QUASI-EXPERIMENTAL STUDY OF THE IMPACT SPACED PRACTICE AND RETRIEVAL HAVE ON MATHEMATICAL FACT FLUENCY IN THIRD-, FOURTH-, AND FIFTH-GRADE STUDENTS

by

Stefanie Louise Breneman-Smith

Liberty University

A Dissertation Presented in Partial Fulfillment

Of the Requirements for the Degree

Doctor of Philosophy

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ABSTRACT

The purpose of this quantitative, quasi-experimental nonequivalent control group study was to determine if there is a difference in mathematical fact fluency among third-, fourth-, and fifth-grade students with varying practice methods and spacing intervals while controlling for pre-test scores. The study filled the literature gap by examining the impact of each variable and their combined additive effect in a third-, fourth-, and fifth-grade student population. Given the importance of math fact fluency on overall math performance, finding the most effective means for building retention is imperative. The sample included 196 third-, fourth-, and fifth-grade students. Data collection was completed prior to the start of the intervention using the fluency instrument from the Mathematics Fluency and Calculation Tests. Following data collection, a two-way analysis of covariance was conducted to measure the differences among the retrieval strategies and the various spacing intervals. The Bonferroni post hoc analysis was used to look for sample group variances. Following data analysis, the main effects of treatment and time demonstrated statistical significance; therefore, null hypotheses one and two were rejected. The researcher was unable to reject the third null hypothesis. The study demonstrated that treatment and time can positively influence students' mathematical fact fluency. It is recommended that future researchers consider alternate levels of the variables, separate the mathematical operations to take a closer look at each independently, study the correlation between fluency and anxiety, and coordinate a longitudinal study looking at how fluency affects later performance.

Keywords: long- and short-term storage, mathematical fact fluency, rehearsal, retention, retrieval, spaced intervals, working memory

Dedication

This study is dedicated to all who supported me along the way. First and foremost, I dedicate this study to the glory of my Heavenly Father. This process tried my faith more than anything else. I have prayed fervently and trusted You to attend to each minute detail. Thank you for Your faithfulness.

Next, I dedicate this study to my husband. You were my biggest cheerleader when the idea of entering the doctoral program blossomed. You were there to encourage me when I had difficult professors or assignments. You pushed me when I wanted to quit. You gave up so much of yourself to keep our family running while I worked on my studies and coursework. I hope that I can be as supportive of you in future endeavors.

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I dedicate this study to my parents, who valued education and always pushed me to do my best. You established my foothold with an undergraduate degree; I am eternally grateful.

I dedicate this study to my colleagues and other teacher participants who have endured the implementation of the intervention. I literally could not have done this without you. I appreciate your sacrificing of yourselves to support my efforts. To all the student participants, you rocked the show. Thank you for your dedication.

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List of Abbreviations

Analysis of Covariance (ANCOVA)

Computer-Assisted Instruction (CAI)

Coronavirus (COVID-19)

Cover, Copy, Compare (CCC)

Heteroskedasticity-Consistent Standard Errors (HC3)

Incremental Rehearsal (IR)

Mathematical Fluency and Calculation Tests (MFaCTs)

National Council of Teachers of Mathematics (NCTM)

No Child Left Behind (NCLB)

Science, Technology, Engineering, Mathematics (STEM)

Simultaneous Prompting (SP)

Statistical Package for Social Sciences (SPSS)

Strategic Incremental Rehearsal (SIR)

United States (US)

CHAPTER ONE: INTRODUCTION

Overview

This quantitative quasi-experimental nonequivalent control group study explored the effect of various practice strategies on mathematical fact fluency. Chapter One provides a background of mathematical performance in the United States. The background includes a historical look at mathematics instruction, an explanation of its impact on society, and an overview of the theoretical basis for this study. The problem statement includes recent literature on rehearsal, retrieval, and spaced practice. The purpose and significance indicate how math fact retrieval and practice affect students' performance. The research questions are presented alongside a list of relevant definitions for the study.

Background

Despite a 1983 report published by The National Commission on Excellence in Education that stated the United States' (US) education system was in a state of demise, forty years later, the country has made few improvements and continues to rank behind other countries in performance (Hussar et al., 2020). In a 2018 report, US students 15 years of age trailed behind eight other countries in reading scores, 30 countries in math, and 11 countries in science. In *The Condition of Education*, fourth-grade math students in the United States scored 15th out of 64 countries, and eighth-grade students scored 11th out of 46 (National Center for Education Statistics, 2021). While not alarmingly behind, US students consistently trail behind Singapore, China, Korea, Russia, Ireland, and Austria. The more significant concern from this report is the disparaging gap between high- and low-achieving US students. Achievement gaps evolve from socioeconomic disparities, coronavirus (COVID-19) response, and diverse instructional practices (Crepeele, 2022). Socioeconomic struggles continue to be prevalent as the United States remains

34th out of the 35 industrial nations in terms of child poverty (Koppelman, 2020). Children born at the turn of the century experience income gaps of up to 40% among high- and low-income families, which translates into an achievement gap double that of Black and White students. According to Koppelman (2020), the negative correlation between income and achievement stems from a lack of resources and educationally motivated experiences, diminished access to nutrition and health care, home insecurity, and insufficient preschool opportunities. In addition to the pronounced achievement gap, when comparing high school graduation rates, the US lags behind the Czech Republic, Lithuania, Poland, Canada, and the Slovak Republic when comparing adult populations (Hussar et al., 2020).

In response to growing concern over the efficacy of the US education system, the National Governors Association Center for Best Practices and the Council of Chief State School Officers (2010) implemented the Common Core Standards. Central in those standards is an expectation that students will maintain a fluent level of mathematical fact retention. Recent studies indicate that many students need help to reach the necessary level of proficiency (Baker & Cuevas, 2018; *NAEP report card*, 2022). The *NAEP Report Card* (2022) report showed a five-point drop in the number of the property and operations category for fourth-grade students from 2019 to 2022. The number property and operations category includes math fact fluency. The same report noted a seven-point drop for eighth-grade students in the same category. These are the lowest scores in nearly two decades, demonstrating a dire need for intervention.

Historical Overview

A popular misconception among the general public is that conceptual math instruction is new. Conceptual math instruction began as early as the 1950s and 1960s in the early Cold War era when it became apparent that Americans were vastly unprepared for the new technological age (Herrera & Owens, 2001; Phillips, 2014). Americans were graduating with a general lack of training and preparation in math and science. Leaders quickly realized that new skills were needed if the US was to remain competitive (Herrera & Owens, 2001). These changes called for a less rigid instructional method, emphasizing reasoning over calculation (Phillips, 2014). The new approach recognized traditional arithmetic calculation as one means of problem-solving but credited other ways.

As early as the 1980s, literature regarding the use of calculators in mathematics education surfaced. While some initially scoffed that calculators would serve as a crutch diminishing students' basic problem-solving abilities, research demonstrated improved problem-solving and basic computation abilities (Hembree & Dessart, 1986). In addition, students reported more positive attitudes toward mathematics and reduced anxiety when using a calculator (Hembree & Dessart, 1986; Idris, 2006; Reys & Reys, 1987). Emphasizing math fact memorization and mental arithmetic while allowing calculator usage allows students greater flexibility in application and problem-solving (Wheatley, 1980). Most literature regarding calculator use in the classroom was published before 2000, likely given the boom of other educational technology; however, Padmi's (2020) study in Indonesia looked at developing a more positive student attitude towards calculators given their usefulness in the classroom. The consensus is that basic math facts must still be committed with automaticity and that students must understand fractions and be able to implement standard algorithms (Ball et al., 2005). Furthermore, teachers must deeply understand mathematics, use engaging instructional methods, and integrate mathematics problem-solving in real-world contexts. Calculators can and should be used to bolster students' computation, but caution must be exercised with younger students so that fluency development is not negatively affected.

Despite the push away from rote problem-solving and towards conceptual math instruction, poor implementation and inadequate teacher training in the new methods led to an unsuccessful implementation (Herrera & Owens, 2001). Teachers were undertrained, and the enthusiasm for conceptual approaches to understanding dissipated (Herrera & Owens, 2001; Phillips, 2014). Student achievement data was inconclusive concerning the effectiveness of the original new math program, which coined new math phrase, referring to the conceptually-based instruction that felt foreign to those trained in standard algorithmic instruction (Phillips, 2014). This new math would later be coined as the new new math in the early 2000s as the need for conceptually-based instruction surfaced again. Abandoning the new math approach forced educational leaders into a back-to-the-basic practice that surfaced and dominated through the 1970s and 1980s (Herrera & Owens, 2001). The success of US public education math instruction remains a debate.

As the US again fell behind other nations, the National Council of Teachers of Mathematics (NCTM) developed new standards to guide curriculum, evaluation, and teaching (Herrera & Owens, 2001). These new standards emphasized higher-order thinking and knowledge construction, thus shifting the instructional pendulum back (National Council of Teachers of Mathematics [NCTM], 1989). The new standards favored the constructivist approach to education, where children construct knowledge from their environment (Kamii & Joseph, 2004). Conceptually based instruction is grounded in Piaget’s three types of knowledge. Students build physical knowledge of objects around them, social-conventional knowledge such as naming devices, and logico-math knowledge of building relationships.

Conceptual instruction has driven much of mathematics instruction over the past two decades; however, it has yet to be accepted by the general public. Many parents were educated under the back-to-the-basics movement, grounded in clear algorithmic problem-solving, and do not understand many of the techniques and strategies taught today (Darragh & Franke, 2022; Remillard & Jackson, 2006). This lack of understanding became abundantly clear during the COVID-19 pandemic shutdown when many parents were forced to take a more hands-on approach to their children's education (Darragh & Franke, 2022). Coupled with negative public attention, teachers fought back, feeling ill-prepared and undereducated in implementing the new curriculum, commencing the math war (Kilpatrick, 2012).

Math fact fluency is a small part of mathematics instruction, and understanding is positioned between more traditional and conceptual instruction (Boaler, 2015). As many schools shift to a conceptual approach to math, basic rote mathematical processes are often overlooked (Malkus, 2021). However, despite the instructional system, some believe that essential math fluency is required for students to develop higher-level problem-solving and thinking skills (Baker & Cuevas, 2018). They propose that fluency in these basic skills opens the working memory to allow for higher-level thinking (Usai et al., 2018). While many argue in favor of math fluency, others claim it is overemphasized and that building true number sense is more critical (Boaler, 2015). They believe that fact practice creates increased student anxiety and lowers student's self-efficacy. With professionals on both sides of the argument, the debate over proper math instruction persists. Educational leaders need to abandon the thought process that "precision and fluency in the execution of basic skills in school mathematics runs counter to the acquisition of conceptual understanding" and recognize that "precision and fluency in the execution of the skills are the requisite vehicles to convey the conceptual understanding" (Wu, 1999, p. 1).

Society-at-Large

A 2005 National Research Council report showed that US science and math performance was declining (Suter & Camilli, 2019). Furthermore, US competitiveness in science, technology, engineering, and mathematics (STEM) declined. The past few decades have shown a flat line in US performance and growth in science and math. The adoption of the Common Core State Standards was intended to strengthen instruction and learning, create coherence across grade levels, and promote an equal emphasis on conceptual understanding, procedures, skills, and applications (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010; Suter & Camilli, 2019). The ultimate goal was to increase the competitiveness of STEM education in America and improve US educational rankings worldwide.

While a divide exists between skill-based and conceptual instruction, US educators should balance skills and understanding (Malkus, 2021; Wu, 1999). Wu (1999) maintained that many proponents favoring skills-based teaching and fundamental algorithmic problem-solving fail to realize that the conceptual understanding of the algorithm is needed for later success in mathematics. A balanced approach to mathematics instruction may be more beneficial. The balanced approach would include an equilibrium of conceptual approaches to teaching and skill-based practice, such as basic math facts.

Nelson et al.'s (2016) study demonstrated that increasing math fact fluency could improve a student's performance on state testing. These results show a general link between fact fluency and overall performance. In conjunction, studies show a correlation between math fact fluency and anxiety in students (Sorvo et al., 2017). Students as young as second grade demonstrate math anxiety even more than their fifth-grade counterparts, and one-third of all students reported anxiety over math calculations. The students with low arithmetic skills also demonstrated increased pressure about math-related situations. Researchers noted a direct correlation between increased math anxiety and poor skill proficiency. Math anxiety appears to be a compounding issue. Students who feel they lack the necessary skills for success have increased stress, and those with increased anxiety worry that they lack the necessary skills (Justicia-Galiano et al., 2017; Sorvo et al., 2017). Given the enormous influence of self-efficacy on student success, it is imperative that students build the necessary skills, such as math fact fluency, to reduce anxiety and develop their self-concept (Hattie, 2018).

Theoretical Background

In 1948, Benjamin Bloom proposed his hierarchy of cognitive skills called Bloom's taxonomy (Anderson et al., 2001). Bloom et al.'s (1956) original taxonomy was divided into three domains: cognitive, affective, and psychomotor. The taxonomy's initial six levels are as follows: knowledge, comprehension, application, analysis, synthesis, and evaluation. Under this system, knowledge, the ability to recall basic information, is the basis for all other levels. Lorin Anderson et al. (2001) updated and republished a new version of Bloom's taxonomy. Regardless of the version, knowledge or remembering remains foundational (Anderson et al., 2001; Bloom et al., 1956). Skills like math fact recall and fluency fall under the knowledge level, which provides the substructure for all other math computations and abilities.

In the 1960s, Atkinson and Shiffrin (1968) proposed the multi-store model of memory. Under their model, information is taken in through sensory modalities and deposited into short-term storage. Data moves from short-term to long-term storage through rehearsal and encoding (Ding et al., 2021). Solidifying math facts into long-term storage allows students an improved working memory or short-term storage for problem-solving and critical thinking (Allen-Lyall, 2018; Ding et al., 2021). If conceptual instruction aims to drive higher-level problem-solving, students must sustain a sound basis of math facts.

Problem Statement

Rehearsal is continued repetition, allowing individuals to keep information in their working memory (Oberauer, 2019). Many individuals participate in rehearsal through verbal repetition of information, which has been shown to improve memory and has been studied in various aspects of math fact fluency (McFarlane & Humphreys, 2012; Oberauer, 2019). For example, Burns et al.'s (2019) study examined intermittent rehearsal with third-, fourth-, and fifth-grade students. Intermittent rehearsal strategies involve slowly adding new materials as mastery of previous material is demonstrated, which is effective overall but progresses slowly. This study compared traditional drill versus intermittent rehearsal and found that intermittent rehearsal is more effective for retention but that the two methods are equivalent in efficiency. The researchers suggested an additional study to examine a sample of students with a broader range of mathematical abilities. Adams and Maki's (2020) study among elementary students looked at traditional drills, intermittent rehearsal, and intermittent rehearsal with visual cues. The researchers looked at the retention of mathematical facts the next day, the cumulative effect, and long-term retention. The traditional drill method seemed most efficient in facts learned per minute, and intermittent rehearsal demonstrated greater effectiveness for long-term retention. The researchers suggested another study utilizing a more significant number of practice sessions.

In contrast to rehearsal, retrieval is effective because it sparks learning by strengthening cues and creating neural pathways in the brain (Brown et al., 2014; Haebig et al., 2021). Retrieval means attempting to reinstate a "prior learning context" (Karpicke et al., 2014, p. 238). In other words, retrieval is "the act of retrieving knowledge from memory," and that act "has the effect of making that knowledge easier to call up again in the future" (Brown et al., 2014, p. 28). The cognitive effort required in retrieval helps memories to "consolidate into a cohesive representation in the brain" (Brown et al., 2014, p. 28).

Many retrieval studies examining spaced versus massed practice surfaced in recent literature. Spaced practice intervals provide a longer lag time between retrieval experiences, while massed practice provides limited time between experiences (Lyle et al., 2020; Nazari & Ebersbach, 2019). In contrast to spaced practice, mass practice is the lumping together of practice sessions with minimal intervals between them (Nazari & Ebersbach, 2019). For example, cramming the night before a test is a mass practice, while studying each night of the week leading up to a test is an example of spaced practice. Within the realm of spaced practice, researchers looked at fixed length intervals versus random length intervals (Çekiç & Bakla, 2019; Latimier et al., 2021). Fixed length intervals are spacings of equal distance, while random length intervals fluctuate in length (Latimier et al., 2021). Research points to mass practice as an effective means of short-term retention (Foot-Seymour et al., 2019). Evidence demonstrates that the spaced method benefits long-term retention (Gordon, 2020). Students retain more with spaced practice (Latimier et al., 2021).

Additionally, delayed retrieval strategies are more impactful than immediate retrieval (Haebig et al., 2021). While several studies looked at spaced versus massed practice, they utilized alternate spacing schedules rather than the proposed ones. Schutte et al.'s (2015) study examined practices spaced throughout the day, while Codding et al. (2019) looked at one-week spacing. As Codding et al. suggested, additional studies should examine varying measures within spaced practice. A more extensive set size might better delineate the impact of spaced practice in a larger population and within a context that more closely resembles a natural school routine.

Schutte et al. suggested further examining longer spacing intervals within longer intervention durations. The problem is that the literature has not addressed the influence of the various practice strategies, alternate spacing intervals, and the additive effect of each on math fact retention in elementary students when controlling for pre-test scores.

Purpose Statement

The purpose of this quantitative, quasi-experimental nonequivalent control group study was to determine if there is a difference in mathematical fact fluency among third-, fourth-, and fifth-grade students with varying practice methods and spacing intervals while controlling for pre-test scores measuring a combined fluency, including addition, subtraction, multiplication, and division. In this study, third-, fourth-, and fifth-grade students participated in nonequivalent control groups based on their previously established class and teacher.

This study had two independent variables. The first variable of the study was the practice method, comprised of two groups: rehearsal and retrieval strategies. Half of the students in the study practiced math fact fluency using rehearsal methods. Rehearsal methods are defined as maintaining information through repetition (Glenberg et al., 1977). The other half used retrieval strategies. Retrieval refers to recalling a previously learned fact (Karpicke et al., 2014).

The second independent variable for this study was various practice spacing intervals. One-third of the sample participated in everyday practice, a third practiced every other day, and the final third practiced every third day. The design allowed for six distinct groups: rehearsal every day, rehearsal every other day, rehearsal every third day, retrieval every day, retrieval every other day, and retrieval every third day.

The dependent variable for this study was student mathematical fact fluency. The researcher controlled which mathematical facts students practiced and designed the experiment to make practice activities similar. The researcher controlled the length of each practice session. Finally, the student's prior ability was considered and controlled through the covariate of a pre-test. The completed study measured the effect of the independent variables on mathematical fluency in a sample of third-, fourth-, and fifth-grade students.

Significance of the Study

Various studies examined spaced and massed interval practice. At this time, many studies explored the benefits of spaced practice on language-based skills. For example, Schmitt et al. (2017) conducted a language acquisition study. In addition, Gordon (2020) conducted a study focused on word learning. Çekiç and Bakla (2019) looked at vocabulary acquisition. Peterson-Brown et al.'s (2019) study looked at the development of mathematics vocabulary through spaced and massed practice. Additionally, many studies on spaced versus massed practice investigated older students and adult learners (Hopkins et al., 2016; Lyle et al., 2020; Pagán & Nation, 2019). The proposed study added to the literature by providing data on spaced practice and acquiring math fact fluency in elementary-aged students.

Several studies examined math fact fluency. Burns et al.'s (2019) study measured incremental retrieval and traditional drill. Incremental retrieval created more long-term retention success, while traditional drill increased the facts completed per minute. Unfortunately, this study looked at a very targeted population of students who struggled in math and did not look at overall math fact fluency. In addition, Adams and Maki (2020) studied incremental retrieval and traditional drill and found that incremental retrieval was better in maintenance over one week, while traditional drill increased short-term retention. The results of this limited study were inconclusive.

Emeny et al.'s (2021) study looked at spacing intervals of one week and found the spaced practice to be more effective than massed practice. Nazari and Ebersbach (2019) examined massed practice with three sessions in one day versus spaced practice with one practice each day for three days with a population of third-grade students. Schutte et al.'s (2015) study of third-grade students looked at spacing intervals within a school day. Some students practiced once a day, while others practiced two, three, or four times in one day. While this study looked at math fact fluency, the spacing intervals all fell within one day. Codding et al.'s (2019) pilot study of second- and third-grade students examined spaced versus massed practice; however, given the nature of the pilot study, the intervention and measurement were only completed once. This current study added to the literature by providing data for an extended period.

Latimier et al.'s (2021) meta-analysis study directly pointed out a need for cross-design research to examine the impact of retrieval practice and spacing intervals, suggesting that a study of this type would look at the additive effect of each variable on math fact fluency. This study filled the gap in the literature regarding the interplay of retrieval practices and spaced intervals on mathematical fact fluency.

Research Question

RQ1: Is there a difference in math fact fluency among third-, fourth-, and fifth-grade students who use alternate practice methods based on various spacing intervals when controlling for pre-test scores measuring addition, subtraction, multiplication, and division fluency?

Definitions

1. *Conceptual Understanding* - Students "draw on knowledge from a wide variety of mathematical topics, sometimes approaching the same problem from different mathematical perspectives or representing the mathematics in different ways" (NCTM, 2000, p. 3).
2. *Conceptually-Based Instruction* - Teachers guide students to explore and make conclusions based on evidence by using several different techniques. Students work under the guidance of their skilled teacher and internally develop their own problem-solving abilities (NCTM, 2000).
3. *Elaborative Rehearsal* - "Learners are involved in constructing generalizations, thinking of personal examples and applications, and responding…on personal levels" (Simpson et al., 1994, p. 267).
3. *Exposure* - Multiple opportunities to interact with content; however, not in a retrieval-based activity. An example is the cover-copy-compare (CCC) method, where students look at an answer to a problem, cover it, copy it down, and then compare it (Stocker & Kubina, 2017).
4. *Fluency* – Fluency is "automaticity with basic facts" (Bray & Maldonado, 2018, p. 92).
5. *Higher-Level Thinking* - The skills of higher-level thinking are defined by the analysis, evaluation, and creative dimensions of Bloom's taxonomy (Anderson et al., 2001).
6. *Long-Term Storage* – Long-term storage is an unlimited, organized storage system that indefinitely maintains semantic, auditory, and visual information (Craik & Lockhart, 1972).

7. *Massed Practice* – Massed practice has few to no rest periods in between (Nazari & Ebersbach, 2019).
8. *Math Fact* – Math facts are single-digit problems (Baker & Cuevas, 2018).
9. *Rehearsal* – Rehearsal is a process that maintains information through rote repetition (Glenberg et al., 1977).
10. *Retention* – Retention is content storage through long-term memory (Rosen-O'Leary & Thompson, 2019).
11. *Retrieval* - Retrieval is "attempting to reinstate a prior learning context" (Karpicke et al., 2014, p. 238).
12. *Short-Term Storage* – Short-term storage is a limited, temporary storage system that holds information in phonemic, visual, and semantic formats (Craik & Lockhart, 1972).
13. *Spaced Intervals* – Spaced intervals are "temporal interval[s] (or lag[s]) between instances of retrieving the same information" (Lyle et al., 2020, p. 278).
14. *Working Memory* - Working memory is a limited capacity resource (Miller-Cotto & Byrnes, 2020).

CHAPTER TWO: LITERATURE REVIEW

Overview

This systematic literature review explored the problem of insufficient mathematical fact fluency amongst elementary students and the impact that spaced practice can have on long-term retention. This chapter presents a review of the literature on the topic. The first section discusses Bloom’s taxonomy related to the need for basic knowledge recall as a foundation for other cognitive skills. Next, it explores the theory of the multi-store memory model as the topic of rehearsal, particularly elaborative rehearsal. In addition, the chapter includes a review of current literature on fact fluency building to show the various methods that have already been studied. The literature review also explores traditional rote study, computer-based review, explicit instruction, and peer tutoring. This literature review reveals the literature gap regarding the spacing and timing of practice sessions and the use of elaborative rehearsal for retention.

Theoretical Framework

Math fact recall and fluency are introductory-level skills, requiring the permanent etching of basic facts and their coordinating answers into long-term storage and the ability to retrieve those quickly and efficiently (Solomon & Mighton, 2017). Bray and Maldonado (2018) defined fluency as “automaticity with facts” (p. 92). The process by which this permanent etch is created focuses on fact fluency and closely ties it to Atkinson and Shiffrin’s (1968) multi-store memory model. Additionally, despite its rudimentary nature, fact fluency is foundational for other mathematic skills when considering Bloom et al.’s (1956) taxonomy. Atkinson and Shiffrin’s multi-story memory model and Bloom et al.’s taxonomy of educational objectives provided the framework for this study.

Bloom's Taxonomy

At the 1948 American Psychological Association Convention, a group of psychologists, including Benjamin Bloom, brainstormed a classification system that would revolutionize instructional rigor for decades (Anderson et al., 2001; Bloom et al., 1956). These scholars aimed to provide a common language by which educators could classify instructional goals for learning and assessment. The creators designed the new framework as a taxonomy to show the contingent order. Bloom et al.'s (1956) taxonomy system was divided into three domains: cognitive, the basis for most curricular work; affective, centered on values and interests; and psychomotor, concerned with motor abilities. Bloom et al. focused on creating a neutral, logical, and consistent system to classify the intended behaviors of students, not creating a structure to assess teacher instruction. These scholars aimed for a comprehensive system that would provoke true thought, be communicated clearly, aid in identifying suitable instructional materials, and be widely used and accepted in the education field to ensure the purpose of their work.

Bloom et al.'s (1956) original taxonomy consisted of six cognitive levels: knowledge, comprehension, application, analysis, synthesis, and evaluation. At its most basic level, knowledge involves cognitive recall of specific bits of information, such as terms, facts, conventions, trends, sequences, classifications, categories, criteria, methods, and principles. While knowledge is located at the bottom-most layer, its presence is eminent in all the domains. The comprehension level, situated above knowledge, is where students must demonstrate a cognitive understanding of what is being communicated. Students must be able to translate, interpret, and extrapolate information to demonstrate successful comprehension. Under application, students must demonstrate cognitive understanding to apply information. Following application is the analysis stage, where students must break information down into parts. Success is measured by a student’s cognitive ability to classify elements, interpret the relationships among parts, and organize the parts. The last two levels are the most complex. Synthesis is a state of creation where students put pieces together. They use their cognitive creativity to communicate, generate a plan, or consolidate abstract relationships. Evaluation combines all lower steps in which students make cognitive judgments based on internal or external qualifications. Despite its wide acceptance, the original Bloom’s taxonomy was later updated (Anderson et al., 2001).

In 2001, Lorin Anderson revised Bloom's original design by separating a knowledge domain consisting of factual, conceptual, procedural, and metacognitive knowledge (Anderson et al., 2001; Radmehr & Drake, 2018). Additionally, the revised edition included separate processes that resemble the original design. The six processes are remembering, understanding, applying, analyzing, evaluating, and creating. The new system separates students’ knowledge and their skills or processes. Like knowledge, the remembering process involves recognizing and recalling. Understanding involves the skills of interpreting, exemplifying, classifying, summarizing, inferring, comparing, and explaining. Applying involves executing and implementing information; analyzing includes differentiation, organization, and attribution. In the updated system, creation and evaluation are transposed. Process five is evaluating and involves checking or critiquing, while process six is creating. Students are expected to generate, plan, and produce. Although similar, the original and revised taxonomies differ in various ways (Anderson et al., 2001; Bloom et al., 1956).

Fluent recall of basic facts is a knowledge level or remembering level skill. However, knowledge recall is required at all proceeding taxonomical levels. Quick recall of basic math facts provides the foundation that students need to move to higher levels of problem-solving, such as application or synthesis (Allen-Lyall, 2018; Baker & Cuevas, 2018). Given its foundational nature, it is vitally important that students develop a solid groundwork of facts that can be recalled efficiently.

Multi-Store Memory Model

Richard Atkinson and Richard Shiffrin (1968) developed the multi-store memory model. In their model, memory is a three-dimensional process. Information is accessed through the senses and becomes part of the short-term store. Visual intake is the most well-known sensory input. Information remains in the short-term store briefly, although the exact time is still debated. The information must undergo continuous rehearsal to remain in the short-term store. The final dimension of the multi-store model is the long-term store, where information remains permanently. The authors proposed that information can move between these overlapping stages. Vital to their model is the idea that information can be recalled from the long term to the short term store for use. Although they were unsure exactly how information moves from the short-term to the long-term store, they believed rehearsal and encoding were necessary, with encoding being the most effective. Encoding involves the process of creating new associations. These can help by lessening the information store, providing order, increasing the amount of information, and protecting against interference. Atkinson and Shiffrin’s short-term store would later become synonymous with working memory (Baddeley & Hitch, 1974).

Baddeley and Hitch (1974) expanded the idea of the short-term store, believing it to be much more complex than initially presented. The newer, more complex working memory system consists of a phonological loop where auditory information is taken in, a visuospatial sketchpad where images are stored, and a control system that oversees the cognitive processes (Baddeley et al., 2019). This system would be updated later to include backup storage known as the episodic buffer (Baddeley, 2000). While the multi-store model became famous for its proposed structure, it also received much criticism. Many research studies led to the evolution of what is known regarding memory and its structures (Atkinson & Shiffrin, 1968; Baddeley & Hitch, 1974; Craik & Lockhart, 1972; Miller, 1956).

Understanding memory is essential to identifying the most effective means for mathematics fact fluency building and retention (Burns et al., 2019). Understanding the best rehearsal and encoding strategies is essential to move facts from the short-term to the long-term store (Ding et al., 2021). In addition, moving basic math facts into the long-term store is essential to complex problem-solving (Baker & Cuevas, 2018). Efficiently accessing math facts from the long-term store generates more space in the working memory for advanced problem-solving (Allen-Lyall, 2018; Ding et al., 2021).

Related Literature

In 1981, the secretary of education formed a commission to evaluate and analyze the American education system. Following their inspection, the National Commission on Excellence in Education (1983) delivered a report that began with an alarming phrase: “Our nation is at risk” (para. 1). This report heightened the nation’s concern over the educational system by emphasizing the decline in global academic status. Despite legislation to improve the nation’s academics, minimal advancement has been made (Hussar et al., 2020; Suter & Camilli, 2019). Concerns persisted while math instruction shifted from a rote algorithmic basis to a conceptual model base. One area that warrants attention is the need for automatic fact fluency. Theoretically, quickly retrieving knowledge-based facts is vital for more complex skills (Bloom et al., 1956). However, finding a balance between rote practice and conceptual knowledge seems critical. While fluency continues to be essential, only some students are demonstrating fact automaticity (Baker & Cuevas, 2018; *NAEP report card*, 2022).

Lack of Fluency

While the *Nation at Risk* report by The National Commission on Excellence in Education (1983) stirred up concern for academic performance in the United States, few changes have been made to close the gap. Funding for STEM initiatives grew exponentially following the report's release, but funding plateaued quickly and began decreasing annually (Suter & Camilli, 2019). More recently, STEM funding has grown by marginal percentages but still falls short of budgeted requests annually (Peterson-Brown, 2022). One alarming indicator of America's sub-par performance is the lack of fact automaticity in math. Baker and Cuevas's (2018) study indicated that students demonstrated an overall proficiency rate of 64.9%, with specific grades attaining even less proficiency. While student automaticity increased in subsequent grade levels within this study, it still failed to meet the requirements of the Common Core State Standards (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). This study mirrored the results of an earlier study by Stickney et al. (2012), who studied a group of over 89,000 students in first through third grades. They found that fewer than 50% of the students met the automaticity required in the Common Core Standards, with struggling students reporting even less success.

While it is easy to speculate that drastic measures could help rectify this problem, a study of 1,651 schools showed that student progress is possible, but completely closing the achievement gap is highly unlikely (Bjorklund-Young & Plasman, 2020). In this study, only 9% of the schools reduced the achievement gap initially presented. Additional data shows that scores have flatlined over the past two decades (Richards, 2020).

In a recent analysis, as many as 40% of students in San Francisco were forced to repeat Algebra I, demonstrating a lack of achievement. Powell et al.'s (2019) study demonstrated that math fact fluency influences college algebra success secondary to a deep understanding of rational numbers. Furthermore, Helfand's (2006) article in the *Los Angeles Times* investigated the algebra dilemma and blamed bottom-up insufficiency. Elementary students move to middle school unprepared, and middle schoolers move to high school unprepared. While many students who fail algebra receive additional support, most fail to receive what is needed: remedial work in basic math. Many foundational arithmetic skills, including basic fact fluency, are required for success in Algebra and, thus, later performance in the STEM field (Harris, n.d.; *Math talk*, 2016).

Furthermore, the gaps between low and high-achieving students continue to widen, with the recent COVID-19 pandemic intensifying this effect (Mervosh, 2022). This lack of success makes one question whether the United States can ever bridge the global achievement gap. In addition to K-12 achievement concerns, inferior academic performance influences the United States workforce. The US workforce continues to lack workers skilled in STEM initiatives who graduate ready to tackle the ever-changing landscape (Belser et al., 2018; Jimenez, 2020; Suter & Camilli, 2019). The gap in educational institutions compounds itself as its impact spreads throughout the greater community in the general workplace. Although proficiency remains a struggle amongst students in the United States, the need for proficiency remains.

Need for Fluency

Even though The National Commission on Excellence in Education's (1983) *Nation at Risk* report did not generate sustained funding for education, specifically in the field of STEM, it did spark policy change (Suter & Camilli, 2019). President Bush's No Child Left Behind (NCLB, 2002) legislation provided additional school accountability. Replacing NCLB, President

Obama's Race to the Top initiative, and the Every Student Succeeds Act (ESSA, 2015) continued to pressure school systems to enact change (Koppelman, 2020). Further supporting these initiatives were the Common Core State Standards, created and first adopted in 2010 (Koppelman, 2020; Suter & Camilli, 2019).

Incidentally, school system changes made under pressure from outside sources are often quick, knee-jerk reactions. Brown et al.'s (2023) case study examined decisions made at the City University of New York, finding that reform leaders used only internal data and, in many cases, were data mining to further their own agendas. The researchers pointed out how data can be misrepresented or misinterpreted to push educational reform that may or may not be in the student's best interest. Some may say that as new educational initiatives are presented, educators are too quick to pendulum jump without looking at "where we've been, where we're at, and where we still need to go" (Lucas, 2013, p. 15). Educational change should be well researched, cautiously approached, and balance traditional practices with new and innovative trends.

The Common Core State Standards laid the groundwork for a more conceptually based approach to mathematics. This novel approach, the new new math, has been criticized significantly (Clements et al., 2019). The new new math is thought to be too academically focused, ignoring the student's developmental needs, and is misunderstood as fast-paced, scripted, and confusing (Clements et al., 2019; Remillard & Jackson, 2006). Conceptually-based math instruction has been poorly received amongst older adults, primarily parents, because of its unfamiliar language, focus on underlining meaning, acceptance of multiple methods to a solution, fast pace, and lack of repetition (Darragh & Franke, 2022; Remillard & Jackson, 2006). Most adults find conceptually based math difficult because it deviates from the strict, algorithmic methods they are accustomed to. While conceptual math fits the constructivist paradigm that dominates public education, a more modest shift may have been better (Schunk, 2020). McGee et al. (2017) found a balanced approach to be more effective in a study aimed at finding the relationship between fact drills and performance and between fact drills and conceptual understanding. They found no direct correlation between automaticity and proficiency or vice versa. They proposed a more balanced approach that paired rote memorization with conceptual instruction. Mathematical fluency, in its most basic form, is when students can utilize various strategies to solve calculations and fluidly and flexibly think about numbers (Thiele et al., 2021).

Regardless of the overarching instructional design, the importance of math fact automaticity is undeniable. Numerous factors influence mathematical performance, but fluency plays a role in student performance. Sorvo et al. (2017) found that poor skill proficiency corresponds to increased anxiety in a math anxiety and performance study. In a longitudinal analysis involving 1,327 students in second to fifth grades, researchers aimed to study the prevalence of anxiety in students, whether there was a correlation between anxiety about failure and anxiety about math, whether skills and anxiety were related, and whether these were dependent on gender or grade. Researchers found that females and younger students were more prone to anxiety. They also noted a correlation between failure anxiety and math anxiety, with math anxiety more closely linked to subpar performance. These results regarding fluency and anxiety closely resemble studies in other areas, such as English language learners. In a study of Chinese university students, Liu (2006) revealed a connection between increased English proficiency and reduced anxiety. Valizdeh’s (2021) study also found that university students who received instruction in English reading comprehension strategies reported decreased anxiety. Increasing skills increases self-efficacy (Celestine, 2019) and improves student performance (Hattie, 2018).

Similarly, Pantoja et al.'s (2020) correlational study found that anxiety can predict performance up to two years later, demonstrating that student emotions are just as important as cognitive development. In a study of 162 students, researchers tracked performance and emotional measures over three years. Researchers had students participate in several number-line activities and found a correlation between anxiety and performance. As anxiety increased, performance decreased. Additionally, as tasks were simplified, anxiety lowered, and performance increased. Given the correlations between anxiety and performance over the three years, the data shows not only a relationship between anxiety and ability but also that anxiety can predict future performance.

Many proclaim that math fact fluency increases students' problem-solving abilities by reducing the cognitive load required (Baker & Cuevas, 2018; Ding et al., 2019). Working memory is vital in higher-order problem-solving and computation. When students exhibit fact fluency, they free up the working memory to use the retrieved facts in a more complex problem-solving manner (Ding et al., 2019; Usai et al., 2018). Strong working memory improved performance on multistep problems; similarly, poor working memory was quickly overwhelmed in multistep problem-solving (Ding et al., 2019). Finally, studies have shown that automaticity can influence performance, even in higher education (Poast et al., 2021). If educational leaders want to improve students' performance and diminish the achievement gap between the United States and other nations, a more balanced approach that combines conceptual understanding and rote memorization may be necessary (Allen-Lyall, 2018; Nahdi & Jatisunda, 2020).

One may counterargue that fact fluency is trivial given the prevalence of calculation tools, like handheld calculators or even smartphone devices; however, these devices should not replace the automaticity of fact fluency or undermine the building of conceptual knowledge.

Despite their historical improvement in students' problem-solving and operational abilities (Ellington, 2003; Hembree & Dessart, 1986), a statement released by NCTM (2015) stated that calculators should not replace students developing fluency and calculation methods. Furthermore, calculators should be used to enhance and supplement problem-solving. Students must still build conceptual knowledge to use calculating tools correctly. In a study of college nursing students, researchers found that calculator use decreased errors in basic arithmetic but increased conceptual errors (Shockley et al., 1989). Similarly, in LaCour et al.'s (2019) study, researchers created a testing environment in which a calculator provided false answers. Very few students had sufficient conceptual understanding to recognize the errors. Few doubted the false answers because many lacked the proper procedures for solving fundamental problems. Calculator use does not negate the need for a solid conceptual foundation and basic computational abilities.

Methods for Building Fluency

As with most trends in education, fluency instructional practices change as educational theory changes. Traditional instruction of the late 20th century trended towards more traditional rote fluency methods, such as flashcards, CCC, taped problems, and timed drills (Mann et al., 2012; McCallum & Schmitt, 2011; Stocker & Kubina, 2017). The influx of technology into the classroom spawned the use of digitally based practices, such as online incentive programs and fact-based games (Berrett & Carter, 2018; Hawkins et al., 2017; Musti-Rao et al., 2015; Rich et al., 2017). Conceptually-based instruction has also recently promoted fact-based strategy instruction, collaborative review, and game-based practice (Greene et al., 2018; Karnes & Grünke, 2021; Morano et al., 2020; Sönmez & Alptekin, 2020). These conceptual strategies help students recognize relationships and patterns between numbers. Despite the chosen method, many researchers believe the most appropriate fact instruction includes a solid understanding of multiple computational strategies (Kling & Bay-Williams, 2021; Solomon & Mighton, 2017).

Experts insist that students be equipped with numerous strategies for computing basic facts and that teachers must be prudent to ensure mastery before moving further in fact instruction (Kling & Bay-Williams, 2021). Additionally, teachers are cautioned against teaching math facts in numerical order, as was done traditionally, but are encouraged to teach math facts in a logical, patterned way, beginning with more accessible factors, such as two and five, and proceeding to more challenging ones (Kling & Bay-Williams, 2021; Solomon & Mighton, 2017). Educators are discouraged from emphasizing speed at the onset of fact practice. Emphasizing speed belittles the importance of strategies and causes undue stress on students (Kling & Bay-Williams, 2021). Educators should consider these three principles: patterns, mastery, and speed, regardless of the chosen practice method, when planning fact instruction.

Traditional Methods. Many of the studies of traditional fact fluency methods are limited in scope. Most report small sample sizes and focus on students with intellectual or learning disabilities (Alptekin, 2019; Bjordahl et al., 2014; Lund et al., 2012; McCallum & Schmitt, 2011). One might assume that these targeted populations indicate the effectiveness of the strategies as they have demonstrated success despite extraneous circumstances. Flashcards are one example of a successful traditional practice method. Flashcards have improved students' fluency and accuracy (Bjordahl et al., 2014; Kromminga & Codding, 2021; Mann et al., 2012), with one study showing as much as a 28% increase in accuracy (Lund et al., 2012). Sleeman et al.'s (2021) study showed increased immediate and delayed fluency rates with students utilizing self-regulated learning practices reaching the highest achievement. Flashcards are a favored method, given their low cost and easy implementation (Bjordahl et al., 2014; Lund et al., 2012; Mann et al., 2012). Teachers find them time-efficient, practical, and easy to implement.

The CCC is a second traditional method to increase fluency (Alptekin, 2019; Stocker & Kubina, 2017). In this method, students look at a math fact and answer, cover it up, copy it from memory, and then compare it to the original. Like traditional flashcards, the CCC method builds upon rote practice. It, too, has demonstrated effectiveness in increasing student accuracy and fluency (Alptekin, 2019; Martin et al., 2019; Stocker & Kubina, 2017). The effectiveness of this method centers on repeated exposure to correct responses and immediate feedback (Alptekin, 2019; Stocker & Kubina, 2017). The easy and practical implementation makes it a popular strategy because students can quickly implement and assess themselves. Studies show that students have successfully maintained their fact knowledge over extended periods following a CCC intervention (Alptekin, 2019).

Another traditional fluency strategy is taped problems. Taped problems involve a student listening to recorded math facts and rushing to write the answer before it is announced on the recording. Like other traditional methods, students can complete this rote practice strategy independently, given its easy implementation and low cost (McCallum & Schmitt, 2011; McCallum et al., 2022; Poncy et al., 2012). Despite its ease of use, this strategy has manifested even more effectiveness than CCC (Poncy et al., 2012). Additionally, students successfully maintained their fact knowledge over time (McCallum & Schmitt, 2011). Although some may consider these outdated strategies, their ease of implementation makes them popular.

Digital Methods. Access to classroom digital devices has increased exponentially over the past two decades. Schools have adjusted away from weekly computer room visits to daily, integrated use of technology for each student through online learning. As many schools have adopted a one-to-one policy for each student, one category of online learning, computer-assisted instruction (CAI), has become more prevalent. CAI is a relatively new and potentially effective method for delivering and monitoring instruction via technology (Berrett & Carter, 2018; Outhwaite et al., 2019). CAI is quickly becoming a popular teaching method in many areas, including mathematics instruction and math fluency practice. Online digital programs are similar to traditional methods in a few aspects. First, they provide immediate feedback, which is vital to retaining math facts (Berrett & Carter, 2018; Hawkins et al., 2017; Kromminga & Codding, 2021). Second, like traditional rote methods, online tools typically provide multiple opportunities for repeated practice (Berrett & Carter, 2018; Hawkins et al., 2017).

While similar to more traditional methods, CAI programs also afford specific advantages over traditional methods. Unlike more conventional methods, CAI programs provide teachers with progress monitoring reports to track student success (Berrett & Carter, 2018; Elmore, 2019; Hawkins et al., 2017). Additionally, many online programs are adaptive and can provide differentiated instruction for each student (Berrett & Carter, 2018; Hawkins et al., 2017). Teachers can adjust preferences as students advance with the click of a few settings, which will alter each student's feedback based on their needs (Musti-Rao et al., 2015). Typically, these customizable features and implementation ease make digital methods preferred (Musti-Rao & Telesman, 2022). Digital programs are praised for being more engaging and enjoyable for students (Berrett & Carter, 2018; Bouck & Long, 2022; Hawkins et al., 2017).

While studies have shown digital CAI programs to be effective in increasing students' fact fluency and maintaining fluency over time (Berrett & Carter, 2018; Bouck & Long, 2022), educators are cautioned to use CAI programs as supplementary and not as a replacement for traditional teacher-led instruction (Musti-Rao et al., 2015; Rich et al., 2017). Students should receive ample training in computer-based programs before implementation, and teachers must maintain strategy-focused instruction based on effective pedagogy for true success (Basar et al., 2021; Maqableh & Alia, 2021; Musti-Rao et al., 2015). Following the COVID-19 pandemic, which shut down educational institutions, many students prefer traditional learning and fail to see a benefit to online learning, including CAI programs (Basar et al., 2021). Many students find motivation to be an issue with online learning. Thus, teachers are encouraged to balance computer-based and traditional paper-pencil practice as students often struggle to transfer skills across different modalities (Basar et al., 2021; Rich et al., 2017). As with most elements of instruction, a healthy balance of methodologies and tools is critical.

Additional Methods. Just as digitally-based practices have slowly encroached upon traditional fact fluency strategies, more constructivist approaches have replaced traditional, rote memorization. As education changes towards a more conceptually based system, the suggested instructional strategies shift; however, despite this shift, research still promotes an integrated, balanced approach between automaticity and strategy focus (Allen-Lyall, 2018; Morano et al., 2020). While proficiency practice helps build fluency and automaticity, strategy practice leads to better application in problem-solving; combining and alternating these two maintains students' interest and motivation (Morano et al., 2020). All fluency-building instruction must be timed in short spurts, be goal-directed, promote accuracy, and drive instructional decisions (Datchuk & Hier, 2019; Morano et al., 2020). The four key components for mastery are flexibility, appropriate strategy use, efficiency, and accuracy (*Math fact fluency*, 2023). Building these components within students ensures proper internalization of the instructed skills.

In addition to traditional and digital fluency methods, newer strategies are being developed and researched. Each of the latest techniques reflects a progressive and constructive philosophy. The first of these strategies is peer and cross-peer tutoring. Tutoring is often an effective method. Peer tutoring is “one student helping another to learn and master some aspect of the curriculum” (Karnes & Grünke, 2021, p. 3). Peer tutoring requires the tutor, someone with knowledge, to impart an explanation to the tutee, someone lacking knowledge (Greene et al., 2018). This collaborative spirit makes tutoring effective (Karnes & Grünke, 2021). Tutoring is often a method of choice given its ease of implementation and the enjoyment and confidence it provides students (Greene et al., 2018; Karnes & Grünke, 2021). When used with typical or struggling students, tutoring can be highly effective (Greene et al., 2018), with one study boasting as high as 600% increases in student accuracy and fluency (Karnes & Grünke, 2021).

Like tutoring, another study looked at a 10-week intervention program where third-grade students participated in various hands-on, cooperative activities to build fact fluency through strategy instruction (Allen-Lyall, 2018). In addition to the active engagement and a confidence boost for students, this strategy had an academic impact that continued into the succeeding school year, with students maintaining their fact knowledge into fourth grade. Research points to the effectiveness of combining strategic instruction with repetition (Morano et al., 2020). Strategic practice highlights patterns for building conceptual understanding, while repetition strengthens memory pathways. Another study examined the simple act of goal setting and its effect on student fluency (Sides & Cuevas, 2020). While the study showed no correlation between goal setting, student motivation, and self-efficacy, a statistical significance was found between goal setting and student fluency performance.

Another group strategy gaining popularity with the push of conceptual learning is the idea of number strings (Bray & Maldonado, 2018). Number string discussions are carefully constructed opportunities led by teachers. In these conversations, teachers use scaffolding to guide students in making connections within their number sense and strategy building. This conceptual approach encourages all students to participate by utilizing various strategies and has influenced overall fluency, understanding of numbers, their relationships, and manipulation (Bouck & Bouck, 2022; Bray & Maldonado, 2018).

Given the intense time and attention required, one final strategy best implemented on a case-by-case basis is simultaneous prompting (SP). Traditionally, a behavior modification technique, SP, was used in extreme cases to help students build fact knowledge (Sönmez & Alptekin, 2020). The teacher presented the facts, which the student immediately repeated, followed by positive reinforcement. The researchers combined corrective feedback, which is not typical for SP, and systematic review. While it is difficult to identify a direct link in this study given the multiple variables, participants made academic gains. As with other studies, the intellectual growth and solidification of facts were due to the continuous exposure to accurate facts (Alptekin, 2019; Sönmez & Alptekin, 2020; Stocker & Kubina, 2017). Student accuracy is driven by repeated exposure to correct answers versus attending to incorrect responses.

There are many factors to consider when planning instructional activities (Parkay et al., 2014). Educators must maintain an accurate picture of students' needs and abilities and match these to the available methods while making the best use of time and resources. Given the success of both rote memory and conceptual strategies, along with their unique purposes, it is suggested that teachers incorporate both. This balance often requires strategic planning but will provide the rehearsal necessary for long-term retention (Allen-Lyall, 2018; Malkus, 2021; McGee et al., 2017; Morano et al., 2020; Nahdi & Jatisunda, 2020).

Math Fact Rehearsal

One facet of memory that has received significant attention is the significance of rehearsal. By its very nature, rehearsal consists of repeating information to keep it within the confines of the working memory (Oberauer, 2019). Studies show that repeating information multiple times improves memorization (McFarlane & Humphreys, 2012; Samdura et al., 2019). At times, rehearsal is synonymous with rote repetition of information. While this is one type of rehearsal (Type I), other types exist. Type I, or low-level rehearsal, is a rote repetition helpful in maintaining information in the working memory when long-term storage is unnecessary (Craik & Lockhart, 1972; Glenberg et al., 1977). The problem with Type I rehearsal is that information can only remain in the working memory for as long as it is rehearsed (Craik & Lockhart, 1972). Proper data processing, including encoding and decoding, is required for information to move past the working memory, for "memory performance is a positive function of the level of processing required by the orienting task" (Craik & Lockhart, 1972, p. 678). Often, individuals will use auditory or articulatory repetition by repeatedly verbalizing the same things, likely because that is required to keep their attention focused on the current task (Oberauer, 2019). Unfortunately, this process has shown to be much more effective for children than adults. Historically, the removal of information from the working memory has been attributed to decay over time, but more likely, the decay is due to interruption from outside sources (McFarlane & Humphreys, 2012; Ricker et al., 2020). Studies show that mere rehearsal does not increase learning but delays forgetting (Craik & Lockhart, 1972).

Significant results have been published regarding the rehearsal of familiar versus unfamiliar information. One might assume that the rehearsal of familiar information is manageable while unfamiliar information is challenging. Research on the rehearsal of familiar and unfamiliar items is inconclusive. Some studies have found that rehearsal of familiar information increases accuracy and reduces false alarm rates (Bisson, 2022; Humphreys et al., 2010). While items of lesser familiarity require more intentional effort, generating greater attention and requiring a more significant cognitive load (Brown et al., 2014; McFarlane & Humphreys, 2012; Paas & van Merriënboer, 2020). Other studies have found that low-frequency or lesser-known information produces higher accuracy rates and knowledge gain despite the distraction of unfamiliar items (Humphreys et al., 2010; Yuan, 2022). Still, other studies indicate little to no difference in rehearsing familiar and unfamiliar items (Ricker et al., 2020).

One specific example of Type I rehearsal is incremental rehearsal (IR). While IR imitates traditional drills, such as flashcards, it supports introducing new information slowly and in a wildly disproportionate ratio to already learned information (Burns et al., 2019). For instance, if a student were practicing ten multiplication fact flashcards under IR, one would contain a new fact while the other nine were review material. Research shows that IR is more effective than traditional retention drills but not efficient, given its slow introduction of new material. Often criticized for its lack of efficiency, IR has also been compared with a sister strategy, strategic incremental rehearsal (SIR). SIR involves quickly adding new information based on students' correct responses (Kupzyk et al., 2011). IR and SIR have demonstrated positive impacts on student retention. Still, a study by Klingbeil et al. (2020) showed neither to be more effective than the other in retrieving or using learned words in generalized reading ability. The study demonstrated that long-term retention was superior with SIR strategies than with IR.

While Type I rehearsal can be effective for short-term storage, it has a negligible effect in the long term. Rehearsal without the ability to organize and make connections is useless (Tulving & Madigan, 1970). Associative or elaborative (Type II) rehearsal requires the formation of a memory trace (Glenberg et al., 1977). The more profound analysis of Type II rehearsal is created by focusing on connecting and finding meaning within the information (Craik & Lockhart, 1972; Oberauer, 2019). While some studies have shown slight memory improvement with Type II rehearsal (Oberauer, 2019), other researchers believe proper processing is required for memory to take shape (Craik & Lockhart, 1972).

In addition to mathematical studies, elaborative rehearsal has demonstrated effectiveness in word recall, vocabulary retention, and test review. Researchers in a study of struggling college students found that rehearsal quality increased test performance (Simpson et al., 1994). Although rote memory rehearsal was more effective for rote memory test items, test questions requiring higher-order thinking were most affected by elaborative rehearsal strategies. Spontaneous elaboration benefited student performance, and interestingly, when given more free time, students tended to engage in elaboration more frequently (Loaiza & Lavilla, 2021). Elaboration strategies include rephrasing, imagery, chunking, and mnemonic devices, and when used, they can lead to significantly better long-term storage, given better encoding (Heerema, 2022).

In several recent university studies, researchers looked at word recall and found that semantic rehearsal was much more effective than phonological when extended maintenance was required (Loaiza & Camos, 2018). Others found that elaborative strategies helped students learn vocabulary and retain information longer with greater motivation in an English language program (Elyas & Al-Zahrani, 2019). While it appears challenging to influence the natural neural processes of the working memory, several studies demonstrate that elaboration is much more favorable for long-term storage (Bartsch et al., 2018). However, Oberauer's (2019) study reported inconclusive results regarding its effectiveness. Researchers noted a weak connection and stated it is hard to separate and test elaboration alone, arguing that tests for elaboration point to effective long-term memory storage more than actual strategies.

A third memory technique, called refreshing, was recently introduced. This newer strategy focuses on bits and pieces of information (Oberauer, 2019). Its positive impact on retrieval and speed has given it notoriety. Unfortunately, refreshing is not a natural technique typical adults use; it requires cueing. Researchers found that spontaneous refreshing does not occur inherently with students (Veragauwe et al., 2021). Children tend to refresh only the last item presented, not the entire information.

Refreshing involves rethinking something while it is fresh in mind. It has been effective in retrieving refreshed information but can negatively affect the decay of the items not refreshed (Bartsch et al., 2018). Many find it challenging to separate refreshing and elaboration because refreshing is often needed for elaboration to occur. However, Bartsch et al. (2019) made clear distinctions between the two, confirming that elaboration positively influences long-term memory storage but not working memory and that refreshing increases memory. Refreshing is effective because it prioritizes information within the working memory realm. Additionally, the authors explained that a third strategy, repeating, is even more effective than refreshing. In a comparison study, refreshing showed a positive short-term impact only on refreshed words with no long-term implications. Elaborative rehearsal was effective for long-term retrieval but not short-term. These differing results indicate that refreshing and elaborative rehearsal are different.

Two additional findings resulted from the studies on rehearsal. First, studies on the impact of rehearsal led researchers to question whether working memory and long-term memory are separate entities (Atkinson & Shiffrin, 1968; Baddeley & Hitch, 1974; Loaiza & Camos, 2018). The argument in favor of the unitary model is that the semantic retrieval cue used in the working memory comes from a search for long-term memory, thus uniting the two parts as one functioning unit (Loaiza & Camos, 2018). Coffman et al. (2019) made an astonishing discovery in their longitudinal study regarding early classroom experiences and memory abilities in older students. The researchers demonstrated that memory strategies could be taught and that students can develop effective memory strategies that translate into more efficient memory in later years using cognitive processing language and cues.

Spaced Practice

One topic of particular interest in memory is the impact of retrieval methods and distributed practice on memory recall. Many believe that repetition and retrieval are necessary for authentic learning. According to Gordon (2020), to demonstrate authentic learning, some learned information must be retained until the next learning opportunity so that it “can be retrieved, strengthened, and enriched by encoding additional information” (p. 956). It is proposed that retrieving bits of information from memory sparks learning by strengthening neural pathways (Haebig et al., 2021). Given its importance, many researchers have sought to study and solidify data on retrieval. Many have compared mass practice to distributed practice (Codding et al., 2019; Hopkins et al., 2016; Peterson-Brown et al., 2019; Schutte et al., 2015). Some have further broken distributed practice into fixed versus interval spacing (Çekiç & Bakla, 2019; Latimier et al., 2021). While many have chosen to study the importance of retrieval practices, various conclusions have been drawn. Several foundational theories have emerged, while other questions remain.

Over the past several years, distributed practice has gained momentum in research. Spaced practice involves repeating material in instructional sessions separated by periods of time. The distributed approach has provided a solid platform for better long-term retention of information (Gordon, 2020; Hopkins et al., 2016; Lyle et al., 2020; Pagán & Nation, 2019; Peterson-Brown et al., 2019; Schutte et al., 2015). Gurung and Burns (2018) studied spaced versus distributed practice and repeated trials versus single trials, finding that university-level students performed best when they encountered the material on class quizzes in spaced timeframes. Interestingly, researchers found that students who were given multiple opportunities on class quizzes and chose to participate in massed practice scored poorly on the final exam. In another study of 11–12-year-old students, researchers found that students who participated in spaced practice could also better predict their performance on the final assessment (Emeny et al., 2021). Researchers hypothesize that students who participated in mass practice replicated problem-solving procedures and were overconfident while lacking proper retention and transfer abilities. While many have shown that distributed or spaced practice influences rote memory-type tasks, few have studied higher-level critical thinking skills retention. However, in one such study, Foot-Seymour et al. (2019) found that students who participated in spaced practice could apply critical thinking skills to a new task. The positive results of these studies lead to the conclusion that spaced practice is optimal for long-term retention.

Various theories are attributed to the outperformance of spaced practice on long-term memory. Some support the study-phase retrieval theory, stating that as information is retrieved in spaced increments, added information is assimilated with old information upon retrieval (Karpicke et al., 2014). Latimier et al. (2021) shared that spaced practice is better because "it makes retrieval more effortful, and as a consequence, this strengthens the trace in the long-term memory" (p. 978). Others argued that spaced practice is most effective because of the depletion of working memory resources. In a study of eight-year-old students, Chen and Kalyuga (2021) found that students performed better on a delayed assessment than on an evaluation directly following the practice. Under the working memory resources depletion theory, the authors argued that an overloaded working memory inhibits long-term retention. Findings from their study support the need for processing time between instruction and assessment, allowing for retention and transfer into long-term storage.

Similarly, Sewang (2021) found distributed practice effective, given the built-in time to process information before encountering it a second time. He argued that distributed practice is effective because mass practice requires longer attention spans, which many cannot sustain. While some might argue its cause, plenty of research supports the spaced method for long-term storage and retrieval. However, one study demonstrated the short-term one-week benefits of spaced practice and the negligible effect of spaced practice on long-term six-week retrieval (Nazari & Ebersbach, 2019).

In contrast to spaced practice, massed practice focuses instruction into one grouped session. Although some may consider massed practice less efficacious, its effectiveness for short-term memory is notable. Many studies indicated that spaced practice is more effective for long-term storage, but massed practice greatly influences short-term memory (Hopkins et al., 2016; Pagán & Nation, 2019; Peterson-Brown et al., 2019). One research study found that spaced practice was less effective for short-term retrieval and damaging to short-term memory, leading to poor student assessment results (Lyle et al., 2020). Logically speaking, massed practice is better for short-term recall, given the proximity between instruction and required recall (Foot-Seymour et al., 2019). Although many studies reveal clear distinctions in the benefits of spaced and massed practice, one study failed to find a statistical significance between the two (Codding et al., 2019).

In addition to studying the distribution of practice sessions compared to massed sessions, researchers examined the impact of fixed interval distribution of practice versus varying interval distribution. Several studies found no significant difference in fixed versus variable intervals (Latimier et al., 2021; Peterson-Brown et al., 2019). Çekiç & Bakla (2019) found fixed intervals more effective than varying ones. In their study of adult language learners, the researchers looked at three groups with differently spaced practice intervals. The authors caution against long time intervals between practices as they noted a negative impact on retention.

Another concern with intentional interventions is that some interventions lead to little change, possibly indicative of a ceiling effect (*Ceiling effect*, n.d.). Highly intensive interventions can negate the independent variable's effect on the measured variable, as demonstrated in a study by Schmitt et al. (2017). The researchers noticed that participants who received high-intensity instruction at high doses performed more poorly than others and attributed this to the threshold effect. Given the high intensity and frequency of instruction, participants failed to form proper representations of information and thus created a ceiling effect for themselves. Researchers found much better results for participants who received low-dose high-intensity or high-dose low-intensity interventions. Similarly, overwhelming students can lead to poor results. DeFouw et al.'s (2023) study found that 10-minute interventions were as effective as longer sessions.

Authentic retrieval activities are beneficial for students, given the effort required. Genuine retrieval requires desirable difficulty, which forces the active participation of students compared to passive study (Gordon, 2020). Despite its superiority, retrieval was found insignificant in studies by Gurung and Burns (2018) and Pagán and Nation (2019); however, the retrieval effects in these studies were skewed. In Gurung and Burns's study, the researchers found that retrieval practice affected grades but not learning. This theory is supported by the fact that students who could complete class quizzes multiple times scored lower than those who took them only once. Some argued that offering multiple attempts at class quizzes reduces the seriousness and effort of students in the first place and thus lessens the need for actual retrieval.

Additionally, Pagán and Nation's (2019) multi-faceted study reported a negative impact of post-assessment retrieval; however, further investigation revealed this report to be misleading. In their study of young adults, the researchers compared distributed versus massed practice, repeated versus diverse viewings, and retrieval versus no retrieval. Following the post-assessment, one group was asked to list all the words they remembered in a simple retrieval exercise, while the other group was given a list of the instructed words and asked to cross out every letter e they found. The e-elimination activity provided additional exposure to the instructional content and thus negated the impact of the valid retrieval exercise. Overall, activities requiring accurate memory retrieval should be considered strong instructional methods.

One overarching theme in literature is the need for repeated exposure to content materials. According to Çekiç and Bakla (2019), adult language learners who received nine repeated exposures performed better than their counterparts who received less exposure. Similarly, in Schutte et al. (2015), students who practiced math facts four times daily performed better than those who practiced two combined or one massed time. Fewer opportunities to respond within instructional sessions lead to reduced performance because multiple opportunities are vitally important for students, as practice alone increases retention (Codding et al., 2019; Gordon, 2020). Interestingly, some may argue that mass practice or the repetition of information only changes the present time and has an insignificant impact on long-term retention (Lyle et al., 2020).

Several themes emerged from the various studies regarding the practice of information. When summarizing the multiple findings, one can surmise that the most effective instruction occurs in long-term spaced practice with fixed intervals (Çekiç & Bakla, 2019). The number of repetitions and spacing between methods appears significant. Repeated, spaced, and delayed retrieval is more impactful than immediate information retrieval (Haebig et al., 2021). One such practice, coined successive relearning by Rawson et al. (2013), incorporated the two critical learning components: spaced practice and repeated retrieval, which have been quite successful. Successive relearning requires repetition until a set level of mastery is acquired (Janes et al., 2020). In a study of 48 college students, successive relearning accounted for an average of 13% gain over traditional instructional activities. Its repeated success lends it the attention it receives today as an instructional model for success.

Memory

Spaced exposure is required to provide the processing time necessary for storing information. In line with the cognitive load theory, working memory performs well when given time to process (Chen & Kalyuga, 2021; Haebig et al., 2021). Working memory is affected by intrinsic load, which is the load required for the task at hand, and extrinsic load, which is the load expressed by outside sources or extraneous factors (Chen & Kalyuga, 2021). Delayed assessment is often more effective than immediate assessment because it provides the necessary processing time and prevents working memory overload (Chen & Kalyuga, 2021; Haebig et al., 2021). Recognizing the impact of the cognitive load theory and working memory depletion is crucial, given the importance of working memory in overall mathematical performance and the correlation between working memory and long-term storage.

Although the depth of the relationship is unknown and the correlation quite complicated, research indicates a connection between mathematical performance and working memory abilities, with strong working memory abilities leading to more robust mathematical performance (Chamander et al., 2019; Raghubar et al., 2010). Caviola et al.'s (2020) extensive study noted verbal working memory as a predictor of math and reading performance, while data supported the positive influence of spatial working memory in older students. Conversely, Chamander et al. (2019) found that low-achieving students maintained poor working memory performance. Liu et al. (2017) stated that improved working memory leads to fast student response time. Additionally, one study recognized a correlational effect between working memory and knowledge. As content knowledge increased, so did the working memory. As the working memory improved, so did overall content knowledge (Miller-Cotto & Byrnes, 2020). The assumed connection between working memory and performance drives the interest in continued studies.

Recognition of the importance of working memory and its limitations for overloading leads to the automatization of lower-level rote skills such as mathematical facts. The demand for short-term memory decreases as basic information reaches automaticity (Miller-Cotto & Byrnes, 2020). Automaticity is especially important for children as their academic skills are less developed than adults. Since mathematic skills are underdeveloped in children, it significantly strains their working memory (Chamander et al., 2019). Still, automaticity helps children supersede their limits (Ding et al., 2021).

Working memory is believed to have defined limits that apply directly to new information. The limitations of working memory do not apply in the case of previously learned information (Chen & Kalyuga, 2021). Thus, moving basic information that is often needed into long-term storage can reduce the demand on working memory and allow unrestricted access to the required content. The transactional model for memory recognizes that memory requires attending to new incoming information and retrieving previously learned information from long-term storage (Miller-Cotto & Byrnes, 2020). As important as working memory is, retrieval from long-term storage without interruption is equally essential. Retrieval fluency is vital in computation amongst students in first through twelfth grades, demonstrating that long-term and short-term storage is critical (Villeneuve et al., 2019). Retrieval from long-term storage can be improved by increasing overall knowledge, integrating knowledge, or continuous repetition (Miller-Cotto & Byrnes, 2020).

Understanding the intricacies of memory can be challenging, and isolating factors that influence or are influenced by memory can be tricky. In summary, long-term and short-term memory are necessary for mathematical performance. Furthermore, short-term working memory is limited and can be overloaded easily. The movement of information from short-term to long-term storage increases the ability to retrieve information freely and helps minimize cognitive load. Thus, committing rote information, such as mathematics facts, to long-term memory is vital for success in more complex tasks.

Retention

Many internal and external factors affect memory retention. Many studies look at the various factors, including strategies, to improve short-term and long-term retention. Some studies have analyzed spaced versus massed practice, others have looked at active retrieval versus passive, and other studies demonstrate the role of anxiety and even general cognition. Despite the interplay of factors, effective retention strategies must depend on the intended goal and the timeframe for retention, as these influence effectiveness (Yeo & Fazio, 2019). One example comes from Kenney and Bailey's (2021) study of college students, in which the sample was divided into four groups. One group participated only in daily class reviews with no exams, one group participated only in exams with no daily in-class assessments, one group participated in both, and one group in neither. As expected, the group participating in the daily reviews and the exams performed best. While this is due to repeated exposure and the active retrieval needed for daily quiz reviews, the group who participated only in daily reviews showed no more gain than those who received no intervention. Similarly, the group who participated in exams had no advantage over the control group. These results speak to the specificity of the intention of each strategy.

Cueing is also an effective strategy to spark retrieval. Proponents of cueing promote open-book assessments versus closed-books, stating that students can perform better when the information is ready for manipulation. Hiller et al.'s (2020) study showed that students who completed an open-book assessment could provide better explanations when allowed to consult the text than students who did not have access to texts during the assessments. Similarly, some argue against using multiple-choice tests, given the nature of their design. However, a study of 38 adult participants revealed that multiple choice questions and answer options provide cues to test takers, which then spark memory about other related content. While these cues dismiss the effectiveness of multiple-choice tests, they remark that answering multiple-choice questions requires active retrieval as test takers eliminate answer options. The cues provided within the test design can be a productive memory aid (Little et al., 2019).

The benefits of spaced practice can increase retention, partly due to the elimination of cognitive load and the time allowed for proper processing. In support of this mental rest is the concept of mindfulness. In two separate university studies, researchers demonstrated the benefits of mindfulness on the retention of information. In one study, students participated in controlled breathing between instruction and assessment, which allowed processing time and increased the short-term retention of the information; however, it did not impact long-term retention (Reuter et al., 2021). Likewise, in another study, students were given a list of words to study. The experimental group then participated in a quiet rest period while the others were instructed to pass the time on social media. Those who rested outperformed their counterparts in short-term and long-term retention (Martini et al., 2020). Social media use impeded retention due to issues with cognitive load and distractors. Extraction of extraneous stimuli prevented consolidation, which would have strengthened memory.

Another typical retrieval structure is the use of embedded visual supports within materials. Students find visual materials helpful as they work to understand topics (Guo et al., 2020). The use of multiple images can be exponentially more helpful. Students reported improved content comprehension and memory retrieval when visuals, such as graphics and animations, are provided (Vanichvasin, 2021). Interestingly, one study looked at the use of pictures within learning content and assessments (Schneider et al., 2020). They found that visual images are effective on assessments when they force students to exert extra effort toward analyzing and explaining, thus requiring higher engagement.

Conversely, images within a text are virtually useless unless those same images are also presented as a cue on the assessment (Schneider et al., 2020). The use of technology allows for the incorporation of multimedia visuals and cues. Ahmad (2019) examined how glossed representations influenced the comprehension of a text. She found that the glossed representations, such as various multimedia representations for unknown words within the text, provided a highly significant impact on the overall understanding of the reading. Visual representations can help clarify content knowledge and vocabulary for students if they are taught how to interact with them appropriately.

Other studies investigated the role of more artistic and less regimented strategies on student retention. Many promising findings have linked creative processes and student design to positive retention. For instance, teachers often utilize graphic organizers to help students organize content. When students fill in teacher-created organizers, it increases retention (Guo et al., 2020). However, the higher-order thinking required when students design their own graphic organizers significantly influences student retention and understanding. This study mimics that of Demirhan and Şahin (2021), who explored the effect of hands-on modeling with teacher candidates and found that unstructured and semi-structured activities were much more effective than structured ones. The students who were given the freedom to create had improved retention over those who followed specified directions. Creation requires higher cognitive ability and makes learning more permanent. In a similar experience, Ebersbach et al. (2020) placed students in various experimental groups. One group was tasked with writing test questions based on the content. The deep processing required to develop questions produced increased retention, over the group who participated in regular testing or those who studied. When students engage with content in higher-level processes, they commit it to long-term storage.

In addition to student creation, several artistic elements have positively impacted memory storage. Students who draw visual representations of material can personalize the information and provide meaning, strengthening understanding (Rosen-O'Leary & Thompson, 2019). The images created are committed to memory and provide mental imagery (McDonald & Vines, 2019). The impact of sketching is excellent when combined with notetaking. As students build multiple representations of information, their understanding is solidified (McDonald & Vines, 2019). In a study by Fiorella and Kuhlmann (2020), 120 college students were tasked with studying a text and then explaining it to someone else using verbal explanations, drawings, or both. Compared to the control group, which did not require an explanation, all groups who participated in explaining the text to one another performed better. Additionally, those who utilized verbal explanation and visual drawings outperformed others due to the combined representations.

Other creative avenues, such as music and narrative storytelling, have also been studied. Many individuals can recite song lyrics or remember certain information through previously drilled mnemonics. Bahrami et al. (2019) demonstrated that songs and mnemonics influence retrieval. When given 14 words to learn, the experimental group outperformed the control group, using music and mnemonics. These strategies demonstrated effectiveness for recall and comprehension. Many believe that music provides a relaxing atmosphere that increases enjoyment and motivation. The melody and rhythm give the repetition necessary for storage.

Narration is also known to be appealing to human nature. One college-level study looked at the impact of a narrated story at the end of a lesson. Researchers wanted to see how the narration influenced retrieval and effectual aspects. The students reported significant approval of the professor following the narration, indicating a general sense of enjoyment from the narration. The students demonstrated increased retention levels, but the effect size was small. Despite reported high levels of attention, the retention of information from the narration was negatively skewed by extraneous details that overwhelmed the cognitive load. While enjoyable, the impact of narrative stories on retention is inconclusive (Kromka & Goodboy, 2019).

Gamification is a more modern strategy that is thought to enhance retrieval. Gamification has increased in popularity as technology has invaded the classroom. Gamification is believed to provide solid instruction and review opportunities due to its adaptability, ease in formative assessment, and repetitive spaced practice. (McGregor et al., 2019). Gamification can affect student performance through repeated exposure to material and immediate feedback (Petrovic-Dzerdz, 2019). It often provides increased motivation, thus sparking increased effort and engagement and improving retention. The factors that influence knowledge retention are endless. The interconnectivity is woven tightly from student-based elements to instruction-based to content-based factors. Balancing these factors to match the intended goal of any instruction is an intense enterprise with substantial potential.

Summary

Successful math performance relies on a student's efficient and accurate problem-solving ability. Efficient and accurate problem-solving depends upon a student's fluency with basic math facts; thus, math fact fluency is crucial to successful math performance. This literature review examined the issue of math fluency, highlighting its importance. At the most basic level of Bloom's taxonomy, recall of math facts provides the basis for all other computations and problem-solving. The literature review looked at traditional methods for fact fluency building and others incorporating digital elements. Additional unique methodologies, such as peer tutoring, were also included. Fact fluency practice aims to move the recall of basic facts from working memory to long-term storage as described under the multi-store memory model, which frees the working memory to focus on in-depth problem-solving. This transfer can be accomplished through the process of rehearsal. A few studies have been conducted looking at the use of spaced practice for effective storage of math facts; however, there remains a gap in the literature regarding spacing the practice across multiple days and incorporating rehearsal.

Educators can ensure the best use of instructional time and implore methods that lead to the best retention of facts by looking for the optimal spacing timeframe.

CHAPTER THREE: METHODS

Overview

The purpose of this quantitative, quasi-experimental nonequivalent control group study was to measure the effect of practice method and spacing interval on student math fact fluency. Chapter three discusses the study's design and rationale, including clearly defined variables. It continues with the research question and null hypotheses. It concludes with detailed information regarding setting, instrumentation, procedural steps, and methods for data analysis.

Design

Like experimental studies, quasi-experimental studies analyze differences among the means of experimental and control groups following a treatment (Gall et al., 2007). Studies of this nature begin with a pre-test, followed by an intervention, and end with a post-test for final data collection (Reichardt, 2019). The quantitative design typically involves data collection from pre- and post-tests and self-report measures (Gall et al., 2007).

Studies of this nature are popular in the social sciences, where true experimentation is nearly impossible. They are effective at providing experimentation that closely resembles real-life situations. The quantitative quasi-experimental nonequivalent control group design was used in this study. This design was the most appropriate method of study as it provided an opportunity to purposefully manipulate a variable for intentional study while using groups that "lack random assignment" (Gall et al., 2007, p. 416). Quasi-experimental studies are intended to study an effect following an intervention and are extremely useful in educational studies, given the use of non-randomized groups. While many studies of this kind include a control, Gall et al. (2007) stated that a nonequivalent control group study does not require a control; rather, two distinct treatment groups can be compared. This study did not have a true control group.

Studies of this design are open to validity concerns relating to history, instrumentation, selection, and attrition (Handley et al., 2018). These concerns were controlled by randomly assigning classrooms to treatment groups and running all treatments concurrently. Bias from participant awareness was controlled by maintaining the integrity of each classroom group and not disrupting standard classroom procedures.

This study contained two separate independent variables. First, the practice method was altered purposefully. Half of the participants used rehearsal strategies to study, while the other half used retrieval strategies. Rehearsal is a process that maintains information through rote repetition (Glenberg et al., 1977). Students practiced math facts through verbal repetition of given equations. Retrieval is “attempting to reinstate a prior learning context” (Karpicke et al., 2014, p. 238). Students in the retrieval groups supplied answers for sets of given equations.

The traditional flashcard method was used because it mimics other traditional methods, such as CCC and timed tests. It provided an easy practice method for the rehearsal and retrieval groups. Fact fluency is a knowledge-level skill (Bloom et al., 1956); therefore, rote memorization techniques, such as flashcards, are appropriate. Additionally, it provided the continuous rehearsal necessary to solidify the facts into short-term storage and, ideally, into long-term storage (Atkinson & Shiffrin, 1968). The second independent variable for analysis was the impact of spaced practice on fact retention. As defined by Lyle et al. (2020), spaced practice allows for intervals between episodes of information retrieval. Students in this study participated in one of three spacing groups: every day, every other day, and every third day.

The dependent variable for the study was math fact fluency. Bray and Maldonado (2018) defined fluency as “automaticity with basic facts” (p. 92). Students completed a pre-test demonstrating initial math fact fluency and a post-test following the intervention. Results on the initial Mathematical Fluency and Calculation Test (MFaCTs) served as the covariate in this study. The researcher included the pre-test as a covariate to adjust for initial differences amongst the treatment groups.

Research Question

RQ1: Is there a difference in math fact fluency among third-, fourth-, and fifth-grade students who use alternate practice methods based on various spacing intervals when controlling for pre-test scores measuring addition, subtraction, multiplication, and division fluency?

Hypotheses

$H_0$1: There is no significant difference in math fact fluency among third-, fourth-, and fifth-grade students who use rehearsal and retrieval practice methods when controlling for pre-test scores as measured by the Mathematics and Fluency Calculation Tests.

$H_0$2: There is no significant difference in math fact fluency among third-, fourth-, and fifth-grade students who practice every day, every other day, and every third day when controlling for pre-test scores as measured by the Mathematics and Fluency Calculation Tests.

$H_0$3: There is no significant interaction between rehearsal and retrieval methods, as measured by the Mathematics and Fluency Calculation Tests, on math fact fluency among third-, fourth-, and fifth-grade students who utilize rehearsal and retrieval methods based on practice every day, every other day, and every third day when controlling for pre-test scores.

Participants and Setting

The following section describes the population of each participating school, the participants, and sampling procedures. The section ends with an explanation of the study's setting.

Population

The population for this study was gathered from seven US schools (See Appendix A). School A is located in south-central Pennsylvania. The students from School A are not fully representative of the local area's demographics, in which 66% of the residents are White, 18% Hispanic, 11% Black, 1% Asian, and 4% multi-ethnic. The diversity in School A is much less than that of the general population. Of the general population, 53% are female and 47% male, with 48% reporting a high school degree and 23% of adults reporting a bachelor's degree or higher. The average income of residents in the area is roughly $35,000, and roughly 81% of residents speak only English.

School B is situated in the northeast part of West Virginia. The school population closely resembles the local demographic. The area demographics are 83% White, 3% Hispanic, 8% Black, 1% Asian, and 3% multi-ethnic diversity. Of the local population, 51% are female and 49% male. Thirty-seven percent of the local population holds a high school diploma, and 23% hold a bachelor's degree or higher. The mean income is $40,000, and 95% of the residents speak English only.

School C is located in northern Maryland, and the school population has a skewed resemblance to the local demographic. The local area hosts a 78% White, 3% Hispanic, 11% Black, 2% Asian, and 4% multi-ethnic population. Forty-nine percent of residents are female, and 51% are male. The average income is $42,000. Thirty-six percent of the local population hold a high school diploma, and 22% have a bachelor's degree or higher. English is the primary language for 92% of the local population.

School D is situated in central South Dakota in the middle of Native American reservation land, and the school population fully represents the local demographic. The local area has an 88% Native American, 8% White, and 1% multi-ethnic population. Fifty-three percent of the local population is female, and 47% is male. Thirty percent of the population holds a high school diploma, and 16% has a bachelor's degree or higher. The average income for the area is $25,000, and 84% of the general population speaks only English.

School E, located in central Minnesota outside of St. Paul, has a close representation of the local population, which is 84% White, 1% Hispanic, 5% Black, 3% Asian, and 6% multi-ethnic. Fifty-three percent of the population is female, and 47% is male. Thirteen percent hold a high school diploma, and 61% have a bachelor's degree or higher. The average yearly income is $67,000, and 88% speak only English.

School F is located in central Texas, just outside of Austin. The school population closely resembles the local population. The local area is 78% White, 2% Hispanic, 5% Black, and 14% other. Fifty-two percent of residents are female, and 48% are male. The average income is around $41,000, 18% of the population hold a high school diploma, and 48% have earned a bachelor's degree or higher. Eighty-five percent of residents speak English only.

School G, located in southeastern New York, is a poor representation of the local population due to the inclusion of an international program. Eighty-five percent of the local population is White, 5% Hispanic, 1% Black, 3% Asian, and 4% multi-ethnic. Females comprise 48% of the general population, while males comprise 52%. Fourteen percent of the population holds a high school diploma, while 62% holds a bachelor's degree or higher. The yearly average income is $92,000, and 88% of the population speaks English only.

Participants

The participants for this study were primarily gathered through convenience sampling relative to the researcher's location. School administrators and teachers were contacted via email regarding participation. Most participants are students in private schools that are geographically local to the researcher. Additional participants were recruited through social media. When interested teachers responded to the participation request, the school administration was contacted for approval. The most significant percentage of participants were students from School A, with smaller samples drawn from the schools.

For this study, the sample size exceeded 166 students, surpassing the recommendation proposed by Gall et al. (2007) for a two-way ANCOVA when assuming a medium effect size with a statistical power of 0.7 and α level of 0.05. The convenience sample was gathered through cluster sampling as students participated in regular classroom groupings.

School A is a private school serving approximately 700 pre-kindergarten through twelfth-grade students. As of fall 2022, 88% of the student population was White, 4% Hispanic, 7% Black, 3% Asian, and 2% other. Approximately 12% of the student population would qualify for free and reduced lunches if they were public school students.

School B, a private school, serves approximately 427 pre-kindergarten through twelfth-grade students. As of fall 2023, 86% of the student population is White, 3% Hispanic, 3% Black, 3% Asian, and 5% multi-ethnic. None of the student population would qualify for free and reduced lunches if they were enrolled in public education.

School C enrolls 244 pre-kindergarten through twelfth-grade students. As of 2024, 98% of the student population is White, 1% Black, and 1% Asian. None of the student population would qualify for free and reduced lunches if they were in public school divisions.

School D serves approximately 275 kindergarten through twelfth-grade students, with a 100% American Indian population. Approximately 50% of the student population is male, and 50% is female. Additionally, 100% of the school students qualify for free lunch.

School E includes 173 kindergarten to eighth-grade students. An average of 82% of the student population is White, 2% Hispanic, 3% Black, 4% Asian, 2% Native American, and 5 % multi-ethnic. Eight percent of the enrolled student body qualifies for free and reduced lunches.

School F enrolls 78 prekindergarten to ninth-grade students. Approximately 81% of the student population is White, while 19% fall into a different ethnic category. Approximately 15% of the population is considered low-income and would qualify for free and reduced lunches.

School G enrolls 143 kindergarten to twelfth-grade students. As of 2023, 16% of the student population is White, 20% Hispanic, 31% Black, 25% Asian, and 8% multi-ethnic.

The study consisted of 16 third-, fourth-, and fifth-grade classrooms from the United States (See Appendix B). Each classroom was a separate treatment group, with multiple classrooms assigned to each treatment variable combination. Each treatment group had between 28-40 participants, with an average group size of 33. Classrooms were assigned their treatment conditions through random assignment.

Classroom A1 provided a sample of 8 students, including five males and three females. All eight students were White. Classroom A1 received daily rehearsal math fact practice. Classroom A2 provided a sample of 22 students, including 10 males and 12 females. The ethnic makeup was as follows: 22 White students. Classroom A2 received daily rehearsal math fact practice. Classroom A3 provided a sample of four students, including two males and two females. The ethnic makeup included one White, two Asian, and one multi-ethnic student. Classroom A3 received daily rehearsal math fact practice.

Classroom B1 provided a sample size of 14, including six males and eight females. The ethnic makeup was 13 White students and one multi-ethnic student. Classroom B1 received rehearsal fact practice every other day. Classroom B2 provided a sample size of 15, including eight males and seven females. The ethnic makeup was 14 White students and one multi-ethnic student. Classroom B2 received rehearsal fact practice every other day.

Classroom C1 provided a sample size of 16, including eight males and eight females. The ethnic makeup was 14 White students and two multi-ethnic students. Classroom C1 received every third-day rehearsal-based practice. Classroom C2 provided a sample size of 15, including seven males and eight females. All 15 participating students were White. Classroom C2 received every third-day rehearsal-based practice. Classroom C3 included nine students, including four males and five females. The ethnicity makeup included four White, two Black, and three Asian students. Classroom C3 received every third-day rehearsal-based practice.

Classroom D1 provided a sample size of 14, including seven males and seven females. The ethnic makeup was 13 White students and one multi-ethnic student. Classroom D1 participated in every day math fact retrieval practice. Classroom D2 included a sample of seven students, including two males and five females. This classroom included five White students, one Asian student, and one multi-ethnic student. Classroom D2 utilized the every day retrieval strategy. Classroom D3 had seven student participants, including one male and six females. The ethnicity makeup was four Black, one Hispanic, and two Asian students. Classroom D3 received every day math fact retrieval practice.

Classroom E1 provided a sample size of five, including one male and four females. The ethnic makeup was 100% Native American. Classroom E1 received every other day retrieval practice. Classroom E2 provided a sample size of three, including one male and two females. All students were White. Classroom E2 received retrieval practice every other day. Classroom E3 included 20 student participants, including nine males and 11 females. The participants included

17 White students and three multi-ethnic students. Classroom E3 participated in every other day retrieval practice.

Classroom F1 provided a sample size of 22, including 10 males and 12 females. The ethnic makeup was 21 White students and one multi-ethnic student. Classroom F1 participated in retrieval practice every third day. Classroom F2 provided a sample size of 15, including seven males and eight females. The ethnic makeup was 14 White students and one multi-ethnic student. Classroom F2 participated in retrieval practice every third day.

Setting

This quasi-experimental study took place within the regular classroom setting under the direction of the classroom teachers. All interventions and data collection occurred in the natural setting throughout the regular school day at times determined by the classroom teacher. Data collection began approximately in marking period two, late fall, to allow student participants time to acclimate to the regular classroom environment and adjust to the new school year.

Instrumentation

The MFaCTs designed by Reynolds et al. (2015) was used to collect data for this study. According to Reynolds et al., the test serves as a screener for mathematical computation issues while removing the compounding factor of higher-level problem-solving and reasoning. In addition, it can be used to assess students' current abilities and monitor progress over time in students ages six to 18. Cecil Reynolds, Judith Voress, and Randy Kamphaus designed the instrument, published in 2015 by Pro-Ed Inc. (Marbach, 2017). Reynolds et al. developed the MFaCTs on the premise that fluency and calculation are essential for math success. MFaCTs was initially intended to identify students who are behind in automaticity, identify students with calculation delays, monitor progress, identify students with advanced skills, and be used in research.

The MFaCTs instrument was created through a norming sample of 1,620 predominantly White, average-ability students. The test consists of two separate sections: fluency and calculation. The calculation subtest is produced in two levels: for children in first to fifth grades and those in sixth through twelfth grades. The subtest has between 50 and 55 items. The fluency test is used with students from first to fifth grades, with a separate test for first to second and third to fifth grades (Reynolds et al., 2015). The fluency subtests consist of 100 items, giving a range for the raw score from 0 to 100. Given this study's focus on fluency, data was only collected from the fluency section of the MFaCTs. The calculation subtest was not administered.

The MFaCTs is intended to be given individually or in groups (Reynolds et al., 2015). Each subtest has three forms: A, B, and C. Forms A and B were used for this study. Professionals of varying levels can implement and score the MFaCTs. Scripted directions describe where to begin testing, when to skip problems, and prompts to pay close attention to each problem's operation. When scoring, one point is given for a correct answer and no points for an incorrect one. The raw scores are then computed into percentile rank, grade or age equivalents, and standard scores.

The MFaCTs has a standard measurement error score of four on the calculation subtest, with a total standard score of 160, and a standard measurement error score of three for fluency, with a total standard score of 136 (Reynolds et al., 2015). The internal consistency exceeds 0.90, with scores for the calculation test at 0.92 and the fluency test at 0.97. The reliability scores are reported using Cronbach's alpha coefficient. The alternate form reliability for age-based analysis is 0.96 for calculation with all age groups, with 0.81 for the fluency of younger students and 0.89

for the fluency of older students. The grade-based analysis demonstrated similar results for alternate-form reliability, with calculations showing a 0.94 coefficient for younger and 0.96 for older students, with fluency coefficients of 0.81 for younger and 0.88 for older students. The test-retest reliability scores for age-based analysis are 0.85 for calculation with younger students, 0.94 for older students, 0.64 for fluency with younger students, and .081 for fluency with older students. Similar results were reported for grade-based analysis with a coefficient of 0.83 for the calculation with younger students, 0.91 for older students, 0.71 for fluency with younger students, and 0.83 for older. While the calculation subtest maintains a higher immediate alternate form coefficient, all scores are above 0.80. The test-retest and delayed alternate form scores are above 0.80, except for fluency in the youngest age group. According to Reynolds et al. (2015), this indicates the rapid pace at which students' fluency abilities change.

The fluency and calculation subtests are not highly correlated, thus demonstrating that they measure separate items (Reynolds et al., 2015). However, researchers report that the three test forms are interchangeable. To test validity, researchers compared the MFaCTs Calculation subtest to the Wide Range Achievement Arithmetic and Comprehensive Mathematical Abilities Test and found high correlations in content. Besides the initial instrument testing, many studies have not utilized the MFaCTs instrument. Most studies on the measured constructs use a multi-skill digital assessment or a simple timed fact test (Allen-Lyall, 2018; Berrett & Carter, 2018; Stocker & Kubina, 2017). One study utilizing the MFaCTs assessment was conducted in North Wales, where researchers sought to measure the impact of continued teacher coaching in a specific flashcard intervention on student fluency. Researchers utilized the MFaCTs to measure the dependent variable of student fluency (Owen et al., 2021). The results of this study were reported using the raw score, given the nature and design of the study itself. Most participants were in second and third grades and were given both fluency sections, the first and second grades section and the third to fifth grades section of the MFaCTs. Given this design, no conversion charts were available to obtain proper standard scores; therefore, the data was reported in raw scores and boasted low means, likely due to the young age of the participants.

The calculated item discrimination is above 0.40, showing that items are of appropriate difficulty (Reynolds et al., 2015). Additionally, the receiver-operating characteristic curve analysis is above 0.80, denoting that the calculation subtest is an accurate way to diagnose learning disorders in students. Finally, researchers report an item bias of less than 1%.

The MFaCTs test was administered as directed in the testing manual. The manual instructed students in first and second grades to be given five minutes to complete the addition and subtraction facts, while older students were given three minutes to complete addition, subtraction, multiplication, and division facts (Reynolds et al., 2015). The MFaCTs developers encourage researchers to use this instrument in their studies (Appendix C) and additionally granted permission for the use in this study (Appendix D).

Procedures

Following the school division’s and school administration’s approvals (See Appendix E), the study’s proposal was submitted for the Institutional Review Board’s approval (See Appendix F). Participating teachers and classrooms were chosen randomly from those willing to participate in the study. Next, the researcher sought permission from the participants’ parents (See Appendix G). Student and parental consent forms were distributed and collected (see Appendix H). After the initial agreements and consents were distributed, the researcher provided each participating teacher with written procedures, data and intervention materials, and consent forms.

The classroom teachers participating in the study were instructed to administrate the initial assessment and the instructional design of the interventions (see Appendix I). Before the intervention, the teachers administered the MFaCTs test Form A for baseline data. Upon completing the pre-test, the researcher contacted the participating teachers again to answer questions and collect initial data and student demographics. Classroom teachers explained the study to students to maintain comfort for all students. They reviewed study procedures and daily practice procedures. The researcher provided a video to train teachers and students in proper procedures.

Following training, teachers implemented the fact practice sessions according to their level of the independent variables. The participants assigned to the rehearsal practice method were provided basic fact flashcards with answers. The flashcard listed the entire problem, such as 6x7 = 42. The student participants were trained in how to rehearse each fact aloud verbally. They verbally repeated the fact and its reversal aloud, for example, 6x7 = 42 and 7x6 = 42. The student flashcard packs contained one card for each fact (minus reversals for addition, subtraction, multiplication, and division) for numerals zero through nine (see Appendix J). Based on their independent variable level, the students completed this activity with a partner for 10 minutes daily, every other day, or every third day.

Practice sessions can be limited in the timeframe to ensure student attention and engagement (Defouw et al., 2023). The rapid flashcard drill helps maintain student engagement (Levin & Bernier, 2011). Teachers timed students for five minutes, and then the partners switched roles. At the end of the time, the students stored their flashcards with the next card on top and completed cards placed at the back of the stack.

The retrieval group followed identical procedures; however, the flashcards did not have the answer on the front of the card. Students were required to retrieve the answer from their memory, and their partners verified it from the back of the card. In all classrooms, teachers were encouraged to review the procedures weekly to remind students of proper implementation. The researcher implemented practice methods for both groups that used flashcards, ensuring that the control and experimental groups had similar student interest and engagement levels.

The study consisted of six groups based on combinations of the two independent variables (see Appendix K). The groups were as follows: retrieval every day, retrieval every other day, retrieval every third day, rehearsal every day, rehearsal every other day, and rehearsal every third day. A third of the groups practiced the facts daily, another third every other day, and the final third of the participants practiced every third day. Practice sessions were conducted for 45 school days, creating 45 sessions for the every day groups, 22 sessions for the every other day groups, and 15 sessions for the every third-day groups.

Midway through the treatment timeframe, the researcher communicated with the teachers again to clarify procedures (see Appendix L). The goal was to ensure the integrity of the research procedure. During this communication, the researcher discussed post-test procedures. Immediately following the post-test, the researcher collected data and thanked the participants.

After 45 days, the MFaCTs assessment Form B was administered for post-test data collection. In some cases, participating schools requested that student identifiers be removed from the data before passing it to the researcher. In these instances, the teacher provided a chart so that student codes could be used and the researcher could still appropriately match the students' pre- and post-test scores. Raw data was collected from the study and stored in a locked box in the researcher's possession. In addition, scores were entered into digital format and stored on the researcher's personal computer under password protection. Study data will be stored for five years following the study's completion.

Data Analysis

Data for this quasi-experimental nonequivalent control group study was analyzed using a two-way ANCOVA test, measuring the impact of the two independent variables, practice method and spacing interval, on the mathematical fact fluency dependent variable. Studies can analyze and compare the variance between the experimental groups and within the groups (Gall et al., 2007). According to Gall et al. (2007), the ANCOVA's goal is to determine if there is a statistical difference in the adjusted means between two or more groups. In this study, the ANCOVA served to identify statistical differences between the six factorial groups created based on the independent variables. The two-way ANCOVA was used, given the two separate independent variables. The two independent variables were categorical and nominal. The first independent variable of the practice method was dichotomous, while the second variable of the spacing interval was trichotomous. Additionally, the dependent variable can be measured on a continuous scale as required by the assumptions for a two-way ANCOVA test (Warner, 2021). Given the presence of the pre-test, the ANCOVA was chosen instead of the ANOVA. The continuous ratio scale pre-test scores served as a covariate for this study. The analysis controlled the initial differences between the nonequivalent control groups.

Initially, the data was visually screened to locate missing, incomplete, and inaccurate entries. Additionally, raw scores were converted into standard scores using the age-based conversion table provided by the test developers in the testing manual (Reynolds et al., 2015). Converting the raw scores accounted for the age differences among the participants. Data can be analyzed using box and whisker plots to search for extreme outliers and a grouped scatter plot to check for the assumptions of linearity and homoscedasticity (Warner, 2021). The linearity test sought to demonstrate a linear relationship between the covariate and the dependent variable.

Homoscedasticity ensured a similar variance in error for all treatment groups (Laerd Statistics, 2018). The plots also provided an analysis of the regression lines to ensure homogeneity. Parallel regression lines ensured no interaction between the covariate and the independent variable (Grande, 2017). An additional Shapiro-Wilks test was done to check for distribution and Levene's normality to ensure group homogeneity (Warner, 2021). All participants were assigned to one intact group, ensuring the independence of observations (Laerd Statistics, 2018).

Data was analyzed using Laerd Statistics and the IBM Statistical Package for Social Sciences (SPSS) software. An α of 0.05 was used. The null hypothesis was rejected at the 95% confidence level. An F-test was performed to analyze the difference between the experimental groups. It was assumed that the F statistic is greater than the F critical to reject the null hypothesis (Warner, 2021). As Warner (2021) suggested, a post hoc test can be used to look for variances among the groups upon rejecting the null hypothesis. Given its conservative nature, a Bonferroni procedure was used to prevent Type 1 errors. The Bonferroni post hoc looked for variances in the levels of the independent variable of time, every day, every other day, and every third day, with the ultimate goal of finding the ideal conditions for fact fluency practice. Finally, the effect size was calculated using a partial eta square given that a small effect size is $\eta^2 = 0.01$, a medium effect is $\eta^2 = 0.06$, and a large effect is $\eta^2 = 0.14$.

CHAPTER FOUR: FINDINGS

Overview

Data from this quantitative, quasi-experimental nonequivalent control group study was analyzed to measure the effect of practice method and spacing interval on student math fact fluency. This chapter presents the descriptive data for each treatment group. In addition, an overview of the ANCOVA assumption testing is included with the results of the data analysis concerning the null hypotheses. Additional information is included detailing further analysis due to violating the homogeneity assumption.

Research Question

RQ1: Is there a difference in math fact fluency among third-, fourth-, and fifth-grade students who use alternate practice methods based on various spacing intervals when controlling for pre-test scores measuring addition, subtraction, multiplication, and division fluency?

Null Hypotheses

$H_0$1: There is no significant difference in math fact fluency among third-, fourth-, and fifth-grade students who use rehearsal and retrieval practice methods when controlling for pre-test scores as measured by the Mathematics and Fluency Calculation Tests.

$H_0$2: There is no significant difference in math fact fluency among third-, fourth-, and fifth-grade students who practice every day, every other day, and every third day when controlling for pre-test scores as measured by the Mathematics and Fluency Calculation Tests.

$H_0$3 There is no significant interaction between rehearsal and retrieval methods, as measured by the Mathematics and Fluency Calculation Tests, on math fact fluency among third-, fourth-, and fifth-grade students who utilize rehearsal and retrieval methods based on practice every day, every other day, and every third day when controlling for pre-test scores.

Descriptive Statistics

There were 196 participants divided into six treatment groups within this quantitative, quasi-experimental, nonequivalent control group study based upon the two-by-three ANCOVA design. Two students began the study but later withdrew. One moved from the participating school, and the other withdrew due to scheduling issues. Group A participated in rehearsal practice every day; Group B participated in rehearsal practice every other day; and Group C participated in rehearsal practice every third day. Group D participated in retrieval practice every day; Group E participated in retrieval every other day practice, and Group F participated in retrieval practice every third day. Table 1 presents the descriptive statistics. Table 2 provides the estimated marginal means for all six treatment groups considering the covariate. Figure 1 provides the estimated marginal means plot. Tables 3 and 4 contain the estimated marginal means for each variable.

Table 1

Descriptive Statistics

Treatment	Time	*M*	*SD*	*n*
Rehearsal	Every day	103.62	16.863	34
	Every other day	96.41	13.067	29
	Every third day	102.75	18.948	40
	Total	101.25	16.898	103
Retrieval	Every day	112.00	15.616	28
	Every other day	106.93	17.252	28
	Every third day	111.46	14.003	37
	Total	110.26	15.509	93
Total	Every day	107.40	16.717	62
	Every other day	101.58	16.031	57
	Every third day	106.94	17.213	77
	Total	105.53	16.827	196

Note: The mean values represent fluency as measured by the MFaCTs post-test.

Table 2

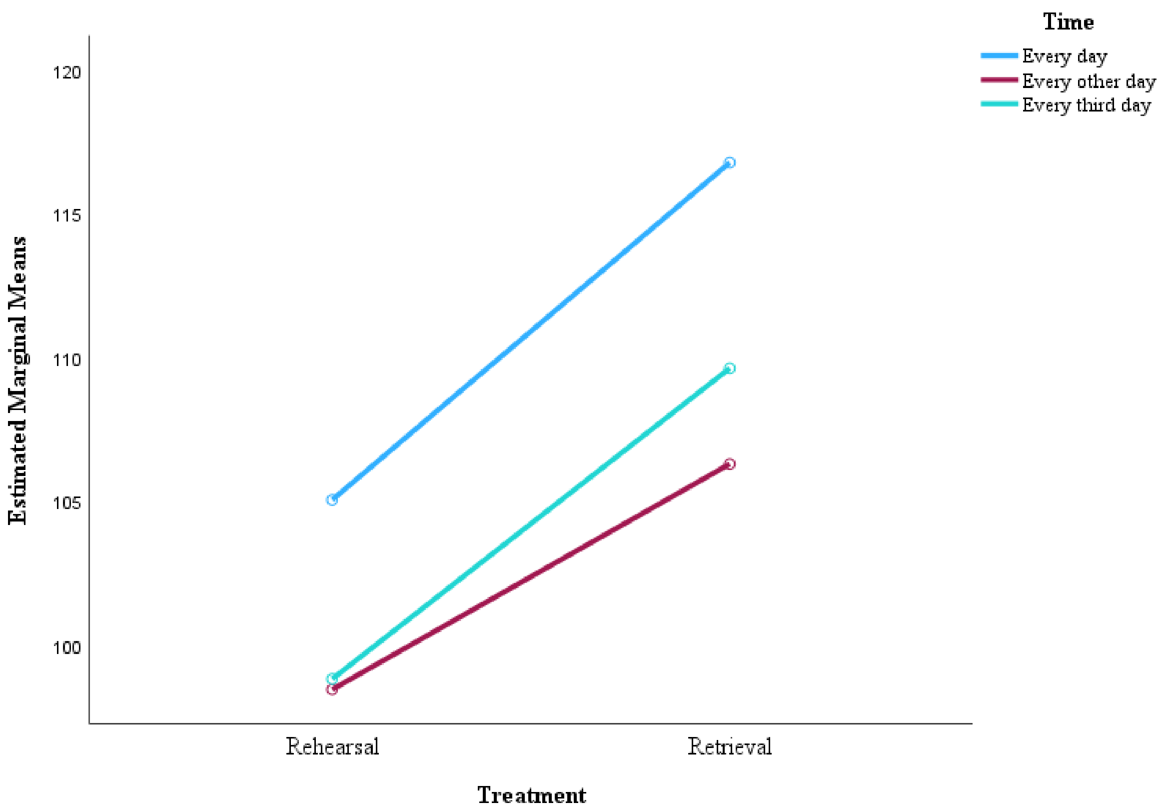
Estimated Marginal Means

Treatment		M^a	*SE*	95% Confidence Interval	
				Lower Bound	Upper Bound
Rehearsal	Every day	105.055	1.542	102.014	108.097
	Every other day	98.465	1.671	95.170	101.761
	Every third day	98.83	1.433	96.004	101.656
Retrieval	Every day	116.776	1.713	113.397	120.155
	Every other day	106.303	1.697	102.954	109.651
	Every third day	109.627	1.479	106.710	112.545

a. Covariates appearing in the model are evaluated at the mean value of 95.50 for the MFaCTs pre-test.

Figure 1

Estimated Marginal Means of MFaCTs Post-Test



Note. Covariates appearing in the model are evaluated on the mean value of 95.50 for the MFaCTS pre-test. MFaCTS post-test scores displayed are standard scores, not raw scores.

Table 3

Estimated Marginal Means for Treatment

Treatment	*M*[a]	*SE*	95% Confidence Interval	
			Lower Bound	Upper Bound
Rehearsal	100.784	0.893	99.023	102.544
Retrieval	110.902	0.940	109.048	112.756

a. Covariates appearing in the model are evaluated at the mean value of 95.50 for the MFaCTs pre-test.

Table 4

Estimated Marginal Means for Time

Time	*M*[a]	*SE*	95% Confidence Interval	
			Lower Bound	Upper Bound
Every day	110.916	1.156	108.636	113.195
Every other day	102.384	1.190	100.036	104.732
Every third day	104.229	1.034	102.190	106.268

a. Covariates appearing in the model are evaluated at the mean value of 95.50 for the MFaCTS pre-test.

Results

After collecting all pre- and post-test data, the tests were scored using a key. Each correct response was worth one point for a total possible raw score of 100. All pre- and post-test scores were verified twice. Raw scores were converted into age-based standard scores using charts provided in the testing manual. The total possible standard score was 136. Prior to data analysis, all assumptions for the ANCOVA were tested. The study's design addressed the first four assumptions, including continuous dependent variable, two or more categorical independent variables, a continuous covariate, and independence of observations.

Data Screening

Data screening was conducted to identify outliers and other unusual points. The box and whisker plots revealed two outliers (see Figures 2 and 3). Additional inspection of the student residuals for the post-test revealed that one outlier was ±3 SD from the mean. Upon further analysis, the researcher determined that the outliers were not data entry errors but rather valid entries. After running analyses with and without the outliers, it was evident that the unusual points had little influence on the overall data set, so the decision was made to include them.

Additional analysis was completed to identify leverage and influential points. Visual inspection of the uncentered leverage values for the post-test data indicated that all data points were within the safe range of less than 0.2 (Laerd Statistics, 2018). Figure 4 provides a histogram of the uncentered leverage values. Finally, the Cook's distance values were analyzed, but no influential points were noted.

Figure 2

Simple Boxplot of Studentized Residual for Mfactpost by Treatment

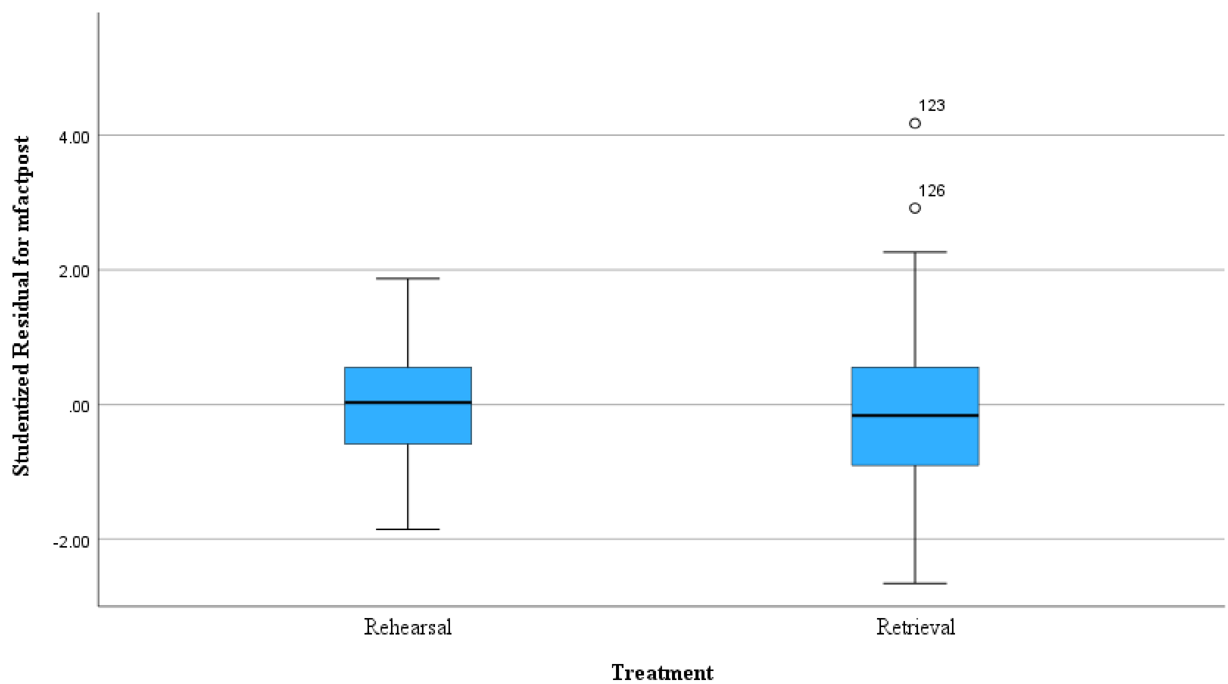


Figure 3

Simple Boxplot of Studentized Residual for Mfactpost by Time

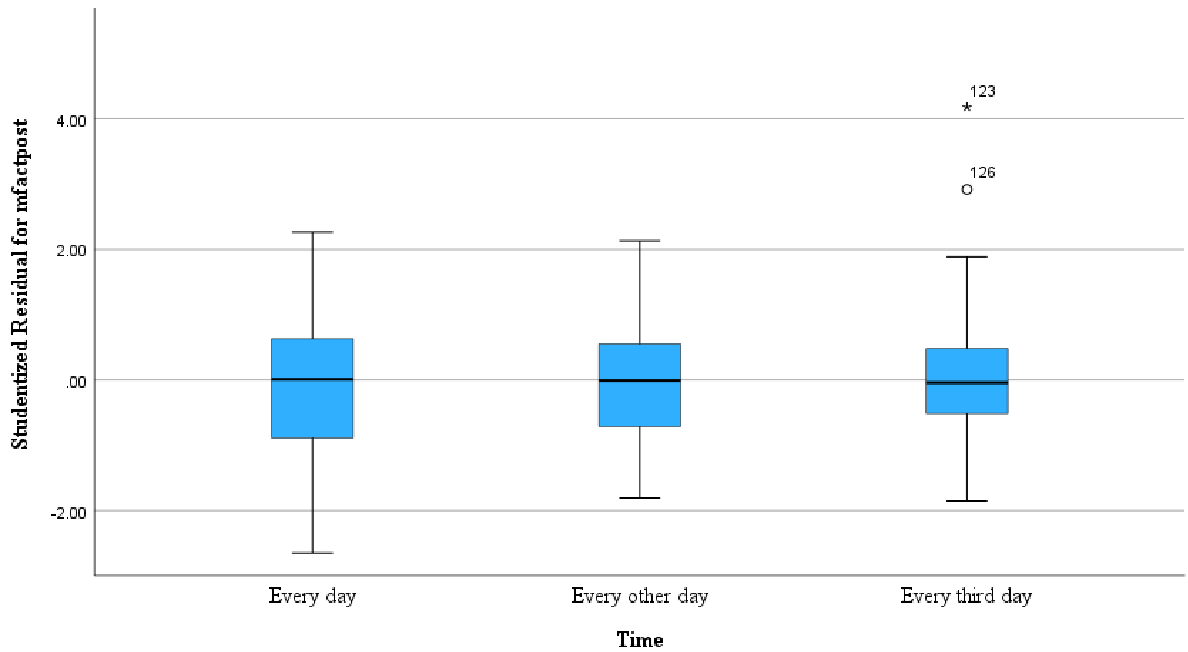
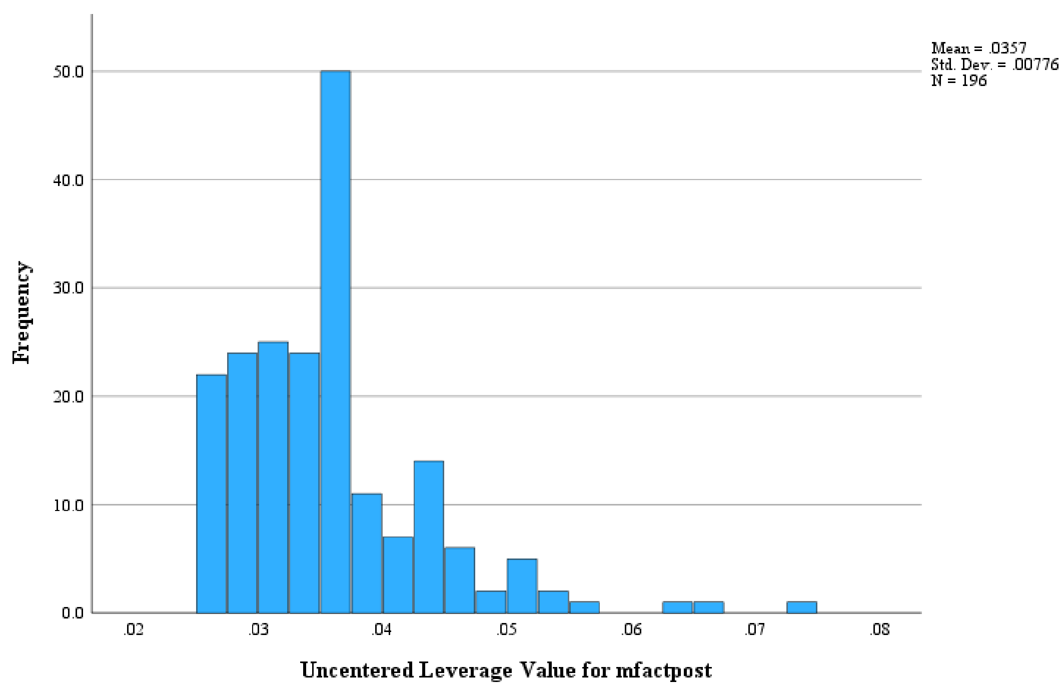


Figure 4

Simple Histogram of Uncentered Leverage Value for Mfactpost



Assumption Testing

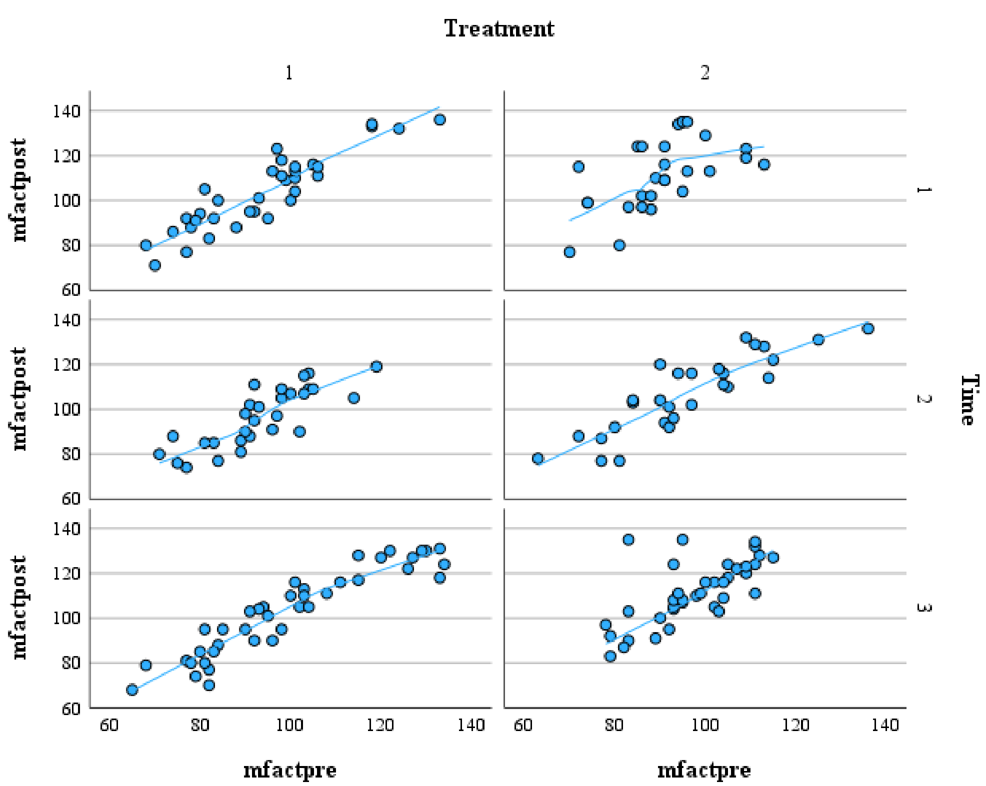
A two-way ANCOVA was used to test for an interaction effect between the two independent variables. According to Warner (2021), the assumptions of linearity, homogeneity of regression, homoscedasticity, homogeneity of variance, and normality must be met.

Assumption of Linearity

Linearity was tested through observation of a scatterplot (see Figure 5). There was a linear relationship between initial pre-test and post-test scores for each intervention group.

Figure 5

Scatter Plot of Mfactpost by Mfactpre by Treatment by Time



Note. mfactpre represents results of the MFaCTs pre-test and mfactpost represents the results of the MFaCTS post-test.

Homogeneity of Regression

The homogeneity of regression slopes was determined by comparing the two-way ANCOVA model with and without interaction terms: $F(5,184) = 0.191$, $p = 0.966$ (see Table 5).

Table 5

Tests of Between-Subjects Effects

Source	Type III *SS*	*df*	*MS*	*F*	Sig.
Corrected Model	40052.129[a]	11	3641.103	44.179	0.000
Intercept	1177.006	1	1177.006	14.281	0.000
treatmentgroups	276.563	5	55.313	0.671	0.646
mfactpre	25639.208	1	25639.208	311.091	0.000
treatmentgroups * mfactpre	78.826	5	15.765	0.191	0.966
Error	15164.744	184	82.417		
Total	2237801.000	196			
Corrected Total	55216.872	195			

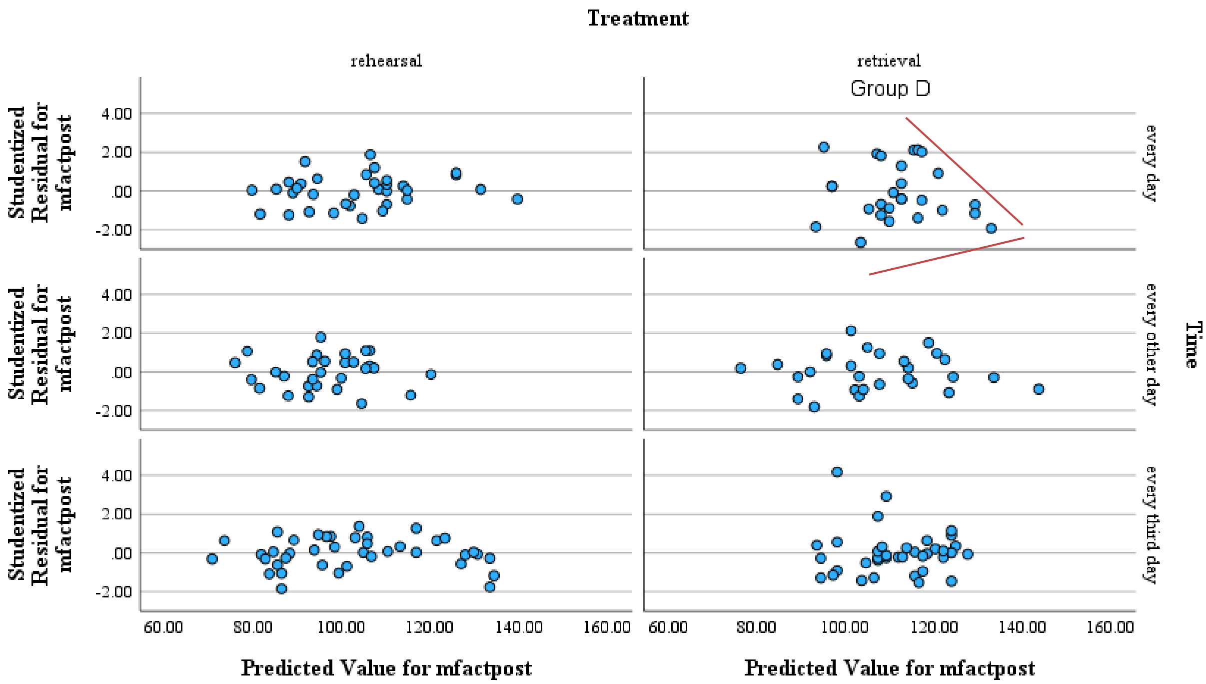
a. R Squared = 0.725 (Adjusted R Squared = 0.709)

Assumption of Homoscedasticity

A visual inspection of a scatter plot showing studentized residuals against the predicted values for each treatment group revealed a relatively scattered plotting for all variable combinations (see Figure 6). This visual leads to the conclusion that the assumption of homoscedasticity was met. However, Levene’s test did not confirm homoscedasticity. Upon further inspection, the scatterplot for Group D, as highlighted, appears to resemble a decreasing funnel shape, accounting for the failed assumption.

Figure 6

Studentized Residual Mfactpost by Predicted Value for Mfactpost by Treatment by Time



Null Hypothesis One: Effect of Treatment

H$_0$1: There is no significant difference in math fact fluency among third-, fourth-, and fifth-grade students who use rehearsal and retrieval practice methods when controlling for pre-test scores as measured by the Mathematics and Fluency Calculation Tests.

Assumption Testing

Homogeneity of variance is a required assumption for a two-way ANCOVA. A *p*-value >0.05 is considered standard for this assumption to be met. This assumption was violated when looking strictly at the variable of treatment ($p = 0.015$), which is noted as a limitation of the study in Chapter 5. See Table 6 for the Levene's test for treatment.

Table 6

Levene's Test of Equality of Error Variances

F	*df1*	*df2*	Sig.
6.051	1	194	0.015

Results for Null Hypothesis One

The researcher rejected null hypothesis one, noting that treatment type played a statistically significant role in student fluency in mathematical facts. The estimated means for the retrieval groups were significantly higher than for the rehearsal groups. At the 95% confidence level, the F-value was (1, 193) = 52.672 with p <0.001 and η^2 = 0.214. This immense effect size indicates that treatment type had a strong effect on student fact fluency. Table 7 shows the between-subjects test for treatment.

Table 7

Tests of Between-Subjects Effects

Source	Type III *SS*	*df*	*MS*	*F*	Sig.	η^2
Corrected Model	37476.588[a]	2	18738.294	203.858	0.000	0.679
Intercept	1999.105	1	1999.105	21.749	0.000	0.101
mfactpre	33512.959	1	33512.959	364.594	0.000	0.654
Treatment	4841.568	1	4841.568	52.672	0.000	0.214
Error	17740.285	193	91.919			
Total	2237801.000	196				
Corrected Total	55216.872	195				

a. R Squared = 0.679 (Adjusted R Squared = 0.675)

Note. Partial eta squared is represented in the text as η^2.

Given the failed assumption of homogeneity of variance, the researcher decided to run the HC3 of robust standard errors to prevent any Type 1 errors. It is not easy to quantify heteroskedasticity's effect on the validity of the statistical analysis (Hayes & Cai, 2007). The robust standard errors are used when concerns over homogeneity exist. They are "derived from an estimate of the variance-covariance matrix of errors" (Hayes & Cai, 2007, p. 712). Given its resilience to small sample sizes, the HC3 was chosen. The results of this first main effect analysis were supported by running the HC3 of robust standard errors (see Table 8).

Table 8

Parameter Estimates with Robust Standard Errors

Parameter	*b*	Robust *SE*	*t*	Sig.	95% Confidence Interval		η^2
					Lower Bound	Upper Bound	
Intercept	25.936	4.500	5.764	0.000	17.061	34.811	0.147
mfactpre	0.888	0.043	20.830	0.000	0.804	0.972	0.692
[treatment=1]	-9.960	1.397	-7.128	0.000	-12.716	-7.204	0.208
[treatment=2]	0[a]						

Note. The HC3 method was used.

a. This parameter is set to zero because it is redundant.

Null Hypothesis Two: Effect of Time

$H_0$2: There is no significant difference in math fact fluency among third-, fourth-, and fifth-grade students who practice every day, every other day, and every third day when controlling for pre-test scores as measured by the Mathematics and Fluency Calculation Tests.

Assumption Testing

Homogeneity of variance is a required assumption for a two-way ANCOVA. A *p*-value >0.05 is considered standard for this assumption to be met. This assumption was met when looking solely at the factor of time (p = 0.245). See Table 9 for the Levene's test for time.

Table 9

Levene's Test of Equality of Error Variances

F	*df1*	*df2*	Sig.
1.416	2	193	0.245

Results for Null Hypothesis Two

The researcher rejected null hypothesis two, demonstrating that time intervals statistically affected student mathematical fluency. At the 95% confidence level, the F-value was (2,192) = 9.991 with p = <0.001. The partial eta square was $\eta^2 = 0.094$. This large effect size indicates that the timing interval had a significant influence on student fact fluency. See Table 10 for the results of the between-subjects analysis for time.

Table 10

Tests of Between-Subjects Effects

Source	Type III *SS*	*df*	*MS*	*F*	Sig.	η^2
Corrected Model	34763.568[a]	3	11587.856	108.778	0.000	0.630
Intercept	1654.480	1	1654.480	15.531	0.000	0.075
mfactpre	33504.185	1	33504.185	314.512	0.000	0.621
time	2128.549	2	1064.274	9.991	0.000	0.094
Error	20453.304	192	106.528			
Total	2237801.000	196				
Corrected Total	55216.872	195				

Note. Partial eta squared is represented in the text as η^2

a. R Squared = 0.630 (Adjusted R Squared = 0.624)

Although the assumption of homogeneity of variance was met when solely considering the variable of time, the HC3 for robust standard errors was run to provide a solid basis for comparison and to maintain consistency across all research questions. The results of the main effect analysis for time were partially supported by the HC3 for robust standard errors (see Table 11).

Table 11

Parameter Estimates with Robust Standard Errors

Parameter	*b*	Robust *SE*	*t*	Sig.	95% Confidence Interval		η^2
					Lower Bound	Upper Bound	
Intercept	17.853	5.071	3.521	0.001	7.851	27.855	0.061
mfactpre	0.903	0.048	18.708	0.000	0.808	0.998	0.646
[time=1]	6.207	1.918	3.236	0.001	2.424	9.990	0.052
[time=2]	-1.780	1.692	-1.052	0.294	-5.117	1.557	0.006
[time=3]	0[a]						

Note. The HC3 method was used.
a. This parameter is set to zero because it is redundant.

Additional analysis of the pairwise comparisons for time indicates a significant difference between every day and every other day, as well as every day and every third day; however, there was no significant difference between every other day and every third day. These were analyzed given the trichotomous nature of the variable (see Table 12). The statistical significance between every day and every other day, as well as every day and every third-day practice, points to previous research on the effectiveness of long-term spaced practice with fixed intervals (Çekiç & Bakla, 2019). The current data shows that the every day practice was most effective and supports the previous work by Miller-Cotto and Byrnes (2020), who advocated for continuous repetition of material. While it seems clear that every day practice was significantly more effective than every other day and every third day, it is unclear why there was no statistical significance between every other day and every third day. One might postulate that every other day provided extra practice sessions per calendar week, but perhaps it was not enough to overshadow the results of every third-day practice.

Table 12

Pairwise Comparisons

(I) Time		*M* Difference (I-J)	*SE*	Sig.[b]	95% Confidence Interval for Difference[b] Lower Bound	Upper Bound
Every day	Every other day	8.532*	1.656	0.000	4.532	12.532
	Every third day	6.687*	1.564	0.000	2.909	10.464
Every other day	Every day	-8.532*	1.656	0.000	-12.532	-4.532
	Every third day	-1.845	1.579	0.733	-5.660	1.970
Every third day	Every day	-6.687*	1.564	0.000	-10.464	-2.909
	Every other day	1.845	1.579	0.733	-1.970	5.660

Note. Values were calculated based on estimated marginal means.
*. The mean difference is significant at the 0.05 level.

b. Adjustment for multiple comparisons: Bonferroni.

Null Hypothesis Three: Interaction Between Treatment and Time

$H_0$3: There is no significant interaction between rehearsal and retrieval methods, as measured by the Mathematics and Fluency Calculation Tests, on math fact fluency among third-, fourth-, and fifth-grade students who utilize rehearsal and retrieval methods based on practice every day, every other day, and every third day when controlling for pre-test scores.

Assumption Testing

Homogeneity of Variance. A Levene's test was run to check for homogeneity of variance. The assumption of homogeneity was violated ($p = 0.001$; see Table 13). Further investigation was done to determine the source of concern upon violating the homogeneity assumption. Levene's tests were run multiple times, excluding each treatment group individually. When the Levene's test was calculated without the inclusion of Group D, the assumption of homogeneity was met (p = 0.691), indicating that the variance of the residuals was not equal in treatment Group D. The violation of the assumption will be noted as a limitation of the study and listed in Chapter 5.

Table 13

Levene's Test of Equality of Error Variances

F	*df1*	*df2*	Sig.
4.290	5	190	0.001

Assumption of Normality. A Shapiro-Wilk's test was run to test for normality. Student residuals were normally distributed (p >0.05), except for the retrieval every third-day group (Group F) (p <0.001; see Table 14). Given the robust nature of the ANCOVA concerning violations of normality, the researcher conducted the two-way ANCOVA while disregarding the violation and accepting approximately normal data (Laerd Statistics, 2018). The violation will be listed in Chapter 5 as a limitation of this study.

Table 14

Tests of Normality

			Shapiro-Wilk		
Treatment			Statistic	*df*	Sig.
Rehearsal	Every day	Studentized Residual for mfactpost	0.977	34	0.671
	Every other day	Studentized Residual for mfactpost	0.981	29	0.867
	Every third day	Studentized Residual for mfactpost	0.969	40	0.346
Retrieval	Every day	Studentized Residual for mfactpost	0.927	28	0.052
	Every other day	Studentized Residual for mfactpost	0.988	28	0.981
	Every third day	Studentized Residual for mfactpost	0.851	37	0.000

*. This is a lower bound of the true significance.
a. Lilliefors Significance Correction

Two-Way Analysis of Covariance

The researcher conducted a two-way ANCOVA to determine whether there was a difference in student mathematical fact fluency based on various treatment practices and timing intervals. The two independent variables were the types of treatment and time-frequency. The dependent variable of mathematical fact fluency can be measured using the MFaCTS (Reynolds et al., 2015). Table 15 provides the Between-Subjects Effects, indicating that the interaction effect was not significant.

Table 15

Tests of Between-Subject Effects

Source	Type III *SS*	*df*	*MS*	*F*	Sig.	η^2
Corrected Model	39973.302[a]	6	6662.217	82.603	0.000	0.724
Intercept	1385.214	1	1385.214	17.175	0.000	0.083
mfactpre	34602.040	1	34602.040	429.019	0.000	0.694
treatment	4912.647	1	4912.647	60.910	0.000	0.244
time	2422.994	2	1211.497	15.021	0.000	0.137
treatment * time	120.797	2	60.398	0.749	0.474	0.008
Error	15243.570	189	80.654			
Total	2237801.000	196				
Corrected Total	55216.872	195				

Note. Partial eta squared is represented in the text as η^2

a. R Squared = 0.724 (Adjusted R Squared = 0.715)

Robust Test/SPSS for Heteroskedasticity- Consistent Standard Errors

To further investigate, in light of the homogeneity violation, the researcher calculated standard robust errors utilizing the heteroskedasticity-consistent standard errors (HC3) as suggested by Hayes and Cai (2007). The results of the robust test support the initial two-way ANCOVA. The third null hypothesis stated that there would be no statistically significant interaction effect between treatment type and timing interval. The researcher failed to reject the null hypothesis at the 95% confidence level. The *F*-value was F (2, 189) = 0.749, p = 0.474, partial η^2 = 0.008, indicating a small effect size. Table 16 contains the results of the HC3.

Table 16

Parameter Estimates with Robust Standard Errors

Parameter	*b*	Robust *SE*	*t*	Sig.	95% Confidence Interval		η^2
					Lower Bound	Upper Bound	
Intercept	21.549	4.710	4.575	0.000	12.258	30.839	0.100
mfactpre	0.922	0.041	22.478	0.000	0.841	1.003	0.728
[treatment=1]	-10.797	2.022	-5.340	0.000	-14.786	-6.809	0.131
[treatment=2]	0[a]						
[time=1]	7.149	3.067	2.331	0.021	1.099	13.199	0.028
[time=2]	-3.325	2.359	-1.410	0.160	-7.977	1.328	0.010
[time=3]	0[a]						
[treatment=1] * [time=1]	-0.924	3.462	-0.267	0.790	-7.752	5.905	0.000
[treatment=1] * [time=2]	2.959	2.947	1.004	0.316	-2.853	8.772	0.005
[treatment=1] * [time=3]	0[a]						
[treatment=2] * [time=1]	0[a]						
[treatment=2] * [time=2]	0[a]						
[treatment=2] * [time=3]	0[a]						

Note. The HC3 method was used.

a. This parameter is set to zero because it is redundant.

CHAPTER FIVE: CONCLUSIONS

Overview

The purpose of this quantitative, quasi-experimental, nonequivalent control group study was to measure the effect of practice method and spacing interval on student math fact fluency. Chapter 5 discusses the significant findings in light of literature and the foundational theories upon which the study was built: Bloom et al.'s (1956) taxonomy of educational objectives and Atkinson and Shiffrin's (1968) multi-store memory model. Additionally, implications for practice are explored before finishing with the stated limitations of this study and recommendations for future research.

Discussion

The purpose of this quantitative, quasi-experimental nonequivalent control group study was to determine if there was a difference in mathematical fact fluency among third-, fourth-, and fifth-grade students with varying practice methods and spacing intervals while controlling for pre-test scores measuring combined fluency. Mathematical fluency is a basic knowledge level skill that includes addition, subtraction, multiplication, and division. As Bloom et al. (1956) proposed, foundational skills, such as knowledge, must be mastered before other higher-level skills can be built. This study analyzed conditions for building students' fact fluency. Solid math fact fluency allows students to move forward with more in-depth problem-solving involving comprehension, application, analysis, synthesis, and evaluation (Allen-Lyall, 2018; Anderson et al., 2001; Baker & Cuevas, 2018; Bloom et al., 1956).

The study served to fill the literature gap identified by Latimier et al. (2021) to look at the interaction effect of practice strategies and spacing interval. Although the researcher failed to reject the null hypothesis, indicating that the interaction was not significant, the data shows marked improvement in student fluency following the interventions, as indicated by the rejection of hypotheses one and two regarding the significance of treatment and time on fluency. The results of this study support the idea that students were able to better develop the basic level skill of rote memory and fluency with mathematical facts. In turn, it is hopeful that they will become better problem-solvers as they apply these facts in more difficult learning situations, as proposed by Bloom et al. (1956).

Atkinson and Shiffrin's (1968) multi-store memory model centers heavily on moving information from the short-term to the long-term store. They proposed that information can remain in the short-term store for only brief periods without active rehearsal. Without active rehearsal, decay begins (McFarlane & Humphreys, 2012; Ricker et al., 2020). The results of this study demonstrate a slight improvement in fluency from rehearsal and support the theory that rehearsal does not truly increase learning but delays forgetting (Craik & Lockhart, 1972). Additional theories suggest that repetition, or rehearsal of information, only changes the present time and has little to no impact on long-term retention (Lyle et al., 2020). As with these theories, the retrieval results of this study supersede those of the rehearsal practice. As Atkinson and Shiffrin proposed, encoding is a much more effective method for building long-term storage. This encoding is similar to the retrieval practice method, where students create new associations and strengthen deeper connections by actively recalling facts.

Group A, who had rehearsal every day, reported a pre-test mean of 93.94 with an estimated post-test mean of 105.055. At the 95% confidence level, that is an increase of 11.115 points. Group B, who had rehearsal every other day, demonstrated growth from the pre-test to the estimated post-test mean of 5.185. Group D, who had retrieval every day, demonstrated an increase of 26.456 points. Group E, who had retrieval every other day, increased by 10.123

points. Group F, who had retrieval every third day, demonstrated an increase of 12.137 from the pre-test to the estimated post-test mean. The only group that failed to show growth due to the intervention was Group C, which had rehearsal every third day. This group's mean, when taking into account the covariate, decreased by -0.92 points. As with other recent research, such as Krominga and Codding (2021), this study demonstrated the overall effectiveness of flashcards with retrieval and rehearsal as a method for building fact fluency. Flashcards should be considered one rote practice strategy option that could supplement conceptually based approaches and establish a balanced approach to mathematics, as Malkus (2021) and Wu (1999) proposed.

Traditionally, many educators have used repeated timed tests to help build fluency (Boaler, 2015). However, these speed-based assessments create increased and unnecessary anxiety (Kling & Bay-Williams, 2021). The success of this flashcard intervention further supported Datchuk and Hier's (2019) and Morano et al.'s (2020) research advocating for short, incremental information reviews. Short reviews of fluency-based information allow students to remain engaged and prevent an overload of information. Interventions for this study were delivered in 10-minute increments, as suggested by DeFouw et al. (2023), and appear to have been effective. Furthermore, the intervention included correction for inaccurate responses, which followed previous research guidelines, ensuring students were not practicing incorrect information and required attention to and repetition with precise responses (Alptekin, 2019; Sönmez & Alptekin, 2020; Stocker & Kubina, 2017).

Null hypotheses one and two referred to the main effects of this study, indicating whether the variables of treatment and time created statistically significant results. Both null hypotheses were rejected, demonstrating that treatment and time statistically affected student fact fluency.

Atkinson and Shiffrin (1968) published their work on the multi-store memory model, which introduced the concept of two separate memory systems: short-term and long-term stores. Successful encoding in the long-term store meant that individuals could regularly pull information from there to the short-term store. This speculated that half of the students further developed neural pathways and encoded the mathematical facts into memory by continuously retrieving facts from the long-term store, as proposed by Haebig et al. (2021). The results fully support retrieval as the preferred method of practice, as suggested by Çekiç and Balka (2019), Codding et al. (2019), and Gordon (2020). Estimated means for the retrieval groups were significantly higher than those of the rehearsal practice, with retrieval every third day (M = 109.627), surpassing even rehearsal every day (M = 105.055), which supports the importance of strengthening neural pathways through purposeful and effortful retrieval of previously learned information by recalling answers to mathematical facts from memory (Brown et al., 2014; Gordon, 2020; Latimier et al., 2021).

Regarding null hypothesis two, the impact of time interval, the data demonstrates that every day practice was best (M = 110.916), followed by every third day (M = 104.229), and finally every other day (M = 102.384). While every day practice seems logically most effective, the transposition of every other day and every third day seems notable. This data could point to Chen and Kalyuga's (2021) and Haebig et al.'s (2021) research regarding the importance of processing time. Research demonstrates that processing time between retrieval practices leads to better results (Sewang, 2021). Additionally, researchers propose that delayed time between practices requires more effortful and purposeful information retrieval (Latimier et al., 2021), further supported by the data from this study, where every third-day practice led to higher estimated means than every other day. Another explanation could be that every third-day practice prevented students' boredom and disengagement. Students may have practiced with more integrity, effort, and attention because the practice had not become monotonous (Chen & Kalyuga, 2021; Latimer et al., 2021; Sewang, 2021). While this may not fully support the every day practice with the highest means, it is possible that the positive effect of practicing every day mitigated the negative effect of possible student boredom.

Implications

The Common Core State Standards were implemented in 2010 (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010). The new standards written to create consistency sparked many school divisions to develop conceptually based approaches to instruction. Unfortunately, many quickly abandoned all practices that appeared to be rote and memory-based. They failed to negotiate the importance of basic fluency in problem-solving. Given the growing decline in math proficiency (*NAEP report card*, 2022), researchers suggest more balanced approaches incorporating conceptual and fluency instruction (Allen-Lyall, 2018; McGee et al., 2017; Nahdi & Jatisunda, 2020). The current study focused on building fluency in students in hopes of creating better problem-solvers.

Bloom et al. (1956) suggested that building fluency was vital to developing student problem-solvers. Bloom et al.'s work illustrated that lower-level skills, such as fact fluency, are essential to higher-level application. The current study examined ways to build fluency and free up the working memory for greater problem-solving. The rehearsal and retrieval strategies utilized in this study mimic the theorized processes of Atkinson and Shiffrin (1968), who implied that retrieval or encoding strategies out top rehearsal strategies and better help solidify information into the long-term store. The current study helped explore further the practices of retrieval and rehearsal in relationship to building repetitive fluency.

The results of this study are a reminder that retrieval strategies are superior to rehearsal, given the prerequisite of attention and intentional thought, as suggested by Brown et al. (2014), Gordon (2020), and Latimier et al. (2021). Teachers are encouraged to provide numerous opportunities to retrieve information, which can be accomplished through class practice, questioning techniques, or even formative assessments. This study demonstrates the effectiveness of flashcards and supports their use, given their ease of implementation, as suggested by Bjordahl et al. (2014), Lund et al. (2012), and Mann et al. (2012). It offers an alternative to timed test practice. As students utilize flashcards and retrieve previously learned information, neural pathways are strengthened, and fluency increases (Haebig et al., 2021).

The study also explored various timing intervals to establish a basis for the frequency of interventions. It helped to fill the gap in the literature regarding spacing intervals by testing alternate spacing intervals, as suggested by Codding et al. (2019) and Schutte et al. (2015). It also served to study the role of spaced practice in the development of math facts in elementary students as compared to previous studies of older students and adults, as suggested by Hopkins et al. (2016), Lyle et al. (2020), and Pagán and Nation (2019).

It is doubtful that educators can dedicate time daily for interventions. This study demonstrated that if every day intervention is impossible, every third day may be equal, if not better, than every other day, which allows educators flexibility in utilizing daily instructional time wisely. Educators are often leery about engaging in repetitive practice daily, as students quickly become bored and disengaged. The results of this study encourage educators to consider spacing repetitive practice sessions to about two times per week, which may prevent students from becoming disinterested.

Over the past decade, schools have been mandated to support struggling students robustly. All states have adopted some version of the Response to Intervention or Multi-Tiered System of Support process (Zhang et al., 2023). The purpose of these approaches is to provide quality instruction to all students, to develop targeted instruction for students who qualify for Tier II supports, approximately 10-15% of the population, and even more in-depth support for those who qualify for Tier III supports, approximately 1-5% of the population. Given the success of the retrieval flashcard strategy in this study and the importance of mathematical fact fluency to overall success, teachers should consider flashcard-based retrieval practice strategies for students struggling with fluency. Additionally, the impact of everyday practice was evident in this study, which further supports the importance of schools utilizing a daily intervention system for the most severely struggling students. Since there was no statistical significance between every other and every third-day practice, further analysis is necessary to determine the most appropriate and effective spacing interval for students who require moderate-level support.

Limitations

As with all research, while factors were controlled to the greatest extent possible, there are limitations to the results presented in this study. There were violations of the assumptions of homogeneity of variance and normality. While the two-way ANCOVA is robust to violations of normality (Laerd Statistics, 2018), one must consider this limitation regarding Type 1 errors. The HC3 procedure was used for robust standard errors to combat the violation of homogeneity; nonetheless, a violation exists. However, the HC3 procedure verified the results of the original two-way ANCOVA and provided the same conclusion.

Additional internal validity concerns exist due to the large number of participating classrooms. Each classroom teacher administered pre- and post-tests and oversaw intervention sessions. Lesson plans and implementation guidelines were provided. A mid-study checklist helped teachers ensure proper procedure and implementation; however, including this many overseers creates possible errors in treatment fidelity (Gall et al., 2007). The variety of classroom settings and participating schools created variability in classroom instruction and practice concerning mathematical fluency, which may have influenced the study results. Teachers were not asked to alter their typical mathematics instruction; thus, some variability may exist if students received additional fluency practice as part of their standard daily instruction. The maturation of student participants given typical classroom instruction may have affected the data of this study to some degree (Gall et al., 2007). However, this also reflects the nature of quasi-experimental studies, often conducted in education, when true experimental studies are nearly impossible.

Recommendations for Future Research

The following research topics would fill gaps in the literature as a follow-up to the current study.

1. This study investigated all fluency operations combined: addition, subtraction, multiplication, and division. It may be helpful to separate these skills and tailor a study to targeted grade bands. For example, addition and subtraction would be tailored to first- and second-grade students; multiplication and division would be tailored to third- and fourth-grade students, preventing younger students from becoming frustrated with operations above their skill level. Additionally, older students would be challenged with operations that are more appropriate for them and would eliminate the threat of the ceiling effect.

2. This study investigated timing intervals of every day, every other day, and every third day. Given that a typical school week is five days, testing the impact of every fourth day and every fifth-day practice may be interesting, which would further allow for an investigation of how many times a week interventions should be implemented.
3. Studying the effect of practice duration on fact fluency may be interesting. This study utilized a ten-minute practice duration. Future researchers could utilize shorter or longer practice durations.
4. Future researchers could conduct a correlational study between mathematical fluency and anxiety and confidence. In hindsight, it would have been interesting to include an anxiety and confidence survey in conjunction with this study to analyze the correlation between increased fluency and student confidence, which could have further supported the work of Sorvo et al. (2017) and Liu (2006), who studied the connection between anxiety and skill fluency.
5. In this study, the intervention protocol utilized partner study practice. Future research could examine the effect of partner or group and individual practice on fact fluency development.
6. A final study of interest is a longitudinal regression study to analyze the impact of fact fluency on future mathematical success and performance. It could demonstrate that fluency influences long-term success in mathematics and dictates the role it should play throughout instruction.

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APPENDIX A

Population Description

School	White %	Hispanic %	Black %	Asian %	Multi-Ethnic %	Female %	Male %	HS Diploma %	Bachelor's Degree %	Income Avg $	English Speaking %
A	66	18	11	1	4	53	47	48	23	35,000	81
B	83	3	8	1	3	51	49	37	23	40,000	95
C	78	3	11	2	4	49	51	36	22	42,000	92
E	84	1	5	3	6	53	47	13	61	67,000	88
F	78	2	5	0	0	52	48	18	48	41,000	85
G	85	5	1	3	4	48	52	14	62	92,000	88
	White %	Native American %			Multi-Ethnic %	Female %	Male %	HS Diploma %	Bachelor's Degree %	Income Avg $	English Speaking %
D	8	88			1	53	47	30	16	25,000	84

APPENDIX B

Treatment Groups	# of Participants	Female %	Male %	White %	Black %	Hispanic %	Multi-Ethnic %	Asian %	Native American %	Treatment
A1	8	38	62	100	0	0	0	0	0	Rehearsal every day
A2	22	55	45	100	0	0	0	0	0	Rehearsal every day
A3	4	50	50	25	0	0	25	50	0	Rehearsal every day
B1	14	57	43	93	0	0	7	0	0	Rehearsal every other day
B2	15	47	53	93	0	0	7	0	0	Rehearsal every other day
C1	16	50	50	87	0	0	13	0	0	Rehearsal every third day
C2	15	53	47	100	0	0	0	0	0	Rehearsal every third day
C3	9	56	44	44	22	0	0	33	0	Rehearsal every third day
D1	14	50	50	93	0	0	7	0	0	Retrieval every day
D2	7	71	29	71	0	0	14	14	0	Retrieval every day
D3	7	86	14	0	57	14	0	29	0	Retrieval every day
E1	5	80	20	0	0	0	0	0	100	Retrieval every other day
E2	3	67	33	100	0	0	0	0	0	Retrieval every other day
E3	20	55	45	85	0	0	15	0	0	Retrieval every other day
F1	22	55	45	95	0	0	5	0	0	Retrieval every third day
F2	15	53	47	93	0	0	7	0	0	Retrieval every third day

APPENDIX C

MFaCTs Encouragement for Use

Acknowledgments

We extend our sincere appreciation to the following professionals, who field-tested the MFaCTs, provided standardization data, or contributed in other ways to the development of this test: Jaisen Bell, Levan Parker (Alabama); Cindy Alavezos, Annette Alberti (California); Brook Olsen (Idaho); Chad Holtz, Sofia Kock (Nebraska); Angela Rodriguez (New Mexico); Lisa Brockhulzen, Virginia Brown, Tracy Larson, Phillipa Laubens (New York); Mark Klein, Sue Nelson (North Dakota); Cresta Ritter (Ohio); Lonneida Alexander, Mary Cole, Mary Gutierrez, Eileen Haney, Taddy Maddox, Susan Rose (Texas).

As authors, we wish to express our appreciation to the staff at PRO-ED for their extraordinary work. They did a superb job with all aspects of design, data-collection support, and follow-up, resulting in a norming sample that lends confidence in the accuracy of the MFaCTs scores. The insightful comments of Don Hammill on the MFaCTs manual are also noted and were beneficial to this process.

Cecil and Randy would like to express their continuing gratitude to their common mentor, Alan S. Kaufman, for his model of practical scholarship. Additionally, Cecil acknowledges his appreciation to Julia for continuing to tolerate his work schedule and providing such balance in his life.

Note

Clinicians and researchers who use the MFaCTs are invited and encouraged to send copies of their work, along with any suggestions for improving the test, to the authors in care of PRO-ED, 8700 Shoal Creek Boulevard, Austin, Texas 78757.

APPENDIX D

MFaCTs Permission

4/19/23, 6:58 PM

[redacted]

MFacts

[redacted]

Tue, Jan 31, 2023 at 10:42 PM

[redacted]

Hi Stefanie,

You do not need our permission to purchase and use the MFaCTS in your research, but we welcome you to do so. The least expensive place to purchase it is through the www.proedinc.com web site. I am not aware of any math tests that only measure multiplication and doubt any would exist since most are designed to follow standard curricula recommended in the field.

Cecil R. Reynolds, PhD

Editor-in-Chief, *Journal of Pediatric Neuropsychology*

Emeritus

Professor of Educational Psychology

Professor of Neuroscience

Distinguished Research Scholar

Texas A&M University

512-656-5075

Former Editor-in-Chief

Psychological Assessment

Archives of Scientific Psychology

Archives of Clinical Neuropsychology

Applied Neuropsychology

APPENDIX E

Approvals

August 21, 2023

Dear Stefanie Breneman-Smith,

After review of your research proposal entitled Quasi-experimental Study of the Impact of Spaced Practice and Retrieval on Mathematical Fact Fluency of Third, Fourth, and Fifth Grade Students, I have decided to grant you permission to conduct your study at Cumberland Valley Christian School.

Check the following as applicable:

☐ I grant permission for you to include 3rd, 4th, and/or 5th grade students from our school building in your study.

☐ I grant permission for the use of a three minute pre-and post-test fact fluency assessment.

☐ I grant permission for teachers to implement the daily flashcard fluency intervention as long as they are willing to participate.

☐ The requested data WILL BE STRIPPED of all identifying information before it is provided to the researcher.

☐ The requested data WILL NOT BE STRIPPED of all identifying information before it is provided to the researcher.

☐ I am requesting a copy of the results upon study completion and/or publication.

Sincerely,

Signature   Date
Building Principal

APPENDIX F

Institutional Review Board Approval

LIBERTY UNIVERSITY.
INSTITUTIONAL REVIEW BOARD

October 12, 2023

Stefanie Breneman-Smith
Nathan Putney

Re: IRB Approval - IRB-FY23-24-298 QUASI-EXPERIMENTAL STUDY OF THE IMPACT OF SPACED PRACTICE AND RETRIEVAL HAVE ON MATHEMATICAL FACT FLUENCY IN THIRD, FOURTH, AND FIFTH-GRADE STUDENTS

Dear Stefanie Breneman-Smith, Nathan Putney,

We are pleased to inform you that your study has been approved by the Liberty University Institutional Review Board (IRB). This approval is extended to you for one year from the following date: October 12, 2023. If you need to make changes to the methodology as it pertains to human subjects, you must submit a modification to the IRB. Modifications can be completed through your Cayuse IRB account.

Your study falls under the expedited review category (45 CFR 46.110), which is applicable to specific, minimal risk studies and minor changes to approved studies for the following reason(s):

7. Research on individual or group characteristics or behavior (including, but not limited to, research on perception, cognition, motivation, identity, language, communication, cultural beliefs or practices, and social behavior) or research employing survey, interview, oral history, focus group, program evaluation, human factors evaluation, or quality assurance methodologies. (NOTE: Some research in this category may be exempt from the HHS regulations for the protection of human subjects. 45 CFR 46.101(b)(2) and (b)(3). This listing refers only to research that is not exempt.)

For a PDF of your approval letter, click on your study number in the My Studies card on your Cayuse dashboard. Next, click the Submissions bar beside the Study Details bar on the Study Details page. Finally, click Initial under Submission Type and choose the Letters tab toward the bottom of the Submission Details page. Your stamped consent form(s) and final versions of your study documents can be found on the same page under the Attachments tab. Your stamped consent form(s) should be copied and used to gain the consent of your research participants. If you plan to provide your consent information electronically, the contents of the attached consent document(s) should be made available without alteration.

Thank you for your cooperation with the IRB, and we wish you well with your research project.

Sincerely,

G. Michele Baker, PhD, CIP
Administrative Chair
Research Ethics Office

LIBERTY UNIVERSITY.
INSTITUTIONAL REVIEW BOARD

December 12, 2023

Stefanie Breneman-Smith
Nathan Putney

Re: Modification - IRB-FY23-24-298 QUASI-EXPERIMENTAL STUDY OF THE IMPACT OF SPACED PRACTICE AND RETRIEVAL HAVE ON MATHEMATICAL FACT FLUENCY IN THIRD, FOURTH, AND FIFTH-GRADE STUDENTS

Dear Stefanie Breneman-Smith, Nathan Putney,

The Liberty University Institutional Review Board (IRB) has rendered the decision below for IRB-FY23-24-298 QUASI-EXPERIMENTAL STUDY OF THE IMPACT OF SPACED PRACTICE AND RETRIEVAL HAVE ON MATHEMATICAL FACT FLUENCY IN THIRD, FOURTH, AND FIFTH-GRADE STUDENTS .

Decision: Approved

Your request to add [redacted] as study sites has been approved. Thank you for submitting documentation of permission from your additional sites for our review and documentation. **For a PDF of your modification letter, click on your study number in the My Studies card on your Cayuse dashboard. Next, click the Submissions bar beside the Study Details bar on the Study Details page. Finally, click Modification under Submission Type and choose the Letters tab toward the bottom of the Submission Details page. If your modification required you to submit revised documents, they can be found on the same page under the Attachments tab.**

Thank you for complying with the IRB's requirements for making changes to your approved study. Please do not hesitate to contact us with any questions.

We wish you well as you continue with your research.

Sincerely,

G. Michele Baker, PhD, CIP
Administrative Chair
Research Ethics Office

APPENDIX G

Parental Permission

Parental Consent

Title of the Project: Quasi-Experimental Study of The Impact Of Spaced Practice And Retrieval Have On Mathematical Fact Fluency In Third, Fourth, And Fifth-Grade Students
Principal Investigator: Stefanie Breneman-Smith, Doctoral Candidate, School of Education, Liberty University

Invitation to be Part of a Research Study

Your child is invited to participate in a research study. To participate, he/she must be a student in 3rd, 4th, or 5th grade. Taking part in this research project is voluntary.

Please take time to read this entire form and ask questions before deciding whether to allow your child to take part in this research project.

What is the study about and why are we doing it?

The purpose of the study is identify how different practice methods and spacing intervals (practice every day, every other day, every third day) impact math fact fluency.

What will participants be asked to do in this study?

If you agree to allow your child to be in this study, I will ask him/her to do the following:

1. Complete a three minute fact fluency pre-test (includes addition, subtraction, multiplication, and division facts).
2. Participate in daily 10 minute flashcard fluency practice with a classmate. This could be rehearsal practice (repetition) or retrieval practice (traditional flashcards). This could be every day, every other day, or every third day and will last for a maximum of 45 school days. Decisions on which practice method and which spacing interval will be used will be decided randomly by class.
3. Complete a three minute fact fluency post-test (includes addition, subtraction, multiplication, and division facts).

How could participants or others benefit from this study?

The repetition provided by the flashcard intervention should allow students to improve their mathematical fact fluency. This, in turn, should boost their self-confidence and self-efficacy.

Benefits to society include a better understanding on best practice in building mathematical fact fluency in upper elementary students.

What risks might participants experience from being in this study?

The expected risks from participating in this study are minimal, which means they are equal to the risks your child would encounter in everyday life.

How will personal information be protected?

The records of this study will be kept private. Published reports will not include any information that will make it possible to identify a subject. Research records will be stored securely, and only the researcher, and members of her doctoral committee will have access to the records.

Participant responses will be kept confidential by replacing names with codes. Data collected from your child may be used in future research studies. If data collected from your child is reused, any information that could identify your child, if applicable, will be removed beforehand.

Data will be stored on a password-locked computer and in a locked box. After five years, all electronic records will be deleted and all hardcopy records will be shredded.

How will participants be compensated for being part of the study?

Participants will not be compensated for participating in this study.

Is the researcher in a position of authority over participants, or does the researcher have a financial conflict of interest?

The researcher serves as a teacher at [redacted] To limit potential or perceived conflicts, your child's teacher may strip all personal identifiers before the researcher receives it. If data is stripped by your child's teacher, a linking list will be given to the researcher with codes to help match pre- and post-test data without revealing student identity. This disclosure is made so that you can decide if this relationship will affect your willingness to allow your child to participate in this study. No action will be taken against an individual based on her or his decision to allow his or her child to participate in this study.

Is study participation voluntary?

Participation in this study is voluntary. Your decision whether to allow your child to participate will not affect your or his/her current or future relations with Liberty University. If you decide to allow your child to participate, he/she is free to not answer any question or withdraw at any time without affecting those relationships.

What should be done if a participant wishes to withdraw from the study?

If you choose to withdraw your child from the study or your child chooses to withdraw, please contact the researcher at the email address/phone number included in the next paragraph. Should you choose to withdraw him/her or should your child choose to withdraw, data collected from your child will be destroyed immediately and will not be included in this study.

Whom do you contact if you have questions or concerns about the study?

The researcher conducting this study is Stefanie Breneman-Smith. You may ask any questions you have now. If you have questions later, **you are encouraged** to contact her at [redacted] or [redacted] You may also contact the researcher's faculty sponsor, Dr. Nathan Putney, at [redacted]

Whom do you contact if you have questions about rights as a research participant?

If you have any questions or concerns regarding this study and would like to talk to someone other than the researcher, **you are encouraged** to contact the IRB. Our physical address is Institutional Review Board, 1971 University Blvd., Green Hall Ste. 2845, Lynchburg, VA, 24515; our phone number is 434-592-5530, and our email address is irb@liberty.edu.

Disclaimer: The Institutional Review Board (IRB) is tasked with ensuring that human subjects research will be conducted in an ethical manner as defined and required by federal regulations. The topics covered and viewpoints expressed or alluded to by student and faculty researchers are those of the researchers and do not necessarily reflect the official policies or positions of Liberty University.

Your Consent

By signing this document, you are agreeing to allow your child to be in this study. Make sure you understand what the study is about before you sign. You will be given a copy of this document for your records. The researcher will keep a copy with the study records. If you have any questions about the study after you sign this document, you can contact the study team using the information provided above.

I have read and understood the above information. I have asked questions and have received answers. I consent to allow my child to participate in the study.

____
Printed Name

____
Signature Date

APPENDIX H

Student Permission

Child Assent to Participate in a Research Study

What is the name of the study and who is doing the study?
The name of the study is Quasi-experimental Study of the Impact of Spaced Practice and Retrieval Have on Mathematical Fact Fluency in Third, Fourth, and Fifth Grade Students, and the person doing the study is Mrs. Stefanie Breneman-Smith.

Why is Mrs. Breneman-Smith doing this study?
Mrs. Breneman-Smith wants to know the best way for students to learn math facts. She is looking to see the best way to practice math facts and how often students should practice.

Why am I being asked to be in this study?
You are being asked to be in this study because you are a student in third, fourth, or fifth grade.

If I decide to be in the study, what will happen and how long will it take?
If you decide to be in this study, you will take two different fact tests – one at the beginning of the marking period and one at the end. These each take 3 minutes. These tests do not impact your school grade at all. You will also work with a partner for 10 minutes to practice math facts using flashcards. This will continue for about one marking period.

Do I have to be in this study?
No, you do not have to be in this study. If you want to be in this study, then tell the researcher. If you don't want to, it's OK to say no. The researcher will not be angry. You can say yes now and change your mind later. It's up to you.
What if I have a question?
You can ask questions any time. You can ask now. You can ask later. You can talk to the researcher. If you do not understand something, please ask the researcher to explain it to you again.
Signing your name below means that you agree to be in the study.

____
Signature of Child Date

Stefanie Breneman-Smith
[redacted]

Dr. Nathan Putney
[redacted]

Liberty University Institutional Review Board
1971 University Blvd, Green Hall 2845, Lynchburg, VA 24515
irb@liberty.edu

APPENDIX I

Intervention Instructional Plan

Lesson Plan for Rehearsal Group

Materials: Flashcard sets (one per every two students) (Note: Flashcard sets are equivalent, but not identical. Each set contains one card of each fact minus reversals to cut down on the total number of cards), five-minute timer

Before beginning go over these instructions with students. Additionally, give each student a partner. These will remain constant over the course of the intervention.

1. Student partners will retrieve their bag of flashcards. Student A will prepare the cards for the day's practice. The teacher will announce that it is time to begin.
2. Student A will display the flashcards one at a time for Student B. Student B will read the fact (including the answer) aloud, followed by its reversal. For example, 6x7 = 42 and 7x6 = 42
3. Student A will then flip to the next card. This will continue until the teacher says time has ended (five-minutes).
4. Students will swap places and continue for another five-minute round.
5. Students should place cards in the storage bag with the next card on top to begin at the following practice session.
6. Complete this ____________________ day for 45 days (_______ sessions total).
7. Once per week, review these directions aloud with all students to reinforce proper procedure.

Pre- and Post-Test Administration Instructions:

1. Hand out booklets to each student. (Pre-test Form A, Post-test Form B)

2. Tell students, **“Open your book to the second page. When I tell you to start, write the answer to each problem. When you finish a row, go on to the next.”** (Demonstrate for students with your finger). **“Be sure to pay attention to the sign and correctly add, subtract, multiply, or divide. If you do not know an answer, skip that problem and leave it blank. Work as fast as you can until I say STOP.”**
3. Tell students to begin.
4. Time for three-minutes and then instruct them to stop. Collect all materials and securely store them until they are retrieved by the researcher.

As adapted from:

Reynolds, C. R., Voress, J. K., & Kamphaus, R. W. (2015). *Mathematics Fluency and Calculation Test. Technical manual.* PRO-ED.

Lesson Plan for Retrieval Group

Materials: Flashcard sets (one per every two students) (Note: Flashcard sets are equivalent, but not identical. Each set contains one card of each fact minus reversals to cut down on the total number of cards), five-minute timer

Before beginning go over these instructions with students. Additionally, give each student a partner. These will remain constant over the course of the intervention.

1. Student partners will retrieve their bag of flashcards. Student A will prepare the cards for the day's practice. The teacher will announce that it is time to begin.
2. Student A will display the flashcards one at a time for Student B. Student B will read the fact aloud and provide the correct response. They will also repeat its reversal. For example, 6x7 = 42 and 7x6 = 42
3. Student A will verify the correct answer and then flip to the next card. If the provided answer is incorrect, Student A will provide the correct answer, at which time, Student B should repeat both the full correct fact and its reversal aloud. If Student A cannot produce the answer, Student B will provide it, and then Student A will repeat the full correct fact and its reversal aloud.
4. This will continue until the teacher says time has ended (five-minutes).
5. Students will swap places and continue for another five-minute round.
6. Students should place cards in the storage bag with the next card on top to begin at the following practice session.
7. Complete this ____________________ day for 45 days (_______ sessions total).
8. Once per week, review these directions aloud with all students to reinforce proper procedure.

Pre- and Post-Test Administration Instructions:

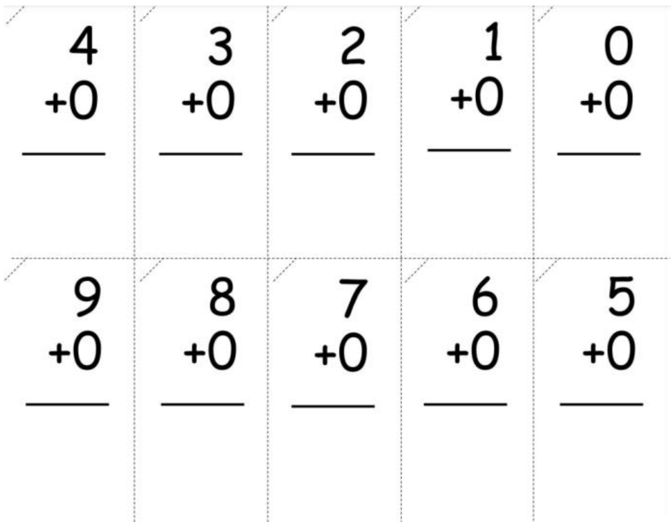
1. Hand out booklets to each student. (Pre-test Form A, Post-test Form B)
2. Tell students, **"Open your book to the second page. When I tell you to start, write the answer to each problem. When you finish a row, go on to the next."** (Demonstrate for students with your finger). **"Be sure to pay attention to the sign and correctly add, subtract, multiply, or divide. If you do not know an answer, skip that problem and leave it blank. Work as fast as you can until I say STOP."**
3. Tell students to begin.
4. Time for three-minutes and then instruct them to stop. Collect all materials and securely store them until they are retrieved by the researcher.

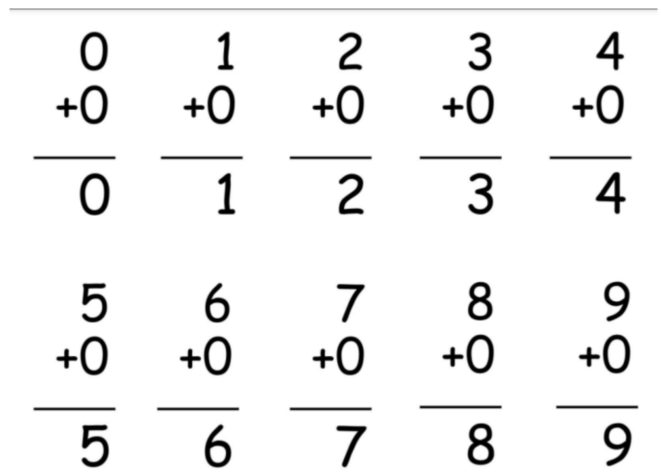
As adapted from:

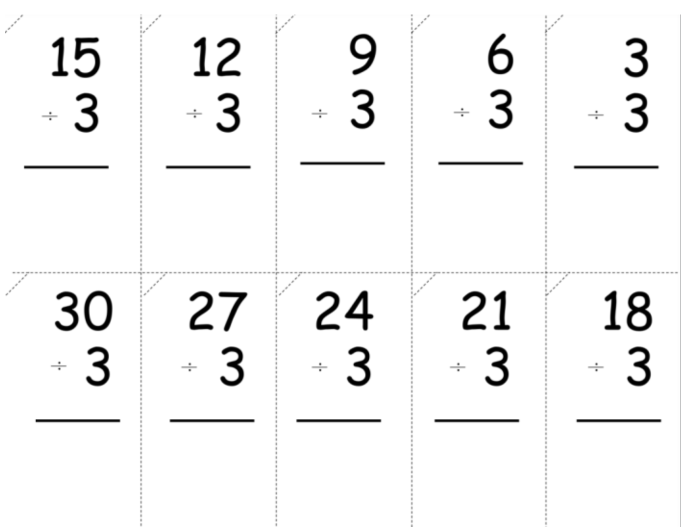
Reynolds, C. R., Voress, J. K., & Kamphaus, R. W. (2015). *Mathematics Fluency and Calculation Test. Technical manual.* PRO-ED.

APPENDIX J

Flashcard Template – Printed Front/Back







APPENDIX K

Factorial Design Chart

Rehearsal Practice			Retrieval Practice		
Group A	Group B	Group C	Group D	Group E	Group F
Every Day	Every Other Day	Every Third Day	Every Day	Every Other Day	Every Third Day
45 sessions	22 sessions	15 sessions	45 sessions	22 sessions	15 sessions

APPENDIX L

Treatment Fidelity Checklist

Directions: To be completed in a meeting between the teacher participants and the researcher.

Pre- and Post-Test:

1. Did you read the directions aloud to students verbatim?
2. Did you give students exactly three-minutes to complete the test?
3. Did you code the assessments to protect student identity, yet provide a means for matching up pre- and post-data?
4. Can you provide demographic information for each student participant?

Treatment:

1. Did students work with one partner daily? If there were an odd number of students, did the teacher serve as the partner for a missing student?
2. Did students work with the same partners daily?
3. Did you accurately time each practice session for five-minutes, switching once daily, for a total of 10 minutes daily?
4. Did students read and respond verbally for each flashcard?
5. Did students provide the reversal for each flashcard fact presented?
6. Did students practice one card at a time?
7. Did students begin with the next card each session?
8. Did you implement the intervention for the correct number of days – 45 consecutive school days, 22 for every other school day, and 15 for every third day?

9. Retrieval only: If the student verbalized an incorrect response, were they corrected by their partner?
10. Retrieval only: If the student could not verbalize a response, were they provided the answer by their partner?