VIRTUAL VERSUS TANGIBLE MATH MANIPULATIVES: A QUASI-EXPERIMENTAL STUDY COMPARING THE IMPACT OF DIFFERENT TYPES ON MATHEMATICAL UNDERSTANDING

by

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Liberty University

A Dissertation Presented in Partial Fulfillment

Of the Requirements for the Degree

Doctor of Philosophy in Curriculum and Instruction

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ABSTRACT

The purpose of this quantitative, quasi-experimental nonequivalent control group design study was to measure the effects of both types of manipulatives on student mathematical understanding of 1st and 4th grade, Title I students. This data is needed for teachers to make informed decisions regarding their instructional choices. 270 participants were separated into three groups based on which type of manipulative their classroom teacher used during instruction: physical, virtual, or both. After 10 weeks of instruction with these manipulatives, student achievement was calculated for each student utilizing the Universal Screeners for Number Sense, and the groups within each grade level band were compared using an analysis of covariance test while controlling for pre-test scores. Both grade levels resulted in statistically significant differences between treatment groups. Physical manipulatives were found to be most impactful for both grade levels, followed by mixed manipulatives, with students using virtual manipulative performing at the lowest levels in both grade levels. It is recommended that future studies consider researching the impact of physical manipulatives first and then virtual manipulatives, repeating this study with a different instrument and/or a longer study period, providing coaching support as part of the study, or researching the impact of an innovative technological tool, such as augmented reality.

Keywords: math manipulatives, student achievement, virtual manipulatives, tangible materials, conceptual understanding

Dedication

Dedicated to my amazing and wonderful little family: to my husband, Hunter, for believing in me and supporting me in this goal, to my daughter, Emma, for inspiring me to show her that women can be nurturing mothers and also chase their educational and occupational dreams, and to my son, Brayden, for keeping me company for countless hours of this process (even though this company mostly consisted of crawling on my keyboard)

Table of Contents

List of Tables

List of Figures

List of Abbreviations

Analysis of covariance (ANCOVA)

Concrete-Representational-Abstract (CFA)

English Language Learners (ELL)

End of Year Universal Screener for Number Sense (EOY USNS)

Illustrative Mathematics (IM)

Institutional Review Board (IRB)

Middle of Year Universal Screener for Number Sense (MOY USNS)

Missouri Learning Standards (MLS)

National Defense Education Act (NDEA)

Open educational resource (OER)

Science, Technology, Engineering, and Mathematics (STEM)

CHAPTER ONE: INTRODUCTION

Overview

The purpose of this quantitative, quasi-experimental study was to determine if there is a difference in mathematical understanding between students who learned using physical manipulatives, virtual manipulatives, or a combination of both. Chapter 1 provides a background for the topic of mathematics education across the United States and math manipulatives, including an overview of the theoretical framework for this study. The problem statement examines the scope of the recent literature on this topic. The purpose of this study is followed by the significance of the current study. Finally, the research questions are introduced, and definitions pertinent to this study are provided.

Background

Today's world is full of rapid technological advancements that are increasingly used in classrooms (Alam, 2021), which can overshadow the use of basic tools (Demarcsek et al., 2021) such as mathematics manipulatives. Mathematics consists of numbers and symbols that often require very abstract thinking, and many students are not hesitant to share that it does not make sense to them or that they are not a "math person." Jo Boaler's (2019) research on math and the brain has made it clear that there is no such thing as a "math person" and that all people are capable of learning math. Math manipulatives provide students with a sensorimotor experience that allows them to explore mathematical concepts and ideas less abstractly (Nikiforidou, 2019). Providing manipulatives to students as they learn aligns with the basic tenets of developmental psychology (Donovan & Alibali, 2021). Virtual manipulatives have grown in popularity since the COVID-19 pandemic as teachers discovered ways to substitute and supplement instruction using them (Livy et al., 2022). However, more research is needed to determine if they can be as impactful as physical manipulatives when it comes to transferring skill knowledge and increasing student achievement (Day & Hurrell, 2017). Teachers need this information to make quality decisions regarding their instruction.

In recent years, several studies have found virtual manipulatives to actually be more effective in teaching certain skill areas (Flevares et al., 2021; Gecu-Parmaksiz & Delialioglu, 2019). In some computer programs, students were unable to practice incorrectly (Flevares et al., 2021) which allowed for more accurate practice that helped the students self-correct when they were not working with the teacher. In other cases, students were exposed to more variations of the same concept (Gecu-Parmaksiz & Delialioglu, 2019) such as geometrical shapes beyond their traditional, regular representations. Other studies have found that students performed better when using physical manipulatives (Nikiforidou, 2019). Each of these described studies focused on a particular mathematical skill or domain with students of a specific age or grade level, narrowing the generalizability of their findings. Existing studies on the use of virtual and tangible manipulatives have brought mixed results (Flevares et al., 2021; Gecu-Parmaksiz & Delialioglu, 2019; Nikiforidou, 2019), and these inconsistencies across studies have made it even more difficult to generalize the findings to guide decision-making in classrooms (Day & Hurrell, 2017).

Historical Overview

Education was not addressed in the Constitution, so many have argued that it should be left up to the states (Smith, 2020). This was true of the American education system for many years until the nation became fearful of losing the Cold War after the Soviets beat the rest of the world to launching Sputnik I during the International Geophysical Year (Cross, 2019; Smith, 2020; Snyder, 2018). This fear quickly turned to blame as Americans experienced a blow to their pride and questioned why their country was not producing top-level engineers and scientists as well (Smith, 2020; Snyder, 2018; Wissehr et al., 2011). Education in the STEM disciplines became a matter of national security (Cross, 2019; Smith, 2020), and the National Defense Education Act (NDEA) of 1958 was signed by President Eisenhower to address this. This act provided funding for schools to support instruction in STEM-related subjects, marking the beginning of a new educational era focused on mathematics and federal policies (Smith, 2020; Wissehr et al., 2011). New standards were developed in the STEM domains, and professional development associations were formed to grow teachers in their STEM instruction (Snyder, 2018). Science summer camps began popping up, and new, rich courses were developed in mathematics. This was one of the most radical shifts that science and mathematics education in the United States have ever seen (Wissehr et al., 2011). Yet, equitable mathematics education is still a crucial concern in the United States today (Honey et al., 2020). Research shows that manipulatives can help students make sense of mathematics (Gecu-Parmaksiz & Delialioglu, 2019; Liggett, 2017; Nikiforidou, 2019) and perhaps they are the answer to these continued achievement gaps. Educators must focus on helping students make sense of mathematics to address this issue across our nation (Dempster, 2022). To do this, educators must first understand the way manipulatives are currently used and why so that they can make informed decisions (Reiten, 2020).

Society-at-Large

Mathematics education has been a focus of the nation for almost 70 years (Graumann, 2019; Smith, 2020; Snyder, 2018). Despite these efforts and the money that has been dedicated to these fields, there is a critical lack of women and minorities in the STEM-related workforce (Honey et al., 2020) indicating that a lack of equitable mathematics education still exists.

Existing studies demonstrate a need to increase the effectiveness of Title I teachers (Rivera Rodas, 2019), especially in the areas of mathematics and science where teacher turnover is 70% higher than in non-Title I buildings (Carver-Thomas & Darling-Hammond, 2019). Carver-Thomas and Darling-Hammond described the negative impact that teacher turnover has on student learning. As theorists such as Dewey pushed for student-centered education, focusing on student interests, mathematics was not seen as an interest and began being taught in a strict, algorithmic way in many classrooms (Raymond, 2018). According to Honey et al. (2020), the demand for humans who can think critically and solve problems logically will continue to grow in the future. While technology continues to advance and become increasingly capable, it is not yet able to exercise logic and probability or make judgement-based decisions.

Additionally, Raymond (2018) argued that mathematics education has been overly focused on its role in society as technology continues to develop. This emphasis on teaching math for the purpose of furthering the economy and technology has affected the way it is taught and limits its uses and application. Raymond (2018) discussed the importance of mathematics in all fields, regardless of future profession. For example, even in the field of civics, one must understand basic statistics in order to analyze data on important social issues, such as crime rates. The methods and curriculum that have been employed over these last 70 years are not meeting expectations of student achievement in math. Instead of continuing with the status quo of mathematics education, educators must evaluate current curricula and practices to improve them and catalyze change in this increasingly important discipline (Hanushek et al., 2019).

Theoretical Background

Piaget's theory of cognitive development places students into four levels of learning. These developmental stages, sensorimotor, preoperational, concrete operational, and formal operational, are often used to align learning activities with age-appropriate abilities. Piaget's theories are founded on an activity-based system of education (Norton et al., 2018) that is often referred to as constructivism. Students at the elementary level cannot cognitively make sense of abstract ideas by simply watching their teacher do something. They must be able to move and touch things themselves to experience hands-on learning. These concrete learning experiences help students to construct their own understanding. Despite criticisms of Piaget's work, his theory of cognitive development provides a solid starting point for educators (Saxena et al., 2019).

Piaget's theory and corresponding constructivist work is very closely tied to the idea of conceptual understanding in mathematics. To be successful, students must develop these understandings at the elementary level before moving on to higher-level math. Children learn through physical experiences (Norton et al., 2018), and they must be exposed to hands-on learning experiences because reasoning through verbal statements alone is too abstract for elementary-age students (Saxena et al., 2019). These experiences must take place before students can imagine the same actions and begin to carry out thinking processes internally. The use of materials and manipulatives can help students make sense of abstract concepts (Norton et al., 2018). For example, students might explore the concept of fair sharing a whole by physically cutting up a shape to share with classmates. This can help them explore the idea tangibly instead of trying to picture splitting a shape and moving its pieces around in their head. When students have repeated exposures to an activity or a material, they begin to internalize the concept (Piaget, 1964), allowing them to carry out those same actions in their imagination, such as later successfully being able to picture how to split the shape into equal parts for classmates without needing to physically cut it. Piaget (1964) stated that these concepts can become interiorized over time, and students no longer need to even imagine running through the actions or activities. Students can work with the idea of split pieces, or fractions, without first needing to envision a shape cut up in their head.

Ultimately, students learn from having physical experiences and, over time, being able to re-imagine those physical experiences. Research has found that students simply seeing tools that were used during these learning experiences can help them locate prior learning in their memory (Norton et al., 2018). Once students no longer need the physical tools, they are ready to make sense of the skill in an algorithmic way. Saxena et al. (2019) studied the impact of concrete materials on student learning about coding and found positive results, despite coding being technological by nature.

The dynamic skill theory is also based on the idea that learning should involve concrete materials and active experiences. Built upon the work of Piaget, this framework recognizes the importance of using developmental ranges as a resource instead of keeping that information in the background (Fischer & Yan, 2002). Fischer and Yan described a skill as a concept that is context-based and task-specific. Learners increase their skill level as their understanding transforms over time and through maturation. Existing skills build upon each other to develop new skills. For example, once a child knows how to walk and knows how to play with a doll, they can combine these skills by having the doll "walk" (Fischer & Yan, 2002). These skills continue to compound into more abstract ideas as students grow older and become capable of higher levels of reasoning. Fischer and Yan (2002) identified transformations that occur through different tiers, which are similar to Piaget's stages. The dynamic aspect of the theory believes that capability is not fixed, and that people have different abilities in different situations. These varying abilities are dependent on the levels of support provided, their emotions at the time, and the specific task. At its core, dynamic skill theory is another constructivist theory that demands active learning experiences for students as they develop their own understandings (Fischer & Yan, 2002).

Problem Statement

Math manipulatives allow students to make sense of mathematics concepts and form their own understandings (Lewis & Colonnese, 2021). Through the constructivist lens of Piaget's theory of cognitive development and the dynamic skill theory, educators facilitate these opportunities and assist students through disequilibrium as they assimilate and accommodate new learning (Piaget, 1964, 1967). Utilizing math manipulatives is developmentally appropriate for students (Donovan & Alibali, 2021) as students internalize concrete representations of abstract mathematical concepts (Day & Hurrell, 2017). While students should develop both procedural and conceptual understandings of math (Özpınar & Arslan, 2022), struggling learners must be scaffolded in their development of mathematical concepts (Burns et al., 2015) with manipulatives. As many of today's teachers utilize virtual versions of math manipulatives, Day and Hurrell (2017) questioned whether these virtual manipulatives provide the same level of conceptual exploration as tangible ones do to the fragile learners that need these scaffolds. Recent studies have investigated the benefits of both types of manipulatives (Flevares et al., 2021; Gecu-Parmaksiz & Delialioglu, 2019; Nikiforidou, 2019), but the extent to which both types of manipulatives specifically support conceptual understandings based on empirical evidence remains unclear. Conceptual understanding is crucial for struggling learners (Burns et al., 2015; Karakonstantaki et al., 2018; Kerschen et al., 2018) and manipulatives play a significant role in developing this type of understanding (Day & Hurrell, 2017; Liggett, 2017; Piaget, 1964, 1967), but many teachers have continued to use virtual manipulatives post-pandemic for various reasons. The problem is that there is a gap in the literature displaying a lack of empirical evidence to support the idea that virtual manipulatives build the same level of conceptual understanding in mathematics that tangible manipulatives do to support their use in the classroom for all learners.

Purpose Statement

The purpose of this quantitative, quasi-experimental nonequivalent control group design study was to measure the effects of both types of manipulatives, in isolation and in combination, on the mathematical understanding of 1st and 4th grade, Title I students. The independent variable in this study was the type of manipulative used in instruction, tangible, virtual, or both in combination. Tangible manipulatives are physical objects and models that students can touch and move around to explore mathematical concepts (Hynes, 1986). Virtual manipulatives are defined as interactive, web-based, dynamic visual representations that allow the construction of mathematical knowledge (Moyer et al., 2002). The dependent variable in this study was the level of number sense students possess. Howden (1989) described number sense as "good intuition about numbers and their relationships. It develops gradually as a result of exploring numbers, visualizing them in a variety of contexts, and relating them in ways that are not limited by traditional algorithms" (p. 11). This study included 1st and 4th-grade participants from the classrooms of Title I schools across a public-school district in southwest Missouri.

Significance of the Study

Past studies have been completed assessing student mathematical achievement of students who used physical manipulatives versus digital manipulatives (Flevares et al., 2021; Gecu-Parmaksiz & Delialioglu, 2019; Nikiforidou, 2019). Similarly, this study explored the impact of manipulatives utilizing scores on the Universal Screeners for Number Sense (USNS)

EOY assessment representing student mathematical understanding. However, this study also incorporated a third, unexplored category: students who learned using a combination of physical and virtual manipulatives in purposeful ways. If hands-on learning helps students build conceptual understanding (Saxena et al., 2019), manipulatives should greatly serve educators in this area. Studies have shown benefits of virtual manipulatives but not necessarily isolated to conceptual understanding gaps (Flevares et al., 2021). This study sought to determine whether these virtual manipulatives, used alone or in combination with physical manipulatives can build those missing foundations as effectively as traditional, hands-on learning while also providing empirical evidence to support the conclusions.

Unlike other studies (Flevares et al., 2021; Gecu-Parmaksiz & Delialioglu, 2019; Nikiforidou, 2019) this study did not focus on one particular grade level. The participants included students in two grades that represented two different stages of Piaget's theory of cognitive development. This is important because not only will it allow the local school district to make instructional decisions using the implications of the results of this study, but it will allow the results to inform future instructional decisions about which types of manipulatives to use with struggling students of various age levels and cognitive developmental stages.

Research Question

RQ1: Is there a difference in number sense between 1st- and 4th-grade Title I students who use physical manipulatives in isolation, students who use virtual manipulatives in isolation, and students who use a combination of virtual and physical manipulatives when controlling for pre-test scores?

Definitions

1. *Conceptual Understanding*- Conceptual understanding is a deeper level understanding of ideas that goes beyond rote memorization and recall and can be applied in next contexts (Dempster, 2022).
2. *Concrete Operational*- Piaget's theory of cognitive development places students between seven and eleven or twelve in this category. This is the beginning of logical development. Students can use objects to draw conclusions and make generalizations, but they cannot yet make meaning of hypotheses that are only verbally expressed (Piaget, 1964, 1967).
3. *Number Sense-* Number sense is "good intuition about numbers and their relationships. It develops gradually as a result of exploring numbers, visualizing them in a variety of contexts, and relating them in ways that are not limited by traditional algorithms" (Howden, 1989, p. 11).
4. *Preoperational*- Piaget's theory of cognitive development places students between the ages of two and seven or eight in this category. This is the beginning of language development and symbolic play and representations, still requiring tangible experiences (Piaget, 1964, 1967).
5. *Tangible Manipulatives*- Tangible manipulatives are physical objects and models that students can touch and move around to explore mathematical concepts (Hynes, 1986).
6. *Virtual Manipulatives*- Virtual manipulatives are interactive, web-based, dynamic visual representations that allow the construction of mathematical knowledge (Moyer et al., 2002).

CHAPTER TWO: LITERATURE REVIEW

Overview

The purpose of this literature review is to present the essential elements of manipulative implementation, to distinguish the types of mathematical understanding students develop and the role that manipulatives play in these processes, and to review prior research on the use of virtual manipulatives. The chapter opens with the theoretical framework. This study is grounded first in Piaget's (1964, 1967) theory of cognitive development which proposes discrete developmental stages that impact thinking and understanding as children progress through each stage. In addition, Fischer and Yan's (2002) dynamic skill theory is also foundational to this research study and is described. Lastly, a thorough review of the literature pertinent to mathematics understandings and manipulatives, both tangible and virtual completes the chapter, which ends with a summary.

Theoretical Framework

Constructivism is a learning theory that states that learners construct their own knowledge through hands-on experiences and actions with their environment (Harlow et al., 2006). While there are many theorists in the constructivism category, the two theories that will drive this study are Piaget's (1964, 1967) theory of cognitive development and Fischer and Yan's (2002) dynamic skill theory. Piaget's (1964, 1967) discrete stages of cognitive development provide context for student capabilities at different ages and stages. Fischer and Yan (2002) build upon the work of Piaget while also pointing out the importance of this developmental context.

Piaget's Theory of Cognitive Development

Piaget (1964, 1967) introduced the construct of disequilibrium and its crucial role in learning. When students encounter new information through exploring an object or an idea, they seek to assimilate this information into their existing schema. When they are unable to assimilate this new information, it creates cognitive disequilibrium. Mental disequilibrium is an uncomfortable feeling that most people try to resolve. This disequilibrium invokes a process where students consider and develop new schemas to accommodate the new learning and information. The new schemas then allow the information to be properly assimilated, thus re-achieving equilibrium and a level of comfort. The process is cyclical in nature and is how humans continue to learn and grow throughout their entire lives (Harlow et al., 2006; Piaget, 1967).

Lovatt and Hedges (2015) argued that this idea of working theory in children's knowledge construction can not only further develop understandings but also intrinsically motivate students to learn. They acknowledge that some modern theorists have disregarded the work of Piaget but that this description of the way students take on new understandings is still relevant and important. Piaget (1964) stated that children can only absorb information from their education when they have a previously existing structure to assimilate that information within. This requires experiences that allow students to build these structures to draw upon during learning. This cycle is continuous each time students encounter new ideas, and it is how understanding is built (Harlow et al., 2006). Harlow et al. emphasized that popularized constructivism tends to focus on passive assimilation and teachers must intentionally facilitate experiences that cause disequilibrium so that students remain active participants in the learning process. For example, Piaget famously tested the understanding of conservation with an experiment involving water in different-sized containers (Miller & Heldmeyer, 1975). For an older student who is capable of understanding conservation and is learning about area or volume, this exact same experiment could provide students with tangible proof that the same amount of space or volume can exist in different combinations of dimensions, an abstract idea that can be hard to make sense of.

Piaget (1964, 1967) divided students into four discrete categories of cognitive thinking, primarily based on their age. His stages include sensorimotor, pre-operational, concrete operational, and formal operational. These categories describe the proposed cognitive capabilities of each developmental group, primarily based on student age, (Piaget, 1964, 1967) and provide educators with insights into the way children develop understandings (Ojose, 2008). Early elementary students fall into the category he described as pre-operational. Pre-operational students can begin participating in verbal exchanges and therefore internalizing words and thoughts (Piaget, 1964, 1967). Internalized ideas at this age are actions carried out in imagination that no longer require muscular movement from students (Norton et al., 2018). These imagined actions are rooted in past physical experiences. These students seek to answer the question "why?" about almost everything they encounter, and they cannot prove their assertions (Piaget, 1967). Their curiosity about the world around them propels their exploration, and pretend play is a prominent element in their world.

As students move through their elementary education, they transition to the concrete operational stage, which marks the beginning of logic and morals (Piaget, 1967). The name concrete operational was derived from the idea that students of this age can operate logic on concrete, tangible objects, but they cannot yet apply this logic to language and verbal statements alone (Piaget, 1964). At this stage, students can begin to consider multiple dimensions at one time. For example, in scenarios of conservation of water, children can reason with both the height of the water and the width of the container. It is crucial that students in this stage have opportunities to access hands-on, concrete experiences (Asoy et al., 2022; Ojose, 2008). Memorizing isolated facts and procedures makes learning higher-level mathematical concepts increasingly more difficult (Asoy et al., 2022). Students who have learned mathematics through “tricks” such as “just invert and multiply” do not develop a coherent understanding of the connections and progressions that build on the conceptual foundations laid in elementary mathematics. For students to properly apply their learning in new mathematical contexts, they must truly understand it in the first place and develop their procedural understandings from there (Miles & Gojak, 2015).

According to Piaget (1967), it is not until the age of 11 or 12 that a student shifts from concrete operational to formal operational thinking. It is at this stage that students become capable of formal, abstract reasoning. Piaget (1967) also specified that these ages are an average and that they can be impacted by environment and experiences. Some countries were found to have different ages, on average (Piaget, 1964), further demonstrating that underlying factors can impact the development of logic and thinking skills. Piaget (1967) claimed that, prior to this age, students need to have access to objects to manipulate to facilitate reasoning with concepts. Reasoning through verbal statements alone is an abstract notion that is detached from reality and as such is not developmentally appropriate for most students in elementary school, especially for struggling learners who have not been exposed to rich educational environments and experiences throughout their lives (Piaget, 1967; Saxena et al., 2019). Students can better make sense of verbal statements when they are paired with physical experiences (Piaget, 1967). This combination of verbal statements and hands-on learning leads to students being able to internalize and interiorize their understandings.

Children learn through physical experiences and, eventually, through imagined physical experiences that they can visualize in their mind. For students to visualize and imagine physical experiences, they must first have lived them and experimented with them (Norton et al., 2018). The implementation of manipulatives in mathematics instruction seeks to create active experiences that allow students to make meaning and provide students with a context to imagine physical experiences in the future. Piaget (1964, 1967) uses the term manipulate in several of his works to specifically describe students having the opportunity to explore tangible objects. He stated that action and social interaction build logic (Piaget, 1967). Constructivism, specifically Piaget's constructivism, directly relates to the purpose behind mathematics manipulatives and the way that students develop conceptual understandings of abstract ideas, or operations (Piaget, 1964, 1967).

Based on their developmental stage, most elementary students do not yet possess the ability to logically reason about mathematical ideas in the context of language and verbal statements alone. The average age at which students become capable of these formal operations falls into grade level bands that are typically housed in middle school environments. In the United States public education system, most students do not turn 11 or 12, reaching formal operations, until around 6th grade. However, elementary students of a younger age can prove capable of logically reasoning through scenarios when they can physically manipulate objects in their learning (Piaget, 1967). Piaget (1964) described an anecdote of a friend of his who had pebbles as a child, and they were fascinated to discover through the movement of the pebbles that the order of the pebbles and the shape the pebbles were moved into did not affect the total number of pebbles. While his friend previously assumed that the new shape had a different number of pebbles, not recognizing the conservation of the amount, he was able to move them with his hands and discover on his own that it did not. That generalization was then able to be applied to other mathematical concepts. As exemplified in this scenario, the constructivism theory drives the development of effective manipulative practices and creates a strong argument for their urgency in the field of mathematics.

Dynamic Skill Theory

Dynamic skill theory, coined by Fischer and Yan (2002) is founded on the idea of concrete and active learning experiences. As students engage with skills and tasks, they build their knowledge base. Fischer and Yan (2002) also identified developmental ranges, or tiers, for students that teachers should consider as they determine their students' capabilities during instruction and learning activities. However, they also noted that capabilities are dynamic, and that people can demonstrate different abilities in different contexts, depending on what kind of support they are provided, their emotions, and what the task itself demands (Fischer & Yan, 2002).

Summary

These constructivist theories (Fischer & Yan, 2002; Piaget, 1964, 1967), amongst others, have led mathematics instruction to incorporate hands-on learning materials, or manipulatives, for students to develop their own understandings on the subject (Liggett, 2017). An abundance of existing literature has sought to identify the impact that math manipulatives and hands-on learning have on mathematics instruction and achievement (Flevares et al., 2021; Gecu-Parmaksiz & Delialioglu, 2019; Lewis & Colonnese, 2021; Nikiforidou, 2019). However, there is still debate as to whether virtual math manipulatives align with constructivist best practices as students are not truly manipulating a physical object with their hands (Day & Hurrell, 2017).

This study addresses technology's role in a constructivist and conceptual-focused mathematics education. This will guide constructivist educators in making decisions about what types of math manipulatives are appropriate to employ and help advance the field of constructivism by establishing whether virtual manipulatives benefit students in the same way as traditional, hands-on materials.

Related Literature

In reviewing the current, scholarly literature that exists concerning mathematics manipulatives, both physical and virtual, a few main themes emerged. The first theme can be defined as manipulative implementation and significance of their usage. Current best practices with manipulatives in the classroom are examined. The effectiveness and significance of manipulatives are analyzed through the context of recent studies. Barriers to manipulative implementation in mathematics are also highlighted in this section. The second theme is focused specifically on virtual manipulatives, which are a popular debate in the world of modern-day math education. Literature on virtual manipulatives will be analyzed for the strengths and weaknesses of this tool. Lastly, existing literature on the mathematical achievement of Title I students will be presented.

Manipulative Implementation

Math manipulative is a term used by educators to encompass any object that can be used to improve mathematics skills (Liggett, 2017). Both Piaget (1964, 1967) and Fischer and Yan (2002) advocated for hands-on learning, especially for elementary-age students in developmental stages making this type of learning necessary. The implementation of manipulatives in a math classroom allows teachers to teach concepts in a constructivist manner. Manipulatives help students move from concrete understandings to pictorial understandings and, eventually, to abstract understandings (Day & Hurrell, 2017), which is the ultimate goal. While there are pedagogical methods to effectively use perceptually rich manipulatives, such as analyzing manipulatives using a task analysis framework (Reiten, 2021), Carbonneau et al. (2020) found that students who used basic, bland manipulatives performed significantly better on a problem-solving task than students who used no manipulatives or perceptually rich manipulatives. These students demonstrated stronger perseverance on the problem-solving task as well. As students see and touch objects and act upon them, they can gain a deeper understanding of mathematical ideas.

Many teachers have also described how the implementation of manipulatives in their classroom increased levels of engagement during math (Şeker, 2020). These tools externally represent what will eventually be internalized by students (Day & Hurrell, 2017). Kesicioglu (2021) interviewed ten preschool teachers on their use of math manipulatives. These teachers revealed several struggles that they experienced implementing these tools, but they also recognized the need for incorporating concrete activities for students to explore concepts with. Quigley (2021) also found that most teachers they interviewed saw manipulatives as an "essential" in mathematics instruction. This was primarily because they saw the importance of helping students develop understanding as they explore different mathematical concepts.

Numerous studies through the years have demonstrated the positive impact that manipulative usage can have on student learning (Day & Hurrell, 2017; Gecu-Parmaksiz & Delialioglu, 2019; Gulkilik et al., 2020; Lewis & Colonnese, 2021; Liggett, 2017; Nikiforidou, 2019). Multiple representations can help students strengthen connections and understandings (Gulkilik et al., 2020; McCullough et al., 2018). Even for high school students taking courses such as physics, manipulatives have been found to improve student learning with statistical significance (Oymak & Ogan-Bekiroglu, 2021). Perhaps one of the most glaring examples of the significance of manipulatives occurred during the COVID-19 pandemic during which teachers were unable to utilize hands-on learning materials with their students. Even once schools reopened, there were many constraints on how close teachers and students could be and what materials they could use that might be touched by more than one person. Since the pandemic, teachers have described increased difficulties with supporting students in math, and manipulative usage is one reason (Barrow et al., 2021).

Current Practices

In today's classrooms, educators use manipulatives in many ways. The common denominator is that they are used to further student understanding of math. Some manipulatives are used to model a mathematical problem-solving scenario visually, such as sticking toothpicks in playdough balls to represent the legs on bugs while trying to determine how many total legs a particular size group has (Quinnell, 2018). Before memorizing how to multiply the number of bugs by the number of legs on each bug, students can visually see why groups can be formed in that way. Methods such as this can help students make sense of a situation and understand the mathematical concepts behind the operation (Lewis & Colonnese, 2021; Liggett, 2017). This hands-on exploration aligns with the developmental stages of elementary students that Piaget (1964, 1967) proposed. While Fischer and Yan (2002) did not identify specific stages, they did emphasize the importance of considering developmental stages to select appropriate learning activities. Teachers must consider the age and stage of their students when determining how best to utilize learning tools such as manipulatives.

Other manipulatives are used to assist with computations. For example, students might put counters into piles of four to help them skip count or multiply (Quinnell, 2018). This practice can scaffold learning for students and can help students to make sense of the process of multiplication while they simultaneously work to memorize their facts. Some teachers even use manipulatives to help students explore above-grade-level skills (Lewis & Colonnese, 2021). Quigley (2021) stated that teachers with a pedagogical belief in constructivism are more likely to implement manipulatives in their classrooms.

Barriers to Implementation

Some teachers have identified barriers to math manipulative implementation (Kesicioglu, 2021) which can prevent educators from using manipulatives in effective ways. More than 30% of teachers in Wilkie and Roche's (2022) study on fraction manipulative preferences indicated that their preference was to not use a model or manipulative at all. If teachers are unsure of how to relate manipulatives to a given concept, they may be less likely to use them. Despite knowing the importance of hands-on learning experiences, given the developmental stages and ages of their students, Kesicioglu (2021) found that preschool teachers used counters and manipulatives often to practice one-to-one correspondence but struggled to practice skills that seemed more abstract with manipulatives.

Additionally, not all teachers have access to ready-made manipulatives and are left trying to find inexpensive supplies or household objects that can be utilized as manipulatives. What makes this even more of a barrier is that there is concern that using household objects as manipulatives can activate unwanted prior knowledge from other experiences with the tool (Donovan & Alibali, 2021) which makes this method of implementation even more difficult. Several teachers indicated a lack of storage space in their classroom for manipulatives (Sanderson, 2021). Using manipulatives effectively also requires strong classroom management and reasonable class sizes. While teachers can be trained and coached to improve their classroom management (Russell et al., 2020), most teachers, and even building principals, do not have control over class size.

Regardless of management methods, it can be hard for teachers to keep students focused on the mathematical properties and exploration abilities of manipulatives when the group is large and classroom routines and procedures have not already been well-established (Kesicioglu, 2021). One teacher described a lesson where students distracted each other by using unifix cubes to build swords and several others mentioned students playing with the manipulatives as toys or getting too loud (Sanderson, 2021). The use of manipulatives also sometimes requires more class time for set-up and clean-up which adds an additional constraint. Interestingly, none of the 18 teachers interviewed by Sanderson (2021) responded that lack of student interest was a barrier to implementation.

Benefits of Mathematics Manipulatives

Manipulatives have been found to have a positive effect on learning (Watt-Douglas & George, 2021). They help teachers see student misconceptions as student thinking becomes visible (Day & Hurrell, 2017), and they can increase student interest and engagement in mathematics (Chiphambo et al., 2020; Fakih, 2023; Roberts et al., 2020). Students at Montessori schools, which are well-known for using manipulatives and hands-on materials in learning, were shown to have sustained, increased achievement in mathematics at the primary level (Basargekar & Lillard, 2021). Pires et al. (2019) also found that students using tangible manipulatives showed more growth on a mathematical assessment than a control group using traditional instruction did. Math manipulatives build students' knowledge base of experiences (Delport, 2021), allowing them to make deeper and richer connections. Providing manipulatives aligns with the principles of developmental and educational psychology (Donovan & Alibali, 2021). This matches the recommendations of both Piaget (1967) and Fischer and Yan (2002). It also provides students with a sense of autonomy, placing them in control of their own learning (Quigley, 2021), and it provides multimodal learning opportunities and differentiation for diverse learners in a classroom (Tjandra, 2023).

Lewis and Colonnese (2021) found that students struggled with realistic, three-act math tasks because they had difficulty thinking creatively. Because of past experiences, their students wanted to be handed a problem to solve and were not easily able to pose problems of their own. Students who have not been asked to examine real-life mathematical scenarios and cannot problem pose often struggle to determine strategies to solve these problems as well as consider how to solve real-world problems that they might encounter using mathematical concepts and strategies. When the researchers implemented manipulatives within these three-act tasks, students were able to better visualize the scenario and find the mathematics involved. Students who had previously struggled to engage with the task could visualize the scenario better with physical tools in front of them, and so the depth of exploration that ensued during these tasks increased greatly.

Use of the manipulative itself does not guarantee that learning will happen, but, when used correctly, it is a powerful tool to facilitate learning (Day & Hurrell, 2017; Hurst & Linsell, 2020). Teachers must help students make the connection between the learning tool and the mathematical concept being studied (Johnson et al., 2021; Ojose, 2008) through explicit explanation during instruction (Asoy et al., 2022). Because of this, Donovan and Alibali (2021) stated that teachers must be intentional with teaching using manipulatives so that students understand the mathematical components. For example, when asked to show what eight looks like using red and blue circles, some students counted out eight circles and some made the counters into the shape of the numeral eight. Some students may not make connections between representations with manipulatives without a teacher probing them to reflect on their inquiry process (Rich, 2023).

Students that were explicitly introduced to manipulatives as mathematical tools scored better on a post-lesson assessment and demonstrated better knowledge transfer than students who did not receive this explicit introduction were more likely to misconstrue the tools as toys (Donovan & Alibali, 2021). When utilized correctly, manipulatives have been found to be effective at increasing student understanding (Kabel et al., 2021; Roberts et al., 2020). Both physical and virtual manipulatives can increase student understanding (Ma et al., 2018). Additionally, manipulatives can be a powerful tool for teachers to quickly assess conceptual understanding (Hurst & Linsell, 2020).

The use of manipulatives can provide students with a chance to develop their fine motor skills, which have been found to be related to early numerical skills in children (Fischer et al., 2018). This makes sense because much of a child's numeracy foundation is rooted in the use of their fingers. Not only is the relationship connected, but Asakawa et al. (2019) determined that the relationship between fine motor skills and early mathematical abilities is casual. Providing fine motor skill intervention to students also increased their performance on a mathematical task. Math manipulatives can serve a dual purpose in that they can provide practice of fine motor skills while simultaneously allowing students to explore mathematical concepts (Asakawa et al., 2019; Fischer et al., 2018).

All 18 teachers that were surveyed by Sanderson (2021) agreed that manipulatives are important to use when teaching math, but only 83% of them stated that they use manipulatives daily to teach mathematical concepts. One teacher stated that manipulatives help bring meaning to written symbols for students. Likewise, the majority of teachers in Tjandra's (2023) study believed that using manipulatives when they teach math helps students retain and recall their learning as well as develop their problem-solving abilities more deeply. Many teachers use manipulatives as a tool to scaffold learning for struggling students and differentiate their instruction for unique learning needs in their classroom (McNeill & Polly, 2023). However, Sanderson (2021) argued that there is a need for additional teacher training in the proper use of math manipulatives.

One novice teacher described her difficulties teaching math using manipulatives in an engaging way until she received mentoring support (Gholam, 2018). Reiten (2020) also found that professional development related to mathematics manipulatives supported teachers in their implementation and feeling comfortable doing so. Additional teacher professional development might prove especially useful in the implementation of virtual manipulatives, a newer but popular development, having been shown to take teachers that are initially unfamiliar with virtual manipulatives and develop them to a point where they are using them regularly in instruction (Reiten, 2021). Participation in professional learning can also increase teacher confidence and self-efficacy in management and implementation of manipulatives, which can lead to their use more frequently during instruction. However, a supportive environment for teachers, where manipulatives are provided, encouraged, and taught is just as important (Ottenbreit-Leftwich et al., 2018).

Tactile Engagement

Price et al. (2021) defined tactile engagement as someone experiencing sensations from physically touching an object. Research has clearly shown that humans learn better when they experience multisensory environments, particularly involving touch (Delport, 2021). Many tangible manipulatives support students in developing understanding through tactile engagement (Sanderson, 2021). This type of engagement can make it easier for students to “just try” and take risks as they work to solve mathematical problems. It increases student interest and excitement around the subject of mathematics (Roberts et al., 2020). This type of multisensory experience can positively impact even the student learning and understanding of adult learners at the university level (Delport, 2021). The size of Delport’s (2021) study does prevent its findings from generalizing to all university students, but it still gives us a picture of the power that tactile experiences might have in exploring mathematical concepts, regardless of the level of students being instructed.

Kabel et al. (2021) interviewed a teacher on the limitations of virtual manipulatives, and this teacher discussed the need for virtual manipulative programs to incorporate a tactile element to be used to their full potential. While not all virtual manipulatives provide this type of support, there are simulation-type manipulatives and programs that allow students to physically maneuver objects during their exploration. Additionally, some districts have begun using augmented reality and 3D printers to create versions of manipulatives that effectively combine technology with tactile experience. (Barrett et al., 2018; Price et al., 2021). Several Montessori teachers agreed that they felt virtual reality manipulatives could remain consistent with their hands-on philosophy (Walkington et al., 2021).

Mathematical Understandings

The types of mathematical understandings that students possess can impact their mathematical abilities (Dempster, 2022; Kerschen et al., 2018; Russell et al., 2020). When students are only asked to memorize rote facts and procedures, they are not given the opportunity to think deeply about the concepts being examined (Dempster, 2022). Current standards and assessments in mathematics demand higher levels of thinking from students that can be transferred to real-world problem-solving (Russell et al., 2020). Teacher questioning also plays a large role in student understanding (Dempster, 2022; Russell et al., 2020), and many students are only exposed to low levels of questioning (Dempster, 2022). This finding is consistent in many Title I classrooms as studies have shown the lack of effectiveness of many Title I teachers (Rivera Rodas, 2019**)**. Dempster (2022) found that teachers often had a skewed view of their own level of questioning. When interviewed, they thought they were asking more depth of knowledge (DOK) level four questions than they were when observers counted, and many thought they were asking fewer DOK level one questions than they were. Studies, such as this one, demonstrate a need for third-party observers to help teachers track the types of questioning they naturally employ. Additionally, exposing students to experiences they may not have had prior to schooling can help to build their conceptual number sense (Kerschen et al., 2018).

Teachers should strive to plan high-quality tasks and facilitate rich discussions that help students make sense of concepts (Dempster, 2022; Russell et al., 2020). Since elementary students are generally not capable of making sense of abstract ideas without physical experiences and visuals because of their cognitive capabilities at this age (Piaget, 1964, 1967), it is crucial that teachers help deepen the conceptual understanding of students. Kerschen et al. (2018) provided students with some of these learning components as part of a four-week summer intervention program and found positive results. While growth between the pre- and post-assessments of the program itself was not statistically significant, students were assessed when they returned to school after the summer program, and the beginning-of-year scores of participants were greater than the scores of non-participants with significance. Further, this difference in average scores remained when students were tested again at the end of that school year indicating that the intervention had a lasting effect and proving the worthwhileness of programs such as these.

Coaching can work to elicit this level of teaching as educators are prompted to think and reflect deeply on both the concepts being covered and the pedagogical methods they employ (Russell et al., 2020). Whether the effect of the coaching itself or the implementation of higher-quality tasks led to student growth in Russell et al.'s (2020) study requires future research, but it is clear that support systems that help teachers implement these rich tasks and discussions show immense promise in closing the achievement gap in mathematics (Kerschen et al., 2018) which is especially crucial for Title I students that are often provided with lower-quality instruction (Rivera Rodas, 2019).

Procedural Understanding versus Conceptual Understanding

Throughout history, mathematics instruction has typically consisted of direct instruction from a teacher covering procedures, followed by student practice, often in the form of a worksheet, on problems involving the procedures just covered by the teacher (Yu & Singh, 2018). Common Core State Standards have since called for the additional development of conceptual understandings that must be used to solve real-world problems (Yu & Singh, 2018). Phuong (2020) explained the need for both types of understandings when solving problems as students must be able to consider the concepts involved to strategize a method to solve the problem but also must possess the procedural ability to carry out the mathematics and determine a solution. Yu and Singh (2018) also argued that there is a need for both types of knowledge to be intertwined when learning mathematics.

Yu and Singh (2018) found that developing conceptual understanding in learners had a positive effect on student achievement compared to developing procedural understanding, which had a negative impact on student achievement. This is significant in that not only did the students provided with conceptual understanding perform better than those provided with procedural instruction, but the students who received procedural instruction were negatively impacted. Many Title I students are not provided equitable access to this type of math instruction (Rivera Rodas, 2019). Yu and Singh (2018) concluded that students who are exposed to instruction that involves developing problem-solving, reasoning, and building mathematical connections between concepts are more likely to score higher on mathematical exams. On the contrary, Phuong (2020) found that more students were proficient in procedural fluency than conceptual understanding. Students in this study struggled to integrate the two skills successfully in problem-solving.

Additionally, Yu and Singh's (2018) study confirmed that students of low socioeconomic status (SES) are less likely to have effective teachers who teach math conceptually, further contributing to the achievement gap. Both studies emphasized the need for teachers to focus on teaching math conceptually (Phuong, 2020; Yu & Singh, 2018). Braithwaite and Sprague (2021) found that students with high procedural knowledge are less likely to rely upon conceptual understanding when solving problems. This further demonstrates the importance of providing students with both types of mathematics learning at the elementary age, not just procedural understanding. These claims also align with the works of Piaget (1964, 1967) and Fischer and Yan (2002), who both advocated for students to make sense of concepts and construct their own meanings. All students must have access to conceptual learning if math education is to be equitable.

Conceptual Understanding and Constructivism

Liggett (2017) argued that the goal of any math instruction should be to help students focus on making sense of and understanding concepts. The use of manipulatives helps students to explore mathematical ideas and to construct those understandings themselves (Liggett, 2017). This active approach to learning shifts teachers from a behaviorism-focused pedagogy to a cognitive-focused pedagogy (Suhendi et al., 2021) working toward higher student achievement. The connections students make between concepts allow them to make sense of their learning in a lasting way (Liggett, 2017). This aligns with the recommendations of Yu and Singh (2018) for instruction.

Conceptual Understanding and Struggling Learners

For students in need of intervention, the focus should be on building number sense. Students with number sense understand what numbers truly represent as a symbol, and this understanding becomes increasingly important as students progress through higher levels of mathematics (Kerschen et al., 2018). Number sense skills are inherently conceptual. These skills compound and students who struggle with one skill, such as understanding the magnitude of fractions, often also struggle with other related skills, such as multiplication (Dyson et al., 2020). This number sense development could transfer to the number line with whole-number magnitudes to eventually make sense of higher-level concepts, such as fraction magnitudes (Barbieri et al., 2020).

Milton et al. (2019), Satsangi et al. (2018), and Özdemir and Kılıç (2023) identified the concrete-representational-abstract progression as an effective way to build conceptual mathematical understanding in students with learning disabilities or other struggling students. In this process, students are first exposed to concrete learning experiences with hands-on materials before moving on to pictures or other representational systems that are related to those hands-on experiences. Only when students are confident with these representations are they transitioned to using abstract symbols and verbal reasoning. Park et al. (2020) found that this method has the potential to still work when the traditional, concrete manipulatives are replaced with virtual manipulatives.

Mills (2019) also studied teachers implementing manipulatives and conceptual teaching for the first time with a set of students struggling with multiplication, a skill often taught procedurally. Students demonstrated a greater understanding of multiplication when the concrete-representational-abstract techniques were employed. These methods are developmentally appropriate for the age of the students based on Piaget's (1964, 1967) theory of cognitive development, placing most elementary students in the pre-operational and concrete operational stages. With this alignment, concept-based interventions like these have been shown to increase student understanding even after time has passed (Barbieri et al., 2020), which further demonstrates their effectiveness. Much of teaching using manipulatives is rooted in this progression.

Virtual Manipulatives

Post-pandemic, many educators have turned toward virtual versions of tangible manipulatives and have since continued using them in their classrooms (Livy et al., 2022). At first glance, they might appear to do the same job, but Day and Hurrell (2017) questioned whether virtual manipulatives can really have the same impact as a concrete object or if they would better serve as a pictorial representation for students who have already used physical tools and are ready to transition to the next stage of thinking. Pires et al. (2019) noted that their two experimental groups, one utilizing tangible manipulatives and one utilizing virtual manipulatives, implemented different types of strategies, alluding to the idea that each type of manipulative might lend itself to a different purpose. Similarly, a teacher with concerns about long-term health effects of virtual manipulatives emphasized the importance of using virtual experiences for learning opportunities that could not otherwise occur, as opposed to digital replacements for hands-on manipulatives (Walkington et al., 2021).

Bumbacher et al. (2018) found that students performed better with each manipulative on a different task, indicating that each of the two skills they were assessing was better suited for one of the types of manipulatives than the other. Kang et al. (2020) found that children in their study did not seem to have a preference between tangible and virtual interactions, while Olympiou and Zacharia (2018) noted higher interest and engagement with physical manipulatives. However, their study observed college students, not elementary students. Alternatively, Weng et al., (2020) found that students had higher engagement when working with virtual manipulatives. These mixed results still leave many decisions up to teacher judgement, based on the dynamics of their students.

Physical manipulatives could range anywhere from a collection of buttons (Quinnell, 2018; Sanderson, 2021) to fraction tiles purchased from a distribution company to homemade slime (Levingston et al., 2019). Any physical material that students can touch and use to make sense of a math concept falls into this category. The key is that students are touching a physical object with their own hands. These manipulatives might be able to be used in versatile ways, such as counters that can be used for several activities and purposes with different grade levels, or they might have a specific function, such as fraction tiles, that are limited to a particular domain of mathematics. For teaching a given concept, such as fractions, it is evident that educators have varied preferences and opinions on the best manipulative to employ (Wilkie & Roche, 2022).

Virtual manipulatives are the same as tangible ones except they are digital versions of them that are typically available online. There are free versions available on websites and some purchased learning platforms, such as Imagine Learning Classroom, have them built in (Bouck et al., 2021). There are also virtual manipulatives that are more supplemental in nature, such as augmented reality (Gecu-Parmaksiz & Delialioglu, 2019) or that are app-based (Bouck et al., 2018). Since these manipulatives do not allow students to maneuver materials with their own hands, the question is whether these manipulatives are appropriate and effective for students in the pre-operational and concrete operational stages, where physical experiences are crucial, as described by Piaget (1964). Fischer and Yan also discussed the importance of active learning, and manipulating tools on a computer screen is a different level of activity. It is important that teachers utilize technology in effective ways as opposed to simply teaching alongside the incorporation of technology (Reiten, 2021).

Teachers who selectively choose which type of manipulative to utilize, not randomly, but for a reason, whether that reason is convenience, cost, being easier for students to use, or something else, are using mixed types of manipulatives. Bouck et al. (2021) provided several examples of how certain situations better lend themselves to a particular type of manipulative. For example, working with concrete manipulatives when learning about money makes sense because students need to learn how to handle physical, real money. However, solving a two-digit by two-digit multiplication problem with partial products might be easier with a virtual manipulative as opposed to laying out 19 groups of 19 blocks on a desk and counting them all one by one.

McCullough et al. (2018) found that many teachers did think strategically to determine whether physical tools or technology better lent themselves to the learning goals they had set for their students. It was recommended by Reiten (2021) that teachers work to find the manipulative tool that aligns with the intended learning goal rather than trying to create a learning goal to match a particular manipulative that they would like to use. Studies on this comparative issue have found mixed results, with some procuring evidence that virtual manipulatives may offer benefits that tangible do not, and some supporting the belief that virtual could never fully replace the benefits of physical (Flevares et al., 2021; Gecu-Parmaksiz & Delialioglu, 2019; Nikiforidou, 2019). Current studies have not sought to encompass the use of both types of manipulatives in combination as an additional category.

Strengths

Day and Hurrell (2017) argue that virtual manipulatives do have a role as a learning tool, but that they may serve a better purpose as a bridge from physical objects to representational and abstract forms. Rich (2023) also theorized that virtual manipulatives can help students visualize the transition from representations to symbols, especially when teachers make intentional choices to support learning goals. As Piaget (1964) described, it is important for students to first have physical experiences that they can eventually interiorize and generalize to other concepts without the need for physical materials.

Other studies have found that virtual manipulatives can be even more effective learning tools than traditional, tangible manipulatives can for several reasons (Flevares et al., 2021; Gecu-Parmaksiz & Delialioglu, 2019; Munfaridah et al., 2021). Virtual manipulatives can help control the way students use them, ensuring they learn the correct methods. For example, a digital ten-frame can prevent students from filling in the bottom row until the top row of the ten-frame is full (Flevares et al., 2021). With a physical ten-frame, it would be possible for a student to fill them in sporadically and practice incorrectly the entire time and not even realize it. Ensuring correct practice is vital in mathematics instruction.

Virtual manipulatives can produce manipulations easier and faster as computer systems generate them (Demir, 2018). Even when students are learning about concepts that involve tactile information, virtual manipulatives still have the potential to impact learning as much as physical manipulatives do (Olympiou & Zacharia, 2018). However, Olympiou and Zacharia (2018) theorized that student age might contribute to this potential, because older students likely have a larger bank of physical experiences to draw from when they are interacting with virtual manipulatives in class. As described earlier, it is important for students to first have prior experiences to make connections with for them to internalize and interiorize abstract understandings (Piaget, 1967), so this theory by Olympiou and Zacharia (2018) is in alignment with educational theory and developmental psychology.

Some types of virtual manipulatives, such as augmented reality, still incorporate the movement of physical objects and even body parts (Kang et al., 2020). Interaction with these augmented reality applications can improve student learning and attitude toward mathematics as many students find them fun to use (Zhufeng & Sitthiworachart, 2023). While both groups in Zhufeng and Sitthiworachart's (2023) study showed significant growth, the experimental group that used augmented reality learning methods grew more than the control group with statistical significance. Students that utilized a digital, adaptive learning tool in Bush's (2021) study gained more fractional knowledge, with statistical significance, than is expected of an average student in 4th or 5th grade, the age of their participants. While Mullen (2020) studied digital versus physical learning modalities and environments as opposed to the effect of manipulatives alone, the study did not find any negative effects on learners in the online environment. This further contributes to the potential promise that digital learning tools might hold as they continue to be explored and implemented.

Some virtual manipulatives can also be more cost-effective for school districts. In teaching geometrical shapes, using wooden blocks limits students to only see traditional versions, while blocks online can be rotated and resized to allow students access to irregular shapes (Gecu-Parmaksiz & Delialioglu, 2019; Prabavathy & Sivaranjani, 2020). This immediate feedback encompassed within interactive applications, which can also be paired with follow-up virtual instruction or assistance, is beneficial for students making sense of a new concept (Bush, 2021; McCullough et al., 2018; Satsangi et al., 2018; Vallejo-Vargas & Reid, 2022), especially in classrooms where students have many different needs but only one teacher. One teacher described the problem with students completing a set of practice problems, only to find out afterward that they did them wrong (Ottenbreit-Leftwich et al., 2018). They can provide versatile opportunities, from modeling for the whole class on the board to students interacting independently during practice on their devices (Satsangi & Raines, 2023). This reliable independent practice allows additional time for teachers to work with other students according to their unique needs (Wen et al., 2019).

These technological tools can also allow students to explore mathematical problems using the actual objects in the described situation, as opposed to symbolic representations in the form of blocks, and they can increase student engagement and satisfaction (Almasri, 2022; Wen et al., 2019). Some teachers have even felt that using virtual learning tools increase their students' sustained attention to the tasks they were completing (Alabdulaziz, 2021). Their implementation supports students in learning math in a way that is relevant to scenarios they might encounter in real-life (Kang et al., 2020).

Virtual manipulatives can simulate mathematical and statistical scenarios that are dynamic in nature (McCullough et al., 2018; Taylor & Hwang, 2021). For example, students can see in real-time how changing the diameter of a shape affects its circumference as well or explore other geometrical formulas where one quantity affects another (Walkington et al., 2021). These concepts can otherwise be difficult to imagine. It can be easier for students to collaborate around a virtual manipulative than with a physical manipulative where one student might be doing much of the work while the other(s) watch (Wang et al., 2020). Some virtual manipulative programs, such as Desmos, are even designed for students to collaborate digitally (McCullough et al., 2018).

These tools can be easier and more intuitive for some students to use (McCullough et al., 2018). Several studies have found that they have been particularly useful in working with students with learning disabilities (Bouck et al., 2018; De Oliveira et al., 2023; Jimenez & Besaw, 2020; Satsangi & Bouck, 2015; Satsangi et al., 2018; Taylor & Hwang, 2021). For example, students who might ordinarily struggle with the logistics of holding and spinning a spinner simply have to click a button to spin one on the computer (Flevares et al., 2021). Even for students who require more effort to manipulate the tools digitally, Kang et al. (2020) observed the associated benefits of additional collaboration and mathematical thinking when students were forced to slow down their process.

Teachers can customize the settings of some virtual manipulatives to differentiate and meet the needs of diverse students, allowing greater flexibility in their classroom usage (Bouck et al., 2021). With this, students might be able to access screen magnifiers, color modifications, or text-to-speech-type features. Yakubova et al. (2023) noted that when using virtual manipulatives with a couple of autistic students, one of whom was minimally vocal, being able to "see" their thinking process with manipulatives helped to identify effective instructional strategies despite not being able to hear the student's explanation. Other virtual manipulative and math programs are designed to be adaptive, meaning that they increase or decrease in difficulty, meeting students' needs according to where they are currently performing. For students who are struggling, a program might provide a more basic level of practice or provide an additional tool or instructional video on the screen (Bush, 2021).

Virtual manipulatives show promise at improving basic arithmetic skills in students with developmental dyscalculia (Prabavathy & Sivaranjani, 2020) and measurement skills in students with autism (Liu et al., 2023). This improvement can not only increase motivation but also enhance the self-esteem of these students and increase their likelihood of being able to engage in grade level learning with their peers (Prabavathy & Sivaranjani, 2020). For older students who might associate manipulatives with young children and therefore feel embarrassed to be seen using them but need additional support, virtual manipulatives can be less obvious (Bouck et al., 2021; Wen et al., 2019). Even for young students, using virtual manipulatives might allow a greater sense of safety as they are not on display (Alabdulaziz, 2021).

Many virtual manipulative applications utilize simultaneous linking, where multiple representations are altered to match when a student completes the process on one of the representations. Students of young ages, such as preschool and kindergarten students, have been found to take advantage of this to make sense of their learning (Litster et al., 2019). Additionally, feedback shows that virtual manipulatives appeal to the social preferences of young students, especially since using them makes them less likely to stand out than needing to use physical tools

(Satsangi & Raines, 2023). Virtual manipulatives can be more conducive to knowledge retention (Kabel et al., 2021; Wang et al., 2020). Kabel et al. (2021) found that students using virtual manipulatives demonstrated higher fact fluency both immediately and two weeks later, compared to students who used physical manipulatives. Some districts have begun exploring the possibilities of creating physical tools with digital fabrication, such as by 3D printing them (Harron et al., 2022) or using a 3D pen (Ng et al., 2018). Students can physically touch their work, and this can provide tactile engagement for students while providing some of the benefits of virtual manipulatives (Harron et al., 2022).

Weaknesses

Other studies, such as Nikiforidou's (2019) study on probability with preschoolers, have found that students performed better after practicing with physical manipulatives, similar to the beliefs of Piaget (1964) and constructivism. Using physical cards provided students in this study with a sensorimotor and tactile experience that allowed them to grapple with the abstract concept of likelihood in realistic scenarios. Litster et al. (2019) found that students who interacted with a particular mathematical app but were missing a key prior understanding did not show improvement because of the app as compared to students who exhibited the necessary prior skills to interact with the app successfully. Since students need prior physical experiences to draw upon in their imagination before transitioning to internalizing and interiorizing their understandings (Piaget, 1964), an implication of this is that teachers may need to consider carefully selecting apps based on the prior knowledge that their students hold and the skills they possess.

It is also possible that without teacher facilitation and support, virtual simulations may not always convey accurate understandings to students. Technology is not a replacement for teaching (Falloon, 2019). Along those lines, while virtual manipulatives can provide a level of independent practice, it is also important for teachers to be able to see their students using manipulatives and sharing their thinking so that teachers know how to guide students further (Wen et al., 2019). Students using virtual manipulatives at their seat eliminates an opportunity for teachers to do this. Several existing studies showing the positive impact of virtual manipulatives only employed the treatment for a small period, so it can be theorized that the novelty of the tool might be an underlying factor (Bush, 2021). Sustained growth with long-term use of digital tools needs to be explored further.

Additionally, while some view dynamic links between representations as a benefit of virtual manipulatives, others worry that when a program is constantly making the connection for students, they may not ever make that connection for themselves, making it more difficult to eventually transition away from the use of the manipulative as is intended (Rich, 2023). Cost is also a barrier for some teachers, as certain virtual programs can be expensive (Fakih, 2023; Ng et al., 2018; Walkington et al., 2021), and access can be an issue in some areas, especially rural ones (Kabel et al., 2021). While many schools have one-to-one Chromebooks or devices now, that is not the case in every district so this could be a barrier to using virtual manipulatives (Reiten, 2020).

Teachers have also cited negative experiences for reasons as simple as technological difficulties (Keldgord & Ching, 2022; Walkington, et al., 2021) as well as the increase of screen time that is associated with virtual manipulatives (Bouck et al., 2021). One teacher described a young student's relief when they took a break from using virtual reality manipulatives because "their eyes were starting to burn" (Walkington et al., 2021). Students spend precious instructional minutes logging onto devices and finding virtual manipulatives (Wen et al., 2019).

For students who do not find technology app usage intuitive, their efforts in figuring out how to work the program or manipulative can distract from the intended learning (McCullough et al., 2018). For others who struggle with reading, many online programs involve more text than using hands-on tools on a table does. However, some programs do connect with assistive technology tools, such as text-to-speech (Wen et al., 2019).

Mathematical Achievement of Title I Students

The federal Title I program designates funding to local school districts according to how many of their students are living in poverty with the goal of improving the academic achievement of these students. Closing the achievement gap with more equitable education is a primary purpose of Title I funding (Kainz, 2019). Despite years of efforts to close the achievement gap, poverty is still a large factor negatively affecting student achievement (Wickham & Mullen, 2021). Children from low socioeconomic status (SES) homes are capable of learning at high levels. However, many teachers hold biases about students of poverty (Wickham & Mullen, 2021), and many teachers do not believe that effort can make individuals better at math (Hall & Yakimowski, 2022). Teachers' beliefs about students and their learning capabilities are important to develop to change and improve instructional practices in the classroom (Hall & Yakimowski, 2022; Wickham & Mullen, 2021). Wickham and Mullen (2021) found that carefully planned professional development had the power to impact teacher's beliefs on poverty.

Other barriers to teaching mathematics to Title I students that were mentioned by teachers were the lack of background knowledge and grade level reading ability that many of their students bring to the classroom (Hall & Yakimowski, 2022). Many of their students are drawing from a shallow pool of prior experience which makes it difficult for them to form connections and prepare to transition to more abstract levels of thinking. This, coupled with the fact that elementary students cannot reason with verbal logic alone (Piaget, 1964), can lead to difficulties in the abstract concepts of math. Additionally, the way that schools choose to spend Title I funding can affect the gains in student achievement, depending whether it is used for professional development, supplemental learning materials, student and staff support through intervention and coaching, or on other, non-academic needs (Kainz, 2019). School leaders in Title I buildings must make decisions accordingly.

Summary

Math manipulatives provide students with the opportunity to make sense of mathematical concepts and form their own understandings regarding them (Lewis & Colonnese, 2021). Through the lens of constructivism, educators facilitate these opportunities and assist students through disequilibrium as they assimilate and accommodate new learning (Piaget, 1964, 1967). The implementation of math manipulatives is developmentally appropriate for students (Donovan & Alibali, 2021) as students internalize concrete representations (Day & Hurrell, 2017). While students should develop both procedural and conceptual understandings of math (Özpınar & Arslan, 2022), it is crucial that struggling learners and those in need of intervention are scaffolded in their development of mathematical concepts (Burns et al., 2015) such as number sense.

As many of today's teachers utilize virtual versions of math manipulatives, Day and Hurrell (2017) questioned whether these virtual manipulatives provide the same level of conceptual exploration that tangible ones do. Recent studies have investigated the benefits of both types of manipulatives (Flevares et al., 2021; Gecu-Parmaksiz & Delialioglu, 2019; Nikiforidou, 2019), but there is a gap in the literature regarding the extent to which both types of manipulatives specifically support conceptual mathematics understandings based on empirical evidence. Wang and Tseng (2018) found the combination of virtual and physical manipulatives to increase student achievement in science, but similar studies have not been conducted with mathematical understanding. Additionally, current studies do not address the third category of using both types of manipulatives in combination. This research is an important issue because conceptual understanding must be developed in students in need of intervention (Karakonstantaki et al., 2018). For educators to make the best instructional choices for their students, they must know the impact of these different teaching materials, including their impact when utilized together.

CHAPTER THREE: METHODS

Overview

The purpose of this quantitative, quasi-experimental nonequivalent control group design study was to measure the impact that both types of manipulatives, tangible and virtual, in isolation and in combination have on the mathematical understanding of 1st and 4th-grade, Title I students. This chapter will discuss the methodology of this study. The design of the study will be described, and the research question will be listed along with the study's hypothesis. The participants and setting of the study will be presented, and then the research instrument will be identified. Lastly, the procedures of both the data collection and data analysis will be described in detail.

Design

This quantitative study utilized a quasi-experimental nonequivalent control group design. An essential characteristic of the nonequivalent control-group design is that students in each of the experimental groups take both a pre-test and a post-test. This accounts for the possible nonequivalence between experimental groups that exists because groups are not formed randomly. Since the instrument for data collection in this study was an assessment that students took as a pre-test to control for existing knowledge and a post-test to demonstrate gained student understanding, this design for a quasi-experiment was most appropriate (Gall et al., 2007; Gopalan et al., 2020). Yuan et al. (2010) utilized this design in a similar study comparing two versions of the same math manipulative. One limitation of this design is that participants were not randomly assigned to groups (Gall et al., 2007; Gopalan et al., 2020).

This study explored the impact of math manipulatives on mathematical understanding. Specifically, the study sought to understand whether virtual manipulatives, used in isolation or in combination with tangible manipulatives, support student mathematical understanding at the same level as tangible manipulatives do. The independent variable of this study consisted of the control group, students in classrooms that solely used tangible manipulatives, and two treatment groups, students in classrooms that used virtual manipulatives instead of tangible ones and students in classrooms that used virtual manipulatives alongside physical ones. Tangible manipulatives are physical objects and models that students can touch and move around to explore mathematical concepts (Hynes, 1986). Virtual manipulatives are defined as interactive, web-based, dynamic visual representations that allow the construction of mathematical knowledge (Moyer et al., 2002).

The dependent variable in this study was the level of number sense students possess with the scores on the MOY USNS serving as a covariate. Howden (1989) described number sense as "good intuition about numbers and their relationships. It develops gradually as a result of exploring numbers, visualizing them in a variety of contexts, and relating them in ways that are not limited by traditional algorithms" (p. 11). This satisfied the requirements of a quasi-experimental nonequivalent control-group design since the dependent variable was continuous and a pre-test was included to control post-test scores (Gall et al., 2007). Each participating classroom was assigned to an experimental group. Since the groups were not randomly assigned, it is possible that their existing understanding on the pre-test prior to the study was nonequivalent. The researcher took statistical measures to help reduce the impact that pre-test differences in scores might have on the post-test scores of control and treatment groups (Gall et al., 2007).

Research Question

RQ1: Is there a difference in number sense between 1st- and 4th-grade Title I students who use physical manipulatives in isolation, students who use virtual manipulatives in isolation, and students who use a combination of virtual and physical manipulatives when controlling for pre-test scores?

Hypothesis

H_0: There is no difference in number sense between 1st- and 4th-grade Title I students who use physical manipulatives in isolation, students who use virtual manipulatives in isolation, and students who use a combination of virtual and physical manipulatives when controlling for pre-test scores as measured by the Universal Screener for Number Sense.

Participants and Setting

This section will begin by describing the participants of this study, including the overall population and its demographics. The demographics of the participants will be provided, and the sampling technique and sample size will be justified. Lastly, the setting of the study will be detailed.

Population

The participants for the study were drawn from a convenience sample of Title I, elementary students located in southwest Missouri during the spring semester of the 2023-2024 school year. The school district that was studied consists of 38 elementary schools, 21 of which are considered Title I schools based on free and reduced lunch qualification rates. Participating classrooms were located at multiple sites in this population. The total population of students in the district at the time of this study was 23,428. 72.3% of students in the district were white, 8.6% were Hispanic, 8.5% were black, 6.2% were mixed-race, and 4.5% were either Asian, American Indian, or Hawaiian/Pacific Islander. 4.93% of students in the district were part of the ELL program, 7.27% of students have been identified as gifted, and 11.3% of students receive special education services. In addition to 46.4% of students in the district qualifying for free and reduced lunch, 5.68% of students were homeless.

Participants

Convenience sampling was utilized for this study. After the receipt of IRB and district approval, the researcher contacted the principal of each Title I school in her district. Once principal permission was received, the researcher contacted the 1st and 4th-grade teachers in that building to introduce the study and its requirements and procedures as well as the compensation offered for participation. A draft of this email can be found in Appendix A. Teachers who responded with interest were selected for the study and their students comprised the group of participants. Teachers were asked which treatment(s) they were willing to provide in their instruction. Teachers who were only willing to utilize one version of manipulative over the other were assigned first. Classes whose teacher had indicated willingness to use either treatment were randomly assigned to their experimental group to fill the remaining open spots using an online random number generator of 1-3, with 1 representing an assignment of physical manipulatives, 2 representing an assignment of virtual manipulatives, and 3 representing an assignment of combined manipulatives.

The number of participants sampled was 222 students, 96 1st-graders and 126 4th-graders, divided among the three experimental groups as equally as possible given the pre-existing classroom assignment constraints. According to Gall et al. (2007), 96 students is the required minimum for an ANCOVA with three groups and a covariate $r = .5$ when assuming a medium effect size with a statistical power of .7 at the .05 alpha level. Since two ANCOVAs were run, one for 1st-graders and one for 4th-graders, 220 students satisfied the minimum requirement of 192 students. The sample came from seven different elementary schools in the district in which teachers volunteered to participate. Within each school, classrooms were selected based on the grade level of the class. Each of the classroom teachers taught mathematics using the IM curriculum and according to the Missouri Learning Standards (MLS). To protect the privacy of all participants involved in the study, pseudonyms will be used for the names of all students, classes, and schools throughout the description of this study.

The control group in this study consisted of three 1st-grade classes and two 4th-grade classes. Each of the two treatment groups in 1st grade consisted of two classes. Each of the two treatment groups in 4th grade consisted of three classes. Classroom teachers of these grade levels in Title I schools within the district were invited to participate in the study. For 1st grade, classes A-C made up the control group using physical manipulatives, classes D and E made up the treatment group using virtual manipulatives in isolation, and classes F and G made up the treatment group using virtual manipulatives in combination with physical manipulatives. For 4th grade, classes H and I made up the control group using physical manipulatives, classes J-L made up the treatment group using virtual manipulatives in isolation, and classes M-O made up the treatment group using virtual manipulatives in combination with physical manipulatives. The demographics of each class are described in Table 1, and each group was similar enough to experience the math manipulatives and lessons in a closely related way. Students in all experimental groups included students at Title I buildings receiving their primary math instruction in the regular education classroom. Because the purpose of this study was to examine the impact of manipulatives on elementary students in general, gifted students, students with disabilities, and ELL students were included in the analyses.

Table 1

Group Demographic Information

	Grade Level	# of Students	% Male	% Female	% English Primary Language	% Other Primary Language	% White	% Black	% Asian	% Other Race	% Free/ Reduced
Class A	01	16	56	44	94	6	75	6	00	19	88
Class B	01	15	47	53	93	7	53	40	00	7	100
Class C	01	10	60	40	90	10	70	00	00	30	100
Class D	01	17	59	41	94	6	53	35	00	12	71
Class E	01	16	44	56	100	00	81	00	6	00	13
Class F	01	13	54	46	100	00	69	15	00	16	92
Class G	01	9	56	44	78	22	22	44	00	33	100
Class H	04	20	50	50	95	5	35	10	10	45	90
Class I	04	15	53	47	93	7	73	00	00	27	93
Class J	04	14	36	64	100	00	86	14	00	00	93
Class K	04	16	44	56	88	12	88	6	00	6	94
Class L	04	14	29	71	100	00	79	21	00	00	71
Class M	04	15	53	47	93	7	87	7	00	8	100
Class N	04	13	23	77	100	00	69	23	00	8	92
Class O	04	19	58	42	95	5	53	21	00	26	100

Setting

The setting of this study consisted of the general education classroom during their 75–90-minute daily mathematics block. Students were instructed by their regular classroom teacher using their specified type of manipulative for 10 weeks. During this time, teachers followed the district's curriculum. These unit and lesson plans were developed by Illustrative Mathematics (IM). Any additional incorporation of manipulatives in lessons by the researcher is listed in tables. The control group used the lesson plans as written, incorporating physical manipulatives when they were directed to by the curriculum or by the researcher in Tables 2 and 3. This ensures all classes utilize manipulatives for the same lessons. Links to sample lesson plans for both grade levels are located in Appendix B. The treatment group using virtual manipulatives in isolation used the lesson plans as written, incorporating the virtual versions of manipulatives that are provided by the district's Imagine Learning software when they were directed to by the curriculum or by Tables 2 and 3. The treatment group that used a combination of both forms of manipulatives, physical manipulatives and the virtual versions of manipulatives that are provided by the district's Imagine Learning software, used the lesson plans as written. Tables 4 and 5 detail which type of manipulative (physical or virtual) for each category of manipulatives (e.g. ten-frame, base ten blocks, etc.) was utilized during this study to ensure that each classroom in this treatment group experienced the same kind of learning. Class sizes vary across buildings in the study but the district's state-reported classroom teacher to student ratio in 2022 was 1:14. The number of classes per grade level within each building ranged from one to 5.

Table 2

1st Grade Supplemental Manipulative Usage

	Type of Manipulative	Description of Usage in the Lesson
Unit 4, Lesson 10	Connecting Cubes	Provide students with connecting cubes when teaching “Write the Number” as a scaffold to determine the next number in the count sequence.
Unit 4, Lesson 16	Connecting Cubes	When teaching “Greatest of Them All”, provide students with connecting cubes to build their numbers to check their answer.
Unit 4, Lesson 22	Connecting Cubes	When teaching “Mystery Number”, provide students with connecting cubes to build their partner’s number as they try to guess.
Unit 6, Lesson 4	Connecting Cubes	When teaching “How Close?” (to 100 version), allow students to use connecting cubes to solve the required addition & subtraction
Unit 7, Lesson 3	Magnatiles	If you have Magnatiles, or similar 2-D shapes available, you might allow students to explore them and make the connection to the picture cards they will be sorting. (before or after Activity 1)

	Type of Manipulative	Description of Usage in the Lesson
Unit 7, Lesson 13	Student Judy Clocks	Allow students to use Judy clocks to match the times on their paper clocks/cards. This is not necessarily to teach how to tell time but to allow them a more interactive way to explore the way the hands move.

Table 3

4^{th} Grade Supplemental Manipulative Usage

	Type of Manipulative	Description of Usage in the Lesson
Unit 5, Lesson 2	Connecting Cubes	Provide students with connecting cubes to build the scenarios described in Activity 2.
Unit 5, Lesson 10	Base Ten Blocks	Since Activity 1 involves converting centimeters, meters, and kilometers, consider having students create a metric conversions slider tool (instructions free online) to help with the conversion process.
Unit 6, Lesson 5	Base Ten Blocks	To encourage the distributive property, use base ten blocks to physically decompose the tens and

	Type of Manipulative	Description of Usage in the Lesson
		the ones, multiply (see each part grow), and then combine the total blocks.
Unit 6, Lesson 10	Base Ten Blocks	Use base ten blocks to visually keep track of the steps of partial products before moving to notation only as well as to see why it works.
Unit 6, Lesson 14	Materials to Create an Array	When solving the mystery number in Activity 2, provide students with materials to create arrays as they explore factors and how factors/multiples can be used to solve this type of problem.
Unit 6, Lesson 22	Counting Cubes or something similar	Allow students to use a counting manipulative material to help them solve for the conversions in Activity 2. Students might also physically cut the classroom outline to help determine where to split the irregular area to solve.
Unit 7, Lesson 15	Base Ten Blocks	Allow students to use base ten blocks to solve for missing angle measurements so that the focus is on the relationships between adjacent angles, etc. and not on computation.
Unit 8, Lesson 6	Protractors & Straightedges	Allow students to use protactors and straightedges to support them as they draw shapes with specific attributes in Activity 2.

Table 4

1st Grade Mixed Manipulative Types

	Type of Manipulative
Counting Objects	Physical
10-Frames & Double Ten-Frames	Virtual
Connecting Cubes	Physical
Two-Color Counters	Physical
Base-Ten Blocks	Virtual
Objects to Measure	Mixture of Both
String	Physical
Geoblocks & Solid Shapes	Mixture of Both
Pattern Blocks	Virtual

Table 5

4th Grade Mixed Manipulative Types

	Type of Manipulative
Connecting Cubes	Virtual
Rulers & Yardsticks	Physical
Pattern Blocks	Virtual
Inch Tiles	Physical
Base Ten Blocks	Virtual

	Type of Manipulative
Rulers	Physical
Patty Paper & Origami Paper	Physical
Protractors	Mixture of Both

Instrumentation

The Universal Screeners for Number Sense (USNS), created by Forefront Education, was used as both a pre- and post-test instrument in this study. All assessments were administered by the researcher at a scheduled time during the given window. Students had up to 60 minutes to complete the written, whole-class assessment, though most did not take the whole hour, and the individual student interviews took about five minutes each (Forefront Education, 2023). To measure this study's variables, the researcher compared each group's scores on the EOY USNS using their scores on the MOY USNS to control for the covariate. The 1st Grade MOY USNS was used as a number sense pre-test for 1st-grade students at the start of the study to control for the covariate. The 1st Grade EOY USNS was used to measure the dependent variable, number sense, of the 1st-grade participants. The 4th Grade MOY USNS was used as a number sense pre-test for 4th-grade students at the start of the study to control for the covariate. The 4th Grade EOY USNS was used to measure the dependent variable, number sense, of the 4th-grade participants.

1st Grade Middle of Year Universal Screeners for Number Sense

The purpose of this instrument is to measure the level of number sense that students possess and to identify students that would benefit from additional supports. The instrument was developed by David Woodward to help educators better understand student learning and specific intervention steps based on assessment (Woodward, 2022). This instrument was used in numerous studies (Wilkins et al., 2021, 2022). The instrument is designed for 1st grade students and is written at an age-appropriate reading level. Validity was assessed by an expert panel and was rated on a Likert scale of one to four, with the results producing a mean of 3.33 and a standard deviation of 0.50, indicating that assessment items do address what they are intended to. The results of this psychometric evaluation provided evidence in three of the five validity categories to show that the USNS is a valid measurement of student mathematical understanding. Rasch reliability scores were utilized as they are more conservative than Cronbach's alpha. The Rasch reliability value of this instrument for people reliability was found to be 0.70, which is considered acceptable, and the item reliability value was found to be 1.00 which is an excellent psychometric (Bostic & Folger, 2023).

The instrument consists of a 5-item interview and 7 written items which are each aligned to a 1st-grade Common Core State Standard (CCSS). The three constructs assessed by this instrument include numerals, words, and sequences, addition and subtraction within 20, and place value. The question types are all open response. The possible scores on the 1st-grade MOY USNS range from a score of 6, meaning that a student has a well below basic understanding of grade level standards to a score of 36, meaning that a student has demonstrated a proficient understanding of grade level standards. A score of at least 30 indicates a student should be deemed proficient. The researcher administered the assessment by having students complete the written part on paper as a whole-group and then pulling students to complete five-minute interviews, one at a time. The researcher and classroom teacher monitored students taking the test to ensure academic honesty but did not assist students. The assessments were all scored by the researcher to ensure inter-rater reliability, and Forefront Education provided in-depth descriptions to assist with scoring on each question (Woodward, 2023). The screeners are open-source and free to anyone interested. A link to download the screener is provided in Appendix B.

1st Grade End of Year Universal Screeners for Number Sense

The purpose of this instrument is to measure the level of number sense that students possess and to identify students that would benefit from additional supports. The instrument was developed by David Woodward to help educators better understand student learning and specific intervention steps based on assessment (Woodward, 2022). This instrument was used in numerous studies (Wilkins et al., 2021, 2022). The instrument is designed for 1st grade students and is written at an age-appropriate reading level. Validity was assessed by an expert panel and was rated on a Likert scale of one to four, with the results producing a mean of 3.33 and a standard deviation of 0.50, indicating that assessment items do address what they are intended to. The results of this psychometric evaluation provided evidence in three of the five validity categories to show that the USNS is a valid measurement of student mathematical understanding. Rasch reliability scores were utilized as they are more conservative than Cronbach's alpha. The Rasch reliability value of this instrument for people reliability was found to be 0.74, which is considered acceptable, and the item reliability value was found to be 1.00 which is an excellent psychometric (Bostic & Folger, 2023).

The instrument consists of a 7-item interview and 5 written items which are each aligned to a 1st-grade Common Core State Standard (CCSS). The three constructs assessed by this instrument include numerals, words, and sequences, addition and subtraction within 20, and place value. The question types are all open response. The possible scores on the 1st-grade EOY USNS range from a score of 6, meaning that a student has a well below basic understanding of grade level standards to a score of 36, meaning that a student has demonstrated a proficient understanding of grade level standards. A score of at least 30 would indicate a student should be deemed proficient. The researcher administered the assessment by having students complete the written part on paper as a whole-group and then pulling students with consent to complete five-minute interviews, one at a time. The researcher and classroom teacher monitored students taking the test to ensure academic honesty but did not assist students. The assessments were all scored by the researcher to ensure inter-rater reliability, and Forefront Education provided in-depth descriptions to assist with scoring on each question (Woodward, 2023). The screeners are open-source and free to anyone interested. A link to download the screener is provided in Appendix B.

4th Grade Middle of Year Universal Screeners for Number Sense

The purpose of this instrument is to measure the level of number sense that students possess and to identify students that would benefit from additional supports. The instrument was developed by David Woodward to help educators better understand student learning and specific intervention steps based on assessment (Woodward, 2022). This instrument was used in numerous studies (Wilkins et al., 2021, 2022). The instrument is designed for 4th grade students and is written at an age-appropriate reading level. Validity was assessed by an expert panel and was rated on a Likert scale of one to four, with the results producing a mean of 3.83 and a standard deviation of 0.41, indicating that assessment items do address what they are intended to. The results of this psychometric evaluation provided evidence in three of the five validity categories to show that the USNS is a valid measurement of student mathematical understanding. Rasch reliability scores were utilized as they are more conservative than Cronbach's alpha. The Rasch reliability value of this instrument for people reliability was found to be 0.91, which is considered excellent, and the item reliability value was found to be 1.00 which is also an excellent psychometric (Bostic & Folger, 2023).

The instrument consists of a 3-item interview and 15 written items which are each aligned to a 4th-grade Common Core State Standard (CCSS). The five constructs assessed by this instrument include numerals, words, and sequences, place value, multiplication and division, fractions, and problem-solving and problem-posing. The question types are all open response. The possible scores on the 4th-grade MOY USNS range from a score of 9, meaning that a student has a well below basic understanding of grade level standards to a score of 54, meaning that a student has demonstrated a proficient understanding of grade level standards. A score of at least 45 would indicate a student should be deemed proficient. The researcher administered the assessment by having students complete the written part on paper as a whole-group and then pulling students to complete five-minute interviews, one at a time. The researcher and classroom teacher monitored students taking the test to ensure academic honesty but did not assist students. The assessments were all scored by the researcher to ensure inter-rater reliability, and Forefront Education provided in-depth descriptions to assist with scoring on each question (Woodward, 2023). The screeners are open-source and free to anyone interested. A link to download the screener is provided in Appendix B.

4th Grade End of Year Universal Screeners for Number Sense

The purpose of this instrument is to measure the level of number sense that students possess and to identify students that would benefit from additional supports. The instrument was developed by David Woodward to help educators better understand student learning and specific intervention steps based on assessment (Woodward, 2022). This instrument was used in numerous studies (Wilkins et al., 2021, 2022). The instrument is designed for 4th grade students and is written at an age-appropriate reading level. Validity was assessed by an expert panel and was rated on a Likert scale of one to four, with the results producing a mean of 3.83 and a standard deviation of 0.41, indicating that assessment items do address what they are intended to. The results of this psychometric evaluation provided evidence in three of the five validity categories to show that the USNS is a valid measurement of student mathematical understanding. Rasch reliability scores were utilized as they are more conservative than Cronbach's alpha. The Rasch reliability value of this instrument for people reliability was found to be 0.80, which is considered good, and the item reliability value was found to be 0.99 which is considered an excellent psychometric (Bostic & Folger, 2023).

The instrument consists of a 5-item interview and 7 written items which are each aligned to a 4th-grade Common Core State Standard (CCSS). The five constructs assessed by this instrument include numerals, words, and sequences, place value, multiplication and division, fractions, and problem-solving and problem-posing. The question types are all open response. The possible scores on the 4th-grade MOY USNS range from a score of 9, meaning that a student has a well below basic understanding of grade level standards to a score of 36, meaning that a student has demonstrated a proficient understanding of grade level standards. A score of at least 30 would indicate a student should be deemed proficient. The researcher administered the assessment by having students complete the written part on paper as a whole-group and then pulling students to complete five-minute interviews, one at a time. The researcher and classroom teacher monitored students taking the test to ensure academic honesty but did not assist students. The assessments were all scored by the researcher to ensure inter-rater reliability, and Forefront Education provided in-depth descriptions to assist with scoring on each question (Woodward, 2023). The screeners are open-source and free to anyone interested. A link to download the screener is provided in Appendix B.

Procedures

The researcher submitted the complete proposal to Liberty University for feedback and approval. Once this approval was obtained, the researcher requested and received approval from the IRB at Liberty University (see Appendix D for the IRB approval letter). After IRB approval was received, the researcher worked with the participating school district to receive permission to conduct the study (see Appendix E). Then, the researcher utilized the sampling method described above to recruit teachers who were willing to participate in the study by teaching lessons according to what was dictated in the tables and allowing the researcher to screen their students with the USNS. These teachers participated in a 30-minute Zoom training with the researcher to go over all procedures for the study and answer any questions they had. An outline of the Zoom meeting can be found in Appendix C. Compensation was offered to participating teachers in the district in the form of $50 gift cards. Informational letters about the study were sent home to the parents of each student in participating classrooms, describing the study. A copy of this letter can be found in Appendix F.

The researcher administered the MOY USNS to each student the week before the 10-week treatment period began. To protect student confidentiality and in alignment with IRB guidelines for the study, scores were logged in a spreadsheet without student names. Scores were coded in numerical order of screening, without any attachment to whose score is whose. The spreadsheet is stored on a password-protected computer and backed up on a password-protected cloud drive. During this process, any hard copies of data were locked in a file cabinet, and the researcher held the only copy of the key. Once data was transferred to the password-protected spreadsheet, paperwork was shredded. The researcher did not keep any identifying information on students in this study.

For 10 weeks, teachers taught math using the district curriculum and adjusting for any manipulative instructions in Tables 2-5. At the completion of the study, the researcher administered the EOY USNS, following all the same collection and storage procedures as with the pre-test, keeping all digital data under password protection and physical data locked in the file cabinet. As with the pre-test, post-test data was shredded once transferred to the password-protected spreadsheet. Scores from both tests were then used for statistical analysis of the data to determine and report findings. Any stored data will be deleted five years after the study has been presented and approved.

Data Analysis

The researcher used a one-way ANCOVA test to compare the post-test scores of the three experimental groups within each grade level band while controlling for variance in the pre-test scores. Because quasi-experiments do not randomly assign participants to treatment groups, Gall et al. (2007) warned that the main threat to internal validity is the possibility that differences in scores on the post-test are due to existing differences in the groups rather than the treatment. They identify ANCOVA as the best statistical analysis to use in quasi-experimental nonequivalent control-group studies to compensate for these differences in groups prior to the study. This study met the criteria for running an ANCOVA statistical analysis according to Gall et al. (2007) because the dependent variable, number sense, was measured at a continuous level. The independent variable, type of manipulative, consisted of three categorical, nominal groups. The covariate variable, pre-test scores, was measured at the continuous level. Independence of observations was present because no participants were in more than one group. Each participant was discretely assigned to one of the three experimental groups according to their regular education classroom.

The researcher conducted data screening including visual screening for missing and inaccurate entries. Warner (2021) stated that data must also be checked for statistical assumptions being met prior to running an ANCOVA. In order to meet each assumption, the covariate was transformed and calculated as the square root of the original covariate. A box-and-whisker plot was used to ensure that there were no outliers affecting the data. After no outliers were identified, a Shapiro-Wilk test for normality was run to ensure normal distribution of each treatment group. Then, Levene's Test of Equal Variance was used to confirm that the population distributions had the same variance. This is the test recommended by Warner (2021) to check for this assumption. The researcher plotted a grouped scatterplot of the post-test scores against the pre-test scores of each treatment group within each grade level band to test for linear relationships between the covariate and the dependent variable. Lastly, SPSS was used to determine whether there was a statistically significant interaction between the independent variable groups and the covariate. This ensured that there was no interaction between the covariate and the independent variable prior to statistical analyses being run.

In addition to reporting the mean and standard deviation for each experimental group, an ANCOVA was run for 4^{th}-graders and for 1^{st}-graders to compare student scores between experimental groups of each grade level band. The alpha level was set at α =.05. Partial eta squared was used to report the effect size. A Bonferroni test was used for post hoc analysis. The researcher analyzed and reported the results of both the ANCOVA for 4^{th}-grade and the ANCOVA for 1^{st}-grade. The implications of each analysis and the implications of the results combined will be discussed further in the following sections.

CHAPTER FOUR: FINDINGS

Overview

The purpose of this study was to determine if there was a statistically significant difference between the number sense of students who were taught using physical manipulatives, students who were taught using virtual manipulatives, and students who were taught using a combination of both. This chapter provides the descriptive statistics that were run by the researcher. The results of the statistical analyses described in Chapter 3 are then detailed. Figures containing charts and graphs from these analyses are shared.

Research Question

RQ1: Is there a difference in number sense between 1st- and 4th-grade Title I students who use physical manipulatives in isolation, students who use virtual manipulatives in isolation, and students who use a combination of virtual and physical manipulatives when controlling for pre-test scores?

Null Hypothesis

H_0: There is no difference in number sense between 1st- and 4th-grade Title I students who use physical manipulatives in isolation, students who use virtual manipulatives in isolation, and students who use a combination of virtual and physical manipulatives when controlling for pre-test scores as measured by the Universal Screener for Number Sense.

Descriptive Statistics

Descriptive statistics were obtained on the covariate (MOY USNS pre-screener scores), dependent variable (EOY USNS post-screener scores), and the adjusted dependent variable (adjusted means for EOY USNS post-screener scores) for each group for both grade levels. Table 6, Table 7, and Table 8 provide the descriptive statistics for 1st grade, and Table 9, Table

10, and Table 11 provide the descriptive statistics for 4th grade.

Table 6

1st Grade Descriptive Statistics: Covariate

Group	*n*	*M*	*SD*
1 - Physical	40	4.56	.94
2 - Virtual	33	5.03	.72
3 - Mixed	22	5.06	.63

Table 7

1st Grade Descriptive Statistics: Dependent Variable

Group	*n*	*M*	*SD*
1 - Physical	40	25.48	9.00
2 - Virtual	33	23.00	7.56
3 - Mixed	22	26.59	6.28

Table 8

1st Grade Descriptive Statistics: Dependent Variable (Adjusted Means)

Group	*n*	*M*	*SE*
1 - Physical	40	28.08	.41
2 - Virtual	33	21.24	.45
3 - Mixed	22	24.49	.55

Table 9

4th Grade Descriptive Statistics: Covariate

Group	*n*	*M*	*SD*
1 - Physical	35	4.77	.95
2 - Virtual	44	5.23	1.03
3 - Mixed	47	4.78	1.01

Table 10

4th Grade Descriptive Statistics: Dependent Variable

Group	*n*	*M*	*SD*
1 - Physical	35	21.23	6.38
2 - Virtual	44	20.84	6.58
3 - Mixed	47	19.09	6.32

Table 11

4th Grade Descriptive Statistics: Dependent Variable (Adjusted Means)

Group	*n*	*M*	*SE*
1 - Physical	35	22.27	.20
2 - Virtual	44	18.97	.18
3 - Mixed	47	20.06	.17

Results

Data Screening

Data screening was conducted on each group's covariate and dependent variable for both grade levels. The researcher sorted the data on each variable and scanned for inconsistencies. No data errors or inconsistencies were identified. Box and whiskers plots were used to detect extreme outliers on each dependent variable. Three outliers (data points 74, 75, and 76) were denoted with an asterisk on the 1st-grade covariate box and whisker plot. The researcher converted the data points to z-scores and they fell within +3 and -3 standard deviations of the sample mean (Warner, 2021). Thus, the data points were not considered extreme scores and were maintained in the data set. There were no other outliers identified. See Figure 1 and Figure 2 for the 1st-grade box and whisker plots and Figure 3 and Figure 4 for the 4th-grade box and whisker plots.

Figure 1

Box and Whisker Plot (1st Grade Covariate)

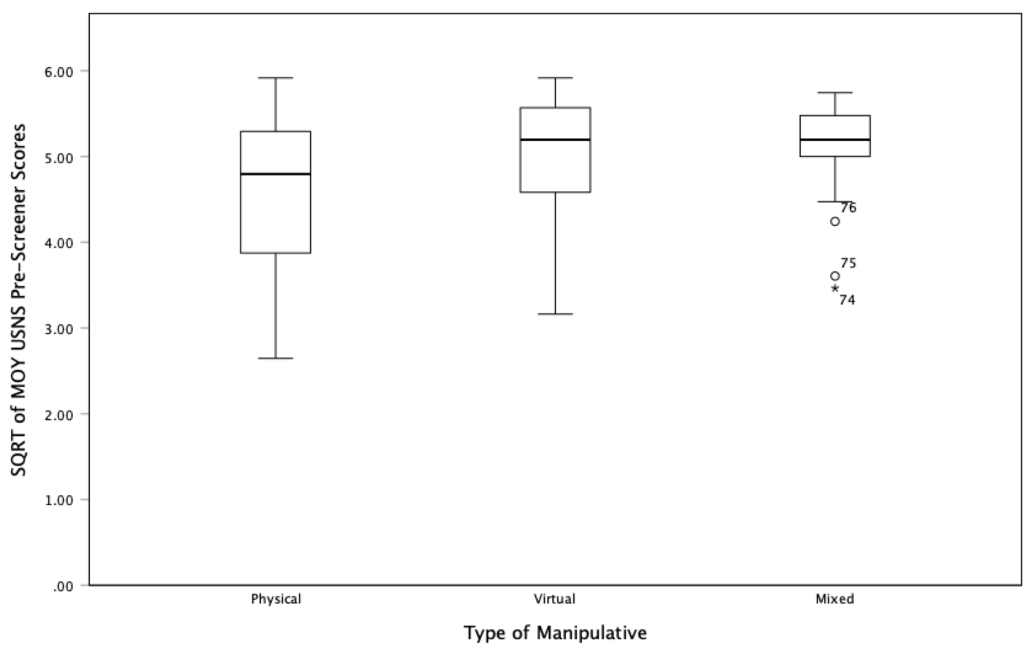


Figure 2

Box and Whisker Plot (1st Grade Dependent Variable)

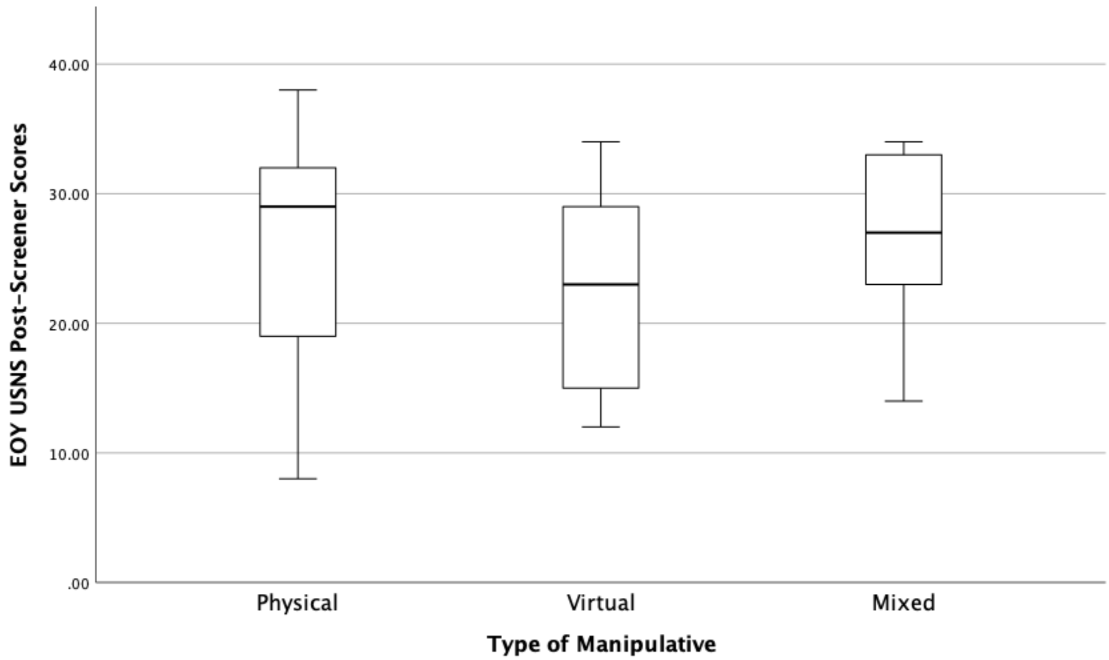


Figure 3

Box and Whisker Plot (4th Grade Covariate)

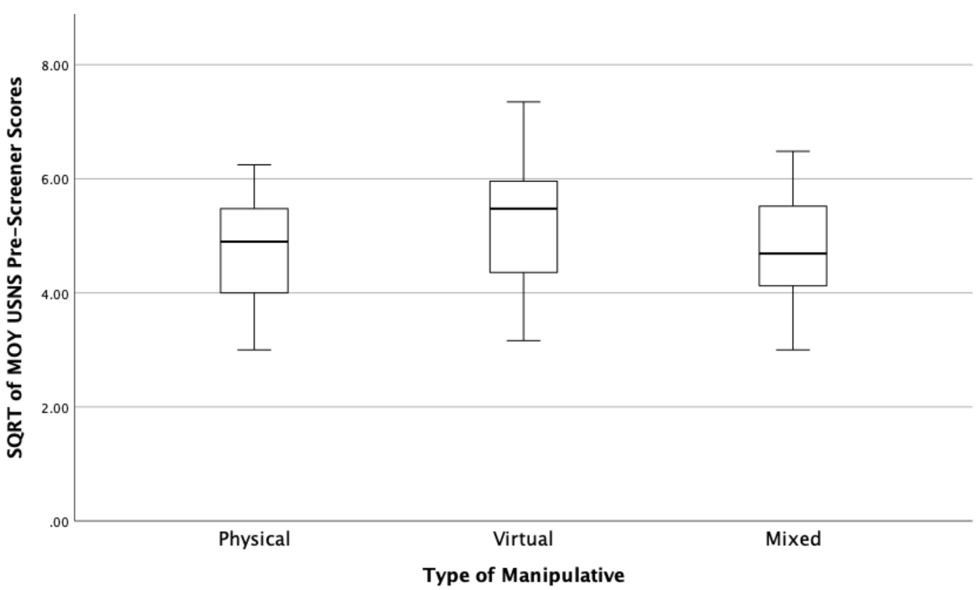
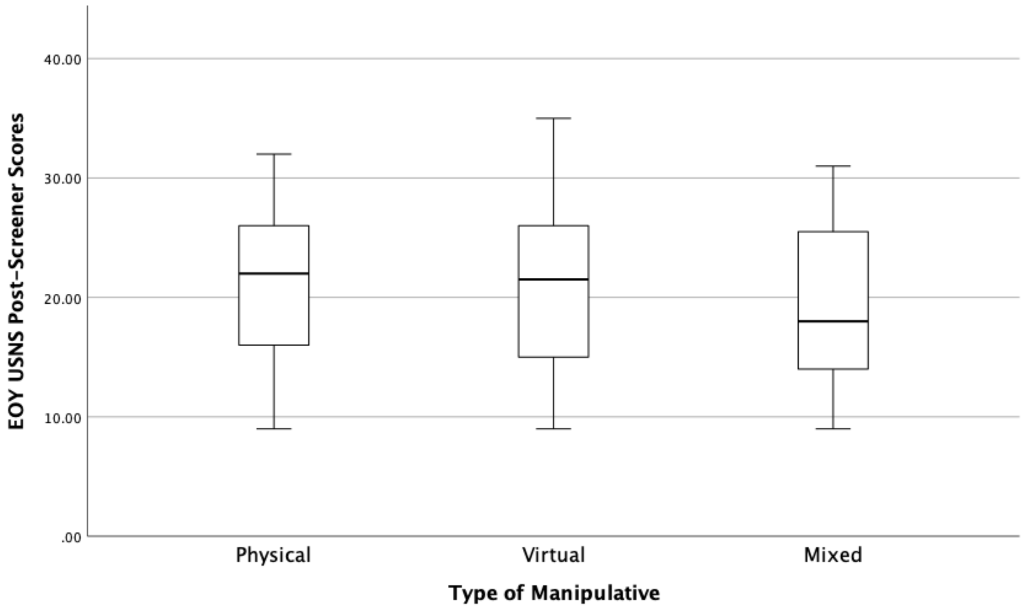


Figure 4

Box and Whisker Plot (4th Grade Dependent Variable)



Assumptions

An analysis of covariance (ANCOVA) was used to test the null hypothesis. The ANCOVA required that the assumptions of normality, linearity and bivariate normal distribution, homogeneity of slopes, and the homogeneity of variance are met.

Normality was examined using a Shapiro-Wilk test. No violations of normality were found. See Table 12 for 1st-grade Tests of Normality and Table 13 for 4th-grade Tests of Normality.

Table 12

1st-Grade Tests of Normality

		Shapiro-Wilk		
	Groups	Statistic	*df*	Sig.
Standardized Residual for post	Physical	.983	40	.790
	Virtual	.955	33	.190
	Mixed	.919	22	.071

Table 13

4th-Grade Tests of Normality

		Shapiro-Wilk		
	Groups	Statistic	*df*	Sig.
Standardized Residual for post	Physical	.977	35	.657
	Virtual	.961	44	.139
	Mixed	.984	47	.761

The assumption of linearity and bivariate normal distribution were tested using scatter plots for each group. Linearity was met and bivariate normal distributions were tenable as the shapes of the distributions were not extreme. Figure 5 includes the scatter plots for each 1st-grade group, and Figure 6 includes the scatter plot for each 4th-grade group.

Figure 5

Scatter Plot for 1st-Grade

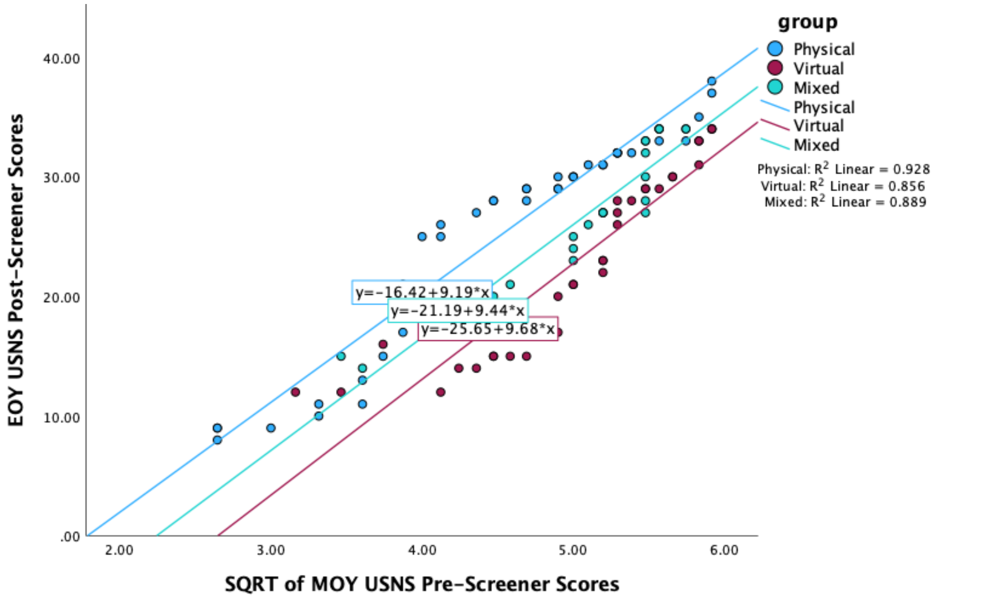
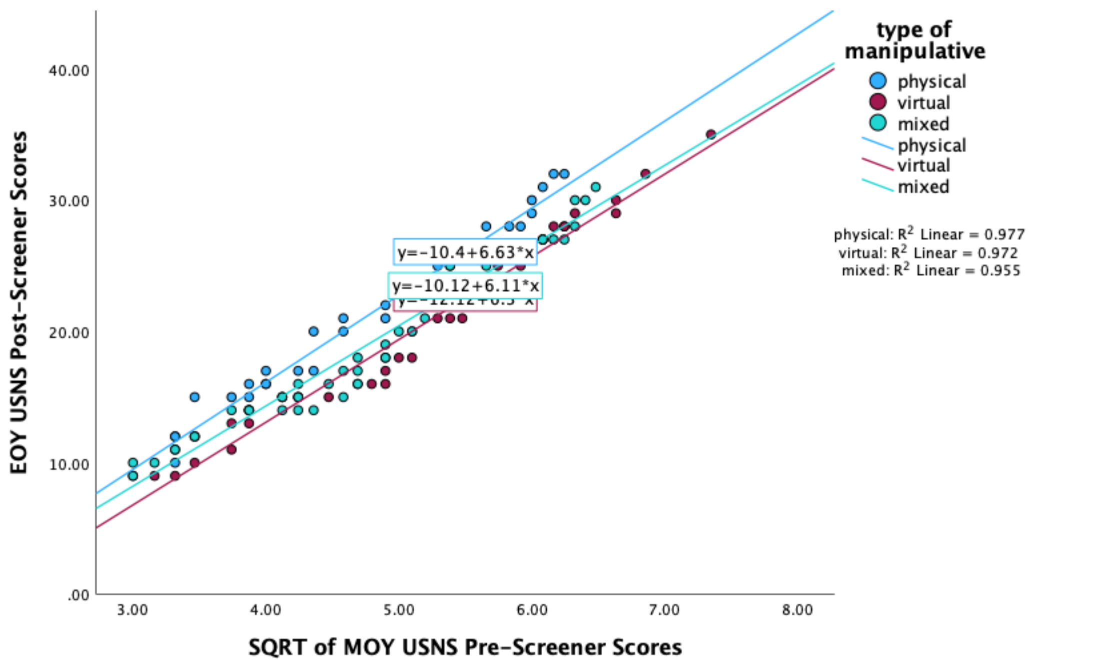


Figure 6

Scatter Plot for 4th-Grade



The assumption of homogeneity of slopes was tested and no interaction was found where $p = .249$. Therefore, the assumption of homogeneity of slope was met. The assumption of homogeneity of variance was examined using the Levene's test. No violation was found where $p = .809$ for 1st-grade and $p = .165$ for 4th-grade.

Results for Null Hypothesis

An ANCOVA was used to test the null hypothesis regarding the level of number sense on the USNS EOY screener 1st-grade and 4th-grade students have after being taught using physical manipulatives, virtual manipulatives, and a mixture of the two while controlling for pre-existing number sense on the MOY USNS screener. The alpha level was set at $\alpha = .05$. The null hypothesis was rejected at a 95% confidence level for both grade levels, where $F(2, 91) = 61.85$, $p < .001$, $\eta_p^2 = .576$ for 1st-grade and $F(2, 122) = 75.16$, $p < .001$, $\eta_p^2 = .552$ for 4th-grade. The effect size was very large for both groups, where $\eta_p^2 = .576$ for 1st-grade and $\eta_p^2 = .552$ for 4th-grade. Because the null was rejected in both cases, post hoc analysis was conducted for both grade levels using a Bonferroni test.

In 1st-grade, there was a significant difference between the Physical group ($M_{adj} = 28.08$, $SE. = .41$) and Mixed group ($M_{adj} = 24.49$, $SE. = .55$) and a significant difference between the Physical group ($M_{adj} = 28.08$, $SE. = .41$) and Virtual group ($M_{adj} = 21.24$, $SE = .45$). There was also a significant difference between the Mixed group ($M_{adj} = 24.49$, $SE. = .55$) and the Virtual group ($M_{adj} = 21.24$, $SE = .45$). See Table 14 for Multiple Comparisons of 1st-Grade Groups.

Table 14

Multiple Comparisons of 1st-Grade Groups

Pairwise Comparisons

Dependent Variable: EOY USNS Post-Screener Scores

(I) group	(J) group	Mean Difference (I-J)	*SE*	Sig.[b]	95% Confidence Interval for Difference[b] Lower Bound	Upper Bound
Physical	Virtual	6.842*	.616	<.001	5.340	8.343
	Mixed	3.597*	.693	<.001	1.907	5.287
Virtual	Physical	-6.842*	.616	<.001	-8.343	-5.340
	Mixed	-3.245*	.698	<.001	-4.947	-1.543
Mixed	Physical	-3.597*	.693	<.001	-5.287	-1.907
	Virtual	3.245*	.698	<.001	1.543	4.947

Based on estimated marginal means

*. The mean difference is significant at the .05 level.

b. Adjustment for multiple comparisons: Bonferroni.

In 4th-grade, there was also a significant difference between the Physical group (M_{adj} = 22.27, *SE.* = .20) and Mixed group (M_{adj} = 20.06, *SE.* = .17) and a significant difference between the Physical group (M_{adj} = 22.27, *SE.* = .20) and Virtual group (M_{adj} = 18.97, *SE* = .18). There was again a significant difference between the Mixed group (M_{adj} = 20.06, *SE.* = .17) and the Virtual group (M_{adj} = 18.97, *SE* = .18). See Table 15 for Multiple Comparisons of 4th-Grade Groups.

Table 15

Multiple Comparisons of 4th-Grade Groups

Pairwise Comparisons

Dependent Variable: EOY USNS Post-Screener Scores

(I) type of manipulative	(J) type of manipulative	Mean Difference (I-J)	*SE*	Sig.[b]	95% Confidence Interval Difference[b] Lower Bound	Upper Bound
physical	virtual	3.303*	.273	<.001	2.641	3.965
	mixed	2.209*	.265	<.001	1.567	2.851
virtual	physical	-3.303*	.273	<.001	-3.965	-2.641
	mixed	-1.094*	.253	<.001	-1.709	-.480
mixed	physical	-2.209*	.265	<.001	-2.851	-1.567
	virtual	1.094*	.253	<.001	.480	1.709

Based on estimated marginal means

*. The mean difference is significant at the .05 level.

b. Adjustment for multiple comparisons: Bonferroni.

CHAPTER FIVE: CONCLUSIONS

Overview

In a post-COVID age of increasing technology usage (Livy et al., 2022), it is crucial that teachers have the ability to make informed decisions about the tools they utilize in their instruction. This chapter will discuss the results of this study which sought to determine the impact of different types of math manipulatives that were previously shared in Chapter 4 as well as the implications these findings have on education moving forward. Limitations to validity will be explored, and the chapter will conclude with recommendations to further this research in the future.

Discussion

The purpose of this quantitative, quasi-experimental nonequivalent control group design study was to determine if the type of math manipulative 1st-grade and 4th-grade students were taught with impacted their number sense according to their Universal Screeners for Number Sense (USNS) scores. The research question was "Is there a difference in number sense between 1st- and 4th-grade Title I students who use physical manipulatives in isolation, students who use virtual manipulatives in isolation, and students who use a combination of virtual and physical manipulatives when controlling for pre-test scores?" The results for this question from both grade levels were very similar. Therefore, they will both be discussed in this section.

Physical Manipulatives

The number sense of students that were taught mathematics using only physical manipulatives was higher than the number sense of students in both of the other treatment groups, even when controlled for the covariate of pre-existing number sense, as measured by the square root of MOY USNS scores. The positive impact of math manipulatives has already been established (Chiphambo et al., 2020; Day & Hurrell, 2017; Delport, 2021; Fakih, 2023; Pires et al., 2019; Roberts et al., 2020; Tjandra, 2023; Watt-Douglas & George, 2021). However, this study confirmed the superiority of tangible materials specifically for students with low socioeconomic status. These findings also align with the constructivist theories of both Piaget (1964, 1967) and Fischer and Yan (2002).

The entire premise of constructivism as a theory is that students learn through hands-on experiences (Harlow et al., 2006). Piaget (1964) believed that children begin to internalize ideas over time, meaning that they are able to carry out actions in their minds when they are repeatedly exposed to them through hands-on experiences. He stated that students must have prior experience in order to absorb and assimilate new content. Once concepts are internalized, continuing to practice them allows them to become interiorized, which is the ultimate goal. When this happens, students are able to work through a concept without needing to even replay actions in their minds. Interiorization is required to achieve the goal of mathematical fluency. The results of this study are in alignment with these learning theories (Piaget, 1964, 1967; Fischer and Yan, 2002) as it was clear that students who had access to physical manipulatives and hands-on experiences developed higher levels of number sense. The tangible materials were a significant support for students in the learning process. These results match the results of other studies on physical versus virtual manipulatives (Nikiforidou, 2019). However, other studies did not incorporate a third group of mixed manipulatives, so the results of this study, which showed that the group using only physical manipulatives developed stronger number sense than even the mixed group is important information for the field of education.

Mixed Manipulatives

While the mixed manipulatives groups performed lower than the physical manipulatives group, they had higher number sense than the students taught using solely virtual manipulatives. Even after controlling for differences in the pre-existing number sense of each experimental group, students taught with mixed manipulatives had higher number sense with statistical significance. Several studies found that virtual manipulatives were more beneficial for particular mathematics domains than physical manipulatives (Flevares et al., 2021; Gecu-Parmaksiz & Delialioglu, 2019; Munfaridah et al., 2021). Pire et al. (2019) even theorized that each manipulative type, physical or virtual, may serve a different purpose. Therefore, it was surprising that this group did not perform the highest because the teachers of these experimental groups had the unique advantage of being able to access the strengths of both types of manipulatives. Since the physical manipulatives group had the highest number sense for both grade levels, it is imperative that all elementary students have access to physical math manipulatives in the classroom.

Virtual Manipulatives

Students in both the 1st-grade and the 4th-grade virtual manipulatives groups scored the lowest on the EOY USNS of the three groups. This finding contrasts with several studies that found virtual manipulatives to be more beneficial (Flevares et al., 2021; Gecu-Parmaksiz & Delialioglu, 2019). However, each of these studies focused on a particular domain or skill, such as probability or place value, and this study was focused on number sense in mathematics at large. This provides a more comprehensive view of the impact these manipulatives had. Russell et al. (2020) noted concerns teachers had with managing the implementation of manipulatives in the classroom. Managing virtual manipulatives was also a concern expressed by one of the 4th-grade teachers in this study who observed that students were often switching tabs and playing games when they were supposed to be using virtual manipulatives during work time.

Day and Hurrell (2017) questioned whether virtual manipulatives can support learners in conceptual exploration as well as physical materials can. They did not for the learners in the Title I schools in this study. While Bouck et al. (2021) and Yakubova et al. (2023) stated that virtual manipulatives might increase equity for students with learning disabilities, this was not the case for the general population of students in the present study. Although special education students were included in this study, as shown in Table 1, these students did not comprise the majority of participants. When addressing the needs of the whole class, the results of this study show that virtual manipulatives were not as effective as physical ones. This is not to say that virtual manipulatives do not have their time and place in unique scenarios. For example, Yakubova et al. (2023) described a student who was mostly nonverbal. In this case, the use of virtual manipulatives allowed the teacher to “see” their thinking and provide more targeted instruction. However, for the majority of Tier I instruction in Title I buildings, physical manipulatives are the strongest choice based on the results of this study. Utilizing the most impactful materials can help schools work toward closing the low socioeconomic status achievement gap in mathematics.

Ages and Stages of Development

A key, unique aspect of this study was that it addressed two grade levels representing different stages of Piaget’s (1964, 1967) theory of cognitive development. It is important to consider the differences between the ages of the participants while also studying the similar impact that the same types of manipulatives had on both groups. First-grade students would be considered pre-operational, according to Piaget (1964, 1967). Students at this age are able to internalize concepts by re-imagining their past experiences and actions (Norton et al. 2018, Piaget, 1964, 1967). Providing students with hands-on experiences with physical manipulatives in the classroom had a positive impact on their number sense development. While students developed better understanding with the use of only physical manipulatives, the use of virtual manipulatives, when combined with those physical tools, was still more beneficial than teaching 1st-graders math with solely digital resources. This is an important finding because even if Title I teachers need to incorporate digital manipulatives, there is empirical evidence to inform them that if they can at least use physical manipulatives some of the time, this combination of both types of manipulatives will still be more beneficial for students than only using the virtual materials.

Piaget (1964, 1967) would label most 4th-grade students as being in the concrete operational stage of development. While these students are able to think logically, they still require tangible objects to help develop their understandings. At this age, they cannot process logically with only language and verbal direction. They need objects and visuals to support this process, thus, supporting the importance of using math manipulatives in elementary classrooms. Providing students with tangible learning materials proved to be more impactful with this age group as well. It is worth noting that while the difference between experimental groups was statistically significant for both grade levels, the numerical differences between each of the 1st-grade groups was larger than the numerical differences between each of the 4th-grade groups. When comparing adjusted means between 1st-grade groups, physical manipulative groups scored an average of 6.842 points higher than virtual groups and 3.597 points higher than mixed groups. In 4th-grade, the difference in adjusted means between physical groups and virtual and mixed groups was 3.303 points and 2.209 points, respectively. This supports Olympiou and Zacharia's (2018) findings that age may play a factor in the effectiveness of virtual manipulatives because the older students are, the larger their bank of experiences to draw upon. As Piaget (1964) stated, the interiorization of concepts requires memories of past physical experiences. Therefore, it is logical to expect that older students would be more capable of accessing a larger amount of memories while using virtual manipulatives in class.

Implications

Teachers across the world are faced daily with the decision of whether to utilize physical or virtual manipulatives with their students . It is vital that educators have the information they need to choose the most effective manipulatives to support struggling learners in developing their conceptual understandings (Burns et al., 2015; Karakonstantaki et al., 2018; Kerschen et al., 2018). Since virtual manipulatives are relatively new in popularity due to the COVID-19 pandemic (Livy et al., 2022), research is still emerging on their effectiveness. There are studies that support the use of virtual manipulatives and have found better results using them (Flevares et al., 2021; Gecu-Parmaksiz & Delialioglu, 2019), but there are also studies that say the opposite (Nikiforidou, 2019). Regardless, these studies have all been targeted to both a specific age, such as preschool students, and a specific concept, such as probability.

Prior to this study, there was a gap in the literature related to empirical data that compared physical manipulatives and virtual manipulatives and their impact on number sense. No studies that compared the two types of manipulatives spanned different stages of cognitive development. There had not been a study that incorporated the effectiveness of combining physical and virtual manipulatives as a comparative treatment. This has prevented teachers from knowing whether this method of instruction is a best practice or not. This study filled that gap by determining that, for both Title I 1st-graders and Title I 4th-graders, physical manipulatives impacted number sense more significantly than virtual manipulatives alone or the combination of physical and virtual manipulatives.

Elementary teachers now have quantitative data to support the decisions they make regarding the types of manipulatives they implement during math instruction. Since Title I students are less likely to have access to conceptual math instruction (Yu & Singh, 2018), which has contributed to the achievement gap, this is also an equity issue. Conducting a study focused on Title I students provides teachers in Title I buildings with data that is generalizable to their specific population. To close the achievement gap, Title I educators must support the development of number sense in their students, which would be best supported through the use of physical materials. Given that the positive impact was highest for physical manipulatives, the more physical materials that are used, the better instructional outcomes will occur for students. It is expected that using mixed materials would have less effectiveness, and using virtual materials only would be least effective based on these findings.

The literature on this subject is diverse, and it is clear that there is no one, right answer when it comes to manipulatives, other than that they should be used in some capacity (Chiphambo et al., 2020; Day & Hurrell, 2017; Delport, 2021; Fakih, 2023; Pires et al., 2019; Roberts et al., 2020; Tjandra, 2023; Watt-Douglas & George, 2021). There are unique advantages to both physical and virtual manipulatives and the dynamics of each student and class must be considered (Flevares et al., 2021; Gecu-Parmaksiz & Delialioglu, 2019; Nikiforidou, 2019). However, even when teachers in this study used a strategic combination of both physical and virtual manipulatives that played on each of their strengths, their students' number sense was lower than the number sense of students who explored all mathematical content with hands-on materials. Technology certainly has its advantages, but when it relates to basic math manipulatives in Title I, general education, elementary classrooms, it cannot replace tangible learning materials due to their positive impact.

Limitations

Selection bias is a potential threat to internal validity. Since students were already assigned to classes, and classes were selected by teacher willingness, the participants were not a truly random sample of the population. Also, some teachers were only comfortable participating in a certain treatment group. It is possible that these teachers were more comfortable with that type of manipulative, which could have impacted their implementation. Data was adjusted for the covariate of the square root of MOY USNS pre-screener scores, but other, extraneous variables could have impacted the results of this study. The researcher took steps to mitigate these effects, such as by providing teachers with explicit instructions on which manipulatives to use and when. This ensured that instruction in each classroom was comparable throughout the study. Some students that participated received math intervention outside of the classroom, but since this occurred in all participating schools, the results should not have skewed the data significantly.

Concerning external validity, this study occurred within one school district in Missouri. This parameter of the study may limit its generalizability to other school districts. To help mitigate this effect, the researcher utilized a diverse set of Title I schools within the district, as the district is one of the largest in Missouri. This can be seen in the demographic data in Table 1. Some schools had heavier populations of ELD students, some had higher free/reduced rates, some had more ethnic and racial diversity. It is also possible that these results could be impacted by time. As technology continues to advance and both teachers and students grow more comfortable with it, virtual manipulatives could become more effective. Additional studies in the future can help to determine if this happens and continue to support teachers in their decision-making.

Recommendations for Future Research

To further the information available to teachers in the field of mathematics education and manipulative implementation, the following are recommendations for future research:

1. Day and Hurrell (2017) believe that virtual manipulatives have a role as a learning tool but that they would be better as a bridge from physical objects to representational and abstract forms. A study that specifically looks for the outcome and impact of using physical manipulatives and *then* virtual manipulatives once students are ready for the transition would provide better insight as to the possible benefits of virtual manipulatives.
2. This study could be replicated with a different instrument. While the USNS is a strong tool to measure number sense, there are many ways to measure different aspects of mathematics. To draw a generalizable conclusion as to which manipulatives are most effective overall in math, one or more studies using other instruments would be beneficial.
3. Similarly, this study could be replicated over a larger time period. 10 weeks was enough time to provide results that were statistically significant even after controlling for the covariate of pre-existing number sense. However, it would be interesting to see the impact of these manipulatives over the course of an entire school year, from start-to-finish.
4. There were numerous pieces of literature that discussed the importance of coaching support being available to teachers while learning to implement math manipulatives, or any teaching strategy, effectively (Russell et al., 2020; Sanderson, 2021). Reiten (2021) stated that additional teacher professional development might be especially

useful for the implementation of virtual manipulatives since they are a newer tool. In this study, coaching support was not provided during this process. It could be worthwhile to determine if the results would be different if coaching support, or additional professional learning on manipulatives, was added for participating teachers.

5. Augmented reality is an emerging technology that provides many new and unique types of virtual interactions. Some of these tools do incorporate the movement of virtual objects and the actual, physical movement of body parts (Kang et al., 2020). Since this could allow for a different type of “hands-on” experience with virtual manipulatives, future studies might consider exploring the use of augmented reality in learning.
6. As mentioned in the limitations section, teachers will likely continue to become more comfortable with technology usage and virtual manipulatives in the coming years. Replication of this study in the future could produce different results.

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APPENDIX A

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------------------,

I am a numeracy interventionist and learning coach in the district, and I am conducting a study on math manipulatives with Title I students for my dissertation. I am looking for 1st- and 4th-grade teachers who are willing to utilize a particular type of manipulative on specified days of math instruction over a 10-week period, and your principal has allowed me to use your site for my study.

If you agree to participate, your students will be the participants, but you will be a crucial component of conducting the study. Consent forms will be sent home with all students in your class with the hope of receiving most of them back. I will screen students who have parental consent using the Universal Screeners for Number Sense (USNS). You would continue to teach math using Illustrative Mathematics (IM) with fidelity during the 10-week treatment. You would be committing to using either only physical manipulatives, only virtual manipulatives, or a combination of both (you would be told which to use for which lessons for consistency purposes) whenever IM calls for them. There may be supplemental manipulative usage involved but all instructions would be provided. After the 10 weeks of instruction, I would return and re-screen your students using the USNS.

You would be trained on all the in-depth details in a 30-minute Zoom meeting and compensated at the end of the study with a $50 gift card. Options for the Zoom meeting are either:

____

OR

____

If you are willing to help me with this study, please respond to this email letting me know which Zoom training you plan to attend and if you are willing to be randomly assigned to an experimental group or are only willing to be in a particular one.

I am happy to send you a copy of my proposal if you would like additional details. If you have any questions for me, please let me know!

Thanks for your consideration,
Haley Gullion

APPENDIX B

1st Grade Sample Lesson Plan

4th Grade Sample Lesson Plan

Universal Screeners for Number Sense Download

APPENDIX C

1. Discuss consent process
    a. Forms will be emailed
    b. Please print and send them home within 3 days
    c. If possible, incentivize their return (whether parents say yes or no, a returned form helps check them off the list)
    d. Keep track of who returns them and send additional copies home, if necessary
    e. I will collect them when I come to give the screeners
2. Sign up for a screening date
    a. On those dates, there will be a 30-minute block for me to administer the whole-class portion
    b. I will then pull students (with consent) individually to assess them
    c. Discuss data recording protocols, storing data safely, etc.
3. Review lesson plans and manipulative charts
    a. Ensure all teachers know where to find the tables, which experimental group they're in, and how to follow the protocol for the 10 weeks
    b. Emphasize the importance of consistency within treatment groups
    c. Copies of these tables will be emailed for convenience
4. Sign up for a post-screening date
    a. On those dates, there will be a 30-minute block for me to administer the whole-class portion
    b. I will then pull students (with consent) individually to assess them
    c. Data procedures will be the same
5. Remind teachers that $50 gift card compensation will be sent at the completion of this process
6. Any questions?
7. THANK TEACHERS FOR THEIR HELP WITH THIS!

APPENDIX D

LIBERTY UNIVERSITY.
INSTITUTIONAL REVIEW BOARD

January 8, 2024

Haley Gullion
Nathan Putney

Re: IRB Exemption - IRB-FY23-24-697 Virtual vs Tangible Manipulatives: A Quasi-experimental Study Comparing the Impact of Different Types on Mathematical Understanding

Dear Haley Gullion, Nathan Putney,

The Liberty University Institutional Review Board (IRB) has reviewed your application in accordance with the Office for Human Research Protections (OHRP) and Food and Drug Administration (FDA and finds your study to be exempt from further IRB review. This means you may begin your research with the data safeguarding methods mentioned in your approved application, and no further IRB required.

Your study falls under the following exemption category, which identifies specific situations in which human participants research is exempt from the policy set forth in 45 CFR 46:104(d):

Category 1. Research, conducted in established or commonly accepted educational settings, that specifically involves normal educational practices that are not likely to adversely impact students' op learn required educational content or the assessment of educators who provide instruction. This includes most research on regular and special education instructional strategies, and research on the of or the comparison among instructional techniques, curricula, or classroom management methods.

For a PDF of your exemption letter, click on your study number in the My Studies card on your Cayuse dashboard. Next, click the Submissions bar beside the Study Details bar on the S page. Finally, click Initial under Submission Type and choose the Letters tab toward the bottom of the Submission Details page. Final versions of your study documents can also be fou same page under the Attachments tab.

Please note that this exemption only applies to your current research application, and any modifications to your protocol must be reported to the Liberty University IRB for verification of continued exe You may report these changes by completing a modification submission through your Cayuse IRB account.

If you have any questions about this exemption or need assistance in determining whether possible modifications to your protocol would change your exemption status, please email us at [redacted]

Sincerely,
[redacted]
Administrative Chair
Research Ethics Office

APPENDIX E

[redacted] **Data Analytics and Accountability**

Research request: Virtual Versus Tangible Math Manipulatives: A Quasi-Experimental Study Comparing The Impact of Different Types on Mathematical Understanding

Requestor: Haley Gullion

Purpose: Doctoral - Dissertation Research

Date: October 26, 2023

Request: Approval Granted

Mrs. Haley Gullion,

I am happy to inform you that you have been approved for conducting your research study **on Virtual Versus Tangible Math Manipulatives: A Quasi-Experimental Study Comparing The Impact Of Different Types On Mathematical Understanding** has been approved. This approval is conditional on receiving permission from both the building principal and the classroom teacher from where your study will take place in.

Thank you,

[redacted]

Coordinator of Accountability

[redacted]

APPENDIX F

Dear Parents/Guardians,

As a doctoral student in the School of Education at Liberty University, I am conducting research as part of the requirements for a Ph.D. degree in curriculum and instruction. The purpose of my research is to see if the type of manipulative (physical, virtual, or a mix of both) teachers use impacts students' mathematical understanding of grade-level standards, and I am writing to inform you of my study. Math manipulatives are considered to include any tool, visual, or object that teachers use to help students build an understanding of mathematics.

Participants will be 1st-grade or 4th-grade students in Springfield Public Schools (SPS). The students will take a number sense screener. It should take approximately 10-15 minutes to complete the number sense screener, and this will occur during the normal school day. This screener will consist of a few questions students answer on paper independently, and a few questions that will be asked of them by a teacher, such as counting out loud, or solving an addition problem with colored counters in front of them. They will then receive 10 weeks of math instruction in their regular classroom. During the instruction, some classes will be taught using physical manipulatives, some will be taught using virtual, and some will be taught using both. After the 10 weeks, the students will be asked to take the screener one more time. This will again take about 10-15 minutes and take place during the normal school day. Screener results and scores will not be used for students' grades or anything else in class. Students will not place their names on the screeners, so I will not be able to link their responses to them. I will use the screener results to see if students performed better because of the type of manipulative their teacher used.

You are receiving this letter so you will be aware of the research project, but no additional steps are needed. Feel free to contact me if you have any questions.

Sincerely,
Haley Gullion
Numeracy Specialist/Learning Coach
[redacted]