compare, question, and experiment to analyze content (Bloom et al., 1956). Students must also be able to appraise, judge, defend, critique, and value cognitive processes to evaluate content (Bloom et al., 1956). At the theory's pyramid peak, students must be able to create, develop, investigate, and participate in conjecture (Bloom et al., 1956). These higher-level Bloom's skills are essential processes to thinking opportunities (Liljedahl, Zager, et al., 2021; Sadeghi et al., 2020).

Critical thinking can also be aligned with philosophy. Buskist & Irons (2008) define critical thinking as accurately explaining decisions, developing, and presenting reasoned and persuasive arguments. Such thinkers must be able to explain rationales meaningfully and transparently. Edwards (2000) refers to critical thinking as interactive development as a basis for learning about self and the world. Overall, critical thinking requires a certain attitude, openness, thoughtfulness, and persistence toward an inquiry (E. Glaser, 1941).

Critical Thinking in Mathematics

Critical thinking and mathematics are deeply intertwined. A student must integrate critical thinking and mathematical knowledge to be successful (Paul & Elder, 2008). Accordingly, Paul and Elder (2008) suggest that there are eight parts to a thinking process: an objective, a set of questions, a body of data, a set of interpretations and interferences, a set of

ideas, a set of assumptions, some potential outcomes, and a point of view. With these eight parts, a student can be a successful thinker (Paul & Elder, 2008). “Critical thinkers engage in the eight parts of the thinking process yet fall into one of six levels of critical thinkers: (1) unreflective thinker, (2) challenged thinker, (3) novice, (4) practicing thinker, (5) advanced thinker, and (6) master” (Nadeak & Naibaho, 2020, p. 4-5). The unreflective thinkers are unaware of their flawed thinking (Nadeak & Naibaho, 2020). The challenged thinker has limited awareness, while the novice thinker begins recognizing the need for critical thinking (Nadeak & Naibaho, 2020). The practicing thinker regularly practices skills, whereas the advanced thinker periodically uses the skills (Nadeak & Naibaho, 2020). The master thinker uses critical thinking as second nature (Nadeak & Naibaho, 2020).

A lack of critical thinking skills can be due to poorly worded problems that, in turn, lead to students needing help to identify what a question is asking (Sarwanto & Chumdari, 2021). In addition, students have misconceptions about mathematic topics and too often rely on memory, not understanding (Sarwanto & Chumdari, 2021). If not developed, critical thinking deficits will affect the quality of future education and learning (Sarwanto & Chumdari, 2021).

Traditional teaching methods cause low critical thinking in students, which may also affect future education and learning (Syahrial et al., 2021). Traditional methods include a significant amount of lecturing and need to have the core values of critical thinking pedagogy (Sarwanto & Chumdari, 2021). Critical thinking is about knowing how and why rather than what (Sarwanto & Chumdari, 2021). Teacher factors such as learning models dominant with lectures, lack of clear problem descriptions, strategies that fail to make students understand, and lack of expertise hinder students from being successful (Sarwanto & Chumdari, 2021). Teachers must allow students opportunities to collaborate, problem-solve, and increase creativity while focusing on developing students’ critical thinking skills (Sarwanto & Chumdari, 2021). Critical thinking

education should encourage collaboration, increase linguistic capabilities, and give learners a sense of engagement (Yaprak & Kaya, 2020).

When critical thinking opportunities are implemented, critical-thinking students are successful (Nahar et al., 2022; Syahrial et al., 2021). When implementing critical thinking models into the lesson, students display increased cognitive and socio-emotional awareness, improved group work, and increased creativity in collaborative problem-solving situations (Nahar et al., 2022). Implementation of tasks increased students' critical thinking and attitudes toward learning (Syahrial et al., 2021). Successful critical thinkers are more prepared for the challenges of 21st-century life (Suratmi & Sopandi, 2022).

Approaches to Teaching Critical Thinking

Critical thinking is reasonable and reflective thinking focused on designing what to believe or do. In addition, Ennis (1987) claims that critical thinking follows three domains: "background knowledge, transfer, and general instruction. Background knowledge is essential for thinking in any domain" (Ennis, 1987, p. 7). Transfer refers to dispositions and abilities from one domain to another (Ennis, 1987). Importantly, transfer is only possible if there is sufficient practice in various domains and instruction that focuses on transfer (Ennis, 1987). The general instruction domain is an explicit critical thinking lesson or course whose skills transfer to the first two domains (Ennis, 1987).

The general approach attempts to teach critical thinking abilities and dispositions within the presentation of the subject matter (Ennis, 1987). The general approach discusses infusion and immersion (Ennis, 1987). The infusion approach uses deep, thoughtful, well-understood subject matter instruction to think critically about a subject (Ennis, 1987). Infusion within critical thinking can be seen in the works of Glaser (1984), Resnick (1987), and Swartz (1984) as a framework and scaffold of how to implement thinking in a classroom. The second topic under

the general approach is immersion (Ennis, 1987). Immersion includes a thought-provoking kind of subject matter instruction in which students get deeply immersed in the subject, but general critical thinking principles need to be made explicit (Ennis, 1987). In addition, a second approach to teaching critical thinking is called the mixed approach (Ennis, 1987). The mixed approach combines the general approach with either infusion or immersion in subject-specific critical thinking instruction (Ennis, 1987). The mixed approach can be seen in the works of Sternberg (1987), Nickerson (1988), and Perkins and Salomon (1989) as an instructional scaffold for the implementation of critical thinking opportunities.

Critical thinking opportunities do not have to be aligned to a specific subject matter (Liljedahl, Zager, et al., 2021; McPeck, 1981; Saltman, 2012). Vagueness of the subject matter is a strength and does not limit the task (Liljedahl, Zager, et al., 2021; Ennis, 1987). Regardless of alignment, critical thinking must be directed towards some end of purpose (Green, 2005). The process of thinking has three essential features: "(1) it gives shape to people's decisions or beliefs, (2) thinking critically encourages people to engage in adequate and accurate thinking, and (3) relevant standards are naturally embedded in critical thinking opportunities" (Green, 2005, p. 37).

There are four areas of competency to succeed in critical thinking: interpretation, communication, knowledge, and technique (Fisher & Scriven., 1997). Strategies that engage students in argumentation and discourse through inductive and deductive logic can positively influence critical thinking (Assante et al., 2022; Rivas & Saiz, 2023). In addition, strategies such as training students to define arguments, evaluate the reliability of sources, identify errors and assumptions, synthesize information, and make inferences have significantly influenced critical thinking (Assante et al., 2022). Strategies that motivate students to recognize and question assumptions have also influenced critical thinking (Brookfield, 2013).

Learning through critical thinking opportunities offers the ability to learn in a student-centered social environment (Brookfield, 2013). Students who are avid critical thinkers enjoy teacher models that explicitly state the expectations, ground rules, and success criteria (Brookfield, 2013; Mezirow, 2000). Concrete, real-world examples aid individuals in thinking critically about a problem (Brookfield, 2013). Notably, the most significant critical thinking moments occur when an unexpected event or idea occurs (Brookfield, 2013; Mezirow, 2000). Theorists call this moment a disorienting dilemma (Mezirow, 2000). When students hear from their peers and consider new perspectives, the power dynamic of the room changes; students find it easier to listen to questions and consider new perspectives when they come from the mouths of their peers rather than the teacher (Brookfield, 2013). Therefore, the classroom must be student-centered to build a thinking classroom (Assante et al., 2022; Brookfield, 2013; Mezirow, 2000; Rehman et al., 2023).

Building Thinking Classrooms

Students spend most of their class time not thinking, leading to a lack of success in mathematics (Liljedahl, Zager, et al., 2021). Implementing critical thinking opportunities requires a thinking classroom (Dempster, 2022; Krall, 2018; Liljedahl, Zager, et al., 2021). The thinking classroom approach offers a unique take on how classrooms should be organized, assigned tasks, and how students learn and work together (Dempster, 2022; Liljedahl, Zager, et al., 2021). A thinking classroom makes a purposeful and consistent habit of thinking (Liljedahl, Zager, et al., 2021; Smith et al., 2020). In the thinking classroom, individuals think collectively, learn together, and construct knowledge and understanding through activities and discourse (Liljedahl, Zager, et al., 2021).

Building a thinking classroom is a radical departure from institutional norms that requires significant changes (Larsen & Liljedahl, 2022). First, educators need to examine what type of

tasks they use (Dempster, 2022; Krall, 2018; Liljedahl, Zager, et al., 2021). Lessons should begin with problem-solving tasks that are highly engaging and allow every student an entry point (Liljedahl, Zager, et al., 2021). In addition, educators should start with non-curricular tasks and then gradually replace them with curricular problem-solving tasks (Krall, 2018; Liljedahl, Zager, et al., 2021). Also, educators must change how they form student groups (Liljedahl, Zager, et al., 2021). Groups should be visibly random groups of three (Liljedahl, Zager, et al., 2021). Educators should focus on creating vertical, non-permanent spaces for students to work Liljedahl, Zager, et al., 2021).

Importantly, how educators answer questions is critical to building thinking classrooms (Herbal-Eisenmann & Breyfogle, 2005; Larsen & Liljedahl, 2022; Smith et al., 2020). Students often ask proximity questions because the teacher is close, stop thinking questions to see if they are correct, and keep thinking questions to keep them working (Liljedahl, Zager, et al., 2021). Only the final type of question should be answered in a thinking classroom (Dempster, 2022; Mellone et al., 2021). Timely use of hints and extensions that balance the challenge of the task with the abilities of the students working on it can be used (Liljedahl, Zager, et al., 2021; Smith et al., 2020).

Student autonomy and communication are critical to building thinking classrooms (Dempster, 2022; Krall, 2018; Liljedahl, Zager, et al., 2021). Students should be allowed to interact extensively with other groups (Liljedahl, Zager, et al., 2021). In addition, consolidation with an educator is crucial (Liljedahl, Zager, et al., 2021). Every group should be allowed to debrief the teacher after finishing the task (Liljedahl, Zager, et al., 2021). Strong and purposeful mathematical discourse is essential to thinking classrooms (Dempster, 2022; Krall, 2018; Liljedahl, Zager, et al., 2021). Sharing ideas, strategies, thoughts, and failures should be encouraged (Dempster, 2022; Krall, 2018; Liljedahl, Zager, et al., 2021).

Thinking classrooms offer summative and formative assessments (Liljedahl, Zager, et al., 2021). What educators choose to evaluate should honor the activities of what the educator values through a learning process more so than the product (Dempster, 2022; Krall, 2018; Liljedahl, Zager, et al., 2021). Formative assessment should focus on where students are academically and where they are going (Liljedahl, Zager, et al., 2021; Smith et al., 2020). Within a thinking classroom, grading should be a student performance report based on data analysis rather than counting points (Liljedahl, Zager, et al., 2021). When the process and thinking during the task are far more critical than the product, it helps to relieve mathematics anxiety and aids in the learning of the task (Boaler & Williams, 2015; Dempster, 2022).

Thinking Tasks

A thinking task is a skilled activity that demands interpretation and evaluation of observation, communication, and other sources of information guided by intellectual standards such as clarity, relevance, adequacy, and coherence (Fisher, 2009). A thinking task is a work undertaken for oneself or others, freely or for some reward (Long, 1985; Fisher, 2005; Liljedahl, Zager, et al., 2021). Tasks can be grouped into three categories:

> (1) the information task in which learners must exchange information with a partner who knows nothing about the related information and needs to collaborate with others to complete the task; (2) the opinion task, in which the learners indicate a personal preference, feeling, or attitude in response to a given situation; (3) the reasoning task in which cognitive skills of inference, reasoning, and deriving new information from the given information occur. (Yaprak & Kaya, 2020, p. 2)

In addition, tasks fall into three different styles (Liljedahl, Zager, et al., 2021). First, a non-curricular task is a highly engaging thinking task used without concern for the curriculum (Liljedahl, Zager, et al., 2021). This type of task promotes building a thinking classroom

(Ballantyne, 2019; Krall, 2018; Liljedahl, Zager, et al., 2021). Second, a scripted curricular task illuminates students' autonomy (Liljedahl, Zager, et al., 2021). A scripted curricular task is introduced as an example and follows steps given by the teacher. A scripted task offers an entry point for every student, allows for mathematical discourse, and focuses more on the task process than the product Dempster, 2022; Krall, 2018; Liljedahl, Zager, et al., 2021). Lastly, an as-is task is a curricular task given through direct instruction to promote mimicking (Liljedahl, Zager, et al., 2021). An as-is task offers little construction of knowledge, and learners often struggle to make meaning of the learning, leading to little understanding (Ballanyte, 2019; Dempster, 2022; Krall, 2018; Liljedahl, Zager, et al., 2021).

A thinking task does not necessarily have to be curricular. A non-curricular thinking task is a problem-solving activity or mental puzzle that is highly engaging, motivates students and gets them to challenge themselves (Liljedahl, Zager, et al., 2021). The traditional approach to learning has become the default technique, leading to less thinking and meaning-making (Herbal-Eisenmann & Breyfogle, 2005; Krall, 2018). Students struggle to find solutions when presented with problems and wait for teachers to step in (Dempster, 2022; Liljedahl, Zager, et al., 2021). Therefore, there is a need for opportunities to be purposeful and consistent with thinking tasks. Starting with non-curricular assignments provides every student with an entry point, encourages mathematical discourse, and offers a chance for success to boost mathematical confidence and limit mathematical anxiety (Boaler & Williams, 2015; Liljedahl, Zager, et al., 2021; Smith et al., 2020).

Teacher Perceptions of Thinking Tasks

Literature on teacher perceptions of critical thinking tasks is needed. Most literature surrounding the teacher perceptions of tasks include negative connotations regarding the lack of experience and knowledge to implement the tasks correctly (Linan-Garcia et al., 2021; Suratmi

& Sopandi, 2022; Tekkumru-Kisa et al., 2018). Teachers claimed that the tasks they implemented were positive; however, the teachers lacked the knowledge and time to acquire more (Linan-Garcia et al., 2021). Teachers sometimes struggle to implement thinking tasks due to a lack of knowledge and confidence (Suratmi & Sopandi, 2022). Those who implemented the tasks also shared they felt limited by lesson plan formats and needed more planning (Suratmi & Sopandi, 2022).

While most research on teacher perceptions is negative due to lack of time and resources, there are some positive instances of implementation of tasks from the teacher's perspective (Barell, 2003; Tekkumru-Kisa et al., 2018). Tekkumru-Kisa et al. (2020) found that teachers who implemented critical thinking tasks began to adopt new pedagogical practices. In addition, teachers believe that critical thinking is essential and can be achieved through implementing a learning curriculum (Barell, 2003).

The Case for Experiential Learning

Experiential learning has become increasingly prevalent in education (Kayes, 2002; Reynolds, 2009; Tomkins & Ulus, 2016). Experiential learning is not a set of tools or techniques but rather a framework for meeting the needs of individuals (Bohon et al., 2017). Teachers across the United States have begun to shift their pedagogies towards experiential learning (Kayes, 2002; Reynolds, 2009; Tomkins & Ulus, 2016). The traditional lecture approach is ineffective as teachers have acknowledged that a more student-centered approach is needed (Tomkins & Ulus, 2016). Real-world encounters with thinking tasks leave students with a more profound impression than textbooks and make learning more meaningful (Chan, 2012).

The shift towards experiential learning allows teachers to create complementary learning experiences for their students, allow for deeper and more meaningful understanding, and explore and evaluate in diverse ways (Reynolds, 2009; Wang & Calvano, 2018). Learning connects

students' emotions and empathy to learning rather than purely on a cognitive level (Chan, 2012). Analytical and critical thinking skills are being enhanced as the immediacy of authentic experience drives instruction (Wang & Calvano, 2018). Experiential learning offers opportunities to develop an understanding based on experiences and reflection (Reynolds, 2009).

Experiential learning transforms teaching philosophy and practice (Bohon et al., 2017). Professional development that offers more reflective time, more engagement in thinking, and activities guided by an experiential model is needed (Bohon et al., 2017). When given ample time to process the change, teachers saw significant implications of success, increased understanding, and improved critical thinking skills (Bohon et al., 2017). In addition, experiential learning enables more observation, reflection, conceptualization, and experimentation (Chan, 2012). Also, this learning style has shown increased skills, group cohesion, self-regulation, and peer self-esteem (Gosen & Washbush, 2004).

Teacher Perceptions of Experiential Learning

Educational pedagogy is unique to the individual. Teachers using experiential learning theory to guide their pedagogy have profoundly influenced their classrooms (Bhusal, 2021; El Sayary, 2021; Hunt & Laverie, 2004; Lewis & DeSantis, 2021). Educators have claimed that experiential learning to guide their pedagogy allows them to have the best scholarly knowledge and practice by blending research and theory with practical application (Lewis & DeSantis, 2021). While some studies outline that this pedagogy style slows instruction, they also agree that slowing it down this way allows for more clearly identifying learning gaps individually (El Sayary, 2021). In addition, educators claimed that experiential learning allowed their students to explore concepts richer and more meaningfully (Bhusal, 2021; Hunt & Laverie, 2021; van Vliet et al., 2015).

Educators using experiential learning saw significant changes in student success (Bhusal, 2021; El Sayary, 2021; Hunt & Laverie, 2014; Lewis & DeSantis, 2021). Educators saw that students could apply knowledge to new situations and compose deeper-level thinking questions (van Vliet et al., 2015). Improvements in metacognition, collaborative learning, creativity, confidence, independence, student autonomy, and student responsibility were found when implementing experiential learning (Bhusal, 2021; El Sayary, 2021). Most importantly, students using experiential learning answered higher-level Bloom's questions correctly at twice the rate of their peers lacking such opportunities (van Vliet et al., 2015).

Political Influence on the Current Status of Education

The United States Congress first signed legislation in 1867, creating an office of education to improve American education (Smith, 2020). Following Sputnik, the nation began to take global competition seriously, and the influence of politics began (Johanningmeier, 2010 Smith, 2020). The National Defense Education Act (NDEA), established in 1958, created federal education programs in mathematics, science, and foreign languages (Johanningmeier, 2010; Smith, 2020). When Lyndon Johnson became president following the assassination of Kennedy, he created the War on Poverty, which led to the passing of the Higher Education Act in 1965 (Smith, 2020). The Higher Education Act led to the landmark creation of the Elementary and Secondary Education Act (ESEA) of 1965, which passed into national law authorizing state-run programs and districts to raise academic achievement (Smith, 2020). The federal government assumed complete control of the direction of education in 1980 when Jimmy Carter created the Department of Education (Smith, 2020).

The influence of political figures on modern-day education is undeniable. The accountability era began with Ronald Raegan's *A Nation at Risk* in 1983 (Bell, 1993; Smith, 2020). Raegan's report formed the accountability movement (Smith, 2020). Raegan's attempt to

fix the education system began by slating the achievement of Americans versus that of the world. Raegan claimed Americans would see it as an act of war if another country had done to our education system what we had done to it (Smith, 2020). In 1991, George H. W. Bush tried to push his academic agenda with a program called America 2000 (Peters, 1992; Smith, 2020). Interestingly, the America 2000 program called for educators to implement national standards and standardized school achievement tests (Peters, 1992; Smith, 2020). However, he was unsuccessful. That is until Bill Clinton took office and changed the name to Goals 2000 (Peters, 1992; Smith, 2020). Goals 2000 was passed in 1994 and required schools to take standardized achievement tests in mathematics and reading according to national standards (Smith, 2020).

One of the most significant influences of the accountability era began in 2001 with the passing of the No Child Left Behind Act (NCLB). ESEA was up for renewal, and following the attacks of 9/11, the nation was eager to pass NCLB unanimously (Smith, 2020; Steinberg & Quinn, 2017). NCLB held schools accountable for how kids learned and achieved (Darling-Hammond, 2011; Smith, 2020). The law was heavily criticized as it penalized schools that did not show improvement (Smith, 2020). The law required Annual Yearly Progress (AYP), and schools that did not meet expectations received penalties (Smith, 2020; Steinberg & Quinn, 2017). NCLB was followed by Obama's Race to the Top in 2009, which adopted new state standards and performance rewards (Chomsky & Robichaud, 2014). Most recently, the Every Student Succeeds Act (ESSA) replaced ESEA and built upon NCLB (Smith, 2020; Steinberg & Quinn, 2017). ESSA is the current reform of education in the United States.

Current Status of Mathematics Education

Mathematics instruction relies too heavily on learning rules and procedures without developing essential conceptual understandings (Wood et al., 1991). Rules and procedures lead to a question of curriculum and instruction. In 2001, the passing of No Child Left Behind

(NCLB) created standards-based reform (SBR). SBR argued that properly implementing rigorous standards, aligned curriculum, and accountability measures would increase student achievement (Pak et al., 2020). However, it became clear that this was different. Teachers claimed that the standards needed to be more ambitious and adequately represent the variety and diversity of learners (Allensworth et al., 2022; Lee, 2021; Tekkumru-Kisa et al., 2020).

The creation of Common Core State Standards (CCSS) identified a centralized set of knowledge and skills students should master while necessitating a fundamental rethinking of student engagement and instructional pedagogy (Allensworth et al., 2022). These standards allowed students to learn to think in a new and more demanding way (Tekkumru-Kisa et al., 2020). However, this requires teachers to develop, adapt, select, and enact tasks that have the potential to reach the envisioning, thinking, and reasoning of the standards (Tekkumru-Kisa et al., 2020).

This task is far easier said than done. Students have yet to be given opportunities to pose mathematical problems as envisioned (Van Harpen & Presmeg, 2013; Van Harpen & Sriraman, 2013). Research has concluded that teachers can pose reasonable problems yet often ignore the topic of mathematical problem-posing (Kar, 2015; Leung & Silver, 1997; Lowrie, 2002; Silver & Cai, 1996). Also, research has shown that there needs to be a coherent, comprehensive account of problem posting as a part of mathematics curriculum and instruction (Lee, 2021). While mathematical problem-posing is essential, it has not received much attention (Lee, 2021).

Statistical Evidence of Mathematics Achievement Decline

Two international groups test mathematics: (1) Trends in International Mathematics and Science Study (TIMSS) and (2) Organization for Economic Cooperation and Development (OECD). According to the most recent TIMSS report in 2019, the mathematics achievement gap continues to grow further and further as the benchmark for international scores has dropped from

the prior three administrations (TIMSS, 2019). The Organization for Economic Cooperation and Development (OECD) comprises 79 countries that take the mathematics assessment. According to a recent report, the United States places 30th out of 79 countries in mathematics (OECD, 2018). More importantly, the OECD (2018) states that scores within the United States have remained steady since 2003.

The U.S. Department of Education releases a national report card every year. Scores nationwide have continued to drop (Marder & Kring Villanueva, 2018; Wen & Dubé, 2022). According to the National Assessment of Education Progress (NAEP), no state or jurisdiction within the United States improved mathematics scores between 2019 and 2022 (U.S. Department of Education, 2022). Moreover, scores of 8th-grade students dropped in 51 out of 53 jurisdictions or states by an average decline of eight points (U.S. Department of Education, 2022). In addition, 4th-grade students dropped in 43 out of 53 jurisdictions or states by an average decline of 5 points (U.S. Department of Education, 2022). All represent the most significant decline since the initial assessments in 1990 (U.S. Department of Education, 2022). From 2013 to 2015, mathematics scores declined by 1.33 points in the previous three cycles (Camilli, 2021). Furthermore, number sense, measurement, algebra, data analysis, and geometry have continuously declined since 2013 (Camilli, 2021).

The Need for More Opportunities

More research is needed to outline and define the current state of critical thinking education. Furthermore, an even smaller amount is known about the exact nature of early childhood educators' knowledge regarding critical thinking knowledge and the implementation of tasks (Linan-Garcia et al., 2021). Four of the most prevalent mathematics journals dealing with education concluded that more attention needs to be paid to the elementary level (Linan-Garcia et al., 2021).

More professional development regarding critical thinking pedagogy, best practices, and implementation should be utilized since there are limited opportunities for teacher professional development in creating critical thinking models for educators to see first-hand (Donnelly et al., 1999; Sarwanto & Chumdari, 2021; Suratmi & Sopandi, 2022). Teachers and educators must be given ample time to learn methods that emphasize students being more active in learning and helping improve skills through critical thinking (Robinson & Kay, 2010). Too often, professional development focuses on knowledge of 21st-century skills rather than implementing those skills (Donnelly et al., 1999). Thus, a gap exists between teachers' knowledge, skills, attitudes, and abilities to implement critical thinking and teachers' need for in-service training to promote critical thinking (Donnelly et al., 1999; Suratmi & Sopandi, 2022).

Inadequacies in Resources and Time

The adequacy of textbooks within the United States is evident (Ornstein, 2010). Textbooks and assessments may not be sufficient for accurately measuring what students have learned (Ornstein, 2010). Many mathematics textbooks are available and used in school districts across the United States. However, with over 15,000 school districts in the United States, textbook publishers aim to please a broad audience and emphasize the breadth of topics versus the depth of subjects (Ornstein, 2010). Textbook companies are tasked with publishing different sets of ever-changing standards in each state while challenging themselves to align continually with new standards and practices every year (Cogan et al., 2015). Compared to other countries, approved textbooks ensure credibility and consistency of topics taught and covered and include more challenging and complex questions (Lo et al., 2001; Ozer & Sezer, 2014). American textbooks tend to present information through vast data with far less problem-solving (Ornstein, 2010; Ozer & Sezer, 2014). On the contrary, texts in other countries focus on fewer topics over a

more extended period to allow for the depth of teaching, allowing students to think about and practice specific procedures (Ornstein, 2010; Ozer & Sezer, 2014).

American students spend significant time watching television, surfing the internet, or using a technological device (Gentile et al., 2012; Locker, 2015; Ornstein, 2010). The average eight-year-old in the United States spends eight hours daily interacting with the media (Locker, 2015). The typical 10-year-old watches an average of 3 hours and 41 minutes of television and spends just over an hour playing video games daily (Gentile et al., 2012). As the students get older, the amount of media time steadily increases to 11 hours a day during their teenage years (Locker, 2015). The excessive time spent on television and other media forms may not positively influence student achievement.

Compared to European and Asian schools, American students have a shorter school day and school year (Ornstein, 2010). School districts within the United States operate on about a 180-day calendar, whereas European countries average a 200-day calendar, and Asian countries have a 220-day calendar (Ornstein, 2010). Adequate usage of time is scrutinized as competitors have longer days and years (Ornstein, 2010). Americans believe the extended school year in European and Asian countries is a core reason international students continue scoring above American students on various tests (Ornstein, 2010).

Inadequate Teacher Preparation & Qualifications

Each state department sets its policies for teacher licensure, and much variation exists from state to state (Norris, 2013; Ornstein, 2010). Teacher licensing and certifications have been structured on how school buildings run rather than how children develop (Bornfreund, 2012). Most states provide licenses for elementary teachers that span K-5 or K-6 after the teacher demonstrates competency through state-required assessments (Bornfreund, 2012; Norris, 2013). Licensure of pre-kindergarten through third grade and another licensure that begins at third grade

and extends through middle school could ensure that teachers become better prepared in their areas (Bornfreund, 2012). Teachers of young children often either understand how students learn but lack subject-area expertise, or they understand content-related knowledge and skills but do not have insight into how students learn (Bornfreund, 2012; Norris, 2013). A restructured licensure may aid in creating more efficient and effective educators.

Teacher preparedness and qualifications must be addressed to increase achievement (Ertekin et al., 2009; Morgan, 2012). Poor school districts already struggling with low student achievement rates often have teachers with little teaching experience and minimal education credentials (Morgan, 2012). In addition, poor school districts face the problem of having a shortage of qualified teachers (Hobbs, 2012). Many school systems worldwide provide better education to less fortunate students (Morgan, 2012; Paine & Scheicher, 2011). For example, countries like Singapore tend to provide students with low socioeconomic backgrounds with opportunities to participate in classrooms with lower student-to-teacher ratios (Morgan, 2012; Paine & Schleicher, 2011). In addition, countries worldwide have their most highly qualified teachers who teach students experiencing the most significant difficulties (Morgan, 2012; Paine & Scheicher, 2011).

Teacher-education systems negatively influence the preparedness of educators (McGee, 2004; Morgan, 2012; Weiss, 2015). In the United States, almost half of all K-12 teachers come from the bottom of their graduating class (Kristof, 2011; Morgan, 2012). Teacher preparation programs are failing to prepare teachers. Teachers lack the training to develop the skills necessary to work in diverse classrooms with students with unique learning and social needs (Stringfellow, 2009). Teachers have yet to receive training based on hands-on methods, higher-level reasons, and other effective methods of instruction during their teacher preparation programs (Ertekin et al., 2009). Countries leading the world in achievement scores prepare their

teachers more extensively than the United States, pay better, and evenly spread their highly qualified teachers through their schools (Darling-Hammond, 2008; Morgan, 2012).

Challenges in Teaching Mathematics

Historically, the teacher's role in mathematics primarily consists of explaining mathematical procedures, while a student's role is to listen and apply those procedures to similar problems (Beswick, 2012; Hiebert & Grouws, 2007; Stigler & Hiebert, 2004;). This common theme across the United States continues to challenge teaching mathematics. Teaching mathematics needs more than just reciting or mimicking strategies and regurgitating facts (Liljedahl, Zager, et al., 2021). Research on the challenges of teaching mathematics yielded three main issues: the national curriculum, linguistic hurdles, and pedagogical concerns.

The national curriculum for mathematics is a part of the problem, as there is no unified or shared approach. Most states within the United States use Common Core Standards, while others have devised their own standards. On a national level, the status of mathematics education could benefit from some changes (Henderson & Ivey, 2023; Hersee, 1993). The national curriculum must say how and what to teach (Hersee, 1993). How educators teach mathematics is far more critical than what we teach in promoting the outcomes educators seek (Hersee, 1993; Koskinen & Pitkaniemi, 2022; Liljedahl, Zager, et al., 2021). Teaching solely what the standards say leads to drilling and practicing rote skills that do not transfer learning (Hersee, 1993; Koskinen & Pitkaniemi, 2022; McTighe & Brown, 2020). In addition, the national curriculum needs to leave space for teacher creativity (Hersee, 1993; McTighe & Brown, 2020).

Also, the national curriculum should challenge students to predict, hypothesize, test hypotheses, and prove results within mathematics (Hersee, 1993; McTighe & Brown, 2020; William, 2003). Notably, the national curriculum should allow for differentiation for diverse students that does not settle for mastery of content but for mastery of constructed knowledge

(Hersee, 1993). Lastly, the mathematics curriculum on a national level should be more than just mechanical skills (Hersee, 1993; McTighe & Brown, 2020; William, 2003). Being a good mathematician requires using mathematics appropriately and being competent with calculations (Hersee, 1993; McTighe& Brown, 2020; William, 2003).

Another challenge in teaching mathematics aligns with the linguistic challenges of students. Achievement in mathematics is highly related to the student's understanding of the math language (Mbugua, 2012). Understanding mathematical language gives students the skills to think about, discuss, and create new math concepts (Chard, 2003). The language of mathematics text, discourse within a classroom, and language used in assessment situations all play critical roles in mathematics difficulties (Sarabi & Gafoor, 2017). When hung up on language, students cannot effectively learn to construct knowledge (Sarabi & Gafoor, 2017). Students who cannot construct knowledge due to language is precisely why some students excel in computation, but their ability to apply those skills suffers when working independently or on word problems (Bruun et al., 2015). The NCTM (2000) Principles and Standards for School Mathematics argues that students with opportunities, encouragement, and support for speaking, writing, reading, and listening in mathematics classes reap dual benefits: they communicate to learn mathematics and learn to communicate mathematically. The role of language undoubtedly offers challenges to teaching mathematics.

Pedagogical techniques and strategies can aid in the difficulties in teaching mathematics. Mathematics instruction must focus on active student learning guidance (Koskinen & Pitkaniemi, 2022). Regardless of the approach, guidance should never hinder the students' creative thinking (Koskinen & Pitkaniemi, 2022). Traditional styles of pedagogy lack student thinking and offer only what the teacher tells them to do. Mathematics instruction should be motivating and goal-oriented (Koskinen & Pitkaniemi, 2022). In addition, mathematics instruction should consist of

relevant context that promotes student interest (Koskinen & Pitkaniemi, 2022). Drawing attention to conceptual links between math content and the natural world prepares students to be 21st-century learners (Koskinen & Pitkaniemi, 2022). Mathematics pedagogy should require well-chosen illustrative tools, whether concrete instruments or digital tools (Koskinen & Pitkaniemi, 2022). Another pedagogical shift that needs to occur revolves around communication. Mathematics communication should focus on the quality of discussion rather than the quantity (Koskinen & Pitkaniemi, 2022). A math environment should be welcoming and supportive and see all students as individuals (Koskinen & Pitkaniemi, 2022).

Student Engagement

Engagement can be viewed from many perspectives. For authentic engagement to occur, students must be engaged on three levels: affective, cognitive, and operative (Fredricks et al., 2004). Student engagement in mathematics is critical during elementary schooling to develop understanding (Attard, 2012). While children can memorize information, engagement allows for curiosity and persistence that will create lifelong learners (Levine, 2012). Classrooms need to shift toward inviting students to solve problems in ways that make sense to them, that allow them to explain and justify their ideas, and to engage in peer discourse (Johnson et al., 2022). Instruction should focus on reasoning and problem-solving, inviting students to be challenged in mathematics (Johnson et al., 2022).

A classroom that engages students positively influences mathematics skills (Bang et al., 2022). More importantly, engaged students are successful students (Bang et al., 2022; Johnson et al., 2022; Pantaleo, 2023). Active students display participation, persistence, and completion of tasks (Miller et al., 1996). An engaged student applies creativity and critical thinking that extends their knowledge and deepens understanding (Pantaleo, 2023). Active students are

validated by being allowed to defend, argue, and justify their explanations of ideas (Johnson et al., 2022).

Project Based-Learning

Project-based learning (PBL) is an investigative strategy that promotes student engagement (Nadeak & Naibaho, 2020; Pan et al., 2023; Rehman et al., 2023). This learning approach is centered on real life and is process-oriented rather than outcome-oriented (Evcimik & Oruc, 2023). PBLs consist of real-life problems that require students to ask questions, make predictions, and complete research (Evcimik & Oruc, 2023). During PBL lessons, students learn by doing (Evcimik & Oruc, 2023). In addition, a sense of ownership is developed as the student solves the problem in their own way (Evcimik & Oruc, 2023).

PBL has successfully promoted critical student thinking skills (Pan et al., 2023; Nadeak & Naibaho, 2020; Rehman et al., 2023.). When implemented correctly, PBL promotes student-centered learning, collaboration, and problem-solving skills (Rehman et al., 2023). In addition, PBL improved student achievement, attitude, and engagement toward learning (Pan et al., 2023; Nadeak & Naibaho, 2020). Furthermore, the PBL strategy improved critical thinking (Abdullah et al., 2022).

PBL experiences are cross-curricular and can be adapted to any situation. However, successful PBL problems shared the following themes: “the problem motivated students, students were required to make reasoned decisions and defend them, the problem incorporated content objectives, the problem required collaboration, and the problem was open-ended" (Duch et al., 2001, p. 13). A PBL experience allows for student-centered mathematics (Grasha, 1996). Student-centered mathematics approaches stray from the traditional experience and enable students to make meaning of their learning (Turner et al., 2013). PBL experiences create an

atmosphere of discourse, thinking, and learning that fosters critical thinking skills (Duch et al., 2001; Grasha, 1996; Liljedahl, Zager, et al., 2021).

Student-Centered Mathematics

Advocacy for student-centered learning continues to grow and move forward (Jacobs & Spangler, 2017; Nasir & Cobb, 2006; Schoenfeld, 2010; Turner et al., 2013). Shifting away from traditional pedagogy and lecture methods requires new trajectories. Student-centered learning can be described as a classroom that provides opportunities for making thinking public, for students to engage with each other's mathematical thinking, and wherein mathematical sense-making, conjecturing, and justifying drive the instruction (Thanheiser & Melhuish, 2023). Conferring to students to understand student thinking and reasoning, structuring mathematical student talk, and orchestrating mathematical discourse are just some of the ways that teachers are changing pedagogically to ensure that mathematics is student-centered (Jacobs & Spangler, 2017; Nasir & Cobb, 2006; Schoenfeld, 2010; Turner et al., 2013).

Student-centered instruction increases academic achievement in mathematics (Chen, 2023; Emanet & Kezer, 2021; Jacobs & Spangler, 2017). Mathematics classrooms offering a student-centered approach have an outstanding impact on affective features such as attitude while decreasing student anxiety levels (Emanet & Kezer, 2021). Student-centered classrooms have also resulted in increases in intrinsic and extrinsic motivation, peer relationships, learning ability, participation, and social skills have also resulted from student-centered classrooms (Chen, 2023; Siswono, 2011; Turgut & Turgut, 2018; Zakaria et al., 2010). Overall, an examination is necessary to understand the mathematics critical thinking process for teachers in primary education, which will aid in contributing to the body of knowledge.

Summary

The literature review identified the empirical gaps and added practical value that

contributes to the body of works that critical thinking is underrepresented in elementary mathematics. Teachers and students need help keeping up with rigorous curricula and standards. Critical thinking is falling through the cracks of education. At the same time, educators continue to push computational thinking through strategies that lack problem-solving skills because of the overwhelming pressures of mandated accountability testing. Using theories of cognitive development from Kolb, Piaget, and Vygotsky, researchers have lit a candle to how education can bridge the gap between expectation and reality (Piaget, 1962; Vygotsky, 1929). Critical thinking is a 21st-century skill that continues to be discussed within research and literature. The General Approach, Mixed Approach, and work of Ennis and McPeck laid the groundwork for educators to implement tasks (Ennis, 1987; McPeck, 1981). Task development must challenge students to use the 21st-century skills educators are trying to teach. They should be open-ended and demand interpretation, observation, communication, and collaboration (Fisher, 2009). Statistical evidence shows that the downward trend of mathematics began as early as 2012 (Camilli, 2021). Worse, the decline is wider than one age group or level. The decline in mathematics achievement can be seen at all levels of education (U.S. Department of Education, 2022). The downward trend we see as a nation must be addressed. Traditional teaching methods and pedagogy are not working or aiding in declining mathematics achievement.

CHAPTER THREE: METHODS

Overview

The purpose of this phenomenological study is to understand the mathematics critical thinking process for teachers in primary education in northern Virginia. The problem is that critical thinking is underrepresented in primary mathematics education. Chapter three outlines the research design of this qualitative hermeneutic phenomenological study. The central research question is also shared with sub-questions regarding critical thinking tasks aligned with the study's theoretical framework. Next, the setting, site, and participants are listed. Next, positionality and philosophical assumptions are shared. The study procedures are then discussed. Following the procedures, the data collection plan is shared, including a data synthesis. Lastly, trustworthiness is discussed.

Research Design

The purpose of this phenomenological study was to understand the mathematics critical thinking process for teachers in primary education in northern Virginia. Qualitative research is a situated activity consisting of interpretive, material practices that make the world visible and transformational (Denzin & Lincoln, 2011). Exploring these experiences yields a qualitative research study (Creswell & Poth, 2018). In addition, several prominent themes can be seen throughout the qualitative research design. This study occurred within the site, using multiple methods and obtained various perspectives while using myself as a critical instrument (Creswell & Poth, 2018). The natural setting was classrooms. The classrooms varied in grade level and location to understand the lived experiences from multiple perspectives. The data collection methods varied between individual semi-structured interviews, a focus group, and document analysis throughout different stages of the study. The participants shared varying perspectives and points of view according to the implementation of the tasks.

A hermeneutical phenomenology is a form of phenomenology in which research is aligned to understand the texts of life and lived experiences (van Manen, 2016, 1990). A lived experience consists of encounters as conscious human beings (van Manen, 2016). A hermeneutic phenomenological study seeks to identify changes in curriculum planning and instruction to understand the lived experiences of mathematics instructors using critical thinking tasks. Hermeneutic studies involve understanding and analyzing shared experiences (van Manen, 2016). Phenomenological studies describe common meanings from individuals who share a lived experience (Creswell & Poth, 2018; van Manen, 2016). Due to the nature of my classroom pedagogy and the use of critical thinking tasks, this study fell under the hermeneutic phenomenological method (van Manen, 2016).

Phenomenology allows the exploration of a single concept or idea (Creswell & Poth, 2018; van Manen, 2016). A phenomenon is a central concept being examined (Creswell & Poth,

2018; van Manen, 2016). The phenomenon of changes in curricular planning and instruction is being investigated. I have chosen to bracket my previous experiences, as I am increasingly employed in qualitative research (Gearing, 2004; van Manen, 2016), to allow personal experiences to be set aside while engaging and exploring the lived experiences of my participants (Creswell & Poth, 2018; Giorgi, 2009).

Research Questions

The proposed research questions are derived from and align with the problem and purpose statement to support this study. The central research question focuses on understanding primary mathematics teachers' lived experiences of implementing critical thinking. The sub-questions align with the theoretical framework upon which this study is guided.

Central Research Question

What are the experiences of primary mathematics teachers implementing critical thinking?

Sub-Question One

What are the concrete experiences of primary mathematics teachers implementing critical thinking?

Sub-Question Two

What are the reflective observation experiences of primary mathematics teachers implementing critical thinking?

Sub-Question Three

What are the abstract conceptualization experiences of primary mathematics teachers implementing critical thinking?

Sub-Question Four

What are the active experimentation experiences of primary mathematics teachers implementing critical thinking?

Setting and Participants

The setting and participants section explains where the study was conducted and who qualified to participate. The following are explanations of the site and participants involved in this study. Permission for both the setting and participants were received. Rationale and justification are included in the following section.

Site

The region of interest in this study will be northern Virginia. Beta Public County Schools (BPCS) is a division hosting approximately 30,400 students and 1,841 educators spanning approximately 277 square miles (National Center for Education Statistics, 2023). The division hosts 17 elementary, eight middle, and five high schools. This school division was chosen due to its size and convenience. The student population demographic consists of 40.6% White, 24.1% Hispanic, 21.4% Black, 8.5% two or more races, 4.9% Asian, and less than a percent of Hawaiian and Native American (National Center for Education Statistics, 2023). In addition, 52% of the students are male, while females account for 48% of the student population (National Center for Education Statistics, 2023). English language learners make up 12.2% of the student population (National Center for Education Statistics, 2023). Approximately 61% of students are eligible for free and reduced lunch (National Center for Education Statistics, 2023). In addition, 43% of teachers hold a bachelor's degree, 51% hold a master's degree, and 1% hold a doctoral degree (National Center for Education Statistics, 2023). The elementary grades have a teacher-to-student ratio of 13.61:1 (National Center for Education Statistics, 2023).

Participants

This study captured the lived experiences of current educators in northern Virginia. The intended final sample size of 12-15 participants was appropriate for a phenomenological study (Creswell & Poth, 2018). All participants must have at least one year of experience in BPCS. Participants are mathematics educators at the primary level and must be at least 18 years of age or older. All participants self-reported using critical thinking tasks, including curricular and noncurricular tasks. There are no degree requirements. Participants came from varying backgrounds.

Recruitment Plan

This study aimed to understand the lived experiences of elementary mathematics teachers and their experiences using critical thinking tasks to inform decisions regarding curriculum and instruction. Before any recruitment was conducted, I obtained approval from the Institutional Review Board (IRB) (see Appendix A). I also requested permission from the site-approving school site agents (see Appendix B) and requested the approval letter be completed (see Appendix C). Following approval from the IRB and the school site agents, I began recruiting participants. In addition, I gained participants by asking mathematics instructional coaches for contact information of teachers who implement thinking tasks.

The proposed study used criterion sampling of 12 - 15 participants (Creswell, 2013). Criterion sampling allowed me to select specific criteria that my participants must meet to ensure that they have experienced the phenomenon of the study (Creswell, 2013). For this study, eligible participants implemented critical thinking tasks pedagogically. Participants were employees of BPCS, teach elementary grades kindergarten through five, and are mathematics educators. Participants must also be 18 years of age or older. Participants self-reported their eligibility through a Google Form screening link within the recruitment letter. Any participants who have not responded to the initial recruitment invitation within one week received a follow-

up recruitment letter (see Appendix E). All participants signed and acknowledged the consent form (see Appendix F) before data was collected through individual interviews, a focus group, or document analysis.

Researcher's Positionality

This chapter section presents my motivation for conducting this study and the foundational frameworks that guide me in the research. The following sections highlight the lens through which my interpretive framework and philosophical assumptions affected the study. Lastly, this section described my role as the researcher in this study.

Interpretive Framework

An interpretive framework identifies the lens through which the researcher brings the inquiry process and how that framework guides the research practice (Creswell & Poth, 2018). The framework that informed this study was social constructivism. I developed meaning from their experiences in a social constructivist worldview (Creswell & Poth, 2018). Social constructivists believe people create knowledge through inquiry and peer collaboration (Dewey, 1916; Vygotsky, 1962). In addition, this framework aided in collecting complex views rather than focusing solely on the narrow meanings of participant experiences (Creswell & Poth, 2018). Applying open-ended data collection questions aligns with the nature of the framework Creswell, 2013; van Manen, 2016). The purpose of this phenomenological study was to understand the mathematics critical thinking process for teachers in primary education in northern Virginia.

Philosophical Assumptions

This section explains my core values' ontological, epistemological, and axiological assumptions. My reality is based on the Bible and the works of Jesus Christ. This view and my stance on the research were applied through the social constructivist lens.

Ontological Assumption

Ontological assumptions in qualitative research question the nature of reality (Creswell & Poth, 2018). Ontological assumptions report different perspectives as themes develop within the findings (Creswell & Poth, 2018). Research states that the discourse within a mathematics classroom is the tool for constructing reality (Acharya et al., 2022). My ontological assumptions surround my faith and belief in the one true God who sent his son, Jesus Christ, to die for our sins so we may be saved. The Bible serves as the ultimate and complete truth (*New International Bible, 1978/2011,* John 14:6). This assumption was seen throughout the study from the perspective that there is one singular reality that comes from the word of God that explains that we are imperfect, without complete knowledge of understanding, and fully loved (*New International Bible, 1978/2011,* Ecclesiastes 7:20).

Epistemological Assumption

Epistemological assumptions in qualitative research questions focus on what counts as knowledge (Creswell & Poth, 2018). I attempted to get as close to the participants as possible in the field where they live or work to understand them (Creswell & Poth, 2018). In addition, epistemological assumptions allow for collecting subjective evidence reliant on quotes in collaboration with the participants (Creswell & Poth, 2018). My epistemological assumptions align with Vygotsky's belief that knowledge is socially constructed (Furinghetti, 2020). In addition, my epistemological assumptions follow the constructivist viewpoint that knowledge is co-created and shaped by experiences (Creswell & Poth, 2018). This assumption was seen throughout the study from the perspective that teachers employ teaching methods that guide students to learn in social situations through collaboration and problem-solving.

Axiological Assumptions

Axiological assumptions in qualitative research question the role of values (Creswell & Poth, 2018). Researchers acknowledge that biases are present and openly discuss values that

shape the study's narrative (Creswell & Poth, 2018). In addition, axiological assumptions allow the researcher to identify their positions, experiences, and professional beliefs (Creswell & Poth, 2018). My axiological assumptions align with the constructivist viewpoint. Individual values are honored and can be negotiated among individuals (Creswell & Poth, 2018). The Bible also influences my assumptions regarding honoring values (*New International Bible, 1978/2011,* Matthew 7:1-5). These assumptions can be seen throughout the study by honoring the values of the participants as well as stating my positions, experiences, and professional beliefs.

Researcher's Role

The researcher is a human instrument in qualitative data collection (Creswell & Poth, 2018). To explore the changes in curriculum and instruction based on the lived experiences of elementary mathematics teachers implementing critical thinking tasks, I aimed to collect data and analyze it as a human instrument. To fully understand the participants' lived experiences, my role as the researcher was to gather information, interpret through coded themes, make observational notes, and analyze data (Creswell & Poth, 2018). To reduce any presuppositions, it is necessary first to define the major themes of the study. As the researcher, I listened respectfully to the participant's descriptions of the study's phenomenon (van Manen, 2016). Using bracketing, I set aside preconceived biases and openly recorded participant descriptions of the experience (van Manen, 2016). The imaginative variation phase following bracketing and reduction allowed me to uncover structural themes from the participants' experiences (van Manen, 2016). I saw the phenomenon from various perspectives to further understand the essence of the participants' experiences (van Manen, 2016). The final step was to develop a statement from the textural and structural descriptions that reveals the phenomenon's essence, understanding that these essences are never truly exhausted but represent my perspective at a particular time and place (van Manen, 2016). This is primarily imparted to van Manen (2016),

who suggests that researchers often need to learn more, which predisposes them to interpretation before understanding the significance of the study. While I will share the participants' lived experiences, it was essential to use bracketing to leave all assumptions and biases out of this proposed study while maintaining my understanding of the topic to fully understand and interpret the data collected from my participants to answer the research questions.

Procedures

The following section will outline the procedures for examining the influences of critical thinking opportunities on primary educators' planning. I discuss securing the IRB approval, receiving site permission, and securing participants. Also, this section outlines the procedures for data collection and analysis.

To begin, IRB approval (see Appendix A) was obtained (Creswell & Poth, 2018) to ensure that my study follows ethical protocol while respecting the participants' welfare (Creswell & Poth, 2018). The IRB application includes the site permission request letter (see Appendix B) and the site permission approval (see Appendix C). IRB and site permission were obtained before recruiting any participants. Participants were recruited using criterion sampling (Creswell & Poth, 2018), meeting the following criteria: 18 or older, holding a current teaching license, teaching a primary grade (K-5) in northern Virginia, and implementing critical thinking classroom opportunities. Developing the lived experience themes, I used multiple phenomenon perspectives (van Manen, 2016) derived from 12 - 15 participants. I recruited participants from the study site using an initial verbal connection script (see Appendix D) that preceded the recruitment letter (see Appendix E) emailed to the participants post-interest expression. After the recruitment letter was sent, a follow-up recruitment letter (see Appendix F) was sent within two weeks to participants who have not yet scheduled an interview with me. Before the individual interview, a consent form (see Appendix G) was given to the participant for approval.

Interviews were conducted using Microsoft Teams. In addition to the Microsoft Teams transcription, the Voice Memo application was used to ensure that all data is recorded accurately (Creswell & Poth, 2018; van Manen, 2016). Interviews were no longer than one hour to ensure the participants' lived experiences were concise, detailed, and of interpretive value (van Manen, 2016). After each interview, immediate transcription began. Immediate transcription allowed for reflection and accuracy (Creswell & Poth, 2018). Transcription allowed me to become well-acquainted with the data. Following the transcription, a document script was completed and shared with the participant for member-checking to ensure validity and accuracy (Creswell & Poth, 2018).

Next, all participants were invited to participate in the focus group. The focus group took place after the individual interviews to ensure all participants could share their lived experiences on a personal level before the group setting (van Manen, 2016). The focus group was on Microsoft Teams. The focus group was recorded using the Microsoft Teams Transcription and the Voice Memo application to ensure the data was accurately recorded (Creswell & Poth, 2018). Immediately following the focus group, transcription began to allow for reflection and accuracy (Creswell & Poth, 2018). Finally, each participant electronically or physically provided a document for analysis. The document detailed a critical thinking opportunity or task used by the participant within the classroom. The document was analyzed using a form that identifies the participant, the task title, the type of task, and any keywords. Each data collection method was analyzed by coding, categorization, and analytic reflection to develop interpreted themes (van Manen, 2016).

Data Collection Plan

The study consisted of semi-structured interviews, focus groups, and document analysis. First, the semi-structured interviews gathered the participants' lived experiences of implementing

critical thinking tasks (Creswell & Poth, 2018; van Manen, 2016). The interviews explored participants' planning and thought processes as they relate to experiential learning theory (Kolb, 1984; van Manen, 2016). Second, all participants were invited to participate in a focus group. The focus group explored participants' lived experiences by examining their planning and thought processes of task development and implementation as they relate to experiential learning theory (Kolb, 1984; van Manen, 2016). Third, participants submitted a document for analysis to explore common themes within the critical thinking opportunities.

Individual Interviews

An interview is a social interaction between the interviewer and interviewee where knowledge is constructed (Kvale, 1996; Rubin & Rubin, 2012; Warren & Xavia Karner, 2015). A phenomenological interview is used to explore and gather experiences that serve as a resource to further an understanding of the phenomenon (van Manen, 2016). Furthermore, this study followed hermeneutic data interpretation to understand further the phenomenon by asking participants to share their lived experiences surrounding the phenomenon (van Manen, 2016).

The interviews took a semi-structured approach, allowing for fluidity of conversation and the ability to ask follow-up questions (Creswell & Poth, 2018; van Manen, 2016). The interviews occurred on Microsoft Teams, based on the researcher's and participants' availability. Interviews were conducted individually on Microsoft Teams to ensure participants feel comfortable, as participants are more inclined to recall their lived experiences when the surroundings are conducive to thinking about the experiences (van Manen, 2016). To prevent participants from feeling rushed, participants selected a time that works best for them (van Manen, 2016). The interviews were recorded, annotated, and kept with ethical considerations (van Manen, 2016).

First, I asked a grand tour question to help establish rapport with participants (van Manen, 2016). Second, I asked questions one by one in an open-ended format. Questions were

grouped into categories to stay focused on the phenomenon (see Appendix H). As the human instrument in this study, I attempted to keep the main research question in mind (van Manen, 2016). Due to the semi-structured nature of the interview, I asked clarifying and prompting questions at any time to ensure the whole experience was shared (van Manen, 2016). To conclude each interview, participants were allowed to add any further experiences that may add to the whole experience that were not discussed (Creswell & Poth, 2018; van Manen, 2016).

Table 1

Individual Interview Questions

1. Please describe your educational background and career through your current position. CRQ
2. In what ways do you develop individualized lesson planning to encourage critical thinking? SQ1
3. How do you teach students to recognize problems for critical thinking implementation? SQ1
4. Describe the opportunities you have had creatively developing lesson plans that create opportunities for critical thinking. SQ1
5. How have you used planning to create situations that require students to develop models to answer a problem? SQ2
6. How do you encourage students to identify a problem that requires a visual interpretation? SQ2
7. What methods do you use to create problem-solving plans that require student think time? SQ2
8. What classroom teaching techniques do you incorporate to define problems? SQ3

9. When you are actively teaching, how do you encourage student decision-making to solve a problem? SQ3
10. Discuss how you support students in developing reasoning skills to solve problems in your curriculum. SQ3
11. How do you relate classroom experiences to real-world settings? SQ4
12. What hands-on opportunities do you provide students to take risks when solving problems? SQ4
13. What instructional techniques do you use to create a product to get things done? SQ4
14. What else would you like to add to our conversation today that we have not yet discussed? CRQ

The interview questions above are coded and grouped into categories. Question one developed a rapport with participants (Creswell & Poth, 2018). Questions two through four align with the concrete experiences of implementing critical thinking opportunities (Kolb, 1984). Questions five through seven align with the reflective observations of implementing critical thinking opportunities (Kolb, 1984). Questions eight through ten align with the abstract conceptualizations of implementing critical thinking opportunities (Kolb, 1984). Questions eleven through thirteen align with the active experimentation of implementing critical thinking opportunities (Kolb, 1984). Lastly, question fourteen allows for participants to share any remaining thoughts.

Focus Group

Focus groups facilitate dialogue and interaction concerning a topic so that experiences and attitudes can be observed (Jarvis & Barberena, 2008). Focus groups allow participants to observe while discussing the research topic (van Manen, 2016). The focus groups were

facilitated in the same manner as the interview process to ensure the participants remain on topic (van Manen, 2016).

All participants were invited to the focus group following individual interviews. The focus group took a semi-structured approach, allowing for fluidity of conversation and follow-up questions (Creswell & Poth, 2018; van Manen, 2016). In addition, the focus group used Microsoft Teams and the Voice Memo application. The interviews were recorded, annotated, and kept with ethical considerations (van Manen, 2016).

First, a grand tour question facilitated dialogue and interaction among participants. The focus group then proceeded with questions individually as they applied to the experiential learning theory and how it connects to providing critical thinking opportunities (see Appendix I). During the focus group, as the human instrument in the study, I took notes on the experiences shared and the attitudes that were observed (Creswell & Poth, 2018; Jarvis & Barberena, 2008; van Manen, 2016). To conclude the focus group, participants were asked a final question that allowed them to share any remaining thoughts surrounding the lived experiences of implementing critical thinking opportunities.

Table 2

Focus Group Questions

1. Tell us about your experiences teaching using critical thinking tasks from your first position to your current position. CRQ
2. What is the most valuable resource you use to help students recognize problems? SQ1
3. What is the most challenging part about including imaginative problem-solving in the classroom? SQ1
4. What is the best model you have used to encourage critical thinking in the classroom? SQ2

5. What are the most valuable strategies to teach patience in problem-solving? SQ2
6. Discuss the challenges associated with logical problem-solving skills in a critical thinking task. SQ3
7. What are the best methods to encourage reasoning skill development? SQ3
8. What are the best strategies to encourage all students to participate in initiating a task? SQ4
9. What are the most challenging aspects of supporting adaptability for problem-solving among students? SQ4
10. What else would you like to add to our discussion about critical thinking in the classroom? CRQ

Question one aligns with the critical research question. Questions two and three align to the concrete experiences of experiential learning theory (Kolb, 1984). Questions four and five align with the reflective observations of experiential learning theory (Kolb, 1984). Questions six and seven align with the abstract conceptualization of experiential learning theory (Kolb, 1984). Questions eight and nine align with the active experimentation of experiential learning theory (Kolb, 1984). Finally, question 10 allows participants to share any remaining thoughts.

Document Analysis

Document analysis is a systematic procedure of reviewing and evaluating documents (Bowen, 2009). In addition, document analysis yields data that can be organized into themes and categories (Bowen, 2009). Thinking tasks that were received aided in uncovering meaning, developing understanding, and discovering insight relevant to the research problem (Merriam, 1988). Importantly, document analysis is efficient, stable, exact, and lacks obtrusiveness (Bowen, 2009).

Following individual interviews, participants submitted a document for analysis used to provide a critical thinking opportunity. Documents that require students to think critically were submitted in either paper or digital format. Documents were analyzed using material reflection (van Manen, 2016). Material reflection asked how participants experience the task (van Manen, 2016). Insights and thematic understandings about the participant experience were articulated by analyzing the documents used to implement critical thinking opportunities (van Manen, 2016). Documents were analyzed using the document analysis form. The form recorded the participant's name to keep experiences tied to the participant. The form's title and any keywords that appear were recorded. In addition, there was an additional section for notes to add my thoughts and reflections regarding the document under analysis.

Table 3

Document Analysis Form

Document Analysis Form	
Participant:	
Form Title:	
Type of Form:	
Keywords	
Notes:	

Data Analysis

Data analysis of t he semi-structured interviews, focus groups, and document analysis used Saldana's (2013) approach to analyzing themes. Saldana (2013) identified the process of analyzing themes by identifying codes, creating categories, generating themes, and applying the themes to the research under study. Interviews, focus groups, and documents submitted for

analysis were transcribed and approached line-by-line. Participants benefited from affirming their experiences using multiple data collection tools to allow for corroboration and saturation to understand and describe the phenomenon (van Manen, 2016). Codes were developed in two cycles using transcription software. The first cycle of codes was assigned data into units (Saldana, 2013). These codes were categorized and grouped by analyzing transcribed lived experiences and the idiosyncrasies of participants (Saldana, 2013). The codes were then placed into categories (Saldana, 2013). Clusters of coded data were refined into subcategories. Following the development of categories, themes were identified, analyzed, and applied to the research study. The themes are the outcome of coding, categorization, and analytic reflection (Saldana, 2013).

Following the analysis of data, conclusions and meanings were made. To keep data organized, coherent, and all in one place, I used Atlas.ti. Atlas.ti allows comments, pictures, videos, audio, transcripts, and more to be organized and coded through chunking and cluttering. Data synthesis and analysis were done simultaneously not to undermine the findings or research study (Merriam & Tisdell, 2016; Suter, 2012). The following steps were taken to make sense of the data: note patterns to generate themes, cluster like groups, compare like groups to understand the phenomenon, note the relationships between variables, and determine theoretical coherence (Miles et al., 2014). These tactics also coincide with the Saldana method of analyzing and synthesizing data (Saldana, 2013). To fully understand the phenomenon, I used these steps to generate overall themes to answer the research questions posited earlier in this chapter.

Trustworthiness

Trustworthiness refers to the rigor of a study, including the degree of confidence in data, interpretation, and methods used (Polit & Beck, 2014). The established protocols and procedures of the study are worthy of consideration by the reader (Polit & Beck, 2014). The foundational

concepts that establish the trustworthiness of a study include credibility, transferability, dependability, and confirmability (Lincoln & Guba, 1985). Lincoln & Guba (1985) later added authenticity. The ability to evaluate the worthiness of a study lies in the techniques used to conduct qualitative research (Lincoln & Guba, 1985).

Credibility

Credibility is confidence in the truth of a study or the extent to which the study's findings accurately describe the reality of the participant's experience of the phenomenon (Lincoln & Guba, 1985). The feasibility of the account the researcher arrives at determines its acceptability to the reader (Bryman, 2016). I achieved credibility in three ways: (a) triangulation, (b) peer debriefing, and (c) member-checking.

Triangulation

Triangulation refers to using multiple data sources in an investigation to produce understanding (Lincoln & Guba, 1985). Method triangulation involves the use of multiple methods of data collection regarding the same phenomenon (Polit & Beck, 2012). Interviews, a focus group, and document analysis were used to create a comprehensive and accurate depiction of the phenomenon. Triangulation of data from three collection methods allowed participants to describe their lived experiences more deeply and allowed me and the participants to refine the nature of their lived experiences collaboratively.

Peer Debriefing

Peer debriefing refers to exposing oneself to a disinterested peer to explore aspects of the inquiry that might otherwise remain within the researcher's mind (Lincoln & Guba, 1985). Using peer debriefing allowed me to discuss emergent findings with colleagues to ensure my analyses would be grounded in data. Due to working within the site and field parameters, participants,

including other educators, coaches, and administrators, were close to the study's nature and location. The peers and current data in the literature provided corroboration for my findings.

Member Checking

Member checking refers to testing data, analytic categories, interpretations, and conclusions with participants (Lincoln & Guba, 1985). Member checking was done informally through conversation throughout the study. Immediate member checking was used during interviews to clear up conceptions from the various perspectives of interviews (Lincoln & Guba, 1985). It was essential to seek to clarify understanding without automatically assuming I understood the participants' lived experiences. In addition, a transcribed transcript of the interviews was provided and reviewed by participants, along with significant points to ensure accuracy.

Transferability

Transferability refers to the researcher showing that the study's findings will be applicable in other contexts (Lincoln & Guba, 1985). Transferability was achieved using thick descriptions when describing research findings (Geertz, 1988). As such, this study aimed to explore the lived experiences of educators using critical thinking tasks and changes made to curriculum planning and instruction. A robust and detailed account of the experiences during data collection was shared. The alignment of participants' lived experiences may apply to educators using critical thinking tasks. This research cannot be applied to all educators as planning curriculum and instruction varies significantly between individuals, schools, and districts.

Dependability

Dependability refers to the researcher showing that the findings are consistent and could be repeated (Lincoln & Guba, 1985). Descriptions of my procedures are thoroughly outlined to

ensure that this study can be replicated. The research questions, researcher role and status within the study, analytic constructs, data collection methods, analysis, and forms were in place (Miles et al., 2014). In addition, the procedures were thoroughly reviewed and deemed sufficient by the committee of this study.

Confirmability

Confirmability refers to the degree of neutrality to which the respondents shape the study's findings, not researcher bias, motivation, or interest (Lincoln & Guba, 1985). Confirmability was achieved in reflexivity, triangulation, and audit trails. First, reflexivity was achieved by being explicit and self-aware about personal assumptions, values, and biases and examining how they may have come into play during the study (Miles et al., 2014). Second, the study employed various aspects of triangulation. Lastly, the methods, procedures, and data collection and analysis were displayed for audit by the reader (Miles et al., 2014). Member checks aided in the neutralization of any biases brought to the study by the researcher.

Ethical Considerations

The following section will discuss the permissions and ethical considerations of participants. A brief overview of permissions will be discussed, including the IRB approval process and site permissions. In addition, an overview of other participant protections will be discussed, including the study's voluntary nature, data collection and protections, and matters of secure details.

Permissions

Gaining permission for my study included IRB approval, site approval, participant solicitation, interview schedules, conduction of interviews, transcription, member checking, and analysis. All plans were reviewed and approved by Liberty University's IRB. The overall purpose of obtaining IRB approval was to provide evidence that the study follows an ethical

design (Creswell & Poth, 2018). Site permission from BPCS was obtained from the gatekeeper of the county mathematics department (Creswell & Poth, 2018). Participants in the study completed an informed consent form agreeing to the provisions of the study before participation (Creswell & Poth, 2018).

Other Participant Protections

Participants were informed of the voluntary study's general purpose and can withdraw participation at any time. Informed consent forms were signed and held in a locked filing cabinet. All participants of the study were treated equally and fairly without partiality. The risks and benefits of the study and mitigation factors were discussed. During data collection, leading questions were avoided (Creswell & Poth, 2018). Participants' and sites' identities are protected using pseudonyms. All digital aspects of the study were protected using a secure computer that is password protected (Creswell & Poth, 2018). All stored data will be secured for three years before destruction, barring future research (Creswell & Poth, 2018).

Summary

This hermeneutic phenomenological study aimed to understand the lived experiences of changes in curriculum and instruction planning in elementary mathematics teachers who implement critical thinking tasks in northern Virginia. This chapter identified the methods for this study, including the research design, as well as explanations for the design selection. Also, this chapter outlines the research question, setting, participants, and my role as the human instrument in the study. Data collection, analysis, and synthesis were outlined and explained. In addition, this chapter outlined my positionality, philosophy, and framework upon which this study will be grounded. Lastly, a section outlining the study’s trustworthiness, including credibility, transferability, dependability, confirmability, and ethical considerations, was discussed.

CHAPTER FOUR: FINDINGS

Overview

The purpose of this phenomenological study is to understand the mathematics critical thinking process for teachers in primary education in northern Virginia. This chapter will present the findings of the study. Included in this chapter are the participant descriptions, data analysis, research findings, and answers to the research questions.

Participants

Twelve mathematics educators within Beta County Public Schools with more than a year of experience implementing critical thinking opportunities participated in this study. Teacher participants for this study were current employees within BCPS and possessed different characteristics concerning age, gender, years of experience, highest degree earned, and teaching philosophy. Two of the participants were male, and 10 were female. Participants' teaching experiences ranged from three to 31 years in the education field. Pseudonyms were utilized to

support participant confidentiality throughout the study. Specific participant details are provided in this section of the chapter.

Kimberly

Kimberly is a gifted education educator who has been teaching for 31 years. As the only gifted education teacher in her school building, Kimberly found that her monthly professional learning communities (PLCs) were incredibly valuable and effective. Collaborative planning with gifted educators from BCPS eased the pressures of creating tasks herself. Kimberly ties her lessons to real-world scenarios with inquiry-based projects, lessons, and tasks. Through her lessons, she has created a culture that promotes thinking. However, she admits it is messy, and teachers must give up some control. "You know, a lot of teachers don't have a tolerance for the mess, but that's how you get them to be those critical thinkers, those creative thinkers, by letting it get messy," said Kimberly. Animated teaching that retains attention aids in building a community that keeps all her students accountable. Kimberly gives her students meaningful learning experiences as she believes it's an effective way for students to learn and remember what they learned. Kimberly shared, "They don't remember because they haven't had those meaningful experiences with those types of things." Kimberly strives to create engaging and meaningful learning experiences that foster independence and encourage students to make meaning of their learning.

Brianna

Brianna is a fourth-grade educator with four years of experience and openly acknowledges the challenge of planning critical thinking tasks due to the necessary differentiation in her classroom. However, she remains committed to these tasks, consistently creating time for these experiences:

> So, I try to have at least one every day, whether it is a real quick question or one that could take a long time. But usually, I spend at least ten minutes every day with something open-ended for them to come up with an idea or answer, not always looking for the right answer, just looking for the process.

Brianna's students can access any material or manipulative they need during these opportunities. She purposefully plans for her class always to draw something to represent their thoughts. Further, she claimed:

> I make it a priority in my classroom. I think it makes a difference because I see my scores this year for their math testing and in number sets, patterns, functions, and algebra. They are all up for all of them.

Brianna is hopeful that her school "buys in" to thinking tasks and offers professional development on planning and implementing tasks.

Joseph

Joseph is a fifth-grade educator with six years of experience. His lesson plans are designed to foster critical thinking by including fun and engaging scenarios. His tasks often begin with numerous answers and encourage discourse. A deep understanding of his student's backgrounds and experiences aids in his planning process as he provides meaningful and relatable opportunities to his student population. Admittedly, Joseph shared, "I feel like there is no time in the day." This admission encourages Joseph to lean on his PLC and team to aid in his planning.

Throughout his years of experience, Joseph has acknowledged that students should lead the discussion rather than him. Joseph has spent substantial time building a community that is encouraging and full of discourse. "I think it is just super, super important to get them talking.

Build their confidence," shared Joseph. Joseph claimed that he would continue to use vertical whiteboard spaces as they have been effective for him.

Owen

Owen is a first-grade educator with seven years of experience. Backward design and collaborative planning allow Owen to teach his students using inquiry-based learning. He finds that planning critical thinking opportunities requires significant work surrounding social skills and fostering independent problem-solving skills. In addition, he uses real-life examples and scenarios in which students can create models through pictures or other non-linguistic representations. Owen shared, “I try to encourage my kids to use pictures because it is a good strategy, and it allows them to catch mistakes that they have made in math.” By allowing his students to catch their mistakes, he fosters independence while encouraging them to think critically.

Owen also shared that he likes to pose the problem and let his students do most of the talking. Student-centered instruction allows his students to work out problems and make their meaning. When discussing tasks, he claimed that he is “hands off” and allows his students to experience failure so that they may learn from it rather than get upset. Owen enjoys teaching mathematics using thinking opportunities and encourages his students to develop independence.

Makayla

Makayla is a fifth-grade educator with four years of experience. While discussing planning, Makayla shared that she begins planning by looking at her student population to serve them best. She declared that questioning her students is critical to implementing critical thinking tasks. Makayla said, “I do not tell them. I just ask more questions.” In addition, she claimed that her PLCs were incredibly helpful to the planning process by being able to talk through it with other educators.

During tasks, Makayla will often let her students lead the task. She shared, “I will just give them a very open-ended question and the manipulatives and be like, OK, go!” Admittedly, she shared that her goal is always to have a product with her tasks. She scaffolds her students and gives them ample thinking time throughout her tasks.

Maria

Maria is a fifth-grade educator with three years of teaching experience. When discussing how she plans critical thinking opportunities, she shared:

> I just wanted to have the students take their background knowledge, bring it, and see if they can connect their background knowledge with what we are about to get into and to see if they can form some type of thinking together.

Asking clarifying questions, asking why, and making her students answer the question in more than one way have been helpful thus far during critical thinking opportunities. She also added that planning with her team and the PLC community has been incredibly helpful.

Maria focuses on the strategies of her students and encourages positive discourse between her students. In addition, she encourages her students to reason and with logic. She told her students, “Do not attack the math. You must process it first. What does this look like logically?” Maria uses vertical whiteboard spaces to encourage collaborative learning and build a community with pre-determined norms.

Talia

Talia is a fifth-grade educator with 18 years of teaching experience. Talia plans individualized lesson plans that implement critical thinking tasks by working collaboratively with her team and the PLC community. She has “flipped the way she has taught math” by leading a student-centered classroom. She shared, “I give minimal instructions and minimal

direction. Here is the problem. Figure it out." Talia believes that flipping her brain to a more student-centered classroom allows her students to make meaning of their learning.

Talia uses numerous problem-solving strategies and constantly asks questions that further the conversation. In addition, Talia described how she speaks with students, saying, "I want you to show me this in multiple ways. I do not just want one representation." Allowing multiple strategies has allowed her students to develop independence, and she continues to use student interest to develop further tasks. Talia was excited to discuss critical thinking opportunities yet admitted that it is "hard to switch your thinking."

Lisa

Lisa is a third-grade educator with 18 years of teaching experience. When planning, she often begins by thinking about her student population. She keeps expectations high and differentiates her critical thinking opportunities. Discourse is essential to the success of the opportunities that Lisa provides her students. Lisa believes students should have "moments of self-discovery" before being taught a strategy. Her lessons are purposeful and intentional while often using vertical learning spaces.

Lisa teaches her students to ask clarifying questions when they are stuck or do not know how to proceed. Students are encouraged to use manipulatives, engage in positive discourse, and follow classroom norms to complete tasks with collaborative groups. Her tasks often use real-world scenarios. "I always try to make it authentic. I do not use worksheets. I do not show procedures. I try to do everything by making connections." said Lisa. Lisa spoke about the importance of providing critical thinking opportunities and moments of discourse and allowing her students to have productive struggles.

Alexandra

Alexandra is an instructional coach with 30 years of teaching experience. Alexandra

claimed that she often puts herself in the seat and brain of multiple students within her classroom to understand better how they would attack the critical thinking opportunity. She also thinks it is essential to consider the “collaborative and communication opportunities” when planning a task. The strategies she uses to teach her students to think independently vary based on the experiences of her students. Her lessons often require minimal talk and direction from her, and she places the focus and thinking on her students.

Anticipating misconceptions or preconceived notions is a powerful tool for Alexandra. She also encourages students to think by allowing them to use multiple strategies. Alexandra shared, “We may all be working toward the same concept, but we are not all going to think about it the same way or arrive at it the same way.” Through visualization, discourse, and justification, Alexandra has created a mathematics community that fosters reasonableness.

Brenda

Brenda is a third-grade educator with 10 years of teaching experience. She said she began planning by considering what questions to ask her students. She also shared, "I think working on starting units and starting new concepts with thinking tasks like more open-ended opens you up to asking more questions." Brenda's most significant pedagogical shift was releasing control by wanting to give them strategies and letting her students guide their thinking. She also claimed that collaborating with her team and the PLC community was beneficial.

Brenda encourages her students to use manipulatives and visualize their learning. Manipulatives and non-linguistic representations often lead to students showing multiple answers and paths toward their answers. While justifying their answer, Brenda has ensured that all students feel safe by establishing a community with a growth mindset and positive attitude. Brenda worries that other educators are too worried about a finished product and the lack of time and are missing the opportunities that critical thinking can offer their students. She

shared, "There are days where I might not get deep into this. But I think because we did this, many of them will be able to make it through." She is excited and looks forward to continuing teaching math through inquiry-based learning.

Aurora

Aurora is a first-grade educator with 16 years of classroom experience. Aurora's teaching philosophy involves the whole person, so she strongly believes in allowing multiple answers and strategies to solve a problem. When planning, she ensures students can access manipulatives and think time. Aurora enjoys using real-life scenarios and discourse to engage her students. She believes that teachers should be more flexible and focus less on the curriculum resources, using them more as tools and less as the law. She feels that "opportunities have not seemed available" to her.

Aurora spoke about building community and ensuring that everyone feels safe. She shared, "They are not afraid. They are not afraid to ask those questions, which is wonderful." In addition to the community she has built, she encourages her students to express their answers in ways that make sense to them. She allows her students to justify their answers using models, kinesthetic activities, plays, and drawings.

Christine

Christine is a third-grade educator with five years of classroom experience. Christine shared that she creates comprehensive plans that include questions she will pose and additional support for her students. Through a cohort and PLC, Christine was able to use collaborative planning and learning to gain knowledge about providing critical thinking opportunities. Christine spoke about the difficulties of releasing control of how she used to teach and trying something new. She shared:

> It is still hard for me to release control sometimes. To allow them to pick these different ways or for me to have more chaos going on because they are critically thinking instead of just you have to do it this way. So it is still really hard sometimes.

Christine enjoys allowing her students to use vertical learning spaces. When actively participating in a task, Christine has established classroom norms and has built a community of respect and passion for mathematics. She shared, "It has been beneficial for them to flip it around and think about how they would define the problem, see math in their everyday life, and solve problems." Christine enjoys teaching math this way and is encouraged by the impact it has had thus far on her teaching, students, and well-being.

Table 4

Teacher Participants

Teacher	Years Taught	Highest Degree Earned	Content Area	Grade Level
Kimberly	31	Masters	Gifted Education	K-5
Brianna	4	Masters	General Education	4th
Joseph	5	Bachelors	General Education	5th
Owen	7	Bachelors	General Education	1st
Makayla	4	Bachelors	General Education	5th
Maria	3	Bachelors	General Education	5th
Talia	18	Masters	General Education	5th

Lisa	18	Bachelors	General Education	3rd
Alexandra	30	Bachelors	Instructional Coach	K-5
Brenda	10	Masters	General Education	3rd
Aurora	16	Masters	General Education	1st
Christine	5	Bachelors	General Education	3rd

Results

In this section, the outcomes of data analysis are presented. The results are grouped into four major themes; each has two sub-themes. Data was ascertained through interviews, a focus group, and document analysis. Individual interviews were conducted using an open-ended approach to allow participants to speak freely and provide information they deemed pertinent to the questions posed. All participants were invited to a focus group and submitted a document for analysis. The key themes that emerged were purposeful planning, building community, and promoting inquiry-based learning.

Table 5

Themes & Subthemes

Themes	Subtheme	
Purposeful Planning	Student-Centered	Questioning
Building Community	Encouraging Discourse	Multiple Strategies
Promoting Inquiry-Based Learning	Students Making Their Own Meaning	Manipulatives & Non-Linguistic Representations

Purposeful Planning

Participants repeatedly described the importance of purposeful planning when discussing the implementation of critical thinking tasks. Over 900 times, participants shared common phrases, ideas, and beliefs about being purposeful during their planning process. Participants consistently shared their ideas and beliefs surrounding planning and implementing critical thinking opportunities by stressing the importance of being purposeful about their questions, the task, and what the students are doing. In addition, all twelve participants spoke about the importance of understanding their student population and planning accordingly. Knowing their students' backgrounds, interests, and learning styles allows participants to purposefully plan a task that will grab their students' attention and offer every student a chance to participate. Lisa stated, “Just be intentional. With what I am saying and what I want them to do.” During the focus group, Brenda shared, “So you kind of have to think all of that through beforehand to know what could go wrong so that you are not stuck in that moment.” All participants spoke about understanding that learners may get to the answer differently therefore planning to allow different learning styles and strategies. Ten out of twelve participants thought about where their students could get stuck and purposefully planned questions and manipulatives to aid them. During her interview, Brianna shared, “I think about the problem and what manipulatives would make them feel more comfortable.” Also, nine of twelve participants shared that an opportunity to think critically should come before direct instruction of the skill at hand.

Student-Centered

Across all three data collection tools, participants discussed the importance of creating a student-centered task over 400 times. A student-centered classroom requires a teacher to give up control. During the focus group, Talia stated, “It was changing my mindset about giving up the

reins in a way and letting them do a lot of the discovering and critical thinking." Half the twelve participants shared that shifting to critical thinking tasks is "messy." However, eleven participants shared that the shift changed their pedagogy and allowed them to release some control over their students. Giving up control had to be purposefully planned to ensure that student's received the best opportunity possible. Participants were purposeful with task design to ensure the students led the learning. "I try my best to talk less than my students," shared Joseph. "Getting out of my way has been difficult. It has been hard just to let go and allow them to do it," said Maria.

Eight participants discussed time as a critical aspect of implementing student-led and centered tasks. Christine shared, " I think it is beneficial for them to flip it around and think about how they would define a problem. How they would see math in their everyday lives and share that thinking." 40 times, participants discussed and shared that implementing tasks takes much time to plan. Brianna shared, "I don't get enough time to plan but I make it a priority." Ten of the twelve participants claimed that collaborative planning with their PLC was helpful, and eight wished they had more time to plan together.

Questioning

All participants spoke about the importance of questions when planning and implementing tasks. Questions, types of questions, pre-planned questions, and other comments regarding questions occurred 237 times across all three data points. "Questioning is huge," said Christine. All twelve participants shared that they ask probing questions requiring their students to think critically before, during, and after a task. Ten of the twelve participants shared that they consistently ask "why" when implementing a task to ensure that students are given the opportunity to think critically about their choices. Alexandra said, " The list of questions you have is important." When preparing and implementing a task, thinking about the questioning

aligned with the task is essential. All twelve documents submitted for analysis included purposeful planning of developing questions that engage, challenge, and support learners. In addition, eight of the tasks included differentiated questions for various learners.

Building Community

From using norms to vertical whiteboard spaces, participants adamantly spoke about the importance of building community. Participants discussed the importance of building a mathematics community 737 times. The community encourages student decision-making, discourse, and usage of manipulatives. “Our environment is a tribe. We do not ever laugh. It is just making it a safe space,” said Aurora.

Building community was discussed three times more during the focus group than every other theme and topic. During her interview, Alexandra shared, “I think about the collaborative piece, the communication, and the discourse opportunities that they are going to have.” In addition, “ Discussions have to have positive and kind words,” shared Christine. Participants clarified that a mathematics community must make students feel safe in their strategies, discourse, and interactions. Classroom norms and a growth mindset were discussed thirty times through data collection. All twelve documents submitted for analysis yielded evidence of building community through collaborative learning in a community setting.

Encouraging Discourse

Encouraging discourse was discussed 307 times across all data collection forms. All participants spoke about encouraging discourse between their students during the implementation of tasks. The word discussion appeared over 200 times across all data collection forms. Discussion during tasks was student-led and allowed the students to take control of the classroom by sharing their ideas, strategies, and thinking. Owen shared, "Students have to have the

opportunity to talk to each other and explain their thinking." Offering opportunities for students to discuss their thinking with peers was discussed 82 times.

Mathematical discourse took many forms throughout data collection. All twelve participants and their tasks required students to work in groups. Most discourse allowed students to think and discuss the task in a collaborative learning environment. Eight of the participants described the usage of vertical whiteboard spaces to engage their learners. These vertical spaces encourage collaborative learning and discourse. Brenda shared, "They are making connections and having those discussions not only about their thinking but other people's thinking and reasoning, how they are connected, and why they make sense." Participants also used the vertical spaces to encourage students to talk to one another to make sense of the learning. In addition, all twelve tasks submitted required some form of discourse between students.

Multiple Strategies

Participants shared they allow their students to use multiple strategies and encourage students to find an answer in their own way 252 times. Joseph said, “Kids have different abilities. We must let them use them.” In addition, Talia shared, “I want my students to represent their answer in multiple ways.” Students are encouraged to answer problems using their thinking rather than the thinking given by instruction. “We may all be working toward the same concept, but we are not all going to think about it the same way or arrive at the same way,” said Alexandra. Allowing students to use many answers is “messy” and requires purposeful planning to be prepared to help all students.

When students are encouraged to use different or multiple strategies, it creates a community of like-minded learners. Brenda shared, “I think that having them show their method of doing it, even if it is different than how I taught it or different than how someone else is doing it, it pushed them to try other ways too.” In addition, Lisa said, “I think the more students share

their different thinking with others, the more flexible they become with what they are thinking as well." During the focus group, allowing students to use their strategies and thinking was discussed 67 times. Most of the conversations centered around allowing students to talk and learn from each other, which is an essential piece of building a mathematics community. Makayla shared, "I think sometimes students get the mindset that they cannot learn from anybody in the class. I am not going to learn from anybody but the teacher. And that is not true." All twelve were open-ended when analyzing the submitted documents, allowing students to think using their own strategies. None of the participants required students to solve the opportunity using a predetermined strategy.

Promoting Inquiry-Based Learning

The third theme concluded from data analysis was promoting inquiry-based learning. Promoting inquiry-based learning occurred 556 times across all three data collection tools. Ten of the participants shared that they started units with an exploratory task. "I try to start with something where they are just exploring to see what they do with it first as a way of introducing concepts," said Brenda. In addition, Kimberly shared, "Opportunities should be much more exploratory." During the focus group, participants shared that allowing their students to learn through inquiry is sometimes tricky. However, participants agreed that inquiry-based learning is effective and essential to critical thinking opportunities. Six participants shared, "Students have to make their meaning of the learning."

Inquiry-based learning appeared differently in each document submitted. Seven documents required students to have some form of manipulative or non-linguistic representation. Brianna shared, "Manipulatives allow my students to make a connection." In addition, Alexandra said, "For these problems we are working on, you get this manipulative. But it is giving them a variety so they can start to make a choice in what makes sense to them." During the focus group,

participants shared that the task design should always promote inquiry-based learning. Four different times throughout the focus group, participants shared, “Just let go and allow them to do it.”

Students Make Their Own Meaning

Participants discussed the importance of students making their own meaning 298 times across all data collection tools. Maria shared, “Students have to be able to make sense of their task.” “Students need the opportunity to turn on their own light bulbs,” said Joseph. Students discovering through inquiry are fostering independence and critical thinking. “It is way more concrete for them because they have discovered it through their learning,” shared Talia. Alexandra shared, “I think that the most learning happens when students are making decisions.”

Using real-world scenarios was discussed by participants 75 times during data collection. Participants shared that it is critical that students can make meaning of their learning through real-world experiences and scenarios. Starting a critical thinking opportunity with something relatable to students is beneficial. “I try to give opportunities, something tangible, that they can relate to,” said Joseph. Tangible experiences that relate to student interests were discussed 70 times. Alexandra shared, “So making those connections to anything they have experienced allows our students to develop a deeper meaning, a deeper understanding.”

Manipulatives & Non-linguistic Representations

Manipulatives and non-linguistic representations were discussed 159 times across all data collection tools. The word manipulative(s) was used 73 times. All twelve participants spoke about using manipulatives during a task. Ten participants shared that they encourage students to use manipulatives during every task. Nine of the documents submitted for analysis required a manipulative or non-linguistic representation. Alexandra shared, “Different types of problems with all kinds of manipulatives start to get them to feel more comfortable because it helps them

make sense of it." Eight participants shared that their students often gravitate toward drawing pictures.

Non-linguistic representations were discussed 86 times across data collection tools. Participants discussed drawing during a task 15 times across data collection tools. Four participants shared that they require a drawing for every critical thinking opportunity they give. Making models appeared 14 times across data collection tools. Lisa shared, "We do a lot of problems where they have to draw or make a model." Allowing for multiple strategies using manipulative or non-linguistic representations promotes inquiry-based learning opportunities, encourages discourse, and builds a mathematics community. "Grab your manipulative, draw the picture, and then we always share the different ways we found it," shared Brenda.

Outlier Data and Findings

Throughout this study, most participant comments were aligned with the research questions. Many participant comments were grouped easily with one another. However, there was one outlying finding regarding testing.

Testing

An outlier found following data collection surrounds standardized testing. Participants ranged from 1st grade to 5th grade. Kindergarten through third grade administers the Phonological Awareness Literacy Screener (PALS) and the Assessing Math Concepts (AMC) test. Third through fifth grade, administer the Standards of Learning (SOL) test for reading, mathematics, history, and science. Standardized testing occurs three times throughout the year for each assessment. "Testing has gotten out of hand," stated Kimberly. Primary grade teachers shared that they try to get students "ready for the SOL," while upper elementary teachers feel "there is never enough time." Brenda shared, "I can't do everything I want, but I try to when I can." However, due to the focus of the study on planning, time was spent discussing productive usage

of time rather than not having enough time.

Research Question Responses

This section provides answers to the research questions presented in this study. All questions are answered and include in-vivo quotes. The goal is to present the essence of the participant group.

Research Question Responses

This phenomenological study aims to understand the mathematics critical thinking process for teachers in primary education in northern Virginia. To further understand the planning and implementation process of critical thinking tasks, I explored the lived experiences of educators implementing tasks. The findings resulted in three themes that align with the research questions of this study.

Central Research Question

What are the experiences of primary mathematics teachers implementing critical thinking? This question reflects all three themes that arose through data collection: purposeful planning, building community, and promoting inquiry-based learning. According to participants, purposefully planning a task, readying questions for all levels of students, and promoting questions that continue critical thinking rather than lead students to an answer are all essential. When asked to discuss his planning process, Joseph said, “I really challenged myself to think about what roadblocks they are going to come across.” During the focus group, Makayla shared, “I decide how I want to facilitate the conversation and decide what direction I want the lesson to go in.” This participant feedback ties to the theme of working purposefully to ensure that lessons are student-centered and that questions are ready in advance for all learners within the classroom.

In addition, building a community was common amongst all participants. Participants described the lengths they went to ensure that their classroom encouraged discourse and allowed

multiple learning methods. Maria said, “The most important thing is to make sure that students feel comfortable.” Allowing multiple learning methods allows students to feel comfortable with discourse and develops a growth mindset for critical thinking opportunities. While sometimes “messy,” the process is often far more important than the product, according to seven participants.

Also, promoting inquiry-based learning was an important part of planning and implementing critical thinking opportunities. Participants created their lessons to allow for student-centered instruction. All 12 tasks involved group work where students were making their own meaning rather than being told a way to solve the problem. In addition, the tasks allowed for manipulatives and non-linguistic representations to be used.

Sub-Question One

What are the concrete experiences of primary mathematics teachers implementing critical thinking? The concrete experiences of primary mathematics teachers implementing critical thinking were found within the purposeful planning and implementation of their tasks (Kolb, 1984). All twelve participants submitted a task and spoke at length about the experiences they have provided. During the focus group, Kimberly shared, “We just have to make the time for these things.” The themes from this study were derived from the opportunities and concrete experiences afforded to the students. Participants shared that “letting go” of the reins allowed their students to flourish. The more they held on to the teaching, the less effective the opportunities were.

Sub-Question Two

What are the reflective observation experiences of primary mathematics teachers implementing critical thinking? The reflective observations and experiences of primary mathematics teachers implementing critical thinking were discussed through data collection

(Kolb, 1984). Throughout the interviews, focus group, and document analysis, participants were allowed to review and reflect on their experiences with critical thinking opportunities. During the focus group, Talia shared, “I wish we had time to meet to discuss what happened.” Participants shared that the focus group was a fun time to bounce ideas off other like-minded educators.

Sub-Question Three

What are the abstract conceptualization experiences of primary mathematics teachers implementing critical thinking? The abstract conceptualizations of primary mathematics teachers implementing critical thinking appear within all three themes found following data collection (Kolb, 1984). Allowing students to make their own meaning allows students to connect the dots, identify patterns, and bridge what they know with what the teacher is trying to teach. The ability to learn from the critical thinking opportunities given allowed participants to shift pedagogically to a more student-centered lesson. “ I have really learned to let go,” said Brianna. In addition, participants shared that manipulatives and other non-linguistic opportunities helped complete the task.

Sub-Question Four

What are the active experimentation experiences of primary mathematics teachers implementing critical thinking? The active experimentation experiences of primary mathematics teachers implementing critical thinking were seen through the submission of tasks (Kolb, 1984). In addition, participants spoke about how each task offers something new and fun for their students to engage in. During the focus group, Christine shared, “Each time gets a little easier for them.” The purposeful planning and implementation of the tasks allowed participants to challenge their students to think critically.

Summary

The major themes of the findings of this study are purposeful planning, building community, and promoting inquiry-based learning. Through the lived experiences of the participants, creating a student-centered classroom that promotes questioning was found to be essential. Also, participants encouraged discourse and used multiple strategies, allowing students to feel more comfortable with the critical thinking opportunities. Participants promoted inquiry-based learning by allowing students to develop meaning using manipulatives and non-linguistic representations. Planning and implementing critical thinking opportunities requires educators to relinquish control of the teaching and give it to the students.

CHAPTER FIVE: CONCLUSION

Overview

The purpose of this phenomenological study was to understand the critical thinking process of mathematics for teachers in primary education in northern Virginia. Participants described their lived experiences of planning and implementing critical thinking opportunities. This chapter presents the summary of thematic findings and elaborates on the answer to the central research question, four sub-questions, and five subsections for discussion, including a summary of thematic findings, interpretation of the findings, implications for policy, implications for practice, theoretical and empirical implications, limitations and delimitations, and recommendations for future studies.

Discussion

This research study examined the life experiences of primary mathematics educators in Northern Virginia who were planning and implementing critical thinking opportunities. Data analysis of individual interviews, focus groups, and document analysis revealed three significant themes and sub-themes to answer the research question and sub-questions. The study's findings revealed the planning and implementation process of primary mathematics educators and the challenges faced.

Summary of Thematic Findings

The data collection methods in this study included individual interviews, a focus group, and document analysis. Eleven participants participated in a focus group interview. This study's central research question was: What are the experiences of primary mathematics teachers implementing critical thinking? This study's sub-questions were as follows: (1) What are the concrete experiences of primary mathematics teachers implementing critical thinking? (2) What are the reflective observation experiences of primary mathematics teachers implementing critical

thinking? (3) What are the abstract conceptualization experiences of primary mathematics teachers implementing critical thinking? and (4) What are the active experimentation experiences of primary mathematics teachers implementing critical thinking? The first theme that emerged was that primary educators use purposeful planning when planning and implementing tasks. Two subthemes emerged from theme one, purposeful planning: ensuring that the lesson is student-led and student-centered and the types of questioning used. The second theme that emerged was building community. Two subthemes emerged from theme two: allowing multiple strategies and encouraging mathematics discourse. The third theme that emerged was promoting inquiry-based learning. Two subthemes emerged from theme three: allowing students to make their meaning and manipulatives and other non-linguistic representations. The themes and subthemes result from the lived experiences of primary mathematics educators in Northern Virginia who planned and implemented critical thinking opportunities.

Interpretation of Findings

This section discusses the interpretation of the findings of this study. The lived experience participants described depended on their unique tools to implement thinking opportunities. This section elaborates on the need for a pedagogical shift. Another interpretation of the study's findings is the need for professional development. The last interpretation of the study's findings expands on the cross-curricula opportunities that critical thinking provides.

Pedagogical Shift

Most participants shared that there was a shift in their pedagogy. In alignment with Kolb (1984), participants allowed their students to experience mathematics. While some participants called it "messy," all participants discussed the need for a shift in pedagogy. In most cases, the shift occurred when teachers began to allow students to lead the lesson. Participants advocated for student-centered learning to continue to grow and move forward (Jacobs & Spangler, 2017;

Nasir & Cobb, 2006; Schoenfeld, 2010; Turner et al., 2013). Most participants had more than three years of experience. Therefore, there was a decent amount of discussion on how different the participants learned versus how they expected their students to learn. Participants wanted their students to learn through engaging lessons requiring them to think and had to change their pedagogical perspective. Therefore, participants allowed students to think and reason, structure mathematical student talk, and orchestrate mathematical discourse (Jacobs & Spangler, 2017; Nasir & Cobb, 2006; Schoenfeld, 2010; Turner et al., 2013). Participants used a traditional style of teaching for most of their careers. Therefore, shifting from a teacher-led lecture style to the experiences shared of allowing students to lead the learning was difficult. A participant elaborated that staying consistent in their new way of thinking was difficult, and they often found themselves sliding back towards their old ways.

Professional Development Needed

Kolb (1984) states that reflection is critical to experiential learning. Participants shared that they often planned alone, rarely planned with others, and learned everything they knew from others. Offering professional development is necessary to further the influences of critical thinking. While some participants shared that they were in a cohort of other teachers, most were doing all this learning and experiencing with little to no guidance and no one to discuss it with. Lacking knowledge and time to acquire the skills needed for implementing critical thinking opportunities was common among participants (Linan-Garcia et al., 2021; Suratmi & Sopandi, 2022; Tekkumru-Kisa et al., 2018). Professional development is needed to aid educators in learning how to release some of their control, build communities of thinkers and collaborators, and, most importantly, implement critical thinking opportunities. Participants wished they had more chances for collaboration, communication, and sharing. While cohorts are an excellent time to work with colleagues, they are often volunteer-based in a career where time is the most

valuable commodity. Participants were vocal in sharing that they sometimes lacked knowledge and confidence in their journey (Suratmi & Sopandi, 2022). Ensuring that professional development aids in the planning and implementation process of critical thinking opportunities may be beneficial.

Cross-Curricular Opportunities

This study focused solely on mathematics instruction. However, an interpretation of this study's findings is that these opportunities could be cross-curricular. Many participants shared how they require their students to build, create, or model something as they would in a library or STEAM classroom (Duch et al., 2001; Grasha, 1996; Liljedahl, Zager, et al., 2021). Participants shared that they need their students to solve problems using pictures, such as in an art classroom. In addition, participants shared that they required their students to write about what they learned from the experience in many ways. The ability to take the ideologies of this study and transfer them to cross-curricular concepts is already underway based on participants' lived experiences. However, with little time to discuss, a lack of professional development, and lost instructional time due to testing, all that cross-curricular work goes unnoticed (Ertekin et al., 2009; Morgan, 2012).

Implications for Practice

The first implication for practice involves planning a critical thinking opportunity. First, opportunities may benefit from being planned in collaborative settings. Many participants spoke about wishing they had others to plan with, to bounce ideas off, and just to hear them talk about the minute things. Second, it may be beneficial to pre-determine what manipulatives students can use. Limiting students to one may limit their thinking. While encouraging discourse is an essential finding for this study, it may also be effective for all aspects of pedagogy. Lastly, promoting inquiry in students may also be practical rather than teaching the strategy when

planning a critical thinking opportunity. Many participants shared that they offered their students the opportunity to think first and then make sense of the strategies.

The second implication for practice would be to build a strong community of learners. While participants created beautiful communities that encouraged discourse, usage of manipulatives, and growth mindsets, it may also be effective for all settings and students. Encouraging one another, keeping all students positive, and letting them do most of the talking may be beneficial. Participants relinquished control of the reins and allowed their students to take ownership of their learning. It may also be practical to let go and let the students learn in a way that makes sense to them.

Empirical and Theoretical Implications

This section addresses the theoretical and empirical implications of this study. The empirical implications are addressed by explaining the lived experiences of primary mathematics educators planning and implementing critical thinking opportunities. The theoretical implications align with Kolb's (1984) experiential learning theory.

Empirical Implications

The participants' lived experiences surrounded three significant themes: purposeful planning, building community, and promoting inquiry-based learning. First, purposeful planning was about creating a student-centered classroom. Participants used unique pedagogical techniques that allowed their classrooms to be student-centered through critical thinking opportunities in agreement with experiential learning theory (Kolb, 1984). Participants share the idea that a student-centered classroom is essential to creating a classroom of influential thinkers (Assante et al., 2022; Brookfield, 2013; Mezirow, 2000; Rehman et al., 2023). In addition, the questioning used by participants aligned with Liljedhah et al. (2021) in that questions required students to further their thinking rather than giving them an answer or a strategy to use.

Second, participants echoed research by building community by allowing multiple strategies and methods and encouraging discourse. Building community looked different for each participant; however, common themes included developing a growth mindset, encouragement, discourse, and using norms to ensure all students participated. Allowing students to use their strategies, multiple strategies, and their thinking allowed for student advocacy and independence and allowed students to feel as if they were part of the community (Jacobs & Spangler, 2017; Nasir & Cobb, 2006; Schoenfeld, 2010; Turner et al., 2013). Encouraging mathematics discourse benefited primary mathematics educators by encouraging collaboration, increasing linguistic capabilities, and giving learners a sense of engagement (Sarwanto & Chumdari, 2021; Yaprak & Kaya, 2020). In addition, participants agreed that student autonomy and communication are critical to building thinking classrooms (Dempster, 2022; Krall, 2018; Liljedahl, Zager, et al., 2021; NCTM, 2015).

Third, participants shared that promoting inquiry-based learning by allowing students to make meaning and use manipulatives was essential to planning and implementing critical thinking opportunities. The critical thinking opportunities afforded to students via participants enabled students to make meaning of their learning (Turner et al., 2013). Pedagogical decisions allowed and sometimes encouraged mathematic manipulatives or other non-linguistic representations that were beneficial (Koskinen & Pitkaniemi, 2022).

Theoretical Implications

This study utilized Kolb's (1984) theory of experiential theory as its theoretical framework to address the influences of critical thinking tasks on the planning and implementation of critical thinking tasks. Kolb's (1984) theory of experiential learning suggests that learning is a continuous process grounded in experiences whereby knowledge is created. There are three factors essential to experiential theory: (1) learning is a continuous process, (2)

learning is a holistic process of adaptation to the world, and (3) learning involves transactions between the person and the environment (Kolb, 1984). In this study, participants' lived experiences of planning and implementing critical thinking opportunities were examined through Kolb's (1984) experiential learning theory.

Continuous Process

Kolb (1984) views learning as a continuous process, claiming that all learning is relearning, and everyone enters each experience with articulate ideas about the topic. When participants implemented critical thinking opportunities, they afforded their students this process. Learning in a group of students where mathematics discourse is encouraged, questions only further thinking, and students are making their meaning allows students to learn and relearn with every opportunity. In addition, participants shared those experiences engaging all learners. Each experience had an entry point for all students, whether the task was related to student interests or the task used real-world scenarios applicable to students. Regardless of the task, each student could articulate ideas about the topic and participate in the opportunity. Also, participants shared that implementing opportunities was more about the process than the product of the critical thinking opportunities. Kolb (1984) believes that learning is a transactional relationship where learning is an active, self-directed process. Implementing critical thinking opportunities engages students in a student-centered way, allowing them to create meaning for the content. Lastly, building a mathematics community that encourages students to collaborate and communicate allows students to learn from each other, offering something for every student.

Holistic Process of Adapting to the World

Kolb (1984) views learning as a holistic process of adapting to the world that describes the emergence of basic life orientations as a function of dialectic tensions between basic modes of relating to the world. All twelve participants spoke about how critical thinking opportunities

engage students in discourse. The ability to talk, listen, react, and collaborate are all afforded within critical thinking opportunities. Participants purposefully planned discourse into their opportunities. This discourse allows the opportunity to be scaffolded by the questioning and guidance of the teacher while appearing to be completely student-led. Students feel ownership over the opportunity as the teacher relinquishes control. The participants spoke at length about relating the opportunities to the interests of students as well as real-world scenarios. Critical thinking opportunities allow students to solve real-world problems in a way that makes sense to them by allowing them to use multiple strategies, build community, and make meaning.

Transactional Relationships

Kolb (1984) posits that learning involves transactions between the person and the environment. The transactional relationship makes learning an active, self-directed process that can be applied in everyday life (Kolb, 1984). Planned critical thinking opportunities purposefully encourage discourse, use of manipulatives and other non-linguistic representations, and allow students to make meaning. Critical thinking opportunities involve transactions between students and manipulatives. Participants shared that often, their students seek out manipulatives and non-linguistic representations to represent their answers. The classroom environment depends on the community in which it is built. Many participants shared that enabling students to have a growth mindset was important. In addition, norms were created and followed to ensure students were engaged in the task. Participants also shared that vertical whiteboard spaces allowed students to engage in their environment.

Limitations and Delimitations

Many limitations and delimitations of this research study were identified. As the researcher, I anticipated and minimized as many limitations as possible during the study. However, this study conveyed three potential limitations: sample size, time constraints from

participants, and reliance on technology. The following delimitations were identified: participants needed to be employed at BCPS, be over 18, have at least one year of experience, and use critical thinking tasks to include curricular and noncurricular tasks. This section of the chapter will explore the limitations and delimitations in further detail.

Limitations

Limitations included potential problems or challenges that arose from the chosen research design influencing the findings in the study (Creswell & Poth, 2018). Three limitations of this study were identified. First, the sample size of twelve participants was used to gather data on the lived experiences of primary mathematics educators. The participants consisted of ten females and two males. Using twelve participants in Northern Virginia may present a challenge in generalizing the findings as it does not accurately represent all primary mathematics educators in the United States. Therefore, this study represents a partial or subjective interpretation of the findings.

Second, participants' time constraints due to work, school, or family responsibilities influenced the findings. While some participants' schedules were open, others found scheduling times for individual interviews and the focus group more difficult. Participants could schedule their interviews but needed flexibility to complete the other data collection forms. The difficulty in time constraints led to problems in gathering data.

Third, this study lacked physical proximity to the participants and relied heavily on technology to navigate and collect data. Proximity to the participants may have allowed for a stronger personal connection, additional contextual understanding of non-verbal communication, and further opportunity to meet candidates for data collection with higher levels of flexibility. Finally, technological constraints and issues were also out of my control, limiting communication time and clarity.

Delimitations

Delimitations are intentional parameters used to define boundaries within a study (Creswell & Poth, 2018). The first delimitation was that BCPS must employ all participants. Second, all participants had to be at least 18 or older. Third, all participants must have at least one year of teaching experience. Lastly, all participants were required to use critical thinking tasks, including curricular or noncurricular tasks. The delimitations of this study excluded other schools and participant perceptions while relying on participant self-reporting of critical thinking lesson development and inclusion.

Recommendations for Future Research

In future research regarding the influences of critical thinking on mathematics instruction, educators should add literature on numerous levels. The first recommendation includes increasing the sample size of the participants to all levels of education. The varying levels of educators may offer more insight into critical thinking opportunities' influences on mathematics instruction. Furthermore, expanding the study participants to a broader geographical location may be beneficial. Virginia uses the Virginia Standards of Learning, while most other states use Common Core Standards. The varying standards may offer more insight.

Another recommendation for future research is to target the impacts of critical thinking opportunities. Exploring the numerical impact of critical thinking opportunities on formative, summative, and mandated assessments may be beneficial. Another recommendation for future research is to use observational data to examine critical thinking opportunities. Research surrounding observations of teachers, as well as students, may be beneficial. The observation of the task may illuminate and deepen the current themes of this study.

Conclusion

This study highlighted an ongoing issue of critical thinking being underrepresented in

primary mathematics education. This is due to mandated accountability testing taking up valuable instructional time, limited opportunities, and a shift in pedagogy (Liljedahl, Zager, et al., 2021; Neumann, 2016; Ridwan et al., 2022; Sumardi et al., 2020). This complex problem required investigation due to holes in the literature, primarily at the elementary level. This phenomenological study aimed to understand the mathematics critical thinking process for teachers in primary education in northern Virginia. This was achieved by employing hermeneutic phenomenology and using data collection methods that support the lived experiences of primary educators implementing critical thinking opportunities. The three data collection methods were individual interviews, a focus group, and document analysis. This study applied digital coding using Atlas.ti to derive key themes of purposeful planning, building community, and promoting inquiry-based learning. The findings of this study indicate that there are opportunities that can be afforded to students when planned purposefully using engaging questioning, encouraging discourse, and allowing students to make their meaning. In addition, the findings of this study have uncovered implications for collaborative and purposeful planning, a shift from teacher-led to student-led instruction, and the use of more experiential learning. Overall, this study indicates that opportunities to engage students in critical thinking opportunities require educators to shift their pedagogical ideologies to allow students to make their meaning rather than giving it to them. Future research is needed to expound on the effectiveness of critical thinking opportunities for all learners.

References

Abdullah, F. M., Mohamed, S. Q., & Jassim, R. H. (2022). Effect of the project-based learning strategy on the development of creative thinking among female students Institute of Fine Arts in interior design. *Journal of Positive School Psychology*, *6*(5), 3178–3192. https://doi.org/10.7771/1541-5015.1649.

Acharya, B. R., Belbase, S., Panthi, R. K., Khanal, B., Kshetree, M. P., & Dawadi, S. D. (2022). Critical conscience for construction of knowledge in mathematics education. *International Journal of Education in Mathematics, Science, and Technology*, *10*(4), 1030–1056. https://doi.org/10.46328/ijemst.2203.

Allensworth, E., Cashdollar, S., & Cassata, A. (2022). Supporting change in instructional practices to meet the common core mathematics and Next Generation Science Standards: How are different supports related to instructional change? *AERA Open*, *8*(1). https://doi.org/10.1177/23328584221088010.

Allington, R. L., (2005). What counts as evidence in evidence-based education? *Reading Today*, *23*(3).

Assante, M. G., Momanu, M., & Enescu, F. (2022). Educational strategies in the development of critical thinking: A grounded theory approach. *Croatian Journal Educational / Hrvatski Casopis Za Odgoj I Obrazovanje*, *24*(4), 1083–1110. https://doi.org/10.15516/cje.v24i4.4593.

Attard, C. (2012). Engagement with mathematics: What does it mean and what does it look like? *Australian Primary Mathematics Classroom, 17*(1), 9–13.

Babakr, Z. H., Mohamedamin, P., & Kakamad, K. (2019). Piaget's cognitive developmental theory: Critical review. *Education Quarterly Reviews*, *2*(3), 517–524. https://doi.org/10.31014/aior.1993.02.03.84.

Ballantyne, A. (2019). Untangling the math debate. *BU Journal of Graduate Studies in Education, 11*(2), 52-56.

Bang, H. J., Linlin, L., & Flynn, K. (2022). Efficacy of an adaptive game-based math learning app to support personalized learning and improve early elementary school students' learning. *Early Childhood Education Journal, 51,* 717-732. https://doi.org/10.1007/s10643-022-01332-3.

Barell, J. (2003). *Developing more curious minds.* Alexandria: Association for Supervision and Curriculum Development.

Barksdale-Ladd, M. A., & Thomas, K. F. (2000). What's at stake in high-stakes testing? Teachers and parents speak out. *Journal of Teacher Education, 51*(5), 384–397. https://doi.org/10.1177/0022487100051005006.

Bell, T. H. (1993). Reflections one decade after "A Nation at Risk." *The Phi Delta Kappan, 74*(8), 592-597.

Beswick, K. (2012). Teachers' beliefs about school mathematics and mathematicians' mathematics and their relationship to practice. *Educational Studies in Mathematics, 79*(1), 127–147. https://doi.org/10.1007/s10649-011-9333-2.

Bhusal, D. R. (2021). English language teachers' perceptions on integrating environmental education. *Journal on English Language Teaching, 11*(3), 11–19. https://doi.org/10.26634/jelt.11.3.17696.

Bloom, B., Englehart, M., Furst, E., Hill, W., & Krathwohl, D. (1956). Taxonomy of educational objectives: The classification of educational goals. Handbook I: Cognitive domain. Longmans, Green.

Boaler, J., & Williams, C. (2015). Fluency without fear: Research evidence on the best ways to learn math facts. *Reflections, 40*(2), 7-12.

Bohon, L. L., McKelvey, S., Rhodes, J. A., & Robnolt, V. J. (2017). Training for content teachers of English Language Learners: Using experiential learning to improve instruction. *Teacher Development*, *21*(5), 609–634. https://doi.org/10.1080/13664530.2016.1277256.

Bornfreund, L. (2012). Preparing teachers for the early grades. *Educational Leadership*, *69*(8), 36-40.

Bowen, G. A. (2009). "Document Analysis as a Qualitative Research Method", *Qualitative Research Journal*, *9*(2), 27-40. https://doi.org/10.3316/QRJ0902027.

Brookfield, S. (2013). Teaching for critical thinking. *International Journal of Adult Vocational Education and Technology*, *4*(1), 1–15. https://doi.org/10.4018/javet.2013010101.

Bruner, J. (1990). *Acts of meaning*. Cambridge, MA: Harvard University Press.

Bruun, F., Diaz, J. M., & Dykes, V. J. (2015). The language of mathematics. *Teaching Children Mathematics*, *21*(9), 530-536. https://doi.org/10.5951/teacchilmath.21.9.0530.

Bryman, A. (2016). *Social Research Methods (5th ed.)*. London: Oxford University Press.

Buskist, W., and Irons, J. (2008). ‘‘Simple Strategies for Teaching.’’ In Teaching Critical Thinking in Psychology: A Handbook of Best Practice, edited by D. S. Dunn, J. S. Halonen, and R. A. Smith, 4958. Oxford: Wiley-Blackwell.

Butler-Bowdon, T. (2007). *50 Psychology Classics.* London: Nicholas Brealey.

Camilli, G. (2021). The 2013-15 decline in NAEP Mathematics: What it teaches us about NAEP and the Common Core. *Measurement: Interdisciplinary Research and Perspectives*, *19*(4), 236–245. https://doi.org/10.1080/15366367.2021.1873062.

Cash, R. M. (2017). *Advancing differentiation: Thinking and learning for the 21st century.* Minneapolis, MN: Free Spirit Publishing Inc.

Chan, C. K. Y. (2012). Exploring an experiential learning project through Kolb's Learning Theory using a qualitative research method. *European Journal of Engineering Education, 37*(4), 405–415. https://doi.org/10.1080/03043797.2012.706596.

Chard, D. (2003). Vocabulary strategies for the mathematics classroom. *Retrieved on October, 21, 2007*.

Chen, B. (2023). Influence of cooperative learning on learners' motivation: The case of Shenzhen Primary School. *Education 3-13, 51*(4), 647-671. https://doi.org/10.1080/03004279.2021.1998179.

Chomsky, N., & Robichaud, A. (2014). Standardized testing as an assault on humanism and critical thinking in education. *Radical Pedagogy*, *11*(1), 3.

Clay, M. M. (2014). *By different paths to common outcomes: Literacy, learning, and teaching.* Portsmouth, NH: Heinemann.

Cogan, L., Burroughs, N., & Schmidt, W. (2015). Supporting classroom instruction. *Phi Delta Kappan*, *97*(1), 29-33. https://doi.org/10.1177/0031721715602233.

Creswell, J. W. (2013). *Qualitative inquiry & research design: Choosing among the five approaches*. Sage.

Creswell, J. W., & Poth, C. N. (2018). *Qualitative inquiry and research design: Choosing among five approaches*. Sage.

Dahl, A. J., Peltier, J. W., & Schibrowsky, J. A. (2018). Critical thinking and reflective learning in the marketing education literature: A historical perspective and future research needs. *Journal of Marketing Education, 40*(2), 101–116. https://doi.org/10.1177/0273475317752452.

Darling-Hammond, L. (2008). Educating teachers: How they do it abroad. *Time, 171*(8), 34.

Darling-Hammond, L. (2011). Testing, No Child Left Behind, and educational equity. In L. M. Stulberg & S. L. Weinberg (Eds), *Diversity in American higher education: Toward a more comprehensive approach* (pp. 36-47). New York: Routledge.

Darling-Hammond, L., & Wise, A.E.J. (1985). Beyond standardization: State standards and school improvement. *The Elementary School Journal, 85*, 315-336. https://doi.org/10.1086/461408.

Dempster, T. S. (2022). Addressing concerns related to low student understanding in mathematics in Manitoba. *BU Journal of Graduate Studies in Education, 14*(1), 4–9.

Denzin, N. K., & Lincoln, Y. S. (2011). Introduction: The discipline and practice of qualitative research. The SAGE Handbook of Qualitative Research (4th ed., pp. 1-19). Sage. https://doi.org/10.4324/9781003154587-intro.

Dewey, J. (1916). *Democracy and education: An introduction to the philosophy of education.* New York: Macmillan.

Donnelly, F. C., Helion, J., & Fry, F. (1999). Modifying teacher behaviors to promote critical thinking in K-12 physical education. *Journal of Teaching in Physical Education, 18*(2), 199–215. https://doi.org/10.1123/jtpe.18.2.199.

Duch, B. J., Groh, S. E., & Allen, D. E. (Ed.). (2001). *The power of problem-based learning.* Sterling, VA: Stylus.

Dudley, D., & Burden, R. (2020). What effect on learning does increasing the proportion of curriculum time allocated to physical education have? A systematic review and meta-analysis. *European Physical Education Review, 26*(1), 85–100. https://doi.org/10.1177/1356336x19830113.

Edwards, A. (2000). ‘‘Looking at action research through the lenses of socio-cultural

psychology and activity theory.'' *Educational Action Research, 8*(1), 195-204. https://doi.org/10.1080/09650790000200104.

El Sayary, A. (2021). Using a reflective practice model to teach STEM education in a blended learning environment. *EURASIA Journal of Mathematics, Science and Technology Education, 17*(2). https://doi.org/10.29333/ejmste/9699.

Emanet, E. A., & Kezer, F. (2021). The effects of student-centered teaching methods used in mathematics courses on mathematics achievement, attitude, and anxiety: a meta-analysis study. *Participatory Educational Research, 8*(2), 240-259. https://doi.org/10.17275/per.21.38.8.2.

Ennis, R. H. (1987). A taxonomy of critical thinking dispositions and abilities. In J. Boykoff Barron & R. S. Steinberg (Eds.), *Teaching thinking skills: Theory and practice* (pp. 11 – 26). Freeman and Company.

Ennis, R.H., & Millman, J. (1985). *Cornell Tests of Critical Thinking.* Midwest Publications.

Ertekin, E., Dilmac, B., & Yazici, E. (2009). The relationship between mathematics anxiety and learning styles of pre-service mathematics teachers. *Social Behavior and Personality, 37*(9), 1187-1196. https://doi.org/10.2224/sbp.2009.37.9.1187.

Evcimik, S., & Oruc, C. (2023). Effects of project-based learning in moral values education. *International Online Journal of Educational Sciences, 15*(3), 495–508. https://doi.org/10.15345/iojes.2023.03.010.

Fisher, A. (2009). *Critical thinking: An introduction*. Cambridge University Press.

Fisher, A., & Scriven, M. (1997). *Critical Thinking. Its Definition and Assessment*. Edgepress: CA, USA/Centre For Research In Critical Thinking: Norwich, UK.

Fredricks, J. A., Blumenfeld, P. C. & Paris, A. H. (2004). School engagement: Potential of the concept, state of the evidence. *Review of Educational Research, 74*(1), 59–110. https://doi.org/10.3102/00346543074001059.

Furinghetti, F. (2020). Rethinking history and epistemology in mathematics education. *International Journal of Mathematical Education in Science & Technology, 51*(6), 967–994. https://doi.org/10.1080/0020739X.2019.1565454.

Gaylor, K. (2005). *How have high school exit exams changed our schools? Some perspectives from Virginia and Maryland*. Washington, DC: Center on Educational Policy.

Gearing, R. E. (2004). Bracketing in research: A typology. *Qualitative Health Research, 14*(10), 1429-1452. https://doi.org/10.1177/1049732304270394.

Geertz, C. (1988). Thick description: Toward an interpretive theory of culture.

Gentile, D. A., Nathanson, A. I., Rasmussen, E. E., Reimer, R. A., & Walsh, D. A. (2012). Do you see what I see? Parent and child reports of parental monitoring of media. *Family Relations, 61*(3), 470–487. https://doi.org/10.1111/j.1741-3729.2012.00709.

Giorgi, A. (2009). *A descriptive phenomenological method in psychology: A modified Husserlian approach.* Duquesne University Press.

Glaser, E. (1941). *An Experiment in the Development of Critical Thinking*. Teachers College, Columbia University.

Glaser, R. (1984). Education and thinking: the role of knowledge. *American Psychologist, 39*, 93-104. https://doi.org/10.1037//0003-066x.39.2.93.

Gosen, J., & Washbush, J. (2004). A review of scholarship on assessing experiential learning effectiveness. *Simulation & Gaming*, *35*(2), 270–293. https://doi.org/10.1177/1046878104263544.

Grasha, A. F. (1996). *Teaching with style: A practical guide to enhancing learning by understanding teaching and learning styles.* Pittsburgh: Allegiance Publishers.

Green, C. (2005). Integrating extensive reading in the task-based curriculum. *ELT Journal, 59*(4), 306-311. https://doi.org/10.1093/elt/cci059.

Henderson, C., & Ivey, A. (2023). Igniting statewide mathematics re-design: A cross-sector, grassroots approach. *New Directions for Community College, 201*, 53-56. https://doi.org/10.1002/cc.20560.

Herbal-Eisenmann, B. A., & Breyfogle, M. L. (2005). Questioning our "patterns" of questioning. *Mathematics Teaching in the Middle School, 10*(9), 484-489. https://doi.org/10.5951/mtms.10.9.0484.

Hersee, J. (1993). National curriculum mathematics: some pros and cons. *Mathematics in School*, 22, 16-17.

Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In J. Frank & K. Lester (Eds.), *Second handbook of research on mathematics teaching and learning* (pp. 371-404). Information Age.

Hobbs, L. (2012). Teaching out-of-field: Factors shaping identities of secondary science and mathematics. *The Journal of Australian Science Teachers, 58*(1), 21-29. https://doi.org/10.1007/s10763-012-9333-4.

Hunt, S. D., & Laverie, D. A. (2004). Experiential learning and the Hunt-Vitell theory of ethics: Teaching marketing ethics by integrating theory and practice. *Marketing Education Review, 14*(3), 1–14. https://doi.org/10.1080/10528008.2004.11488874.

Inhelder, B., Sinclair, H., & Bovet, M. (1974). *Learning and the development of cognition.* Cambridge, MA: Harvard University.

Jacobs, V., & Spangler, D. A. (2017). Research on core practices in K–12 mathematics teaching. In J. Cai (Ed.), *Compendium for research in mathematics education* (pp. 766–792). National Council of Teachers of Mathematics.

Jarvis, S., & Barberena, L. (2008). Focus group. In *Encyclopedia of Survey Research Methods* (Vol. 0, pp. 287-289). Sage Publications, Inc., https://doi.org/10.4135/9781412963947.

Johanningmeier, E. V. (2010). "A Nation at Risk" and "Sputnik": Compared and reconsidered. *American Educational History Journal, 37*(2), 347-365.

Johnson, B., Reschly, A. L., Martin, R. P., Pinzone, C., & Appleton, J. J. (2022). Student engagement in elementary school: Profiles and associated characteristics. *Journal of Education for Students Placed at Risk (JESPAR), 29*(2), 83–104. https://doi.org/10.1080/10824669.2022.2112196.

Jones, M. G., Jones, B. D., Hardin, B., Chapman, L., Yarbrough, T., & Davis, M. (1999). The influence of high stakes testing on teachers and students in North Carolina. *Phi Delta Kappan, 81*(3), 199–203.

Joubert, I., & Harrison, G. D. (2021). Revisiting Piaget, his contribution to South African early childhood education. *Early Child Development and Care*, 191(7–8), 1002–1012. https://doi.org/10.4324/9781003120216-3.

Kamii, C. (1982). *Number in preschool and kindergarten*. Washington, DC: The National Association of Young Children.

Kar, T. (2015). Analysis of problems posed by sixth-grade middle school students for the addition of fractions in terms of semantic structures. *International Journal of Mathematical Education in Science and Technology, 46*(6), 879–894. https://doi.org/10.1080/0020739x.2015.1021394.

Kayes, D. C. (2002). Experiential learning and its critics: Preserving the role of experience in management learning and education. *Academy of Management Learning & Education 1(*2), 137–149. https://doi.org/10.5465/amle.2002.8509336.

Kolb, D. A. (1984). *Experiential learning: Experience as a source of learning and development*. Prentice-Hill.

Krall, G. (2018). *Necessary conditions: Teaching secondary math with academic safety, quality tasks, and effective facilitation*. Stenhouse.

Kristof, N. (2011, March 13). *Pay teachers more*. The New York Times. Retrieved from http://www.nytimes.com/2011/03/13/opinion/13kristof.html.

Komariyah, S., & Laili, A. F. N. (2018). The effect of critical thinking ability on mathematics learning outcomes. *Jurnal Penelitian Pendidikan Dan Pengajaran Matematika*, *4*(2), 55–60.

Koskinen, R. & Pitkäniemi, H. (2022). Meaningful learning in mathematics: A research synthesis of teaching approaches. *International Electronic Journal of Mathematics Education.* 17. https://doi.org/10.29333/iejme/11715.

Kurfiss, J. G. (1988). *Critical Thinking: Theory, Research, Practice, and Possibilities. ASHE-ERIC Higher Education Report No. 2, 1988*.

Kvale, S. (1996). Interviews: An introduction to qualitative research interviewing. Sage.

Larsen, J., & Liljedahl, P. (2022). Building thinking classrooms online: From practice to theory and back again. *Adults Learning Mathematics (ALM-IJ)*, *16*(1), 36–52.

Lee, S.-Y. (2021). Research status of mathematical problem posing in mathematics education journals. *International Journal of Science and Mathematics Education*, *19*(8), 1677–1693. https://doi.org/10.1007/s10763-020-10128-z.

Lewis, K. D., & DeSantis, J. (2021). Amplify experiential learning with cloud video. *College Teaching*, *69*(3), 173–179. https://doi.org/10.1080/87567555.2020.1845598.

Leung, S. S., & Silver, E. A. (1997). The role of task format, mathematics knowledge, and creative thinking on the arithmetic problem posing of prospective elementary school teachers. *Mathematics Education Research Journal*, *9*(1), 5–24. https://doi.org/10.1007/bf03217299.

Levine, M. (2012). *Teach your children well: Parenting for authentic success*. New York: Harper.

Liljedahl, P., Clarke, A., & Morrison, N. (2021). Building thinking classrooms: A conversation with Dr. Peter Liljedahl. *Journal of Mathematics Education at Teachers College*, *12*(1), 1–7.

Liljedahl, P., Zager, T., & Wheeler, L. (2021). *Building Thinking Classrooms in Mathematics: 14 teaching practices for enhancing learning, grades K-12*. Corwin. https://doi.org/10.1007/978-3-319-28023-3_21.

Liñán-García, M. del M., Muñoz-Catalán, M. C., Contreras, L. C., & Barrera-Castarnado, V. J. (2021). Specialized knowledge for teaching geometry in a primary education class: Analysis from the knowledge mobilized by a teacher and the knowledge evoked in the researcher. *Mathematics*, *9*(21), 2805. https://doi.org/10.3390/math9212805.

Lincoln, Y. S. & Guba, E. G. (1985). *Naturalistic Inquiry*. Newbury Park, CA: Sage Publications.

Lo, J., Cai, J., & Watanabe, T. (2001). A comparative study of the selected textbook from China, Japan, Taiwan, and the United States on teaching ratio and proportion concepts. Paper presented at the North American Chapter of the International Group for the Psychology of Mathematics Education, Snowbird, Utah.

Locker, M. (2015). This country just made it illegal to give kids too much screen time. Time.Com, N.PAG.

Lourenco, O., & Machado, A. (1996). In defense of Piaget's theory: A reply to 10 common criticisms. *American Psychological Association, 103*(1), 143-162. https://doi.org/10.1037//0033-295x.103.1.143.

Lowrie, T. (2002). Young children posing problems The influence of teacher intervention on the type of problems children pose. *Mathematics Education Research Journal, 14*(2), 87–98. https://doi.org/10.1007/bf03217355.

Marder, M., & Villanueva, C. K. (2018). Rise and fall of Texas STEM education: Evidence from the nation's report card. In *Center for Public Policy Priorities*. Center for Public Policy Priorities.

Mbugua, Z. K. (2012). Influence of mathematical language on achievement in mathematics by secondary school students in Kenya. *International Journal of Education and Information Studies,* 2, 1-7. https://doi.org/10.4314/joten.v10i1.35715.

McGee, G. (2004). Closing the achievement gap: Lessons from Illinois' golden spike high poverty, high performing schools. *Journal of Education for Students Placed at Risk, 9*(2), 97-125. https://doi.org/10.4324/9781315046099-2.

McPeck, J. E. (1981). *Critical Thinking and Education*. Martin Robertson.

McTighe, J., & Brown, P. (2020). Standards are not curriculum: Using understanding by design to make standards come alive. *Science and Children, 58*(1), 76-81. https://doi.org/10.1080/08872376.2021.12291448.

Mellone, M., Pacelli, T., & Liljedahl, P. (2021). Cultural transposition of a thinking classroom: To conceive possible unthoughts in mathematical problem solving activity. *Mathematics Education, 53*(4), 785–798. https://doi.org/10.1007/s11858-021-01256-z.

Merriam, S. B. (1988). *Case study research in education: A qualitative approach*. Jossey-Bass.

Merriam, S. B., & Tisdell, E. J. (2016). *Qualitative research: A guide to design and Implementation (4th ed.)*. John Wiley & Sons.

Mezirow, J. (2000). *Learning as transformation: Critical perspectives on a theory in progress*. Jossey-Bass.

Miles, M. B., Huberman, A. M., & Saldaña, J. (2014). *Qualitative data analysis: A methods source-book (3rd ed.*). SAGE.

Miller, R., Greene, B., & Montalvo, G. (1996). Engagement in academic work: The role of learning goals, future consequences, pleasing others, and perceived ability. *Contemporary Educational Psychology, 21*(4), 388-422. https://doi.org/10.1006/ceps.1996.0028.

Morgan, H. (2012). Poverty-stricken schools: What we can learn from the rest of the world and from successful schools in economically disadvantaged areas in the US. *Education, 133*(2), 291-297.

Nadeak, B., & Naibaho, L. (2020). The effectiveness of problem-based learning on students' critical thinking. *J Din Pendidik, 13*(1):1–7.

Nahar, S., & Suhendri, & Zailani, & Hardivizon. (2022). Improving students' collaboration thinking skill under the implementation of the quantum teaching model. *International Journal of Instruction, 15*(3), 451–464. https://doi.org/10.29333/iji.2022.15325a.

Nasir, N. S., & Cobb, P. (2006). *Improving access to mathematics: Diversity and equity in the classroom.* Multicultural Education Series. ERIC.

National Center for Education Statistics. (n.d.). *Search for public school districts*. National Center for Education Statistics (NCES) Home Page, a part of the U.S. Department of Education. https://nces.ed.gov/ccd/districtsearch/district_detail.asp?ID2=5103130.

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and standards for school mathematics*. Reston, VA.

National Council of Teachers of Mathematics (NCTM). (2015). *Principles to actions: Ensuring mathematical success for all*. The National Council of Teachers of Mathematics.

Neumann, J. W. (2016). Examining mandated testing, teachers' milieu, and teachers' knowledge and beliefs: Gaining a fuller understanding of the web of influence on teachers' classroom practices. *Teachers College Record*, *118*(2), 1–50. https://doi.org/10.1177/016146811611800209.

New International Bible. (2011). Zondervan. (Original work published 1978).

Nickerson, R. S. (1988). On improving thinking through instruction. In Ernst Z. Rothkopf (Ed.), *Review of research in education* (pp. 3-57). American Educational Research Association.

Norris, J. (2013). Some challenges in assessment for teacher licensure, program accreditation, and educational reform. *Modern Language Journal, 97*(2), 554-560. https://doi.org/10.1111/j.1540-4781.2013.12022.x.

OECD. (2018). 2018 PISA Results. https://www.oecd.org/pisa/PISA-results_ENGLISH.png (accessed on 5/28/2023).

Ornstein, A. (2010). *Achievement gaps in education*. Society, 47(5), 424-429. https://doi.org/10.1007/s12115-010-9354-y.

Ozer, E., & Sezer, R. (2014). A comparative analysis of questions in American, Singaporean, and Turkish mathematics textbooks based on the topics covered in 8th grade Turkey. *Educational Sciences: Theory and Practice, 14*(1), 411-421. https://doi.org/10.12738/estp.2014.1.1688.

Paine, S., & Schleicher, A. (2011). What the U.S. can learn from the world's most successful education reform efforts. http://www.mcgrawhillresearchfoundation.org/wp-content/uploads/pisa-intl-competitiveness.pdf.

Pak, K., Polikoff, M. S., Desimone, L. M., & Saldívar García, E. (2020). The adaptive challenges of curriculum implementation: Insights for educational leaders driving standards-based reform. *AERA Open*, *6*(2). https://doi.org/10.1177/2332858420932828.

Pan, A.J., Lai C.F., & Kuo H.C. (2023). Investigating the impact of a possibility-thinking integrated project-based learning history course on high school students' creativity, learning motivation, and history knowledge. *Think Skills Creat*, *47*,101-214. https://doi.org/10.1016/j.tsc.2022.101214.

Pantaleo, S. (2023). Elementary students' engagement in transduction and creative and critical thinking. *Literacy*, (1), 1-14. https://doi.org/10.1111/lit.12350.

Paul, R., & Elder, L. (2008). Critical Thinking: The Nuts and Bolts of Education. *Optometric Education*, *33*, 88-91.

Peters, R. (1992). *America 200 and the "New" Geography.*

Perkins, D. N., & Salomon, G. (1989). Are cognitive skills context bound? *Educational Researcher*, *18*(1), 16-25. https://doi.org/10.2307/1176006.

Piaget, J. (1962). Comments on Vygotsky's critical remarks concerning *The Language and Thought of the Child*, and *Judgement and Reasoning in the Child*. The Massachusetts Institute of Technology, 1-14. https://doi.org/10.1016/s0732-118x(00)00012-x.

Polit, D.F., & Beck, C.T. (2014). *Essentials of nursing research: Appraising evidence for nursing practice (8th ed.).* Philadelphia, PA: Wolters Kluwer/Lippincott Williams & Wilkins.

Rehman, N., Zhang, W., Mahmood, A., Fareed, M. Z., & Batool, S. (2023). Fostering twenty-first century skills among primary school students through math project-based learning. *Humanities & Social Sciences Communications, 10*(1), 1–12. https://doi.org/10.1057/s41599-023-01914-5.

Resnick, L. B. (1987). *Education and learning to think.* Washington, DC: National Academy Press.

Reynolds, M. (2009). Wild Frontiers—Reflections on experiential learning. *Management Learning, 40*(4), 387–392. https://doi.org/10.1177/1350507609335848.

Reynolds, M., & Vince, R. (2007). *The Handbook of Experiential Learning and Management Education*. Oxford: Oxford University Press.

Ridwan, R., Retnawati, H., Hadi, S., & Jailani. (2022). Teachers' perceptions in applying mathematics critical thinking skills for middle school students: A case of phenomenology. *Anatolian Journal of Education, 7*(1), 1–16. https://doi.org/10.29333/aje.2022.711a.

Rivas, S. F., & Saiz, C. (2023). Evaluation of the effectiveness of the ARDESOS-DIAPROVE critical thinking training programme. *Thinking Skills & Creativity*, 48, N.PAG. https://doi.org/10.1016/j.tsc.2023.101306.

Robinson, S. P., & Kay, K. (2010). *21st century knowledge and skills in educator preparation.* United States of America: Partnership for 21st Century Skills.

Romberg, T. A., Zarinnia, E. A., & Williams, S. R. (1989, March). *The influence of mandated testing on mathematics instruction.* National Center for Research in Mathematical Science Education, University of Wisconsin-Madison.

Rottenberg, C., & Smith, M. L (1990, April). Unintended effects of external testing in elementary school. Paper presented at the annual meeting of the American Educational Research Association, Boston.

Rubin, H. J., & Rubin, I. S. (2012). *Qualitative interviewing: The art of hearing data* (3rd ed.). Sage.

Sadeghi, F., Adel, S. M. R., Zareian, G., & Davoudi, M. (2020). Iranian EFL teachers' and learners' perceptions of the principles of critical thinking: A constructivist grounded theory study. *Iranian Journal of Language Teaching Research*, *8*(2), 63–81.

Saldana, J. (2013). *The coding manual for qualitative researchers* (Second ed.). SAGE.

Saltman, D. C., Tavabie, A., & Kidd, M. R. (2012). The use of reflective and reasoned portfolios by doctors. *Journal of Evaluation in Clinical Practice*, *18*(1), 182–185. https://doi.org/10.1111/j.1365-2753.2010.01514.x.

Sarabi, M. K., & Gafoor, K. (2017). Linguistics challenges and its influence on the perceived difficulty in mathematics learning of elementary school students of Kerala. *North East Regional Institute.*

Sarwanto, F., & Chumdari. (2021). Critical thinking skills and their influences on elementary school students. *Malaysian Journal of Learning and Instruction*, *18*(2), 161-188. https://doi.org/10.32890/mjli2021.18.2.6.

Schoenfeld, A. H. (2010). Research methods in (mathematics) education. *In Handbook of international research in mathematics education* (pp. 481–533). Routledge.

Sharkins, K., Newton, A., Causey, C., & Ernest, J. M. (2017). Flipping theory: Ways in which children's experiences in the 21st century classroom can provide insight into the theories of Piaget and Vygotsky. *International Journal of Early Childhood Education and Care*, *6*, 11–18. https://doi.org/10.37134/saecj.vol6.2.2017.

Shepard, L. A. (1990). Inflated test score gains: Is the problem old norms or teaching the test? Educational Measurement: *Issues and Practice, 8*, 15-21.

Shepard, L. A., & Dougherty, K. C. (1991). *Effects of High-Stakes Testing on Instruction*.

Silver, E. A., & Cai, J. (1996). An analysis of arithmetic problem posing by middle school students. *Journal for Research in Mathematics Education, 27*(5), 521–539. https://doi.org/10.5951/jresematheduc.27.5.0521.

Siswono, T. Y. E. (2011). "Level of Student's Creative Thinking in Classroom Mathematics." Educational Research and Reviews 6 (7): 548. https://doi.org/10.22342/jme.1.1.794.17-40.

Smith, S. J. (2020). *Windows into the history and philosophy of Education*. Kendall Hunt.

Smith, M., Steele, M., & Sherin, M. (2020). *The 5 practices in practice: Successfully orchestrating mathematics discussions in your high school classroom*. Corwin.

Steinberg, M. P., & Quinn, R. (2017). Education reform in the post-NCLB era: lessons learned for transforming urban public education. *Cityscape: A Journal of Policy Development and Research, 19*(1), 191-216.

Steffe, L. P., Cobb, P., & von Glasersfeld, E. (1988). *Construction of arithmetical meanings and strategies.* Springer-Verlag.

Sternberg, R. J. (1987). Questions and answers about the nature and teaching of thinking skills. In J. B. Baron & R. J. Sternberg (Eds.), *Teaching thinking skills: Theory and practice* (pp. 251-259). Freeman.

Stigler, J. W., & Hiebert, J. (2004). Improving mathematics teaching. *Educational Leadership, 61*(5), 12-17.

Stringfellow, J. (2009). Designing and implementing mathematics instruction for students with

diverse learning needs. *Intervention in School and Clinic, 45*(2), 142. https://doi.org/10.1177/1053451208321450.

Su, H. F., Ricci, F. A., & Mnatsakanian, M. (2016). Mathematical teaching strategies: Pathways to critical thinking and metacognition. *Journal of Research in Education and Science (IJRES), 2* (1), 190-200. https://doi.org/10.21890/ijres.57796.

Sumardi, L., Rohman, A., & Wahyudiati, D. (2020). Does the teaching and learning process in primary schools correspond to the characteristics of 21st century learning? *International Journal of Instruction, 13*(3), 357–370. https://doi.org/10.29333/iji.2020.13325a.

Suratmi, S., & Sopandi, W. (2022). Knowledge, skills, and attitudes of teachers in training critical thinking of elementary school students. *Journal of Education and Learning (EduLearn), 16*(3), 291–298. https://doi.org/10.11591/edulearn.v16i3.20493.

Suter, W. N. (2012). *Introduction to educational research: A critical thinking approach* (2nd ed.). Sage Publications. https://www.doi.org/10.4135/9781483384443.

Swartz, R. J. (1984). Critical thinking, the curriculum, and the problem of transfer. In D. N. Perkins, Jr., J. Lochhead, & J. Bishop (Eds.), *Thinking: Progress in research and teaching* (pp. 261-284). Erlbaum. https://doi.org/10.4324/9781315802015-21.

Syahrial, Asrial, Kurniawan, D. A., Perdana, R., & Pratama, R. A. (2021). Implementing inquiry-based ethno-constructivism learning module to improve students' critical thinking skills and attitudes towards cultural values. *Eurasian Journal of Educational Research, 95*, 118–138. https://doi.org/10.14689/ejer.2021.95.7.

Thanheiser, E., & Melhuish, K. (2023). Teaching routines and student-centered mathematics instruction: The essential role of conferring to understand student thinking and reasoning. *The Journal of Mathematical Behavior, 70,* 1-19. https://doi.org/10.1016/j.jmathb.2023.101032.

Tekkumru-Kisa, M., Stein, M. K., & Coker, R. (2018). Teachers' learning to facilitate high-level student thinking: Influence of a video-based professional development. *Journal of Research in Science Teaching, 55*(4), 479–502. https://doi.org/10.1002/tea.21427.

Tekkumru-Kisa, M., Stein, M. K., & Doyle, W. (2020). Theory and research on tasks revisited: Task as a context for students' thinking in the era of ambitious reforms in mathematics and science. *Educational Researcher, 49*(8), 606–617. https://doi.org/10.3102/0013189x20932480.

TIMSS. (2019) U.S. Highlights Web Report (NCES 2021-021). U.S. Department of Education. Institute of Education Sciences, National Center for Education Statistics. https://nces.ed.gov/timss/results19/index.asp.

Tomkins, L., & Ulus, E. (2016). 'Oh, was that "experiential learning"?!' Spaces, synergies, and surprises with Kolb's learning cycle. *Management Learning, 47*(2), 158–178. https://doi.org/10.1177/1350507615587451.

Topçiu, M. (2015). Vygotsky theory on social interaction and its influence on the development of pre-school children. *European Journal of Social Science Education and Research, 2*(3), 172-179. https://doi.org/10.26417/ejser.v4i1.p172-179.

Turgut, S., & Turgut, I. (2018). The effects of cooperative learning on mathematics achievement in Turkey a meta-analysis study. *International Journal of Instruction, 11*(3), 663–680. https://doi.org/10.12973/iji.2018.11345a.

Turner, E., Dominguez, H., Maldonado, L., & Empson, S. (2013). English learners' participation in mathematical discussion: Shifting positionings and dynamic identities. *Journal for Research in Mathematics Education, 44*(1), 199–234. https://doi.org/10.5951/jresematheduc.44.1.0199.

U.S. Department of Education. Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2022 Mathematics Assessment.

Van Harpen, X. Y., & Presmeg, N. C. (2013). An investigation of relationships between students' mathematical problem-posing abilities and their mathematical content knowledge. *Educational Studies in Mathematics, 83*(1), 117–132. https://doi.org/10.1007/s10649-012-9456-0.

Van Harpen, X. Y., & Sriraman, B. (2013). Creativity and mathematical problem posing: an analysis of high school students' mathematical problem posing in China and the USA. *Educational Studies in Mathematics, 82*(2), 201–221. https://doi.org/10.1007/s10649-012-9419-5.

van Hover, S., & Pierce, E. (2006). "Next year will be different": Two first-year history teachers' perceptions of the influence of Virginia's accountability reform on their instructional decision making. *Journal of Social Studies Research, 30*(2), 38–50.

Van Kerkhove, B. & Van Bendegem, J. P. (2012) The many faces of mathematical constructivism. *Constructivist Foundations 7*(2): 97–103.

van Manen, M. (2016). *Phenomenology of practice: Meaning-giving methods in phenomenological research and writing*. Routledge Taylor & Francis Group.

van Vliet, E. A., Winnips, J. C., & Brouwer, N. (2015). Flipped-class pedagogy enhances student metacognition and collaborative-learning strategies in higher education, but effect does not persist. *CBE - Life Sciences Education, 14*(3). https://doi.org/10.1187/cbe.14-09-0141.

Vogler, K. E. (2005). Influence of a high school graduation examination on social studies teachers' instructional practices. *Journal of Social Studies Research, 29*(2), 19–33.

Vogler, K. E. (2008). Comparing the influence of accountability examinations on Mississippi and Tennessee social studies teachers' instructional practices. *Educational Assessment, 13*, 1–32. https://doi.org/10.1080/10627190801968158.

Voigt, J. (1985). Patterns and routines in classroom interaction. *Recherches en Didactique des Mathematics, 6*, 69-118. https://doi.org/10.4324/9780203053140-9.

Vygotsky, L. S. (1929). The problem of the cultural development of the child II. *Journal of Genetic Psychology*, (36), 415-32. https://doi.org/10.1080/08856559.1929.10532201.

Vygotsky, L. S. (1962). *Thought and Language*. The Massachusetts Institute of Technology. https://doi.org/10.1037/11193-000.

Wang, L., & Calvano, L. (2018). Understanding how service-learning pedagogy influences student learning objectives. *Journal of Education for Business, 93*(5), 204–212. https://doi.org/10.1080/08832323.2018.1444574.

Warren, C. A., & Xavia Karner, T. (2015). *Discovering qualitative methods: Ethnography, interviews, documents, and images* (3rd ed.). Oxford University Press.

Watson, G., & Glaser, E.M. (1980). "*The Watson-Glaser Critical Thinking Appraisal*." Cleveland: Psychology Corporation.

Weiss, S. (2015). Room for improvement. *State Legislatures, 41*(3), 26-30. https://doi.org/10.1787/888933258555.

Wen, R., & Dubé, A. K. (2022). A Systematic review of secondary students' attitudes towards mathematics and its relations with mathematics achievement. *Journal of Numerical Cognition, 8*(2), 295–325. https://doi.org/10.5964/jnc.7937.

William, D. (2003). National curriculum assessment: how to make it better. *Research Papers in Education, 18*(2), 129-136. https://doi.org/10.1080/0267152032000081896.

Wood, T., Cobb, P., & Yackel, E. (1991). Change in teaching mathematics: A case study. *American Educational Research Journal*, *28*(3), 587–616. https://doi.org/10.3102/00028312028003587.

Yaprak, Z., & Kaya, F. (2020). Improving EFL learners' oral production through reasoning-gap tasks enhanced with critical thinking standards: Developing and implementing a critical TBLT model, pre-task plan, and speaking rubric. *Advances in Language and Literary Studies, 11*(1), 40–50. https://doi.org/10.7575/aiac.alls.v.11n.1p.40.

Zakaria, E., L. C. Chin, and M. Y. Daud. (2010). The effects of cooperative learning on students' mathematics achievement and attitude towards mathematics. *Journal of Social Sciences, 6*(2), 272–275. https://doi.org/10.3844/jssp.2010.272.275.

Zhang, J. (2022). The influence of Piaget in the field of learning science. *Higher Education Studies, 12*(3), 162–168. https://doi.org/10.5539/hes.v12n3p162.

Appendix A

Liberty University IRB Approval

LIBERTY UNIVERSITY.
INSTITUTIONAL REVIEW BOARD

March 25, 2024

Glen Miller
Heather Strafaccia

Re: IRB Exemption - IRB-FY23-24-388 AN EXAMINATION OF THE IMPACT OF CRITICAL THINKING TASKS ON MATHEMATICS INSTRUCTION: A QUALITATIVE STUDY

Dear Glen Miller, Heather Strafaccia,

The Liberty University Institutional Review Board (IRB) has reviewed your application per the Office for Human Research Protections (OHRP) and Food and Drug Administration (FDA) regulations and finds your study to be exempt from further IRB review. This means you may begin your research with the data-safeguarding methods described in your IRB application, and no further IRB oversight is required.

Your study falls under the following exemption category, which identifies specific situations in which human participants research is exempt from the policy set forth in 45 CFR 46:104(d):

Category 2.(iii). Research that only includes interactions involving educational tests (cognitive, diagnostic, aptitude, achievement), survey procedures, interview procedures, or observation of public behavior (including visual or auditory recording) if at least one of the following criteria is met:
The information obtained is recorded by the investigator in such a manner that the identity of the human subjects can readily be ascertained, directly or through identifiers linked to the subjects, and an IRB conducts a limited IRB review to make the determination required by §46.111(a)(7).

For a PDF of your exemption letter, click on your study number in the My Studies card on your Cayuse dashboard. Next, click the Submissions bar beside the Study Details bar on the Study Details page. Finally, click Initial under Submission Type and choose the Letters tab toward the bottom of the Submission Details page. Your information sheet and final versions of your study documents, **which you must use to conduct your study,** can also be found on the same page under the Attachments tab.

This exemption only applies to your current research application, and any modifications to your protocol must be reported to the Liberty University IRB for verification of continued exemption status. You may report these changes by completing a modification submission through your Cayuse IRB account.

If you have any questions about this exemption or need assistance in determining whether possible modifications to your protocol would change your exemption status, please email us at irb@liberty.edu.

Sincerely,
G. Michele Baker, PhD, CIP
Administrative Chair
Research Ethics Office

Appendix B

Site Permission Request Letter

November 22, 2023

[redacted]

As a graduate student in the School of Education at Liberty University, I am researching to understand better the changes made to instructional planning and curriculum development due to the underrepresentation of critical thinking in mathematics. The title of my research project is An Examination of the Influences of Critical Thinking Tasks on Mathematics Instruction: A Qualitative Study and the purpose of this phenomenological study is to understand the mathematics critical thinking process for teachers in primary education in Northern Virginia.

I request your permission to conduct my research at [redacted].

The data will be used to study the lived experiences of mathematics instructors. Participants will be presented with informed consent information before participating. Participating in this study is entirely voluntary, and participants are welcome to discontinue participation at any time.

Thank you for considering my request. If you grant permission, please provide a signed statement on official letterhead indicating your approval. A permission letter document is attached for your convenience.

Sincerely,

Glen E. Miller III
Liberty University Ph.D. Candidate

Appendix C

Site Approval

[redacted] [redacted]

To: Mr. Glen Miller
From: [redacted].
Re: Research Request
Date: 2/07/24

Mr. Glen Miller,

Your request to conduct the study, AN EXAMINATION OF THE IMPACT OF CRITICAL THINKING TASKS ON MATHEMATICS INSTRUCTION: A QUALITATIVE STUDY, has been approved by [redacted] with the following conditions and understandings:

(1) The researcher will not be naming our school district or any individual classrooms in the published doctoral report; pseudonyms will be used as necessary, and anonymity of individual participants, classrooms, and the division at large will be maintained.
(2) The researcher has agreed to provide a division-level data comparison for the compared programs within [redacted] to benefit the division. A digital copy of the broader research study will also be provided.
(3) All data collected/analyzed will remain entirely confidential and will not be provided to anyone outside of the student's supervising faculty/staff in accordance with the researcher's IRB and the Research Review Committee for [redacted].
(4) This approval does not constitute a commitment of resources or endorsement of the study or its findings by [redacted] or the School Board.
(5) The researcher agrees to abide by all the policies and regulations of [redacted] and will conduct this study within the stipulations accompanying any letter of approval.

Please note that approval by the central office does not guarantee participation by [redacted] schools or staff, and [redacted] reserves the right to withdraw from the study at any time should circumstances change. If you have any questions about the approval, please let me know.

Best wishes on your research.

Sincerely,

[redacted]

Appendix D

Initial Verbal Recruitment

Hello Potential Participant,

As a doctoral candidate in the School of Education at Liberty University, I am researching to better understand changes in mathematics instruction and planning following implementing critical thinking opportunities such as Liljedahl's Thinking Tasks, noncurricular tasks, or any other task that makes your students think. The purpose of my research is to examine the effects of implementing critical thinking tasks on instructional and curriculum planning, and I am writing to invite you to join my study.

Participants must be 18 years or older and have at least one year of experience in the field. Participants must implement the usage of critical thinking tasks. There are no degree requirements. Participants will come from varying backgrounds. Participants will be asked to participate in an in-person semi-structured interview, a focus group, and provide a document for analysis. It should take approximately two total hours to complete the procedures listed. Participation will be anonymous, and no personal, identifying information will be collected.

Would you like to participate? [Yes] Great, [could I get your email address so I can send you the link to the survey? [No] I understand. Thank you for your time. [Conclude the conversation.]

A consent document will be emailed if you meet the study criteria one week before the interview. The consent document contains additional information about my research. The consent document contains additional information about my research. Because participation is confidential, you do not need to sign and return the consent document unless you would prefer to do so. After you have read the consent form, please click the link to complete and return the survey. Doing so will indicate that you have read the consent information and would like to participate in the study.

Thank you for your time. Do you have any questions?

Appendix E

Recruitment Letter

Dear Potential Participant,

As a doctoral candidate in the School of Education at Liberty University, I am researching to better understand changes in mathematics instruction and planning following the implementing of critical thinking opportunities such as Liljedahl's Thinking Tasks, noncurricular tasks, or any other task that makes your students think. The purpose of my research is to examine the effects of implementing critical thinking tasks on instructional and curriculum planning, and I am writing to invite you to join my study.

Participants must be 18 years or older and have at least one year of experience in the field. Participants must implement the usage of critical thinking tasks. There are no degree requirements. Participants will come from varying backgrounds. Participants will be asked to participate in an in-person semi-structured interview, a focus group, and provide a document for analysis. It should take approximately two total hours to complete the procedures listed. Participation will be anonymous, and no personal, identifying information will be collected.

To participate, click here to complete the screening survey. If you meet my participant criteria, I will work with you to schedule a time for an interview.

A consent document will be emailed if you meet the study criteria one week before the interview. The consent document contains additional information about my research.

Because participation is confidential, you do not need to sign and return the consent document unless you would prefer to do so. After you have read the consent form, please click the link to complete and return the survey. Doing so will indicate that you have read the consent information and would like to participate in the study.

Sincerely,

Glen E. Miller
Liberty University Ph.D. Candidate
[redacted]

Appendix F

Recruitment Follow-Up Letter

Dear Potential Participant,

As a doctoral candidate in the School of Education at Liberty University, I am conducting research to better understand changes in mathematics instruction and planning following the implementation of critical thinking opportunities such as Liljedahl's Thinking Tasks, noncurricular tasks, or any other task that makes your students think. Last week an email was sent to you inviting you to participate in a research study. This follow-up email is being sent to remind you to complete the survey if you would like to participate and have not already done so. The deadline for participation is May 18th, 2024.

Participants must be 18 years or older and have at least one year of experience in the field. Participants must implement the usage of critical thinking tasks. There are no degree requirements. Participants will come from varying backgrounds. Participants will be asked to participate in an in-person semi-structured interview, a focus group, and provide a document for analysis. It should take approximately two total hours to complete the procedures listed. Participation will be anonymous, and no personal, identifying information will be collected.

To participate, click here to complete the screening survey. If you meet my participant criteria, I will work with you to schedule a time for an interview.

A consent document will be emailed if you meet the study criteria one week before the interview. The consent document contains additional information about my research.

Because participation is confidential, you do not need to sign and return the consent document unless you would prefer to do so. After you have read the consent form, please click the link to complete and return the survey. Doing so will indicate that you have read the consent information and would like to participate in the study.

Sincerely,

Glen E. Miller
Liberty University Ph.D. Candidate
[redacted]

Appendix G

Consent

Title of the Project: An Examination of the Impact of Critical Thinking Tasks on Mathematics Instruction: A Qualitative Study
Principal Investigator: Glen Miller, Doctoral Candidate, School of Education, Liberty University

Invitation to be Part of a Research Study

You are invited to participate in a research study. To participate, you must have at least one year of experience in the field of education and teach using critical thinking tasks. Participants must be at least 18 years of age or older. There are no degree requirements. Taking part in this research project is voluntary.
Please read this entire form and ask questions before deciding whether to participate in this research.

What is the study about and why is it being done?

The purpose of the study is to examine the underrepresentation of critical thinking opportunities to understand how using critical thinking tasks can impact mathematics instruction.

What will happen if you take part in this study?

If you agree to be in this study, I will ask you to do the following:

1. Participate in an in-person, semi-structured, audio-recorded interview that will take no more than 1 hour.
2. Participate in an in-person focus group that will take no longer than 1 hour.
3. Submit a task that requires students to think critically.

How could you or others benefit from this study?

Participants should not expect a direct benefit from participating in this study.
Benefits to society include research surrounding thinking tasks on an elementary mathematics level.

What risks might you experience from being in this study?

The expected risks from participating in this study are minimal, which means they are equal to the risks you would encounter in everyday life.
I am a mandatory reporter. During this study, if I receive information about child abuse, child neglect, elder abuse, or intent to harm self or others, I will be required to report it to the appropriate authorities.

How will personal information be protected?

The records of this study will be kept private. Published reports will not include any information that will make it possible to identify a subject. Research records will be stored securely, and only the researcher will have access to the records.

- Participant responses will be kept confidential by replacing names with pseudonyms.
- Interviews will be conducted in a location where others will not easily overhear the conversation. Data collected from you may be [used in future research studies] [and/or] [shared with other researchers]. If data collected from you is reused or

shared, any information that could identify you, if applicable, will be removed beforehand

- Data will be stored on a password-locked computer in a locked cabinet. After five years, all electronic records will be deleted, and all hardcopy records will be shredded.
- Recordings will be stored on a password-locked computer for five years until participants have reviewed and confirmed the accuracy of the transcripts and then deleted. The researcher and his doctoral committee members will have access to these recordings.

Is study participation voluntary?

Participation in this study is voluntary. Participation will not affect your current or future relations with Liberty University or Stafford County Schools. If you decide to participate, you are free not to answer any questions or withdraw at any time.

What should you do if you decide to withdraw from the study?

If you choose to withdraw from the study, please contact the researcher at the email address/phone number included in the next paragraph. Should you choose to withdraw, data collected from you will be destroyed immediately and will not be included in this study.

Whom do you contact if you have questions or concerns about the study?

The researcher conducting this study is Glen Miller. You may ask any questions you have now. If you have questions later, you are encouraged to contact him at [redacted]. You may also contact the researcher's faculty sponsor, Dr. Heather Strafaccia, at [redacted].

Whom do you contact if you have questions about your rights as a research participant?

If you have any questions or concerns regarding this study and want to talk to someone other than the researcher, you are encouraged to contact the IRB. Our physical address is Institutional Review Board, 1971 University Blvd., Green Hall Ste. 2845, Lynchburg, VA, 24515; our phone number is 434-951-5530, and our email is irb@liberty.edu.

Disclaimer: The Institutional Review Board (IRB) is tasked with ensuring that human subjects research will be conducted in an ethical manner as defined and required by federal regulations. The topics covered and viewpoints expressed or alluded to by student and faculty researchers are those of the researchers and do not necessarily reflect the official policies or positions of Liberty University.

Your Consent

By signing this document, you are agreeing to be in this study. Make sure you understand what the study is about before you sign. You will be given a copy of this document for your records. The researcher will keep a copy with the study records. If you have any questions about the study after you sign this document, you can contact the study team using the information provided above.

I have read and understood the above information. I have asked questions and have received answers. I consent to participate in the study.

The researcher has my permission to audio-record me as part of my participation in this study.

_____
Printed Subject Name

_____
Signature & Date

Appendix H

Interview Questions

Individual Interview Questions

1. Please describe your educational background and career through your current position. CRQ
2. In what ways do you develop individualized lesson planning to encourage critical thinking? SQ1
3. How do you teach students to recognize problems for critical thinking implementation? SQ1
4. Describe the opportunities you have had creatively developing lesson plans that create opportunities for critical thinking. SQ1
5. How have you used planning to create situations that require students to develop models to answer a problem? SQ2
6. How do you encourage students to identify a problem that requires a visual interpretation? SQ2
7. What methods do you use to create problem-solving plans that require student think time? SQ2
8. What classroom teaching techniques do you incorporate to define problems? SQ3
9. When you are actively teaching, how do you encourage student decision-making to solve a problem? SQ3
10. Discuss how you support students in developing reasoning skills to solve problems in your curriculum. SQ3
11. How do you relate classroom experiences to real-world settings? SQ4
12. What hands-on opportunities do you provide students to take risks when solving problems? SQ4
13. What instructional techniques do you use to create a product to get things done? SQ4

14. What else would you like to add to our conversation today that we have not yet discussed? CRQ

Appendix I

Focus Group Questions

Focus Group Questions

1. Tell us about your experiences teaching using critical thinking tasks from your first position to your current position. CRQ
2. What is the most valuable resource you use to help students recognize problems? SQ1
3. What is the most challenging part about including imaginative problem-solving in the classroom? SQ1
4. What is the best model you have used to encourage critical thinking in the classroom? SQ2
5. What are the most valuable strategies to teach patience in problem-solving? SQ2
6. Discuss the challenges associated with logical problem-solving skills in a critical thinking task. SQ3
7. What are the best methods to encourage reasoning skill development? SQ3
8. What are the best strategies to encourage all students to participate in initiating a task? SQ4
9. What are the most challenging aspects of supporting adaptability for problem-solving among students? SQ4
10. What else would you like to add to our discussion about critical thinking in the classroom? CRQ