How Prerequisite Skills Training Affects Math Achievement Scores for Students with Learning Disabilities:

A Quasi-Experimental Control Group Study

By

Timothy Ryan Reedy

Liberty University

A Quantitative Dissertation Presented in Partial

Fulfillment of the Requirements for the Degree

Doctor of Philosophy

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Approved By:

Laura J. Mansfield, Ed.D., CCC-SLP, Committee Chair

Susan Stanley, Ed.D., Committee Member

Abstract

The purpose of this quantitative, quasi-experimental study with a pretest-posttest control group design, while controlling for math achievement pretest scores, was to investigate the impact prerequisite skills instruction has on math achievement scores for ninth-grade pre-algebra students with specific learning disabilities (SLDs) in math. While there have been several studies on how prerequisite skills affect students with learning disabilities in elementary school, very little research has extended into high school. Convenience sampling was used in the study and included 70 students from a high school that serves students across the state of Pennsylvania. The Mathematics Achievement Test (MAT) was used as the primary instrument for the study. Data was collected throughout the 8-week study with a pretest, an exit ticket at the end of the class, and a posttest. The researcher used a one-way Analysis of Covariance (ANCOVA) and observed a significant difference between the test scores of the control group and the experimental group. Recommendations for future research included conducting research with a larger, more diverse group of students, and conducting the research using other methods of instruction involving prerequisite skills.

Keywords: prerequisite skills, intervention, working memory, specific learning disability

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Dedication

This document is dedicated to Sister Mary Perpetua and Joseph Murphy, who guided me on my path by teaching me the true value of education.

Acknowledgments

First, I would like to thank God for providing me with a guiding light as I navigated through this journey in my life. I could not have completed this dissertation without the love and support of my family. I would like to acknowledge Ms. Moodie, Ms. Duke, and Ms. Ahonen for their friendship and inspiration from the first day I met them. I would also like to thank Dr. Romanelli for his advice throughout the dissertation process. To all my professors, colleagues, and students who have challenged me throughout this process to reach my full potential, thank you.

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List of Abbreviations

Analysis of Covariance (ANCOVA)

Competency Based Education (CBE)

Education for All Handicapped Children Act (EHA)

Evidence Based Practice (EBP)

Every Student Succeeds Act (ESSA)

Free and Appropriate Education (FAPE)

Functional Communication Training (FCT)

Individualized Education Program (IEP)

Individuals with Disabilities Education Improvement Act (IDEA)

Least Restrictive Environment (LRE)

Mathematics Achievement Test (MAT)

Multi-Tiered Systems of Support (MTSS)

National Assessment of Educational Progress (NAEP)

No Child Left Behind Act (NCLB)

Response to Intervention (RTI)

Science, Technology, Engineering, and Mathematics (STEM)

Specially Designed Instruction (SDI)

Specific Learning Disability (SLD)

Statistical Package for the Social Sciences (SPSS)

Chapter 1: Introduction

Overview

Building mathematical skills in secondary education strengthens problem-solving, memory, and organizational skills that can benefit daily living and careers. Students with specific learning disabilities (SLDs) struggle in various domains of mathematics due to a deficit in the proper organization of memory schemas, which prevents the successful recall of prerequisite knowledge. The present study addresses a gap in the literature by introducing an instructional strategy to strengthen prerequisite skills before requisite skills are taught. This chapter provides an overview of the study, including background, problem, and purpose statements; the significance of the study for stakeholders vis-a-vis literature; and research questions. The chapter concludes by defining special terms used in the present study.

Background

Mathematical thinking extends into nearly every aspect of modern life (Fuchs et al., 2020; Mukhni et al., 2021; Namkung & Bricko, 2021; Wilkey et al., 2020), from personal health to daily accounting skills. While math pervades much of our daily lives, students with SLDs struggle with math and are in danger of struggling to learn these skills without proper instruction (Lein et al., 2020). Additionally, students with disabilities consistently perform lower on state and national assessments when compared to their peers without disabilities (Gilmour & Henry, 2018; Myers et al., 2022; Rose, 2020). In 2016, the Institute for Education Sciences noted that students with disabilities in fourth grade scored 0.87 SD below their peers without disabilities on the 2015 National Assessment of Educational Progress (NAEP).

An SLD is a neurological learning disorder that can affect reading, writing expression, and/or mathematical expressions (Alloway & Carpenter, 2020). Stevens and Schulte (2017) noted that almost 40% of students classified under the 13 categories available in the Individuals with Disabilities Education Act (IDEA) have SLDs. Students with SLDs in math struggle to discover and utilize strategies for achieving learning goals, leading to lower math achievement scores (Johnson et al., 2020). Without support, students with SLDs in math may struggle with essential daily functions, such as banking, calculating the cost of travel, and understanding interest on car loans and mortgages (Wilkey et al., 2020). Seitz and Weinert (2022) also noted that these students are less likely to graduate high school, attend college, or find sustainable employment.

Specially Designed Instruction (SDI) for students with disabilities adapts teaching methods as appropriate to meet a student's needs (Hedin et al., 2020). SDI goes beyond accommodations and scaffolding (Rodgers & Weiss, 2019) and is tailored to assist each student in their educational goals once they are determined eligible for services through an Individualized Education Program (IEP). Even with procedures to help students with disabilities, several of them leave school without functional math skills (Dueker & Day, 2022). Research has found that identifying and teaching missing math skills reduces learning gaps for students with disabilities, thereby suggesting that teaching prerequisite skills is an essential component of practical instruction for students with learning disabilities in math (Bertrand et al., 2021; Dueker & Day, 2022; Namkung & Bricko, 2021).

Historical Overview

SLDs have impacted the education system for over a hundred years. According to McDowell (2018), SLDs can be traced back to 120 years ago when a British doctor was perplexed by a patient with high intelligence who had difficulty reading. In the United States, Section 504 of the Rehabilitation Act of 1972 began to address the need for access to and accommodation in general education for students with learning disabilities. This was soon followed by the Education for All Handicapped Children (EHA) Act of 1975, comprising requirements for Free and Appropriate Education (FAPE) for all children. The IEP for students who qualified as having a disability under the legislation was also introduced. The EHA of 1975 was renewed with the passage of IDEA in 1990, which strengthened the demands to provide inclusionary services for students with disabilities. The passage of legislation, such as IDEA and the Every Student Succeeds Act (ESSA), emphasized higher levels of thinking and problem-solving (Marita & Hord, 2017); hence, students with disabilities could achieve greater success in higher education and employment.

According to Scammacca et al. (2020), efforts to bridge reading and math gaps have been a priority for America’s education system and were further supported by Supreme Court rulings, such as Endrew F. vs. Douglas County School District. According to Yell and Katsiyannis (2019), the Supreme Court strengthened the need for more rigorous goals, adopting a progress monitoring system, and collecting data to drive instruction. The academic domains of reading and math remain strong focus areas in education. Hence, students with disabilities have more comprehensive access to employment opportunities and acceptance in higher education institutions or vocational training. According to Rodgers and Weiss (2019), students with SLDs receive most of their instruction in a general education setting but are still receiving lower achievement scores.

Society-at-Large

Students with SLDs are often overlooked by colleges, universities, and employment agencies, due to their continued struggle in math and reading during post-secondary education. According to Smith and White (2019), Science, Technology, Engineering, and Mathematics

(STEM) graduates are crucial to the future growth of communities, but finding an adequate number of workers with appropriate skills has become a matter of growing concern. Dreyer et al. (2020) noted that colleges and universities had adopted platforms to promote access for students with learning disabilities. However, this remains a complex issue, given the aspect of readiness after high school. Plasman and Gottfried (2018) also reported that over 3 million individuals aged 16-24 were not enrolled in school or did not have a high school credential to advance on their career paths. Without learning opportunities, students with disabilities will struggle throughout their lifetimes to gain meaningful employment and find purpose within communities. Overall achievement during secondary education for students with learning disabilities remains a high predictor of success in higher education and jobs, according to Doren et al. (2014).

Theoretical Background

The theoretical framework of this study is based on Bloom’s Taxonomy. In 1956, Bloom published a framework for categorizing educational goals into those focused on memory, comprehension, application, and evaluation. According to Dochy et al. (2002), Bloom’s work stressed a hierarchy of learning that included one’s prior knowledge. Dochy et al. also emphasized that generative knowledge was built upon a learner’s previous knowledge. Bloom (1956) believed that knowledge served as the foundation of the learning pyramid and as a precondition for strengthening skills to achieve mastery across different learning levels. Radmehr and Drake (2017) viewed Bloom’s Taxonomy as a learning structure that described students’ abilities to master content knowledge as they moved through the pyramid. Bloom’s learning theory builds upon a continuum of prerequisite knowledge and understanding of the importance of the skill. According to Jones et al. (2019), Bloom’s Taxonomy strengthens higher order thinking for students, tapping into skills such as creativity, information generation, goal setting, and memory.

More robust instructional strategies emphasizing math are crucial to addressing high school dropout rates and low employability for students with SLDs (Namkung & Bricko, 2021; Rose, 2020; Wilkey et al., 2020). While laws and court rulings have furthered special education over the last several decades, much focus has been left to administrative levels instead of classroom instructional styles and pedagogy. Based on Bloom's Taxonomy learning theory, strengthening generative knowledge with support from prerequisite skills promotes more substantial academic success for students in secondary education (Agarwal, 2019; Ramlawati et al., 2020).

Problem Statement

Kiss et al. (2019) noted gaps in national achievements for students with math-based disabilities, despite the increase in research calling for improved instruction and curricula. While the effect of prerequisite skill instruction on students has been researched and has been positively associated with gains in mathematics (Dueker & Day, 2022; Hardy & Hemmeter, 2019), very little research has focused on the impact of this instruction on high school students with disabilities who continue to struggle with essential mathematical skills. Since teaching prerequisite skills has been shown to improve student outcomes (Apanasionok et al., 2021), it is critical to determine whether teaching these skills will positively impact math achievement scores at the state and national levels for high school students with math-based disabilities.

Apanasionok et al. (2021) highlighted the need for early numeracy programs teaching prerequisite skills. As a result of such initiatives, students had more successful outcomes; however, such studies did not address the need to reinforce prerequisite skills at the high school level. Other research studies have addressed the idea that teaching prerequisite skills in early childhood math might have successful outcomes (Dueker & Day, 2022; Martin et al., 2019). However, very little empirical research has been conducted on the need to teach prerequisite skills at the high school level for students with math-based disabilities. Thus, the research problem is that there exists a gap in existing literature regarding the teaching of prerequisite skills to high school students with math-based disabilities, even though research consistently indicates the importance of such skills in developing more complex and abstract mathematical strategies (Ardiansari & Wahyudin, 2020; Bertrand et al., 2021; Martin et al., 2019; Namkung & Bricko, 2021).

Purpose Statement

The purpose of this quantitative, quasi-experimental study with a pre-test-posttest control group design, while controlling for math achievement pretest scores, is to investigate the impact prerequisite skills instruction has on math achievement scores for ninth-grade pre-algebra students with SLDs in math. Gasparetti (2022) defined the independent variable, prerequisite skills, as lower-level educational concepts already mastered, which students could apply to more higher order thinking and advanced skills. The prerequisite skills evaluated before presenting new material included awareness of exponents, fractions, decimals, variables, and inequalities. These skills were measured using the pretest and posttest scores, and throughout the study, with exit tickets measuring students' skill levels, so that teachers were better informed on progress (IXL Learning, 2023) using the IXL program. The dependent variable of student math achievement is defined as the "student's knowledge and skills in mathematics that can be applied to problem solving situations" (National Assessment of Educational Progress, 2022, "What Does the NAEP Mathematics Section Measure?", paragraph 1). To ensure all students in the intervention and control groups were at the same mathematical skill level before the intervention was introduced, a pretest was given to measure the covariate in this study. The research study included 70 students identified as having an SLD in math, enrolled in ninth-grade pre-algebra, and receiving intervention services through an IEP.

Significance of the Study

The current study will add to the scholarship in the field by determining if direct, explicit instruction in prerequisite skills for students with math-based disabilities has a positive impact on their math achievement. While previous research has shown successful outcomes vis-a-vis mathematical achievement scores due to prerequisite skills interventions (Apanasionok et al., 2021; Dueker & Day, 2022; Poast et al., 2021), this study has focused mainly on either early childhood education or students in higher education. Dueker and Day (2022) found that student post-test scores improved as a result of a video modeling intervention to acquire prerequisite skills in early childhood education. Apanasionok et al. (2021) also found that additional training in mathematical prerequisite skills helped improve early numeracy skills among students with disabilities. The current study is similar to studies conducted by Dueker and Day (2022) and Apanasionok et al. (2021); however, it relates to Poast et al.'s (2021) study, which concluded that teaching prerequisite skills in math at the high school level could increase success rates in intermediate algebra classes. The current study will extend to students with disabilities in 9^{th}-grade pre-algebra math courses.

This study will also add to the existing knowledge of prerequisite skills interventions, especially for students with math-based disabilities in a high school setting. Prerequisite knowledge has been found to improve student achievement in math and other academic areas such as science and engineering (Apsari et al., 2021; Mukhni et al., 2021; Namkung & Bricko, 2021; Vandenbussche et al., 2018). This study will help better inform educators and stakeholders about what instructional strategies work, so that students with learning disabilities are more successful in graduating from secondary education, being accepted into higher education institutions, and finding sustainable employment. The results of this study will help general education teachers, special education teachers, and other education professionals better understand the impact of prerequisite skill instruction on math achievement skills for students with learning disabilities in math.

Research Question

RQ: Is there a difference in math achievement scores for students with learning disabilities in math who had received prerequisite skills training before new content instruction and those who had not, when controlling for math achievement pretest scores as measured by the Mathematics Achievement Test (MAT)?

Definitions

1. *Bloom's Taxonomy Learning Theory*: This includes higher order thinking design, tapping into skills including creativity, production of information, and reinforcement of lower order thinking including interpreting, explaining, and comparing (Jones et al., 2019).
2. *Explicit Instruction*: It is a combination of direct and unambiguous teaching behaviors to support learning across all curriculum content areas (Gunn et al., 2021).
3. *Learning*: Learning is defined as a phenomenon involving the construction of problem-solving knowledge essential to long-term learning situations (Trigueros, 2019).
4. *Math Achievement*: It refers to a student's knowledge and skills in math that can be applied to problem-solving situations (National Assessment of Educational Progress, 2022, "What Does the NAEP Mathematics Section Measure?", paragraph 1).

5. *Prerequisite Skills*: These indicate lower-level educational concepts already mastered, which students can apply to further higher order thinking and advanced skills (Gasparetti, 2022).
6. *SDI*: It comprises adaptations to content, methodology, or delivery of instruction to address students' needs (Hedin et al., 2020).
7. *Working Memory*: Working memory is an executive functioning skill that allows students to plan and adjust to new environments. Disruptions in working memory can lead to inattentiveness, challenges in implementing plans, and obstinate thinking (Vasquez & Marino, 2020).

Chapter 2: Literature Review

Overview

A systematic review of the literature was conducted to investigate the effects of teaching prerequisite skills in math to students with SLDs. This chapter reviews the current literature on teaching prerequisite skills and their impact on math summative assessment scores. This chapter briefly discusses the historical evolution of special education and prerequisite skills, the methods and strategies of implementing prerequisite skills, and the role prerequisite skills play in math. Literature related to the factors that lead to the development of prerequisite skills in students with disabilities is addressed. A gap in the literature has been identified, thereby establishing the contemporary relevance of this study.

Theoretical Framework

Bloom (1956) published a higher order thinking framework for categorizing educational goals that included remembering, comprehension, application, analysis, synthesis, and evaluation. Zaidi et al. (2018) described this framework as a cognitive hierarchy, requiring students to recall and comprehend information before it was synthesized and evaluated for transmission into memory storage. Bloom's Taxonomy is sometimes referred to as a hierarchy of learning or the goals of the learning process. Knowledge forms the foundation of Bloom's Taxonomy, followed by comprehension, application, analysis, synthesis, and evaluation (Agarwal, 2019), and it can be further broken down into factual, conceptual, procedural, and metacognitive knowledge. According to Bloom (1956), the most common objective of education, for teachers, is students' acquisition of knowledge or information. Bloom viewed knowledge as something that had to be necessarily taught for students to develop problem-solving skills and higher order thinking. He also believed that knowledge was the basis for solving problems.

Bloom's Taxonomy has been influential for over half a century and is still relevant in contemporary education (Bloom, 1956; Burns et al., 2018; Davies et al., 2021).

In 2001, Bloom's Taxonomy was revised to highlight learning in verb tense (Agarwal, 2019). Anderson and Krathwohl (2001) redesigned Bloom's Taxonomy at the beginning of the 21st century to account for new theories, such as metacognition and constructivism. Bloom's Revised Taxonomy was divided into two dimensions: knowledge and cognition, considering varied levels of expertise and the fact that students need to build higher order thinking skills (Agarwal, 2019). Remember, understand, apply, analyze, evaluate, and create are the six major categories that form this updated taxonomy (Agarwal, 2019; Ramlawati et al., 2020). Remember is the lowest level on the Revised Bloom's Taxonomy hierarchy. However, it still plays a crucial role in students' recall of factual information, eventually leading to higher success in solving higher order questions (Davies et al., 2021). Lau et al. (2018) described Bloom's Revised Taxonomy as a cognitive process combining the knowledge dimension with other dimensions of higher order thinking. Bloom's Taxonomy significantly impacts teacher pedagogy because of its simplicity, the idea that foundational knowledge precedes higher order thinking skills for students and that the retrieval process plays a critical role in advancing through the taxonomy (Agarwal, 2019).

Bloom (1956) described knowledge as the foundation of the pyramid. Knowledge was a precondition for students to move through different levels of learning and academics, acquiring mastery of skills while generating or creating new thoughts. Bloom primarily focused on learning support throughout the classroom. This included teaching prerequisite skills before existing requisites, with an emphasis on understanding the model and how professionals could apply it for lesson planning in education. Bloom's Taxonomy has been classified as a learning structure that describes students' abilities to master content knowledge as they move through the pyramid, creating prerequisite knowledge as they continue into different levels (Agarwal, 2019; Radmehr & Drake, 2017; Ramlawati et al., 2020). Bloom's Taxonomy approaches learning through a higher order thinking method: it begins with prior knowledge being brought back into working memory (Spindler, 2020) and allows students to solve problems, as well as to experience and model different techniques and concepts of math.

Bloom (1956) delved further into the concept of knowledge in his original publication, *Taxonomy of Educational Objectives: The Classification of Educational Goals, Handbook 1: Cognitive Domain.* Bloom noted that knowledge formed the basis of the taxonomy because knowledge would be used in all the other categories, wherein remembering, recognizing, or recalling ideas, material, or phenomena would be essential to building higher order thinking skills. Bloom realized the importance of knowledge due to its strong foundation in the taxonomy, but he also learned how students applied new information to new situations and problems. Bloom viewed knowledge as a prerequisite for problem-solving skills in academic areas such as math and reading comprehension but also understood the importance of these skills in vocational settings. Anderson and Krathwohl (2001) expanded on Bloom's original foundation of knowledge, placing it into a separate domain and adding remembering to the subcategories of recognizing and recalling. Bloom's Taxonomy learning theory constructed a foundation for how prerequisite knowledge skills could assist students with SLDs in retaining information, recalling the information later, and manipulating the information to create more rigorous learning (Burns et al., 2018). Thus, Bloom's Taxonomy builds concepts for schemas and knowledge that support the manipulation of prerequisite skills to form requisites.

Memory based on prior knowledge and experience can be activated throughout several levels within Bloom's Taxonomy (Radmehr & Drake, 2017), making the theory applicable to all levels of learning. Zaidi et al. (2018) explained that Bloom's hierarchy was based on the belief that, to achieve higher order levels of learning, such as synthesis and evaluation, students must be able to apply lower levels of learning, such as recalling information or simple comprehension of information acquired. Mastering lower levels in the taxonomy is a prerequisite for mastering the next higher levels (Ullah et al., 2020). Bloom's (1956) original taxonomy included a foundation for knowledge followed by comprehension, application, analysis, synthesis, and evaluation. Anderson and Krathwohl (2001) revised the taxonomy by adding the foundation of remembering that followed understanding, applying, analyzing, evaluating, and creating. Bloom's Taxonomy provides a simplistic and straightforward scheme for evaluating learning tasks and achieving specific goals in math (Alayont et al., 2022). Alayont et al. (2022) described knowledge as the recall of specific methods and processes in math. Comprehension involves the use of the material learned, whereas application comprises the formation of ideas, rules, and procedures used in concrete situations. Alayont et al. (2022) further described the final steps of the analysis as an understanding of relationships between ideas, synthesis as putting elements together to form a whole, and evaluation as understanding the value of materials and methods for varied purposes.

Bloom's Taxonomy was also designed to assist with curriculum development, and it has been widely used to inform and guide assessment practices in schools today (Zaidi et al., 2018). Bloom's Taxonomy has also been used in school systems to build metacognitive skills essential to domains such as math (Radmehr & Drake, 2017). In the high school curriculum, mathematical computation requires extensive prerequisite knowledge from previous grades to ensure that each student reaches mastery levels, following a continuum past secondary education and into job development (Zaidi et al., 2018). According to Radmehr and Drake (2017), Bloom's Taxonomy has been helpful in areas such as music, art education, and other electives. However, mathematical equations produce a range of solutions and strategies that involve cognitive processes, which activate prior knowledge to solve and master newly acquired skills through the creation of evaluation and synthesis skills. Radmehr and Drake (2017) further explained that, within Bloom's Taxonomy learning theory, the category of knowledge followed a low complexity level, but it intersected with the other five categories that used high complexity knowledge levels, including factual, conceptual, procedural, and metacognitive, thereby proposing prerequisite skills as the foundation of acquisition.

Related Literature

This section reviews the history of special education and the modern trends that highlight the importance of teaching prerequisite skills. The related literature reviews the importance of prerequisite skills for students with SLDs. It also highlights the models and strategies of instruction that utilize prerequisite skills and how they fit into the goals and objectives of math. This section also highlights various types of interventions in special education, and how prerequisite skills are utilized in the math curriculum. Finally, the role of teaching strategies in the math curriculum is outlined, along with how they play a role in prerequisite skills training.

History of Special Education

According to Coviello and DeMatthews (2021), schools in the United States were initially designed to exclude students with disabilities through their institutionalization and isolation in self-contained settings as the primary places of education. Section 504 of the Rehabilitation Act was one of the first civil rights laws passed to protect the rights of individuals with disabilities (Murphy, 2020). This law protected students with disabilities from exclusion in general education settings, with its primary purpose being the prohibition of discrimination. In 1975, the Education for All Handicapped Children Act (EHA) established an obligation for public school systems: the creation of an IEP for students with disabilities that enabled them to receive a free and appropriate education (Rozalski et al., 2021). EHA also provided federal money to state and local school systems to assist them with the inclusion of students with disabilities within school systems.

The Supreme Court ruling set the stage for providing more robust intervention strategies, which used higher order thinking strategies and differentiated instruction modeling Bloom's Taxonomy for students with disabilities. According to Wilson et al. (2019), teachers have been tasked with implementing the Least Restrictive Environment policies set forth by IDEA over the last 30 years. Wilson et al. (2019) also explained that, according to IDEA, students should be educated with their peers to the maximum extent appropriate, and students with disabilities should only be removed from general education classrooms if they cannot achieve their goals successfully with the maximum support available. Rowley v. Board of Education became the first case heard on special education, which developed a two-part test, including setting reasonable educational benefits for students and checking if the school system had provided FAPE as understood by IDEA (Yell & Bateman, 2019).

Inclusive education has become the cornerstone of several special education laws and policies around the globe (Buchner et al., 2021). Federal laws such as IDEA require schools to consider that children with disabilities receive SDI in regular education settings enabling the student to make progress in the local school system, and that removal from regular education settings should occur if the student cannot be successful even when SDI is being provided to the student (Wehmeyer et al., 2021). Buchner et al. (2021) noted that, after World War II, most students with disabilities were educated in special education settings that were not inclusive of the general education curriculum. In 1975, the EHA guaranteed all students a free and appropriate education, regardless of their disabilities (Wehmeyer et al., 2021). The Act has been reauthorized multiple times since 1975, according to Wehmeyer et al. (2021), and as of 2004, it is titled the Individuals with Disabilities Education Improvement Act (IDEA). All states fully implemented the IDEA in 1978, due to parental dissatisfaction with the lack of access to the general curriculum for students with disabilities. Brock (2018) noted that IDEA strongly emphasized a preference for the placement of students with disabilities in general education settings, to ensure they had access to the curriculum of their peers.

Since the EHA of 1975, special education has focused on the access of students with disabilities to the curriculum instead of the quality of education (Yell, 2019). Most recent court rulings, such as Endrew F. v. Douglas County School District, have addressed the amount of educational benefit needed for school districts to fulfill the requirement of free and appropriate education under IDEA. Yell (2019) discussed a new standard deemed the Rowley/Endrew test, which provided both a procedural test and a reasonable calculation to enable a student with disabilities to make progress in their educational goals. The Rowley/Endrew test stressed the need for more substantial interventions to ensure that students with disabilities progressed toward their educational goals.

Modern Trends in Special Education

Students with disabilities have fared better in schools that provided opportunities for inclusion, according to Ashby et al. (2020). In recent years, legislation and research have moved towards promoting inclusion for students with disabilities and the importance of literacy instruction with extensive support for students (Toews & Kurth, 2019). The No Child Left Behind Act (NCLB) was signed into law in 2002, attempting to increase inclusion and accountability in special education services by requiring state testing for students with disabilities and highly qualified teachers (Ford et al., 2020). However, the NCLB has had only minimal impacts over the last 20 years. The NCLB was succeeded by the ESSA in 2015, to ensure that schools implemented a high-quality accountability system for students, and to ensure that both academic and non-academic variables were being monitored to reinforce the inclusion of special education students within general education settings (Grapin & Benson, 2019).

More recent Supreme Court rulings, such as Endrew F. v. Douglas County School District, supported the idea that a standard higher than the more-than-de-minimis standard should be embraced and that an IEP must offer an opportunity for the student to make progress while considering the disability. Kressler and Cavendish (2020) pointed out that most school districts in the United States have responded to court rulings such as Endrew F. with equity-focused provisions. However, they also noted that several teachers continue to struggle with the implementation and understanding of techniques for including students with disabilities in their classrooms. Differentiated instruction has been more generally accepted in general education classrooms over the past several years due to its inclusive components that use higher order learning with reference to Bloom’s Taxonomy (Stollman et al., 2022)

SLDs

An SLD is a neurological and learning disorder affecting reading, writing, and mathematical expressions for students at least six months of age (Alloway & Carpenter, 2020). SLDs are not newly discovered; according to McDowell (2018), they can be traced back to 120 years ago when a British doctor was perplexed by a patient with high intelligence who had difficulty reading. Classifications of learning disorders have changed from DSM IV to DSM V (Peterson et al., 2021). Learning disabilities were grouped together into one category instead of several categories and then splintered off to clarify the types of learning disabilities (Peterson et al., 2021). According to Wen et al. (2020), in 2019, approximately 5% of students in secondary education had an SLD; however, many believe the numbers could be even higher. Learning disabilities in math can be observed by educators as early as kindergarten; however, interventions as early as elementary and middle school have been known to improve skills (Namkung & Bricko, 2021; Rogers et al., 2020; Rose, 2020).

Models of Instruction

Explicit and implicit instructional methods have been used in special education classrooms. Explicit instructional methods aim to promote intentional learning (Ahmadian, 2020), primarily by gaining the student’s attention, providing rules as guidelines, using anticipatory sets to rehearse prerequisite skills needed for the current lesson, and providing scaffolds during the instruction. Prerequisite skills are verified at the beginning of instruction (Foxworth et al., 2022; Morris et al., 2022), rather than being taught all over again. Instead of simply stating what the prerequisite skills are, the teacher can use several strategies to demonstrate student mastery of prerequisite skills, including but not limited to utilizing technology to draw concepts (Foxworth et al., 2022). In the body of the instruction, students are presented with the lesson’s purpose and guided instruction that includes modeling and follows with scaffolds throughout, except for independent practice (Long et al., 2021). Explicit instruction typically begins with what Bloom (1956) described as “what is knowable” so students can relate previous prerequisite skills to newly acquired skills (p. 31). On the other hand, implicit instruction was described by Spit et al. (2022) as a method that teaches students without their knowing it, wherein the student does not know they are learning but still uses a component of previous knowledge resulting in the stronger acquisition of new skills being taught in the classroom. Explicit and implicit instructional methods both have positive impacts on students with disabilities and include components of prerequisite skills (Chen & Kalyuga, 2020; Foxworth et al., 2022; Long et al., 2021).

Explicit Instruction

Explicit instruction is a general term used to describe direct instruction provided to students with disabilities, and is often used to teach specific behaviors and curricula such as math (Bouck et al., 2022; Foxworth et al., 2022; Gunn et al., 2021; Long et al., 2021). Johnson et al. (2019) defined explicit instruction as a highly effective model for students with disabilities, which has been supported by nearly 50 years of research. Explicit instruction is an Evidence Based Practice (EBP) (Bouck et al., 2022). It can be used to teach across varied curriculum types, including math, science, language arts, and social studies. Explicit instruction relies on teachers providing a solid structure and explaining each step thoroughly as students move through a lesson (Foxworth et al., 2022; Long et al., 2021). The instruction starts with a lesson introduction that includes reinforcing prerequisite skills, stating goals and objectives, and stating the relevance of the instruction (Foxworth et al., 2022). The body of the instruction includes the "I do", "we do", and "you do" sections, where the lesson is modeled, guided instruction is provided along with prompts, and independent practice occurs (Long et al., 2021). Foxworth et al. (2022) also noted that explicit instruction has a concluding part wherein a preview, review, and independent work are assigned.

Lesson Introduction for Explicit Instruction. The lesson introduction for explicit instruction begins with gaining the student's attention, stating the lesson's goals, discussing the relevance of the lesson, and reviewing prerequisite skills (Morris et al., 2022; Petermann & Vorholzer, 2022). Gaining attention from the student should be the priority for teachers (Foxworth et al., 2022), with verbal prompts being primarily used. According to Ahmadian (2020), learners' attention is a direct target feature of explicit instruction because it begins the thinking process, especially for students with disabilities. Teachers should then state the lesson plan goals to focus on the essential content for students with disabilities, according to Foxworth et al. (2022). Real-life connections should be made for students to understand the lesson's relevance (Bertrand et al., 2021; Bouck et al., 2022; Foxworth et al., 2022). Verifying prerequisite skills occurs during the opening of the lesson in explicit instruction. Reviewing prerequisite skills requires more than just restating previous information (Foxworth et al., 2022), and does not require reteaching of material. Verifying prerequisite skills entails multiple opportunities for each student to respond so teachers can be assured that learning new skills can be completed promptly with maximized engagement (Bertrand et al., 2021; Foxworth et al., 2022; Namkung & Bricko, 2021).

Lesson Body for Explicit Instruction. The body of the lesson for explicit instruction offers several models for students to master academic content (Bouck et al., 2022; Foxworth et al., 2022; Long et al., 2021). Teachers often use demonstrations, guided practice, and models to assist students with learning new skills, including reading and math (Gunn et al., 2021; Morris et al., 2022). The body of the instruction is classified into "I do", "we do", and "you do" sections. The body of the lesson in explicit instruction also includes the essential components of modeling or demonstrating the skills or behavior, providing guided practice with prompts that would eventually be reduced over the lesson, and the teacher checking for understanding several times (Morris et al., 2022). During this section, the students integrate new information learned during guided instruction along with the prerequisite skills and prior knowledge previously reviewed at the lesson's opening (Chen & Kalyuga, 2020). Also, teachers provide examples and non-examples as well as rules for academic behaviors so that students only undergeneralize or overgeneralize a particular skill or concept (Foxworth et al., 2022).

Teachers often offer several model lessons and allow students to respond, so as to ensure high levels of engagement, decrease off-task behavior from students, and increase academic success and outcomes (Doabler et al., 2021; Foxworth et al., 2022; Long et al., 2021). In the "I do" section, students are shown problems through modeling from the teacher (Long et al., 2021). During the "we do" section, students often solve problems along with the instructor, with prompts and instant feedback provided (Foxworth et al., 2022; Long et al., 2021). Teachers can deliver the lesson to the whole class or separate the students into smaller groups in the "we do" component of the instruction (Bouck et al., 2022). After guided instruction, prompts, and feedback have been delivered, students are tasked to complete independent practice in the "you do" section (Long et al., 2021). Long et al. noted that this section could also include formative assessments to assess mastery of skills. This section also allows the students to practice new skills learned during the lesson (Foxworth et al., 2022; Long et al., 2021).

Lesson Closing for Explicit Instruction. The lesson closing includes a review, preview, and independent practice to prove student mastery of skills (Morris et al., 2022). The review section allows students to reflect on the lesson, with learning objectives being restated and crucial points being verbally expressed to the students (Foxworth et al., 2022). Instructors can also ask questions to understand the extent of learning of the students. The preview section provides a glimpse into the next lesson and helps facilitate motivation and cognitive thinking about how prerequisite skills can be used to cross over into a different curriculum (Foxworth et al., 2022; Morris et al., 2022). At the end of the lesson, independent work is assigned to students. Independent work can help generalize the skills learned and strengthen working memory and short-term memory (Foxworth et al., 2022), thereby assisting students with stronger prerequisite skills with their recalling skills in the future.

Impact of Explicit Instruction on Math Curriculum

According to Bouck et al. (2019), explicit instruction is a common approach in math for students with learning disabilities as well as students who are determined to be eligible for Tier 3 and Tier 4 Response to Intervention (RTI) models. It is an EBP for teaching math to students with disabilities or those considered at risk (Long et al., 2021). Explicit instruction has been used successfully to teach students how to solve problems, conduct number strategies, and use cognitive strategies (Bouck et al., 2022; Chen & Kalyuga, 2020). Conceptual skills, procedural skills, and declarative knowledge are reinforced when teachers provide explicit instruction in math, according to Long et al. (2021). Chen and Kalyuga (2020) also noted the effectiveness of explicit instructions in problem-solving during math instruction, wherein prerequisite skills were required for successful outcomes. Students with disabilities and those considered at risk require more intensive EBPs such as explicit instruction (Morris et al., 2022), which will assist them with acquiring more skills.

Math using the explicit instructional model typically begins with a graphic organizer used during the instruction’s opening to present the lesson’s purpose and relevance (Bouck et al., 2022; Long et al., 2021). According to Bouck et al. (2022), the graphic organizer helps students establish real-world connections to retain attention and make the lesson meaningful so that the recall of information is stronger in the future. In the body of the lesson, students are typically guided through the instruction with the “I do”, “we do”, and “you do” sections (Foxworth et al., 2022; Long et al., 2021). Teachers typically demonstrate or model the skill or behavior in the “I do” section, pairing the strategies with think-aloud and explaining each step (Long et al., 2021). The “we do” section of explicit instruction allows students to demonstrate their skills through practicing (Doabler et al., 2021), while the teacher provides prompts and immediate feedback. Students typically solve problems with the teacher, as prompts help correct mistakes and provide feedback on accuracy and strategies (Long et al., 2021). Teachers sometimes return to the model or demonstration phase in math (Long et al., 2021). However, explicit instruction leads to better achievement in math for students with disabilities and those considered at risk (Bouck et al., 2022; Doabler et al., 2021; Long et al., 2021).

Benefits of Explicit Instruction

Explicit instruction is rarely deployed in the classroom, even though it has been proven effective myriad times (Petermann & Vorholzer, 2022). A growing body of research indicates explicit instruction is essential for students with disabilities who struggle with basic skills (Bouck et al., 2022; Gunn et al., 2021; Kruit et al., 2018; Long et al., 2021). Doabler et al. (2021) also noted that explicit instruction leads to better achievement for students who are considered at risk, including students who do not have IEPs and need extra support to learn basic and more advanced skills. Students in kindergarten and second grade have shown the most improvement when explicit instruction was deployed in the classroom as the primary intervention (Doabler et al., 2021). When skills are taught using the explicit instructional model, students often obtain more successful outcomes in future grades and employment in adulthood (Ahmadian, 2020; Doabler et al., 2019; Fuchs et al., 2020; Petermann & Vorholzer, 2022).

Explicit instruction is a systematic approach to learning that can be used across different types of curricula (Doabler et al., 2021). It is a common approach in math, especially for students with disabilities (Bouck et al., 2022). It has been found to be effective in teaching varied math skills, including problem-solving, to students with disabilities (Chen & Kalyuga, 2020). Explicit instruction often uses teacher demonstrations with a clear, concise, and consistent style of instruction (Gunn et al., 2021). These can assist students in practicing math skills. Positive gains have also been identified after using the explicit instructional model in high school mathematics, particularly when prerequisite skills were reinforced during the instruction (Bertrand et al., 2021; Foxworth et al., 2022; Namkung & Bricko, 2021). These skills can also be used across varied curricula. Further, they assist students with conducting experiments, identifying and controlling variables, observing changes in an experiment, measuring data, and utilizing devices to organize information, record results, and present the data in scientific experiments (Bouck et al., 2022; Kruit et al., 2018; Naude et al., 2022). Prerequisite skills in explicit instruction are needed for learners to engage fully and can be verified by stating what was learned several days earlier or through the reconciliation of previous skills via reminders in the general education setting (Bertrand et al., 2021; Foxworth et al., 2022).

Implicit Instruction

Implicit instruction is a type of teaching method geared towards incidental learning, wherein learners may not be aware of specific features of the lesson. Here, the job of the teacher is to remain in the background and provide a creative learning environment that enriches cognition without the explicit instructional method of instruction (Ahmadian, 2020). The instructional method begins by attracting the students to the lesson, similar to explicit instruction, but students use prior knowledge to build on previous knowledge through spontaneous or natural learning (Peltekov, 2020). Implicit instruction can sometimes be described as spontaneous and unobtrusive, allowing learners to draw conclusions on their own rather than using the systematic approach that explicit instruction uses (Ahmadian, 2020). The instructional model was also described as "learning with background knowledge" (Guivarch et al., 2017), wherein students' prerequisite skills are not reinforced but occur naturally, and are then generalized via incidental learning throughout the instruction.

The primary purpose of implicit instruction is for students to learn independently without the specific structure followed by explicit instruction. The implicit instructional model utilizes an instructional strategy that links prior knowledge with current knowledge at the beginning of the instruction (Ahmadian, 2020; Hunt & Silva, 2020). Implicit teaching does not follow a specific structure or pattern for learning but instead promotes listening to and shadowing of the teacher (Peltekov, 2020). This instructional model also promotes feedback as recasting, wherein a teacher restates what the student communicates but with more detailed information.

Differences in Explicit and Implicit Instruction

Implicit instructional methods differ from explicit instruction in several ways. Implicit instruction relies on the idea that students will learn naturally, and only partial teaching guidance from educators is necessary for students to acquire new skills and knowledge (Gunn et al., 2021). Explicit instruction relies on a structured approach to learning that caters to intentional learning, wherein learners are made aware, and the teacher guides the instruction throughout the lesson (Ahmadian, 2020; Foxworth et al., 2022; Gunn et al., 2021; Long et al., 2021). Implicit instruction also encourages learners to infer information while learning new skills without awareness of rules that may guide the instruction (Peltekov, 2020). While several studies have shown implicit instruction to be effective, explicit instruction is still considered more effective for students with disabilities, because it promotes more work production, problem-solving, and comprehension of information (Ahmadian, 2020; Bouck et al., 2022; Chen & Kalyuga, 2020;

Long et al., 2021). This could be because explicit instruction utilizes a student-centered, teacher-directed approach to learning new skills and generalizing them across diverse environments and times (Foxworth et al., 2022; Gunn et al., 2021; Long et al., 2021).

Stages of Math Competency

Mathematics is an essential competency for everyday life. Students learn math in school from pre-kindergarten through high school and continue to learn it in college and workforce training (Mukhni et al., 2021). Math serves as a foundation for several other curricula and academic disciplines in school, including engineering, business, and economics (Apanasionok et al., 2021; Bertrand et al., 2021; Martin et al., 2019; Poast et al., 2021). Research has also indicated that students who perform at or above proficiency in math in secondary school are more likely to succeed in everyday life skills, in college, and in STEM courses (Apanasionok et al., 2021; Bertrand et al., 2021; Dueker & Day, 2022; Kiss et al., 2019; Namkung & Bricko, 2021; Stocker & Kubina, 2021). Students in elementary school learn basic arithmetic and continue with more concrete mathematical operations, such as decimals, fractions, rational numbers, and multi-step equations (Rose, 2020). After elementary and middle school, students are expected to learn complex operations on unknowns and equivalent transformations on both sides of equations (Sharpe & Marsh, 2022). Math is a cumulative curriculum (Rose, 2020) that is used right from the most fundamental life skills to the most advanced skills in the workforce and college. Math is also a subject that students begin to learn in pre-kindergarten, continuing through their adulthood (Andini & Prabawanto, 2021; Mukhni et al., 2021; Rose, 2020).

Early and Elementary Math

Toddlers begin learning math through basic numeracy skills, including the early perception of what numbers look like, followed by processing and reasoning in numbers (Seitz &

Weinert, 2022). Introductory math courses in pre-kindergarten through elementary school have been classified as a gateway or beginning for courses in advanced math (Bertrand et al., 2021), making them prerequisite math skills for middle and high school. In kindergarten and elementary school, students begin with the most basic arithmetic skills, such as adding, subtracting, multiplying, and dividing, which are close calculations that need only a few direct links among previous math courses (Andini & Prabawanto, 2021; Gliksman et al., 2022; Kiss et al., 2019). Although students in elementary school do not need direct links from previous courses, arithmetic is needed for students to advance into middle school and eventually into algebra and other areas of math (Mukhni et al., 2021). Teachers should focus on conceptual understanding, fluency in numbers, and computations of arithmetic skills for students to be successful in later grades (Andini & Prabawanto, 2021; Gliksman et al., 2022; Stocker & Kubina, 2021).

Middle School Math

Middle school continues with transferring math skills from concrete beginnings, wherein students learn basic mathematical operations, to more traditional skills and connecting previous skills learned with new ones (Mukhni et al., 2021). Fifth- and sixth-grade mathematics is highly critical to students' success in seventh and eighth grades (Rose, 2020). Expectations for students in middle school increase with reference to math, requiring them to work with complex computations and more challenging problem-solving (Stocker & Kubina, 2021). Students begin middle school by performing simple operations and connecting them with integers and rational numbers (Rose, 2020). Students continue learning skills in seventh grade by exploring basic operations with rational numbers, followed by learning multi-step equations in eighth grade that are reinforced in high school pre-algebra and algebra courses (Namkung & Bricko, 2021; Rose, 2020). By the end of middle school, students moving into high school, pre-algebra, and algebra should be able to work with radical numbers, understand the connections among proportional relationships, lines, and linear equations, and be able to analyze and eventually solve linear equations (Namkung & Bricko, 2021; Stocker & Kubina, 2021). At this stage, students should be able to take concepts and interconnect them with concepts already learned, thus creating a coherent progression in their math curriculum (Rose, 2020).

Deficiencies in Elementary and Middle School Math

Several factors affect student achievement, but lack of prerequisite knowledge is the most prevalent one in moving through math succession (Apsari et al., 2021; Bertrand et al., 2021; Kiss et al., 2019; Rose, 2020). Students who struggle with middle school math have difficulty in understanding the order of operations and sometimes need more fluency with basic arithmetic learned in elementary school (Namkung & Bricko, 2021). Students with disabilities often need help with the order of operations and operation groups taught in middle school, including commutative and distributive properties (Ardiansari & Wahyudin, 2020), because they begin to connect basic arithmetic with more formal operations. Unfortunately, when students fall behind in math, they stay behind throughout their careers, and most students begin to experience problems with math starting in middle school (Rose, 2020; Stocker & Kubina, 2021). Mathematical skills go unlearned for several reasons, including a lack of prerequisite skills, uneven teaching in early grades, and chronic absenteeism, followed by continuing gaps (Kiss et al., 2019; Rose, 2020). Teachers often try to reteach previous years' content, but it often leads to students falling further behind than before (Rose, 2020). As a result, several students attend high school needing more skills for algebra and future math courses (Apsari et al., 2021; Ardiansari & Wahyudin, 2020; Namkung & Bricko, 2021; Rose, 2020).

High School Math

Several students begin pre-algebra and algebra in high school, needing more fluency in the math curriculum, primarily because they need more prerequisite skills training (Apsari et al., 2021; Namkung & Bricko, 2021; Reynolds & Joseph, 2022). Pre-algebra and algebra are the math courses students begin in high school, continuing to geometry, pre-algebra, and higher stages of math, including pre-calculus and trigonometry. Algebra can be defined as doing arithmetic with letters (Ardiansari & Wahyudin, 2020). A new mathematical language is learned as part of algebra by high school students, and it is expected to take time to learn new and more enhanced concepts, especially for students with disabilities (Sharpe & Marsh, 2022). Algebra focuses on the relationships between prior skills learned and new skills being taught (Andini & Prabawanto, 2021).

Several strategies can be used in the classroom to remediate prerequisite skills needed for pre-algebra and algebra classes. Class remediation has been successful for students struggling to connect prerequisite skills with current skills taught in the classroom (Apsari et al., 2021; Rose, 2020; Sharpe & Marsh, 2022). Students should explicitly practice skills that combine responding accurately and with appropriate speed, so as to acquire fluency with the new skills (Stocker & Kubina, 2021). Explicit instruction is essential for students to understand principles related to equality that constitute a central link among basic arithmetic learned in elementary school, foundations of algebra learned in middle school, and pre-algebra and algebra needed for higher math content (Ardiansari & Wahyudin, 2020). Computer instruction and applications are being used more often to review prerequisite skills and support students in classroom instruction (Reynolds & Joseph, 2022). Strategies using technology provide visual representations, which often assist in instruction for several concepts, including number patterns in algebra and pre-algebra (Apsari et al., 2021; Reynolds & Joseph, 2022).

In recent years, algebra has become more of a gatekeeper than a gateway for students with disabilities (Sharpe & Marsh, 2022). Algebra has become a central concern for educators and stakeholders across the United States because of recent test scores (Alloway & Carpenter, 2020; Reynolds & Joseph, 2022). Students often need help in algebra because of a poor understanding of structural concepts such as operations and how to use variables (Ardiansari & Wahyudin, 2020). Algebra becomes more difficult for students because it requires a more substantial level of reasoning than typical foundational math found in previous school years (Reynolds & Joseph, 2022). It requires mastery of hierarchical expression in math, which often takes more work for students to achieve (Stocker & Kubina, 2021). Namkung and Bricko (2021) suggested that proficiency in algebra at high school is critical for competing successfully in the American job market, due to its increasing use of technology. Algebra is considered necessary for students to enter college (Reynolds & Joseph, 2022), and is often considered a prerequisite for continuing the high school curriculum. Without these skills, students, especially those with disabilities, will struggle to create paths of achievement in their future academic careers and the workforce.

Teaching Math to Students with Disabilities

As more special education students continue to enter general education classrooms due to the belief in greater inclusion, mathematics and special education have become more interwoven (Sheppard & Wieman, 2020). Math is a critical subject in secondary education and is directly related to several fields, including science, technology, and engineering (Martin et al., 2019; Reynolds & Joseph, 2022; Vostanis et al., 2020). Students with disabilities continue to struggle in math as they advance through school, especially as schools are beginning to shift towards providing more accommodation for students with disabilities instead of modification of the curriculum, due to the Supreme Court ruling in Edward F. vs. Douglas County Board of Education (Lein et al., 2020). Eighty-four percent of students with disabilities scored lower than the basic level on the 2017 NAEP, compared to 56% of students without disabilities. Students with SLDs typically performed lower on standardized math tests, and achievement was further hindered due to a lack of skill sets essential for mastery over the content area (Johnson et al., 2020; Kiss et al., 2019; Mutlu, 2019; Myers et al., 2022; Namkung & Bricko, 2021; Reynolds & Joseph, 2022; Rose, 2020).

Predictors of Math Success

Problem-solving, calculations, and recognizing number facts play an essential role in the continuum of math skills that eventually lead to high school. Mathematics has become a critical component that crosses into other curricula, including STEM (Hsieh et al., 2021; Stocker & Kubina, 2021). Students who do not graduate from high school are seen to have notably poor math skills when deciding to unenroll (Wilkey et al., 2020), and those who graduate also need help with post-secondary education such as trade school or college and with maintaining steady employment. Early childhood skills in math are strong indicators of future academic achievement (Martin et al., 2019; Silver et al., 2021), with only one in four high schoolers performing at or above proficiency levels (Hsieh et al., 2021). Students with learning disabilities in math account for three to six percent of the population and often display difficulty in numerical processing at the early stages (Wilkey et al., 2020). If deficiencies are recognized, they can be appropriately reinforced before the formation of more significant educational gaps, especially at the early stages of education.

Numeracy Skills as Predictors

Developing early numeracy skills when students are in kindergarten and first grade is essential to success in later grades, including algebra, typically studied in ninth grade (Park & Nelson, 2022). Numeracy skills at an early stage include simple counting, being able to tell time, measuring and weighing specific items, and recognizing basic graphs and operations in mathematics (Alallawi et al., 2022; Kiss et al., 2019). Aunio et al. (2021) also noted that numeracy skills could include understanding number lines, naming symbols in mathematics problems, counting, and being able to understand simple word problems at the appropriate reading level. Students who have successfully ascertained numeracy skills can count numbers verbally, identify numerals using visuals, and compose quantities when completing simple addition and subtraction problems (Park & Nelson, 2022).

According to Park and Nelson (2022), many of these skills should begin to form before students enter any type of schooling. Achievement gaps have been noted as early as preschool for students who struggle to develop math skills in later grades (Dueker & Day, 2022; Kiss et al., 2019; Park & Nelson, 2022). Deficits in language skills early in secondary education have also been noted by educators for students who continue to struggle in math in future grades, according to Aunio et al. (2021). EBPs can help alleviate the challenges students with math disabilities face (Aunio et al., 2021). For example, Park and Nelson (2022) noted that interventions with EBPs had assisted students in third grade with overall math performance and students in seventh grade with identifying rational numbers and manipulating them in problems. Students who have had intensive interventions based on EBPs have also been found to produce independent functioning in math during later grades in secondary education (Alallawi et al., 2022; Bertrand et al., 2021).

Problem-Solving Skills as Predictors

Word-problems are written descriptions that assist students in developing skills for problem situations that are to be solved mathematically (Myers et al., 2022). Word-problem solving skills are critical to developing math skills because they reflect an understanding of concepts and how students can apply specific skills in their daily lives (Fuchs et al., 2020). Students utilize math skills in other curricular areas, including science, and use them in their personal lives, such as while using technology (Apanasionok et al., 2021; Fuchs et al., 2020; Wilkey et al., 2020). Students begin to work with word-problems as early as kindergarten, according to Myers et al. (2022), and strengthen skills throughout secondary education, including high school. Myers et al. (2022) noted that students with disabilities in the ninth grade are 9% proficient in solving word-problems in math, compared to 34% of their general education peers.

For students to be successful in word-problem solving, they will need several prerequisite skills, including conceptual skills and linguistics of math (Naude et al., 2022). Other prerequisite skills include understanding what the word-problems say and setting up the correct equations based on the information (Fuchs et al., 2020). Kong et al. (2021) also noted that students need linguistic skills to construct the appropriate number of sentences to solve the problem accurately. Students considered at risk have noted difficulty in solving word-problems, as they begin to move away from basic math skills in elementary school (Kong et al., 2021). EBPs have been shown to be successful, and more of these need to be identified for students with disabilities so that proficiency levels can rise (Morris et al., 2022; Myers et al., 2022; Namkung & Bricko, 2021; Reynolds & Joseph, 2022).

Prerequisite Skills in Math

According to Hardy and Hemmeter (2019), early math instruction has been an area of focus for researchers due to the belief that early math skills are highly predictive of later academic achievement for students with learning disabilities. Lewis and Fisher (2018) pointed out the importance of understanding what the student already knew and how those skills were connected with what they also needed to learn. Bloom (1956) also pointed out the importance of gauging a student’s knowledge of the prerequisites, and of the belief that educators should strike a balance vis-a-vis what the student needs to know to fulfill the taxonomy levels. Several prerequisite skills are required for students to achieve goals in math, including English proficiency and linguistics (Kong et al., 2021; Martin & Fuchs, 2019; Naude et al., 2022). Prerequisite skills are critical to other areas where math is applied, such as STEM programs. Deeken et al. (2019) identified various aspects of mathematical content knowledge and processes required for students to succeed in college STEM settings.

According to Vasquez and Marino (2020), prerequisite knowledge and skills assist students in planning and organizing information. Reynolds and Joseph (2022) proposed computer-based instructional models that can assist students with completing prerequisite knowledge and forming working memory to develop more vital analytical and creative thinking skills. According to Lein et al. (2020), school districts across the United States are aware of the problems students with disabilities face, and many have implemented Multi-Tiered Systems of Support (MTSS), with supports typically aligned to more tutoring, individualized instruction, and differentiated instruction inside the classrooms. Still, students with disabilities struggle with word-problem solving even with extra support throughout their academic careers (Lein et al., 2020).

Namkung and Bricko (2021) emphasized that mathematical skills were fundamental to strengthening the complex thinking skills required for advancing education and demonstrating abstract and quantitative reasoning. Teachers have been under significant pressure to ensure that students achieve success, especially in the areas of reading and math. Hardy and Hemmeter (2019) stated that there existed sufficient evidence in early childhood intervention practices that math skills were highly predictive of later academic achievement in content curriculum areas such as science and social studies. Math skills are complex, chained behaviors that have implications for students with disabilities in other content areas, such as science and reading (Bertrand et al., 2021; Hardy & Hemmeter, 2019; Mukhni et al., 2021). Without these skills that cross into other curriculums, gaps in education for students with disabilities will continue to widen (Rose, 2020; Wilkey et al., 2020)

Prerequisite Skills for Algebra Competency

Several prerequisite skills are needed for students to be successful in algebra and other math curricula. Some students have already attained the specific prerequisite skills needed for high school math in ninth grade, whereas others, especially students with disabilities, are unprepared to enter algebra due to the lack of prerequisite skills established in previous grades (Namkung & Bricko, 2021). Understanding basic numbers, numerical operations, algebraic equations, graphing, using a graph, and functions are all prerequisite skills required for students to understand algebraic concepts (Namkung & Bricko, 2021; Permata & Wijayanti, 2019). Permata and Wijayanti (2019) noted that students would need to master the concept of number operations before they could understand algebraic concepts. However, students would also need to understand the concepts; they need prerequisite skills before they begin learning through metacognitive processes. Hurst and Cordes (2018) also noted several prerequisite skills in fourth through seventh grades, including fractions and decimal concepts, which were critical to higher-order math, such as pre-algebra, geometry, and algebra. Remembering and recalling critical information in algebra is the first step to success (Bertrand et al., 2021; Namkung & Bricko, 2021), which leads to the further steps of understanding, applying, analyzing, evaluating, and creating, as per Bloom's Revised Taxonomy.

Prerequisites in Math Assessments

Math assessments are vital parts of a curriculum that provide an overview of the effectiveness of teaching strategies and methods mainly used in the classroom (Himmah et al., 2019). It is generally accepted that students find math challenging to master (Apsari et al., 2021; Husna & Johar, 2018; Reynolds & Joseph, 2022), especially the topic of algebra, which is a prerequisite itself for other math content areas such as calculus and geometry. Mathematics, especially algebra, plays an essential role in daily life because these skills relate to higher order thinking skills that can be used across curriculum contents and daily living (Bertrand et al., 2021; Husna & Johar, 2018; Namkung & Bricko, 2021; Reynolds & Joseph, 2022). Bloom's Taxonomy has been used to show student outcomes on assessments because they need to use knowledge in remembering to build other higher order thinking skills, such as understanding and applying (Himmah et al., 2019). Students who can use prerequisite knowledge to acquire skills, such as the application of distributive property rules, and to apply these skills to various assessment questions will be more successful in their educational goals.

Teaching Strategies

Strategies in the classroom can be broken down into two categories: student-centered and teacher-centered. Teacher-centered strategies, including the standard lecture practice, have had a long tradition in the classroom (Martina & Pepin, 2022). However, student-centered strategies, which include problem-based learning and collaborative learning, are quickly becoming popular in the classroom, so as to better engage students in modern classrooms (Ruiz-Ortega et al., 2019). Martina and Pepin (2022) described student-centered strategies as collaborative work among students while the teacher facilitates learning. According to Ruiz-Ortega et al. (2019), school systems have replaced traditional teacher-centered strategies with student-centered ones, including problem-based or peer-collaborative learning. Prerequisite skills are still essential for student-centered and teacher-centered strategies, mainly student-centered, wherein teachers need to plan and implement lessons based on prerequisite knowledge, learning styles, interests, motivations, and competencies in other areas (Atchia, 2021).

Student-Centered Teaching Strategies

Benabentos et al. (2021) defined student-centered teaching strategies as active, evidence-based teaching practices, which engage participants in their learning, thereby often reducing course failures and enhancing student performance. Characteristics of student-centered teaching strategies include an active role on the part of the student and the teacher taking the role of a tutor during the learning process, according to Ruiz-Ortega et al. (2019). During student-centered strategies, students often work on open-ended problems (Martina & Pepin, 2022), which are usually based upon the four types of knowledge in Bloom's Revised Taxonomy: factual knowledge or the prerequisites already known to the students, conceptual knowledge, procedural knowledge, and metacognitive knowledge. Self-reflection is a tool often used in student-centered strategies; it connects prerequisite skills with the current curriculum being taught (Nagro, 2020). When prerequisites were taught along with reflective activities in school, students gained a better understanding of the lesson, including how to improve upon their current knowledge, according to Nagro (2020).

Competency Based Education (CBE), a student-centered outcome-focused curriculum delivery system, remains critical for students to become work-ready while in school, according to Richardson et al. (2021). According to Dueker and Day (2022), students with disabilities who do not complete essential reading and math skills in school often have difficulty accessing employment opportunities and basic living conditions. Richardson et al. (2021) noted that teaching prerequisite skills was an area of primary focus for students to achieve mastery over content and skills. Student motivation, active participation, thorough preparation, collaboration with peers, learning style, and time management were all prerequisite skills needed for deeper learning and to provide stronger learning outcomes for students with learning disabilities (Grover et al., 2018). Student-centered teaching strategies remained highly effective and were found to be more effective when prerequisite skills were reinforced at the beginning of the instruction (Nagro, 2020; Richardson et al., 2021). Thus, student-centered strategies have become more prevalent in the classroom; they follow Bloom’s (1956) belief that recall and memory are fundamental skills for all other ends and purposes in education.

Teacher-Centered Strategies

Mameli et al. (2020) described teacher-centered strategies as environments wherein teachers utilized the traditional lecture process, conducted readings for students, proctored tests, and led discussions with students. Student-to-student interaction was limited during teacher-centered strategies, with knowledge being directly transferred to the student from the teacher (Retscher et al., 2022). Mameli et al. (2020) further explained a teacher-centered strategy as the teacher guiding the student’s actions and activities. Teacher-centered strategies use a traditional approach to teaching; here, the teacher acts as the principal instructor and often uses guided instruction that closely follows explicit instructional methods (Retscher et al., 2022).

Teacher-centered strategies usually involve the process of scaffolding to include the “I do”, “we do”, and “you do” process that follows a standard-lecture type of lesson, with the teacher overwhelmingly commanding the classroom. According to Schall-Leckrone (2018), scaffolded instruction advances new content and knowledge by building upon students' prior knowledge, and can be presented in multiple ways to the students during classroom instruction. Further, according to Schall-Leckrone (2018), scaffolding utilizes an expert teacher who temporarily supports students until they master new skills and concepts. Subsequently, the scaffolds are removed until the student can complete the work independently, thereby proving their mastery over the skill or concept. Bloom suggested that successful educational strategies should connect the building blocks of a taxonomy to strengthen varied cognitive levels (Sanchez et al., 2020). Traditional scaffolding provides such a connection but with the teacher at the helm of learning instead of the student. According to Sanchez et al. (2020), such teaching strategies comprise the classical methods but follow the critical levels of complexity in thinking, with prerequisite knowledge forming the basis of higher-order thinking in classrooms.

Summary

The theoretical framework of Bloom's Taxonomy begins with a foundation of prerequisite knowledge, providing a successful path for academic achievement. Since the Brown vs. Board of Education ruling, special education has created a path to stronger inclusion in general education classrooms. The Rowley test serves as a litmus test for inclusion in the general education classroom, and more recent cases such as Endrew F. vs. Board of Education, along with laws, have encouraged both inclusion and a rigorous learning environment.

SLDs in reading and math account for a majority of new cases in the United States. The current models for instruction include both explicit and implicit teaching. Special education employs explicit instruction primarily because it uses guided instruction, scaffolds, and prompts to achieve learning success.

Math can be found at all grade levels in secondary education (Apanasionok et al., 2021; Mukhni et al., 2021; Rose, 2020), and math skills are successive from each level. Students with disabilities struggle with math in upper grades because they need to interconnect arithmetic with more sophisticated areas such as algebra and geometry. Students begin to learn the foundations of algebra and geometry in middle school and often begin to display difficulty at this stage (Rose, 2020). Math predictions can often be observed in middle school, including numeracy skills and problem-solving skills learned in earlier grades. Prerequisite skills for students entering algebra include basic numbers, numerical operations, algebraic equations, graphing and how to use a graph, and functions (Namkung & Bricko, 2021; Permata & Wijayanti, 2019). Teaching these skills through remediation strategies has been seen to have the most successful outcomes for students with disabilities.

The literature review has identified gaps in studies about prerequisite skills usage in educational standards. The most significant of these gaps is the lack of empirical research examining the relationship between prerequisite skills and achievement scores for students with SLDs who have been placed in remedial classes for math in ninth grade that include pre-algebra. Current research has focused on prerequisite skills in early childhood elementary education but lacks focus on students in high school education, particularly ninth-grade math students with learning disabilities. Therefore, there exists a current need to investigate whether students can strengthen their achievement scores in math through the attainment of prerequisite skills via interventions in ninth grade (Sidney, 2020).

Chapter 3: Methods

Overview

The purpose of this quantitative quasi-experimental research design was to provide an understanding of how the prerequisite skills intervention benefited students with SLDs in math. The understanding thus obtained was used to address the research question and its corresponding null hypothesis. This chapter describes the methods used to address the research question and the corresponding null hypothesis. The details of the design, the setting, the participants, and the procedures, including the data analysis, are described below.

Design

A quantitative, quasi-experimental, non-equivalent control group design was used to compare math achievement scores for students with SLDs and without prerequisite skills, prior to learning new content, when controlling for pretest scores, in a public school in the northeastern United States. The quasi-experimental, non-equivalent control group design was chosen for this quantitative study. According to Gall et al. (2007), a quasi-experimental, non-equivalent control group design is the most used design in educational research, wherein the research participants are not randomly assigned to control and experimental groups, and both groups take a pretest and posttest. The quasi-experimental, non-equivalent control group design was chosen for several reasons for this research study.

The purpose of quasi-experimental research studies is to show strong causal inferences without randomized control experiments (Bärnighausen et al., 2017). Quasi-experimental research studies are often used as a reference while conducting a randomized control trial in a study. A quasi-experimental research design was found to be most appropriate for this study because the students in the study sample were not randomly assigned to the groups. A nonequivalent control group design was deemed to be most appropriate because the participants could not be randomly assigned in the study, and both groups were taking a pretest and posttest (Gall et al., 2007). Several limitations do exist for this type of design, including both groups being exposed to similar environments, thereby making it difficult to understand if the intervention had an effect (Miller et al., 2020). Miller et al. (2020) noted that threats to internal validity could also occur because preexisting differences between the groups could be attributed to the intervention and lack of randomization. The quasi-experimental non-equivalent control group design was also chosen because data could be collected through instrumentation instead of observation (Gopalan et al., 2020). Similar studies have also used a quasi-experimental approach (Choo et al., 2021; Clarke et al., 2022; Dehghani et al., 2022; Plasman & Gottfried, 2018).

This research study compared the achievement scores (dependent variable) between two groups of students, with one group receiving prerequisite skills training (independent variable), using the pretest as the covariate. Gasparetti (2022) defined the independent variable, prerequisite skills, as lower-level educational concepts already mastered, which students could apply to higher-order thinking and advanced skills. The prerequisite skills evaluated before presenting new material included awareness of exponents, fractions, decimals, variables, and inequalities. The dependent variable of student math achievement was defined as “student’s knowledge and skills in mathematics that can be applied to problem-solving situations” (National Assessment of Educational Progress, 2022, “What Does the NAEP Mathematics Section Measure?”, paragraph 1), and was measured using the MAT.

Research Question

RQ: Is there a difference in math achievement scores for students with learning disabilities in math who received prerequisite skills training before new content instruction and those who did not when controlling for math achievement pretest scores as measured by MAT?

Hypothesis

H_0: There is no difference in math achievement scores for students with learning disabilities in math who received prerequisite skills training before new content instruction and those who did not when controlling for math achievement pretest scores as measured by MAT.

Participants and Setting

This section reviews the population, participants, and setting for the study conducted. The type of sampling used during the study, the location of the school selected, and the school year in which the study was conducted are stated in this section. The number of participants and demographics are also specified. The grade level of students is also stated in this section. The setting is described in detail to ensure that the study can be replicated. A pseudonym is used to identify the name of the school for purposes of confidentiality.

Population

The participants in this study were students from Smith County School District located in Pennsylvania, studying during the first semester of the 2023-2024 school year. The school district comprises a lower- to middle-class income population in Pennsylvania. It has 6,978 students in grades PK-12, with a student-teacher ratio of 15:1. The population is 54% Caucasian, 22% African American, 15% Hispanic, 1% Asian, and 8% from other races.

Participants

Convenience sampling was used to select participants for the study. The sample size came from a high school in Pennsylvania that serves students across the state via required synchronous online learning. Participants for the study were identified as ninth-grade students in cotaught classrooms with an IEP and were classified as having an SLD in math. The students were selected from pre-algebra classes in the ninth-grade curriculum. Pre-algebra classes in the district teach skills including but not limited to, integers, fractions, decimals, negative numbers, basic equations, variables, and properties of operations. The researcher selected the students based on the student's disability, math class, and grade level. Students were introduced to the study through a letter sent to parents detailing the study and how it might support students in math. Three Zoom sessions were set up to answer parents' questions before their wards participated in the study. Parental consent was required for this study because the students were minors.

The number of participants sampled was 70, which exceeded the required minimum of 66 for a one-way Analysis of Covariance (ANCOVA) with two groups when assuming a medium-size effect with a statistical power of .7 and an alpha level, a= .05 (Gall et al., 2007). Each group had to consist of 35 participants to keep the group sizes similar. Thirty-six participants were 14 years of age, 31 participants were 15 years of age, and three participants were 16 years of age. Thirty-eight participants were identified as Caucasian, 17 participants were identified as African-American, and 15 participants were identified as Hispanic. The sample consisted of 37 males and 33 females in the ninth grade.

The control group consisted of 35 students in ninth grade from among the study participants, who were non-randomly assigned. This group consisted of 19 males and 16 females in the ninth grade. Eighteen participants were 14 years of age, 16 participants were 15 years of age, and one of the participants was 16 years of age. Twenty participants were identified as

Caucasian, 11 participants were identified as African-American, and four participants were identified as Hispanic.

The experimental group consisted of 35 students in ninth grade from among the study participants, who were non-randomly assigned. The group consisted of 18 males and 17 females in the ninth grade. Eighteen of the participants were 14 years of age, 15 of the participants were 15 years of age, and two of the participants were 16 years of age. Twenty-one of the participants were identified as Caucasian, nine of the participants were identified as African-American, and five of the participants were identified as Hispanic.

Setting

For the present study, the high school chosen was given the pseudonym "Smith". This high school services students within the state of Pennsylvania as a public charter school in an online setting. The students are required to attend synchronous ninth-grade pre-algebra math lessons daily as part of their curriculum. The participants were selected from a classroom for pre-algebra students in the ninth grade who had received synchronous education in the first semester of the 2023-2024 school year.

Instrumentation

The MAT instrument, a measurement of students' comprehension of various math skills (Havard et al., 2018), was used to measure students' math achievement (see Appendix A for the instrument). The purpose of the MAT is to demonstrate the knowledge, application, and mastery of skills as aligned with the common core standards chosen from the NAEP. According to the NAEP (2022), MAT assesses math knowledge and the student's ability to apply this knowledge in problem-solving scenarios, resulting in a long-term view of skills, knowledge, and performance. The instrument is used for various subjects in grades four, eight, and 12. NAEP assessments were authorized by Congress in 1990 and have been used as the nation's report card since 1969. The NAEP's math test was designed to measure several math skills throughout students' development, including conceptual, procedural, and problem-solving abilities (Ockey, 2007). These assessments have become a data collection tool to monitor the progress of skills, including math skills, for secondary education students. The instrument has been used in a number of studies (Abedi et al., 1998; Barker, 2003; Havard et al., 2018; Peterson & Fennema, 1985; Young et al., 2018). Cronbach's alpha was run to determine the reliability of the instrument. The MAT is a norm-referenced test with well-documented reliability (0.97) and validity (0.87) (Mbwiri, 2017).

The instrument consisted of 20 multiple-choice questions with four possible selected responses for each question. There were four questions that measured skills in exponents, five questions that measured fractions, four questions that measured decimals, three questions that measured inequalities, and four questions that measured variables. The scores ranged from 0 to 40 points. A score of 0 indicated the lowest possible score, meaning the student performed poorly on the test. A score of 40 points indicated the highest score, meaning the student performed the best possible on the test. The assessment was completed under the researcher's supervision using a lockdown browser (see Appendix B for instructions). The approximate time given for completing the assessment was 60 minutes. The researcher scored the assessment and graded it, based on 2 points per question. Permission to use the MAT in the present study was requested and received on (May 12, 2023; see Appendix C).

Procedures

Approval for conducting the current study was received from Liberty University's Institutional Review Board (see Appendix D). After approval was received, course instructors were notified, and consent from parents and students was obtained (see Appendix E). Parent permission was required due to participants being under the age of 18. A virtual training class was conducted for the instructors involved in the study. A presentation of the materials was provided to detail how the intervention should take place. The instructors received procedural training detailing the steps of the intervention. The instructors were all special education teachers with dual certifications and knowledge in implementing intervention processes, including but not limited to Functional Communication Training, RTI protocols, and proctoring of state-mandated tests.

The intervention followed the explicit instructional model. The instructor explained what materials, websites, or technology the students would need. Students had time to collect materials or open a website/software. The teacher presented a hook for the lesson of the day that would change based on the lesson plans (see Appendix F for sample lesson plans). The objective(s) were clearly explained to the students. Results from the previous prerequisite check for understanding were reviewed by the instructor and the students. The teacher reviewed a problem for each prerequisite needed for the particular lesson on a virtual board. The students assisted with the completion of another problem on the virtual board. At the conclusion of the two examples, the students signed into IXL for a check of understanding that consisted of two to four problems related to the specific prerequisite skills reviewed. The class continued with regular instruction, using the explicit instructional model.

The data was collected throughout the 8-week intervention strategy that coincided with the existing standards being taught at the school. Data was collected during the pretest on the first day and also during the posttest, which took place on the last day of teaching the four standards that had lasted for two weeks each. Data was also collected on a daily basis for the checks of understanding in IXL. At all stages of data collection, all information that could identify the participants was protected. Data were stored securely, and only the researcher had access to records. Data were stored on a password-protected computer and a password-protected cloud storage device. When not being utilized, the computer was stored in a locked security safe. The data will be retained for a period of five years after the completion of this research study.

Data Analysis

A one-way ANCOVA was used to analyze the data collected during the research study. ANCOVA is appropriate for this study because data was collected from two categorical groups: a control group receiving typical intervention strategies and an experimental group receiving prerequisite skills intervention prior to their typical instruction. ANCOVA helps provide the means to work with measurement error in the covariates and to establish a cause-effect relationship among the variables (Gall et al., 2007). ANCOVA can also help control Type 1 errors that might reject the null hypothesis when it is, in fact, accurate. The dependent variable was measured on a continuous scale, and the independent variable consisted of two independent groups. The research study used a pretest as a control variable on a continuous scale. ANCOVA aims to adjust the pretest scores to show the effect of the posttest measure (Oakes & Feldman, 2001). The data thus collected was analyzed and reported using ANCOVA in Statistical Package for the Social Sciences (SPSS) software.

The data was visually screened for any missing entries and inaccuracies. A box and whisker plot was used for each group to screen for extreme outliers. The Shapiro-Wilk Test was used to check for normal distribution of the results. Levene’s Test of Equality of Error Variance was used to check for equal variances for all samples. An ANCOVA requires that the assumption of linearity be met. Linearity was examined using a grouped scatter plot of the independent variable against the covariate for each independent variable group. The assumption of homogeneity of regression slopes was applied by visually inspecting the regression lines on similarly grouped scatterplots. The alpha level for the hypothesis was set at $p < .05$. Partial eta squared was used to report the effect size.

Chapter 4: Findings

Overview

This chapter begins with the research question and null hypothesis, followed by a presentation of the descriptive statistics of the dataset. Data screening procedures for the ANCOVA follow the descriptive statistics. The final section presents the results of the null hypothesis, which includes a discussion of the ANCOVA for the research study.

Research Question

RQ: Is there a difference in math achievement scores for students with learning disabilities in math who had received prerequisite skills training before new content instruction and those who had not when controlling for math achievement pretest scores as measured by MAT?

Null Hypothesis

H_0: There is no difference in math achievement scores for students with learning disabilities in math who had received prerequisite skills training before new content instruction and those who had not when controlling for math achievement pretest scores as measured by MAT.

Descriptive Statistics

Descriptive statistics were obtained on the covariate (pre-test) and the dependent variable (student math achievement). Table 1 and Table 2 provide the descriptive statistics.

Table 1

Descriptive Statistics: Covariate: Pretest

Group	n	M	SD
1 - Control	35	16	4.028
2 - Experimental	35	16.47	4.058

Table 2

Descriptive Statistics

Dependent Variable: Posttest, Student Math Achievement

Group	n	M	SD
1 - Control	35	17.43	3.709
2 - Experimental	35	18.89	3.919

Results

Data Screening

Data screening was conducted on covariate and dependent variables. The researchers sorted the data on each variable and scanned for inconsistencies. No data errors or inconsistencies were identified. Box and whisker plots were used to detect extreme outliers on each dependent variable. No extreme outliers were identified. Please refer to Figure 1 and Figure 2 for box and whisker plots.

Figure 1

Box and Whisker Plots (Covariate)

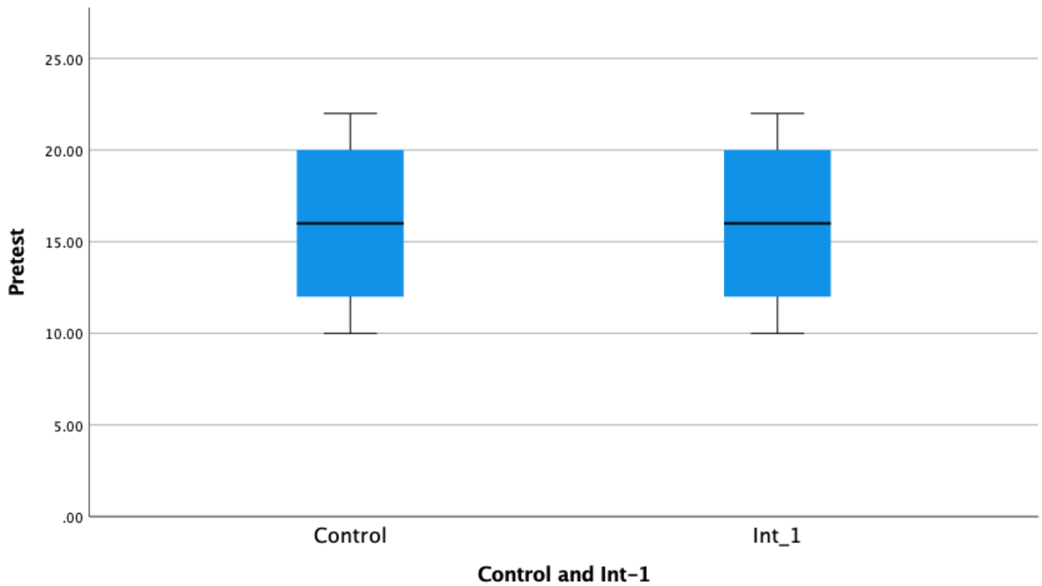
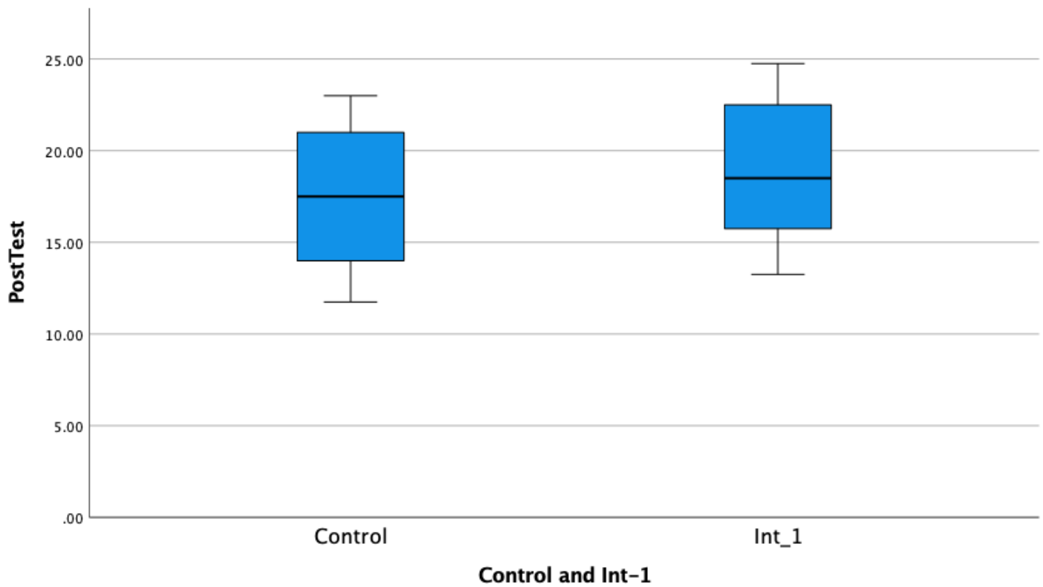


Figure 2

Box and Whisker Plots (Dependent)



Assumptions

An ANCOVA was used to test the null hypothesis. The ANCOVA required that the assumptions of normality, linearity, bivariate normal distribution, homogeneity of slopes, and homogeneity of variance were met.

Normality was examined using a Shapiro-Wilk test because the sample size was less than 50. No violations of normality were found. Table 3 depicts the Tests of Normality.

Table 3

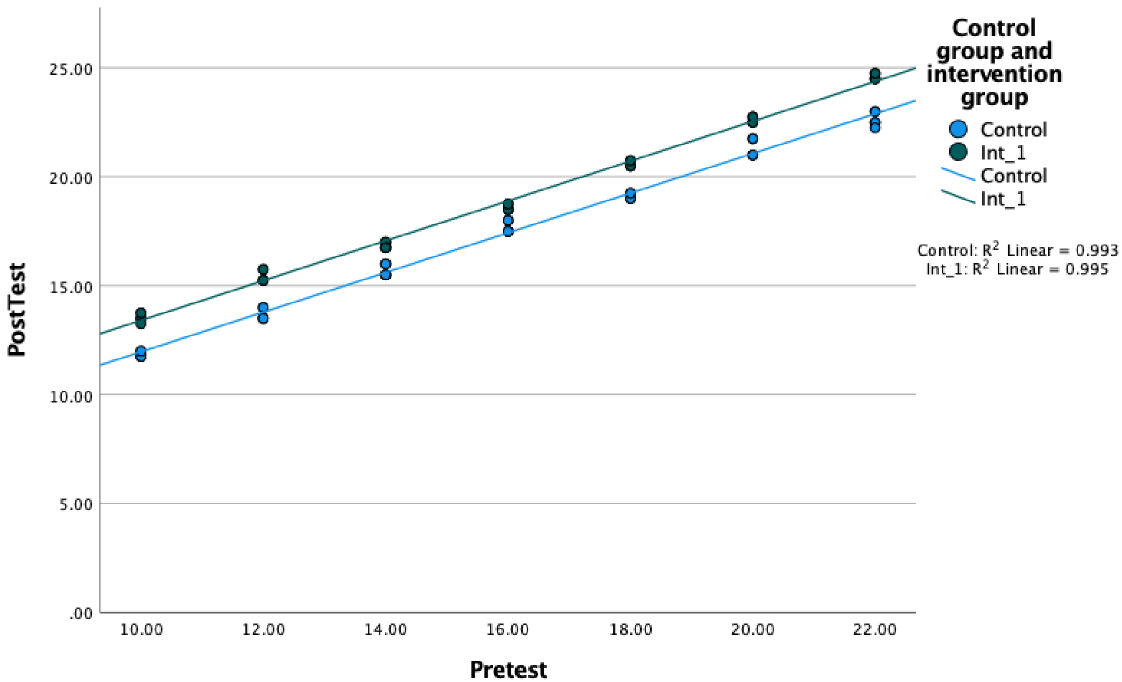
Tests of Normality

	Shapiro-Wilk		
Groups	Statistic	df	Sig.
Control	.945	35	.079
Intervention	.963	35	.272

The assumptions of linearity and bivariate normal distribution were tested using scatter plots for each group. Linearity was met and bivariate normal distributions were tenable, as the shapes of the distributions were not extreme. Figure 3 includes the scatter plot for each group.

Figure 3

Scatter Plot for Posttest by Groups



The assumption of homogeneity of slopes was tested, and no interaction was found, with p = .837. Therefore, the assumption of homogeneity of slope was met. The assumption of homogeneity of variance was examined using Levene's test. No violation was found, with p = .331. The assumption of homogeneity of variance was met.

Results for Null Hypothesis

An ANCOVA was used to test the null hypothesis regarding the math achievement scores for students with learning disabilities in math, who had received prerequisite skills training before new content instruction and those who had not, while controlling for pre-test scores. The null hypothesis was rejected at a 95% confidence level, with $F(1, 67) = 454.43$, p = <.001, $\eta_p^2 = .872$. The effect size was very large. Since the null hypothesis was rejected, post hoc analysis was conducted to identify differences in pairs using a Bonferroni post hoc test.

There was a significant difference between the control group (M_{adj} = 17.43, *SE.* = .049) and the experimental group (M_{adj} = 18.89, *SE.* = .049). Table 4 depicts multiple comparisons of groups.

Table 4

Multiple Comparisons of Groups

Pairwise Comparisons
Dependent Variable: Posttest

(I) Control Group and Intervention Group	(J) Control Group and Intervention Group	Mean Difference (I-J)	*SE*	Sig.[b]	95% Confidence Interval for Difference[b]	
					Lower Bound	Upper Bound
Control	Int-1	-1.464*	.069	<.001	-1.601	-1.327
Int-1	Control	1.464*	.069	<.001	1.327	1.601

Based on estimated marginal means
*. The mean difference is significant at the .05 level.
b. adjustment for multiple comparisons: Bonferroni

Chapter 5: Conclusions

Overview

This chapter begins with a discussion of the results of the study and presents a comparison of the results with the literature presented in previous chapters. The discussion is followed by sections dealing with the implications and limitations of this research study. The final section of the chapter presents recommendations for future research.

Discussion

The purpose of this quantitative, quasi-experimental study with a pre-test-posttest control group design, while controlling for math achievement pre-test scores, was to investigate the impact of prerequisite skills instruction on math achievement scores for ninth-grade pre-algebra students with SLDs in math. The study used MAT to measure the independent variable of prerequisite skills. The pre-test results were the covariates in the study. The dependent variable, student math achievement results, was derived from two groups of students: a control group and an experimental group.

The researcher sought to measure the impact of teaching prerequisite skills on students with learning disabilities in ninth-grade pre-algebra. Bloom’s (1956) learning theory formed the theoretical framework for the study. Bloom emphasized the importance of a foundation in learning, through a taxonomy that begins with a foundation of knowledge and moves on to strengthen additional layers of learning, including understanding, applying, analyzing, evaluating, and creating. Davies et al. (2021) related the learning of prerequisite skills to Bloom’s foundation of knowledge that eventually led to success in solving higher-order questions. The results of the study found a strong difference between the control group and the experimental group when prerequisite skills were taught before requisite skills. The study demonstrated that the sample of students significantly benefited from prerequisite skills instruction and the memorization of facts before instruction on current skills took place. The theoretical framework is closely related to this study because it discovered that the foundation of knowledge, or prerequisites, reinforces requisite math skills: a centerpiece of Bloom's (1956) theory for mastering skills while using a taxonomy for learning.

The hypothesis for the current research study was that there were no differences in math achievement scores for students with learning disabilities in math who had received prerequisite skills training before new content instruction compared to those who had not when controlling for math achievement pre-test scores as measured by the MAT. This hypothesis was rejected, as it was found that high school students benefitted significantly from explicit instruction about prerequisite skills in ninth-grade algebra. Other research studies have also identified the need for prerequisite skills training (Apanasionok et al., 2021; Dueker & Day, 2022; Poast et al, 2021).

The current research is similar to Dueker and Day's (2022) findings that prerequisite skills reinforce the mastery of skills currently being taught on the grading standards. They found that students strengthened their post-test scores because of an intervention focusing on prerequisite numeracy and mathematical skills, thereby allowing them to perform at or near their typical peers. During their study, students in elementary school education acquired missing skills that were identified and helped improve upon currently taught skills. This new study expands this finding into high school education, reinforcing the idea that students need to memorize basic skills before mastering more complex ones.

Apanasionok et al. (2021) found that students involved in guided instructional delivery methods, including warm-ups and prerequisite skills training improved early numeracy skills essential for successful student outcomes in math. This investigation supports the finding that explicit instructional methods, including guided instruction and the correct organization of explicit instruction, increased student achievement in skills. These skills are important for instruction in math classes and for independent living among students with disabilities. This study also found that students strengthened their skills in requisite knowledge while reviewing prerequisite skills during instructional time via highly structured and guided instruction typically delivered when teachers used the explicit instructional method.

Poast et al.’s (2021) study expanded teaching prerequisite skills to higher education and increased success rates for students in math. The study also indicated that prerequisite skills played an essential role in students’ success in instructional and curricular practices while learning math. This study established a need for stronger fact retrieval skills among students who studied math and those with disabilities. Both Poast et al.’s (2021) study and this investigation have found that memorizing math facts improved student achievement scores in math.

Apanasionok et al. (2021), Dueker and Day (2022), and Poast et al. (2021) found that prerequisite skills were essential to the success of students in early elementary education and higher education. The current research study expanded into a high school setting and also found that prerequisite skills increased student achievement scores when they were taught using highly structured and guided explicit instructional methods. Both previous research and the current study have connected the retrieval of facts to success in teaching requisites, a centerpiece of Bloom’s (1956) theory for mastering skills through a taxonomy for learning.

Implications

This study has identified that students with learning disabilities benefit from explicit instructional methods, which review prerequisite skills before current skills are taught using various instructional strategies in a math class. The prerequisite skills taught in this study included recognizing the importance of instruction of these skills and the importance of developing more robust instructional methods in the classrooms. With gaps noted in national achievement levels for students with math-based disabilities (Kiss et al., 2019), utilizing research-based instructional methods is critical to students' success in their future endeavors, including STEM education, the workforce, and higher education. Explicit instructional methods follow a structure that includes providing a hook, stating the objectives, and reviewing prerequisite skills with guided instruction following an "I do, we do, and you do" structure. This organization and structure reinforce the required math skills so that students can access future instruction in other areas of computation. Prerequisite skills training in high school that strengthens math abilities could result in a decrease in high school dropout rates and low unemployment rates for students with SLDs (Namkung & Bricko, 2021; Rose, 2020).

Understanding the context of specific strategies to address gaps in education is critical to the success of students. There is an overwhelming amount of research to support the need for teachers to support students with guided instruction, including prerequisite skills (Ahmadian, 2020; Foxworth et al., 2022; Long et al., 2021). Successful instruction in the classroom that leads to a more substantial interpretation of math skills contributes to students' confidence and engagement in their education (Reynolds & Joseph, 2022). As a result, communities and educational institutions will see high graduation and employment rates, especially for those with SLDs (Rose, 2020). This study has emphasized the need for more explicit instruction with prerequisite skills training because expanding these skills into the high school setting will help support teachers who are looking for more research-based strategies. Teaching prerequisite skills in high school will also support the higher graduation and employment rates desired by communities.

Limitations

This study has been limited by several key factors. First, a true experiment was not used because the students were already assigned to the classrooms. Threats to internal validity can occur due to the lack of randomization in the experiment (Miller et al., 2020). The research study could not be conducted using randomization, due to the school setting and predetermination of classes before the beginning of the school year.

A second limitation of the study was the sample size and demographics. A single high school was used to collect the data in a ninth-grade mathematics classroom. This may affect the generalization of the results, primarily since a single school was utilized and located within a specific region of the United States. The results of the current study cannot be generalized beyond the population. Additionally, a limited number of students were available for the study, because the study focused on students with mathematics-based learning disabilities in the ninth grade.

A third limitation of the study was its 8-week timeframe. While the study did produce results, questions still need to be answered on whether or not a more extensive study could result in varying data. Having students work on prerequisite skills throughout the entire semester could yield different results, due to the amount of information learned and prerequisite skills reviewed.

Recommendations for Future Research

The following recommendations may be offered for future research in this domain.

1. A similar experiment may be conducted with a larger, more diverse sample. The students in the study lived in a specific region of the United States and were enrolled in one school. Future research should expand across varied regions in the United States and to different schools within those regions.

2. An investigation that includes other areas of math and various grades may be pursued. The current research study was limited to ninth-grade pre-algebra. Students in other areas of mathematics, including geometry, trigonometry, precalculus, and algebra 2, may also benefit from explicit instructional methods that utilize prerequisite skills training before current skills are taught in the classroom. Future research should also expand into other grade levels in high school.
3. A similar investigation may be focused on students in general education classrooms without IEPs. The current study included students who had an IEP and a learning disability in the area of math. Students without IEPs also continue to struggle in the area of math and continue to face difficulties throughout their academic careers. Studies that compare results for students in the general education classroom who have prerequisite skills training before requisites in math can also close research gaps.
4. Varied levels of taxonomy may be implemented in future research studies. The current study focuses only on knowledge, the first step in the taxonomy. Future studies can also help understand how to strengthen core levels of the taxonomy, including understanding, applying, analyzing, evaluating, and creating. In turn, these studies can also help understand how effective the first level of knowledge is in education.
5. Other methods of instruction need to be researched. The current study utilized an explicit instructional method. This type of method is not always feasible in every classroom, especially those that use alternative strategies. Other methods of instruction, such as implicit instructional methods, should be utilized for future research studies.

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APPENDICES

Appendix A

Instrument

Today you will be taking a math assessment that measures skills for algebra. This test has 20 questions. Each question is worth 2 points for a total of 40 points. You will have enough time to read and answer all the questions below. Each question is multiply choice. Select the best answer for each question. You cannot skip any questions. You may go back to questions and review your work. You may also use scratch paper to solve problems. You may not use a calculator or any outside sources including the internet. Your web browser will be locked down while you take this assessment. Scroll down to answer each question until you answer all 20 questions and see the submit button. When you come to the end of the test, please click submit and let your instructor know you have finished. If you have any questions during the assessment, please ask the instructor.

* Indicates required question

1. Place your first and last name in the box below. *

2. * 2 points

1. What are all the whole numbers that make 8 - □ > 3 true?

A. 0, 1, 2, 3, 4, 5

B. 0, 1, 2, 3, 4

C. 0, 1, 2

D. 5

Mark only one oval.

- A
- B
- C
- D

3. * 2 points

2. Sally is shorter than Ronnie. Sally is taller than Michael. Denise's height is between Sally's height and Ronnie's height. Who is the shortest person?

A. Denise

B. Michael

C. Ronnie

D. Sally

Mark only one oval.

- A
- B
- C
- D

4. * 2 points

3. What are all values of n for which $-2n \geq n + 6$?

A. $n \leq -2$

B. $n \geq -2$

C. $n \geq 0$

D. $n \leq 6$

E. $n \geq 6$

Mark only one oval.

5. * 2 points

4. If $3 + w = b$, then $w =$

A. b/3
B. $b \times 3$
C. $b + 3$
D. $3 - b$
E. $b - 3$

Mark only one oval.

- A
- B
- C
- D
- E

6. * 2 points

5. In which of the following are the three fractions arranged from least to greatest?

A. $\frac{2}{7}, \frac{1}{2}, \frac{5}{9}$
B. $\frac{1}{2}, \frac{2}{7}, \frac{5}{9}$
C. $\frac{1}{2}, \frac{5}{9}, \frac{2}{7}$
D. $\frac{5}{9}, \frac{1}{2}, \frac{2}{7}$
E. $\frac{5}{9}, \frac{2}{7}, \frac{1}{2}$

7. * 2 points

6. In the past year and a half, Alfred's dog gained an average of $\frac{\ }{4}$ pound each month. Today, Alfred's dog weighs 75.5 pounds. How much did the dog weigh a year and a half ago?

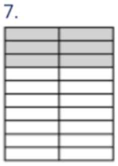
A. 57.5 pounds
B. 71.0 pounds
C. 71.5 pounds
D. 74.0 pounds
E. 79.5 pounds

Mark only one oval.

- A
- B
- C
- D
- E

8. * 2 points

7.



What fraction of the figure above is shaded?

A. $\frac{1}{4}$
B. $\frac{3}{10}$
C. $\frac{1}{3}$
D. $\frac{3}{7}$
E. $\frac{7}{10}$

9. *

2 points

8. Jim has 3/4 of a yard of string which he wishes to divide into pieces, each 1/8 of a yard long. How many pieces will he have?

A. 3
B. 4
C. 6
D. 8

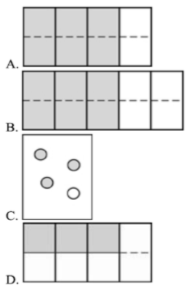
Mark only one oval.

- A
- B
- C
- D

10.

2 points

9. Which picture shows that $\frac{3}{4}$ is the same as $\frac{6}{8}$?



Mark only one oval.

- A
- B
- C
- D

11. *

2 points

10. What is 4 hundredths written in decimal notation?

A.0.004
B.0.04
C.0.400
D.4.00
E. 400.0

Mark only one oval.

- A
- B
- C
- D
- E

12. *

2 points

11. The diameter of a red blood cell, in inches, is 3×10^{-4}. This expression is the same as which of the following numbers?

A. 0.00003
B. 0.0003
C. 0.003
D. 3,000
E. 30,000

Mark only one oval.

- A
- B
- C
- D
- E

13. * 2 points

12. If $x = 2n + 1$, what is the value of x when $n = 10$?

A. 11

B. 13

C. 20

D. 21

E. 211

Mark only one oval.

- A
- B
- C
- D
- E

14. * 2 points

13. The formula $d = 16t^2$ gives the distance d, in feet, that an object has fallen t seconds after it is dropped from a bridge. A rock was dropped from the bridge and its fall to the water took 4 seconds. According to the formula, what is the distance from the bridge to the water?

A. 16 feet

B. 64 feet

C. 128 feet

D. 256 feet

E. 4,096 feet

Mark only one oval.

- A
- B
- C
- D
- E

15. * 2 points

14. Which of the following equations is NOT equivalent to the equation $n + 18 = 23$?

A. $23 = n - 18$

B. $23 = 18 + n$

C. $18 = 23 - n$

D. $18 + n = 23$

E. $n = 23 - 18$

Mark only one oval.

- A
- B
- C
- D
- E

16. * 2 points

15. Which of the following is equal to $6(x + 6)$?

A. $x + 12$

B. $6x + 6$

C. $6x + 12$

D. $6x + 36$

E. $6x + 66$

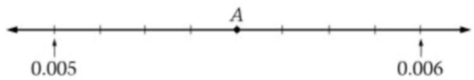
Mark only one oval.

- A
- B
- C
- D
- E

17. *

2 points

16.



What number is represented by point *A* on the number line above?

A. 0.0010

B. 0.0054

C. 0.0055

D. 0.006

E. 0.055

Mark only one oval.

- A
- B
- C
- D
- E

18. *

2 points

17. Which of the following numbers is twenty-three and eight-thousandths?

A. 230.8

B. 23.8

C. 23.08

D. 23.008

E. 23.0008

Mark only one oval.

- A
- B
- C
- D
- E

19. *

2 points

18. An airplane climbs at a rate of 66.8 feet per minute. It descends at twice the rate that it climbs. Assuming it descends at a constant rate, how many feet will the airplane descend in 30 minutes?

A. 96.8

B. 133.6

C. 1,002

D. 2,004

E. 4,008

Mark only one oval.

- A
- B
- C
- D
- E

20. *

2 points

19. The mean distance from Venus to the Sun is 1.08 X 10[illegible] kilometers. Which of the following quantities is equal to this distance?

A.10,800,000 kilometers

B.108,000,000 kilometers

C.1,080,000,000 kilometers

D.10,800,000,000 kilometers

E.108,000,000,000 kilometers

Mark only one oval.

- A
- B
- C
- D
- E

21. *

2 points

20. A reasonable prediction of the distance d in feet, that a car travels after the driver has applied the brakes can be found by using the formula $d = 0.055r^2$, where r is the speed of the car in miles per hour.

If Mario is driving at 60 miles per hour and applies the brakes, then according to the formula, how many feet will Mario's car travel before it stops?

A. 330

B. 198

C.10.89

D.6.6

E.3.3

Mark only one oval.

- ◯ A
- ◯ B
- ◯ C
- ◯ D
- ◯ E

Appendix B

Instructors: Please Read the following directions to the students.

Today, you will take a math assessment measuring your algebra skills. This test has 20 questions. Each question is worth 2 points for a total of 40 points. You will have enough time to read and answer all the questions below. Each question is a multiple choice. Select the best answer for each question. You cannot skip any questions. You may go back to questions and review your work. You may also use scratch paper to solve problems. You may not use a calculator or any outside sources, including the Internet. Your web browser will be locked down while you take this assessment. Scroll down to answer each question until you answer all 20 questions and see the submit button. When you finish the test, please click submit and let your instructor know you have completed it. If you have any questions during the assessment, please ask the instructor.

Examination Conduct

You must only attempt this exam once. Any additional attempts should only be made when a severe technical issue has occurred and supporting evidence is required.

You are not permitted to obtain assistance by improper means or ask for help from or give help to any other person.

You are not permitted to take screenshots, record the screen, copy and paste questions or answers, or otherwise attempt to take any of the content of this exam out of the exam for any purpose.

You are not permitted to post any requests for clarification of exam content. Answer all questions to the best of your ability and perception of the questions' intent, and make reasonable assumptions, if necessary, to answer all questions.

Test Rules

- You must use a functioning webcam and microphone.
- No cell phones or other secondary devices in the room or test area
- Your desk/table must be clear of materials except your test-taking device.
- No one else can be in the room with you.
- No talking
- The testing room must be well-lit, and you must be visible.
- No dual screens/monitors
- Do not leave the camera.
- No use of additional applications or internet

Appendix C

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Appendix D

LIBERTY UNIVERSITY.
INSTITUTIONAL REVIEW BOARD

July 19, 2023

Tim Reedy
Laura Mansfield

Re: IRB Approval - IRB-FY23-24-73 How Prerequisite Skills Training Affects Math Achievement Scores for Students with Learning Disabilities: A Quasi-Experimental Control Group Study

Dear Tim Reedy, Laura Mansfield,

We are pleased to inform you that your study has been approved by the Liberty University Institutional Review Board (IRB). This approval is extended to you for one year from the following date: July 19, 2023. If you need to make changes to the methodology as it pertains to human subjects, you must submit a modification to the IRB. Modifications can be completed through your Cayuse IRB account.

Your study falls under the expedited review category (45 CFR 46.110), which is applicable to specific, minimal risk studies and minor changes to approved studies for the following reason(s):

7. Research on individual or group characteristics or behavior (including, but not limited to, research on perception, cognition, motivation, identity, language, communication, cultural beliefs or practices, and social behavior) or research employing survey, interview, oral history, focus group, program evaluation, human factors evaluation, or quality assurance methodologies. (NOTE: Some research in this category may be exempt from the HHS regulations for the protection of human subjects. 45 CFR 46.101(b)(2) and (b)(3). This listing refers only to research that is not exempt.)

For a PDF of your approval letter, click on your study number in the My Studies card on your Cayuse dashboard. Next, click the Submissions bar beside the Study Details bar on the Study Details page. Finally, click Initial under Submission Type and choose the Letters tab toward the bottom of the Submission Details page. Your stamped consent form(s) and final versions of your study documents can be found on the same page under the Attachments tab. Your stamped consent form(s) should be copied and used to gain the consent of your research participants. If you plan to provide your consent information electronically, the contents of the attached consent document(s) should be made available without alteration.

Thank you for your cooperation with the IRB, and we wish you well with your research project.

Sincerely,

G. Michele Baker, PhD, CIP
Administrative Chair
Research Ethics Office

Appendix E

Combined Parental Consent and Student Assent

Title of the Project: *How Prerequisite Skills Training Affects Math Achievement Scores for Students with Learning Disabilities: A Quasi-Experimental Control Group Study*
Principal Investigator: Timothy Reedy, Doctoral Candidate, School of Education, Liberty University

Invitation to be Part of a Research Study

Your student is invited to participate in a research study. To participate, he or she must be a 9th grade student at [redacted] taking 9th grade algebra and listed as having an IEP and be classified as having a specific learning disability in mathematics. Taking part in this research project is voluntary.

Please take time to read this entire form and ask questions before deciding whether to allow your student to take part in this research project.

What is the study about and why are we doing it?

The purpose of the study is to investigate the impact prerequisite skills instruction has on math achievement scores for 9th-grade pre-algebra students with SLDs in math.

What will participants be asked to do in this study?

This study will use a control group (those who do not receive the intervention) and an experimental group (those who will receive the intervention). Students will not be randomly assigned to these groups, and participants may or may not receive the intervention as part of their participation.

If you agree to allow your student to be in this study, I will ask her or him to do the following:

1. Participate in a pre-test that will take approximately 1 hour.
2. Participate in a prerequisite skills instruction (if in the treatment group) OR traditional instruction (if in control group) during regular classroom time for an 8-week period.
3. Participate in a post-test that will take approximately 1 hour.

How could participants or others benefit from this study?

The direct benefits participants in the treatment group should expect to receive from taking part in this study are an increase in understanding of prerequisite skills needed to be successful in algebra and possibly future math classes. Participants in the control group should not expect to receive a direct benefit.

Benefits to society include a stronger understanding of how specific teaching methods in prerequisite skills may increase student readiness in math classes.

What risks might participants experience from being in this study?

The expected risks from participating in this study are minimal, which means they are equal to the risks your student would encounter in everyday life.

How will personal information be protected?

The records of this study will be kept private. Research records will be stored securely, and only the researcher will have access to the records.

-Participant responses will be anonymous.

-Data will be stored on a password-locked computer, and physical data will be stored in a locked cabinet. After five years, all electronic records will be deleted, and all hardcopy records will be shredded.

How will participants be compensated for being part of the study?

Participants will be compensated for participating in this study. At the conclusion of the study, participants will receive a $25.00 Visa gift card. Students must complete the study to be eligible for the gift card. Email addresses will be requested for compensation purposes; however, they will be collected by email at the conclusion of the survey via a separate link to maintain your

Is the researcher in a position of authority over participants, or does the researcher have a financial conflict of interest?

[redacted]. To limit potential or perceived

researcher receives it/etc. This disclosure is made so that you can decide if this relationship will affect your willingness to allow your student to participate in this study. No action will be taken against an individual based on her or his decision to allow his or her student to participate in this study.

Is study participation voluntary?

Participation in this study is voluntary. Your decision whether to allow your student to participate will not affect your or his or her current or future relations with Liberty University. If you decide to allow your student to participate, she or he is free to not answer any question or withdraw at any time without affecting those relationships.

What should be done if a participant wishes to withdraw from the study?

If you choose to withdraw your student from the study or your student chooses to withdraw, please contact the researcher at the email address/phone number included in the next paragraph. Should you choose to withdraw her or him or should your student choose to withdraw, data collected from your student will be destroyed immediately and will not be included in this study.

Whom do you contact if you have questions or concerns about the study?

The researcher conducting this study is Timothy Reedy You may ask any questions you have now. If you have questions later, **you are encouraged** to contact him at [redacted]. You may Laura J. Mansfield, Ed.D., CCC-SLP, at [redacted]

Whom do you contact if you have questions about rights as a research participant?

If you have any questions or concerns regarding this study and would like to talk to someone other than the researcher, **you are encouraged** to contact the IRB. Our physical address is Institutional Review Board, 1971 University Blvd., Green Hall Ste. 2845, Lynchburg, VA, 24515; our phone number is 434-592-5530, and our email address is irb@liberty.edu.

Disclaimer: The Institutional Review Board (IRB) is tasked with ensuring that human subjects research will be conducted in an ethical manner as defined and required by federal regulations. The topics covered and viewpoints expressed or alluded to by student and faculty researchers are those of the researchers and do not necessarily reflect the official policies or positions of Liberty University.

Your Consent

By signing this document, you are agreeing to allow your student to be in this study. Make sure you understand what the study is about before you sign. You will be given a copy of this document for your records. The researcher will keep a copy with the study records. If you have any questions about the study after you sign this document, you can contact the study team using the information provided above.

I have read and understood the above information. I have asked questions and have received answers. I consent to allow my student to participate in the study.

____

____

Parent/Guardian Date

____

Date

Appendix F

Name:
Week of:

Course: **9th Grade Math**	Unit/Lesson Title: Semester A				Reflections and Modifications (based on STs needs)
Objectives	**Linear Equations:** Write, solve, and/or graph linear equations using various methods. Write, solve, and/or graph systems of linear equations using various methods.				
Standards:	**CC.2.2.HS.D.10 Represent, solve and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically.** **CC.2.2.HS.D.8 Apply inverse operations to solve equations or formulas for a given variable.**				
	Monday Two Step Equations	Tuesday Two Step Equations	Wednesday Two Step Equations	Thursday Two Step Equations	
Student Hook	What is a two-step equation song video	Who Remembers Vocab Game	Game Review in Kahoot	Crossword Puzzle Math Terms	
Objectives	Understand the meaning of variables, constants, and	Solve two step equations using addition, subtraction,	Solve two-step equations with decimals by isolating the variable.	Solve two step equations with fractions by isolating the	

	coefficients. Simplify expressions with addition, subtraction, multiplication, and division	multiplication. Solve word problems by forming and solving a two-step equation	Solve word problems by forming and solving a two-step equation	variable. Solve word problems by forming and solving a two-step equation	
Prerequisite/ Anticipatory Sets (Intervention)	1. Students will be shown how to solve a one-step equation on the board with a variable A+3=10. The teacher will complete the problem on the board. The teacher will explain what a variable is and why they are used in math. 2. Students will assist with solving an equation with teacher. 3. Students will solve 3 problems using IXL Prerequisites: variables, addition, subtraction	1. Students will be shown how to solve a one-step equation a variable in a different part of the equation 9=5+A. The teacher will review rules for variables. 2. Students will assist instructor with solving an equation 3.Students will solve 3 problems using IXL Prerequisites: variables, addition, subtraction	1.Students will be shown a decimal problem on the board. 1.425+7.38. Teacher will complete the problem on the interactive board. 2. Students will assist teacher with a decimal problem 73.77-12.63 3.Students will solve 3 problems using IXL Prerequisites: Addition, subtraction, decimals, variables	1.Students will be shown a fraction problem on the board. The teacher will complete the problem on the interactive board. 2.Students will assist the teacher with completing another problem on the board 3.Students will solve 3 problem using IXL Prerequisites: Addition, subtraction, fractions, variables	
Instruction/ Interactive Activities	Manipulatives Activity 1.Students will be asked to gather tiles to represent the numbers, signs, and variables 2. Teacher will	Card Game Activity 1.Students will be shown different placements of the variable on the interactive board	Work Group Activity 1.Students will be shown an example of an equation with decimals.	Team and Teach 1.Students will be shown how to complete a two-step equation with a fraction on the	

	demonstrate to the students how to create a math problem with the manipulatives. 3.Teacher will model how to put together a math problem and solve while students perform the action. The vocab terms will be reinforced in this section. 4.Students will create a math problem, solve it, and show the instructor the answer when complete 5.Student will write down the example in their notebook	2. Teacher and students will solve an equation on the interactive board equation using previously learned principles. 3.Students will select a card and complete the equation. 4.Students will share their equation with results on a padlet. 5.Students will review the padlet	2.Teacher will demonstrate how to solve the problem 3.Teacher and students will complete a problem together with the use of decimals 4.Students will be paired into groups of four to solve two problems as a group. 5.Students will share the results.	interactive board 2.The teacher will assist the students in completing one problem 3.Students will be broken up into teams of five and asked to complete 1 problem on the worksheet 4.A representative from the group will teach the class how to complete the problem.	
Formative Assessment	IXL Exit Ticket	IXL Completion of Worksheet	IXL Exit Ticket	IXL Completion of Worksheet	
Vocab/Key words	Expression, equation, variable, coefficient, base, value, constant	Expression, equation, variable, coefficient, base, value, constant	Expression, equation, variable, coefficient, base, value, constant	Coefficient, combining like terms, equation, variable, value, constant	
Extra Materials Needed	IXL Notebook	IXL Notebook Online Worksheet	IXL Notebook	IXL Notebook Online Worksheet	