ELEMENTARY TEACHERS' EXPERIENCES OF IMPLEMENTING CONSTRUCTIVIST MATH STRATEGIES: A QUALITATIVE STUDY

by

Jenny Rae Wielinski

Liberty University

A Dissertation Presented in Partial Fulfillment

Of the Requirements for the Degree

Doctor of Education

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Abstract

The purpose of this transcendental phenomenological study was to describe elementary teachers' lived experiences of implementing constructivist strategies, specifically the concrete, representational, and abstract approach (CRA) when teaching mathematics in a midwestern school district by asking: What are elementary teachers' lived experiences implementing constructivist strategies? The theory that guided this study is Bruner's constructivist learning theory, as the enactive-iconic-symbolic learning he described remarkably parallels the concrete, representational, abstract (CRA) framework for teaching and learning mathematics. This qualitative study consisted of 10 elementary math teachers who instructed kindergarten through fifth grades in Montgomery City Schools in suburban Ohio. Data from interviews, focus groups, and journal entries were coded and placed into emerging themes. A detailed descriptive analysis of the data was included, and a complete description of the participants' lived experiences were integrated. Four themes developed through data analysis using Moustakas's modified method of analysis in this phenomenological study and include (a) mathematical understanding with concrete objects, (b) mathematical concepts with representations, (c) providing inquiry with abstract problems, and (d) recognizing needs with abstract concepts. These themes corresponded to the theoretical framework of the study. This study confirmed Bruner's learning theory through participants shared lived experiences teaching math with constructivist strategies. Teachers experienced success using CRA to help teach students conceptual understanding of mathematics.

Keywords: constructivist, concrete, representational, abstract, CRA, manipulatives, teachers' experiences

Copyright Page

Dedication

This endeavor would not have been possible without the love, guidance, support, and protection from God. The LORD is my rock, my fortress, and my deliverer; my God is my rock, in whom I take refuge, my shield and the horn of my salvation, my stronghold. Psalm 18:2. Thank You! Praise You, Jesus!

To my precious children and parents, who prayed for me, encouraged me, believed in me, and sacrificed right along with me. I will forever be thankful for your love, support, and patience. You all mean the world to me, and I love you to the moon and back.

To the memory of my sister Jody, who was my biggest cheerleader and best friend. Thank you for showing me how to push through the hard things in life and to finish strong.

To my dear friends, thank you for your prayers, support, and laughter through this journey. You helped make this possibility come to fruition.

Acknowledgments (Optional)

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List of Abbreviations

Common Core State Standards (CCSS)

Common Core State Standards Initiative (CCSSI)

Concrete, Pictorial, Abstract (CPA)

Concrete, Representational, Abstract (CRA)

English Learners (EL)

Programme for International Student Assessment (PISA)

National Assessment of Educational Progress (NAEP)

National Center for Education Statistics (NCES)

The Trends in International Mathematics and Science Study (TIMMS)

CHAPTER ONE: INTRODUCTION

Overview

Mathematics is a challenge for many students and adults in America and throughout the world. Globally, 600 million students are not competent in basic math skills at the end of primary schooling (UNESCO, 2017). Countries recognize the importance of improving mathematical competence in young children (Kärki et al., 2022; Palatas et al., 2016). Researchers agree that early mathematics education is an essential component of education in all countries (Räsänen et al., 2019; Urban et al., 2021). In the United States of America, teachers are required to teach according to Common Core State Standards (CCSS) and other state standards that emphasize conceptual and procedural understanding and the application of reasoning when problem-solving (CCSSI, 2010; Mancl et al., 2012). Despite the change in standards, students are still struggling with mathematics (Loveless, 2020; NAEP, 2019). One constructivist method of teaching that is examined in this study is the concrete, representational, and abstract framework (CRA). CRA is based on Bruner's learning theory (1966). Research supports the benefits of this teaching method (Bouck et al., 2017; Bouck et al., 2018; Flores et al., 2016; Flores, Meyer, & Moore, 2020; Hinton & Flores, 2019); however, there is scant research on teachers' experiences when utilizing this framework. In this chapter, I present the background of the historical problem American students have with mathematics regardless of the methods and standards teachers use to teach it. I explain the problem and purpose of the study and the gap in literature relating to the problem. Research questions, definitions, and a summary conclude Chapter One.

Background

An estimated 6% of students in the United States have a mathematics learning disability (Devine et al., 2018; Powell et al., 2013). Only 17% of fourth-grade students with disabilities scored at or above proficient on the National Assessment on Educational Progress (NAEP, 2019). The majority of school-aged students do not have a specific math disability; however, roughly 14% of students in North America and Europe struggle to learn basic math competencies (UNESCO, 2017). Elementary students with no disabilities yet struggle with math require assistance beyond their general education teaching, such as more teacher support, differentiation, and remediation with Tier II intervention services (Flores, Hinton, & Meyer, 2020). Additionally, some teachers, as well as students, find mathematics difficult. Newton (2018) examined the mathematical understanding of teachers and math coaches and posited that further development of their conceptual understanding was needed. Learners face several challenges in mathematics; these are outlined in the historical, social, and theoretical contexts.

Historical Context

American schools have a history of implementing various mathematical approaches with limited student success. Studying the quality of teaching and learning mathematics in America was relatively new during the past century (Kilpatrick, 2020). In 1930 America, progressivism became popular through the Activity Movement. The Activity Movement heavily focused on subject integration versus teaching specific subjects. Proponents of this progressive movement believed that learning basic multiplication tables was not a worthwhile endeavor (Klein, 2003). Unfortunately, this program led to many army recruits in the 1940s who could not do basic math and had to be taught by the United States Army (Raimi, 2004). As a result, a new educational math philosophy emerged to help improve math competency called the Life Adjustment Movement. The focus of the Life Adjustment Movement was on everyday math skills needed for daily living, and students learned practical math skills with no focus on higher levels of math, such as geometry and algebra (Klein, 2003). The Life Adjustment Movement failed to prepare the majority of students for college-level math and was no longer accepted by society, leading to the New Math of the 1950s and 1960s. New Math also faded quickly in popularity because of lackluster results and difficult theoretic language not suited for classroom teaching (Klein, 2003). The progressive Open Movement took its place in the 1970s. The Open Movement encouraged children to decide what they wanted to learn; regrettably, this also had poor results for math achievement (Klein, 2003). In the 1970s, elementary teacher preparation in universities began to include math education courses and pedagogical qualifications. Still, the focus was on the general teaching of all subjects, including math, but not mathematics specialization (Schubring & Karp, 2020). The focus on getting back to the basics in the 1970s led to focusing on problem-solving in the 1980s (Li & Schoenfeld, 2019). Many students' difficulties in math continued into college and even impacted college courses. According to *A Nation at Risk* (National Commission on Excellence in Education, 1983), remedial math courses in colleges increased by a staggering 72%. Because of the increase in math remediation in colleges, new math standards in K-12 were introduced, and by 1997 most states in America adopted standards similar to those from the NCTM (Raimi, 2004). The Curriculum Standards Movement started in the 1990s (Li & Schoenfeld, 2019) and is still in place today.

Student math achievement on national mathematics assessments in the United States remained low, despite years of work to improve mathematical learning and new standards (Loveless, 2020). Scores on the National Assessment of Educational Progress (NAEP) in 2019 showed that 41% of fourth-grade and 34% of eighth-grade students scored at or above the proficient level in mathematics (NAEP, 2019). In addition, 40% of fourth-grade students scored at the basic level, and 19% earned scores below basic. Poor math achievement scores were also evidenced on international assessments. In 2018, American eighth-grade students ranked 25th out of 79 nations participating in The Programme for International Student Assessment (PISA) in math (OECD, 2019). American fourth-grade students ranked 15th out of 64 participating educational systems, and eighth-grade students ranked 11th out of 46 on the Trends in International Mathematics and Science Study (TIMSS) assessment in 2019 (U.S. Department of Education. Institute of Education Sciences, National Center for Education Statistics, 2020-2021). The TIMMS assessment measures the trends in math and science achievement for fourth and eighth-grade students worldwide every four years. Based on these results, there is room to improve teaching and student achievement in the U.S.

In the past, there have been two main pedagogical perspectives used when teaching mathematics: a constructivist approach, where teachers challenge students through critical thinking and student-centered activities (Bature & Atweh, 2020), and another that focuses on procedures. A constructivist approach uses representations and requires applying knowledge learned to problem-solve, increasing conceptual understanding (Cunningham, 2004). Conceptual understanding is a student's ability to reason, generate examples of concepts, connect models, pictures, concrete objects, and representations, and apply the principles used to represent concepts (NAEP, 2003). Conceptual understanding often contrasts a procedural focus used in math instruction that stresses rules to be followed and memorized through a standards-based progression (Reys, 2001). Procedural knowledge focuses on sequencing actions and knowing a series of steps to follow when solving math problems (Schneider et al., 2011). Procedural knowledge also includes understanding basic number concepts and calculations (Bachman et al., 2015). Procedural fluency is the ability to use procedural knowledge and apply the most appropriate strategy for a given problem (NCTM, 2014a).

Over the years, schools in the U.S. tried to increase student achievement with programs, movements, and by using various concepts and strategies. In the past five decades, there has been an increased emphasis on mathematical processes and practices such as problem-solving; however, the major focus in math remains on the *content* of what students are to learn (Li & Schoenfeld, 2019). A change in standards was recently tried again. Rigorous national standards were created to improve mathematical achievement through the Common Core State Standards Initiative (NAEP, 2019; CCSSI, 2010). Despite the recent change in standards again, students still scored poorly on national and international tests (Loveless, 2020, OECD, 2019).

Social Context

Students who struggle with mathematics become adults who struggle with mathematical concepts. Researchers used data from the Clinic for Learning Disorders in Finland and tracked children with math and reading disabilities into adulthood. They argued that adults with a developmental math disability may have higher unemployment and increased use of antidepressants than those without math disabilities (Aro et al., 2019). Low numeracy may impact adults financially, physiologically, and socially (Bruine de Bruin & Slovic, 2021). Additionally, Parsons and Bynner (2006) found that adults’ employment and income were negatively impacted for those with low numeracy skills, even if literacy skills were satisfactory. In a quantitative correlational study, Nguyen et al. (2022) found that a family history of self-reported math difficulties also negatively impacted children’s mathematic performance in elementary in first through third grades. These examples highlight how difficulty with math impacts society in various ways and may continue into adulthood.

To address difficulties with mathematics, the majority of states in America adopted the Common Core State Standards Initiative (CCSSI) in 2010, and teachers were required to teach according to the new standards. The National Governors Association Center for Best Practices and the Council of Chief State School Officers (CCSSO) were key in the early draft of the state-led CCSSI and were developed to establish commonality in learning expectations for students in Grades K–12 (Porter et al., 2011). These standards increased the rigor required in mathematics and moved away from traditional memorization and procedural drill and instead increased focus on conceptual understanding, procedural knowledge, and application of math skills (CCSSI, 2010; Mancl et al., 2012). The increase in coherence and increased focus in the CCSS aimed to help students understand concepts more deeply to solve real-world problems (CCSSI, 2010). Teachers needed to change instructional practices, move away from rote memorization, and instead emphasize a heavier focus on teaching mathematical conceptual understanding and reasoning, a rapid change for both students and teachers.

As these new standards were taught, concerns mounted that teachers might need more preparation to teach them (Griffin & Ward, 2014). Swars and Chestnutt (2016) found that teachers felt they had to instruct students who were not prepared for the increased mathematical rigor and had difficulty teaching more rigorous material to English Learners. The shockwaves from higher mathematical demands reverberated to students, current educators, and even teachers in training. Poor national and international student assessment scores only added to this concern. The new standards have yet to raise test scores even years after adoption (Liu & Jacobson, 2022; Loveless, 2020).

Teachers are the key to student learning and achievement, and student learning depends on individual teacher knowledge and proficiency (Schmidt & Houang, 2012). Teachers' knowledge influences instructional practices (Yang et al., 2020). Students' mathematical understanding, problem-solving skills, confidence, and attitudes about math result from their school learning (NCTM, 2014b). *How* teachers teach math may impact student achievement

(Kaskens et al., 2020). Therefore, it is imperative that teachers understand the meaning of math standards, how to implement them, and have the support and materials for teaching them (Schweig et al., 2020). It is important to remember that even though standards change, standards do not teach students; teachers teach students (NCTM, 2020). States adopted new rigorous standards and expected them to be taught and assessed across the country. Still, states need to provide resources and professional development for how best to teach the new standards. In The United States of America, it is up to state and local school districts to provide for those needs; there is no “one way” to teach mathematics. Such variables in teaching may be problematic because, according to Räsänen et al. (2019), how to identify students who struggle with math, how best to support them, and how to provide appropriate interventions are challenges for many teachers. Requirements for educators to teach to higher proficiency standards have not transformed student achievement (Loveless, 2020).

In addition to the nationwide struggle of elementary math students, some elementary teachers are anxious about teaching mathematics (Schaeffer et al., 2021). Like students, teachers can have math anxiety and spend less time on mathematics instruction (Trice & Ogden, 1986). Even the indirect messages teachers convey as they teach mathematics can impact student learning (Ramirez et al., 2018). Several researchers recently developed a Math Anxiety Scale for teachers to continue to study this topic (Ganley et al., 2019). Ganley et al. posited a correlation between teachers with heightened anxiety and the following: lower mathematical content knowledge, more procedural and traditional mathematical instruction, being a primary teacher, and the absence of math certifications. Math anxiety is due to a variety of reasons, but one of the biggest is the lack of a strong math background, which is greatest at the elementary teaching level. Less than 5% of elementary teachers have a math degree, and less than 2% have a math teaching certification (Malzahn, 2002). Anxiety has not only been found in math teachers; many pre-service teachers have anxieties about teaching math to others (Gresham, 2018).

Theoretical Context

Several theories for teaching and learning math have been used in the past. In early American schooling, Kilpatrick was a follower of Dewey and was a progressive who believed that mathematics should be strictly practical in nature, taught when needed, and based on student interests (Kilpatrick, 1925). In 1960, Bruner theorized that children learned best when they could construct their knowledge. Continued research of children’s cognitive stages of development showed that they moved through three modes of thinking: enactive action-based, iconic image-based, and symbolic language-based learning (Bruner, 1966). Teachers who use the constructivist theory shift instruction from a rigid, structured, teacher-controlled sequence and pace to a more student-centered and participative teaching and learning style (Bature & Atweh, 2020). With all of the expansion of elementary schools in the 1960s and the increased focus on the “three R’s” (reading, writing, and arithmetic), mathematics finally became a curricular focus; however, primary teachers were not well prepared to teach mathematics (Schubring & Karp, 2020).

Following Bruner’s constructivist theories, in the1980s, Vygotsky’s sociocultural approach and social constructivism, particularly the Zone of Proximal Development (Vygotsky, 1978), became popular. As a result, teachers began to encourage cooperative learning. Both Bruner and Vygotsky believed that scaffolding helped children achieve higher learning goals (Vygotsky, 1978). Ball and Bass (2000) looked at constructivism in mathematics separately, not socially or individually, but with a mathematical lens that focused on the specifics of mathematics and what mathematicians do. They posited that the focus should be on students’ reasoning to support claims, which is also very important for teachers. Although many theories were posited for teaching mathematics, standards-based learning, and heavy accountability became the norm in America in the 1990s.

Newton (2018) posited that until math teachers are explicitly taught the math they teach, students will continue to struggle. The problems students have in mathematics and the decades of work to try and improve students' mathematical learning need to be addressed. This study explored Bruner's constructivist theory of enactive, iconic, and symbolic learning by examining teachers' experiences using constructivist strategies such as the CRA framework. The results of this qualitative study added to the body of knowledge by examining an area that has not been specifically studied before.

Problem Statement

The problem is that despite the various strategies teachers used to teach mathematics over the years, elementary students struggle with mathematics in America. In 2019, only 41% of fourth-grade students in America scored at or above proficient on the National Assessment of Educational Progress (NAEP, 2019). American preschool children from low-income homes enter school with low knowledge of mathematics; this is a strong predictor of their later math achievement in fifth grade (Rittle-Johnson et al., 2017). Students who are identified with math disabilities and difficulties and who have low achievement showed measured growth in math but did not catch up to their peers in math content (Nelson & Powell, 2018). Additionally, students who struggled with math found generalizing math skills arduous (Nelson et al., 2020).

Many math teachers teach by using procedures (Holm & Kajander, 2020). Students often learn mathematics procedurally without conceptual understanding (Agrawal & Morin, 2016; Braithwaite et al., 2018; Dubé & Robinson, 2018). Learning procedurally without deep understanding is especially true for students with learning difficulties and/or disabilities in math (Witzel et al., 2008). Students with math disabilities need individualized and specialized support (Zhang et al., 2020). Bouck et al. (2018) suggested using manipulatives assists students with disabilities. However, Schweig et al. (2020) found that 48.95% of teachers spent little to no time using hands-on manipulatives to explore mathematical problems and concepts during 10 days of logging instructional practices. The adoption of the CCSS in 2010 increased the rigor of elementary mathematics; however, heavy reliance on procedural knowledge is different from what current standards demand. Students are now required to understand problems conceptually and procedurally and then apply that knowledge to new problems, generalizing what they know (CCSSI, 2010; Mancl et al., 2012). Students are also required to solve problems using multiple strategies (Flores & Milton, 2020). Despite new standards, student test scores have remained the same (Loveless, 2020).

While teachers believed the new standards have value, many had limited content knowledge to teach those standards and need more resources to do so (Kaufman et al., 2018; Schweig et al., 2020; Swars & Chestnutt, 2016). Having limited content knowledge needed to teach the CCSS is problematic for teachers and students. Additionally, prospective elementary math teachers are required to assess students’ procedural fluency and conceptual understanding (Association of Mathematics Teacher Educators (AMTE), 2020). Instructional practices to help build mathematical, conceptual understanding include adding problem-based instruction and increased dialog and discussion in the classroom (Myers et al., 2020).

CRA is an evidence-based practice that helps students understand math skills and why those skills work by highlighting the connections between concrete manipulatives, representational pictures, and abstract numbers (Yakubova et al., 2020). CRA can benefit general and special education students when taught by those who understand this framework (Mahayukti et al., 2019). Kim (2020) researched the importance of concreteness fading, used in CRA, and argued that teachers should be shown how to change their instructional practices to teach the connections between each stage to increase students' conceptual understanding.

There was little research on teachers' lived experiences using constructivist strategies to teach math despite the movements, strategies, and standards used for math instruction. The voices of educators who teach mathematics with constructivist strategies, such as the CRA framework, were missing from research. There was a gap in the literature. This study examined elementary teachers' lived experiences using constructivist strategies, specifically the CRA framework, for math instruction. This study provided a better understanding of this specific population's experiences regarding mathematics. Listening to the voices and experiences of elementary math teachers who use constructivist strategies added valuable information to qualitative research in math education, future professional development, teaching strategies, and pre-service training for educators.

Purpose Statement

The purpose of this transcendental phenomenological study was to describe elementary teachers' lived experiences of implementing constructivist strategies, specifically the concrete, representational, and abstract approach (CRA) when teaching mathematics in a midwestern school district. Using a constructivist strategy of the CRA framework is generally defined as a three-stage evidence-based approach for students learning computation in addition, subtraction, multiplication, and division (Bouck et al., 2018). CRA is an explicit method for teaching math that utilizes several representations of concepts (Flores & Hinton, 2022). Bruner's constructivist learning theory embraced enactive-iconic-symbolic learning (Bruner, 1966), which was the basis of CRA and the theory this study was grounded upon.

Significance of the Study

American teachers have been teaching mathematics with disappointing results, even with increased rigor and accountability since the implementation of CCSS (Loveless, 2020; NAEP, 2019; OECD, 2019). In Singapore, however, mathematics teaching and student learning have been successful, as evidenced by high international math scores (OECD, 2019). Teachers in Singapore use constructivist strategies when teaching and learning mathematics (Kaur, 2019). In research, there was a lack of teachers' voices using constructivist strategies such as the CRA method. By focusing on teachers' lived experiences when using constructivist strategies to teach math, this phenomenological study illuminated real-world, insightful experiences when teaching CRA strategies. This study made several contributions (empirically, theoretically, and practically). While many studies examined CRA strategies and their impact on teacher instruction and student learning, researching instructors' experiences added a new layer of understanding to constructivist learning.

Theoretical

This study described elementary teachers' lived experiences implementing constructivist strategies, specifically the CRA approach when teaching mathematics in a midwestern school district. I focused on the experiences of teachers and highlighted actions in teaching that trigger the enactive, iconic, and symbolic stages of learning based on Bruner's seminal work and learning theory (1966). I studied the experiences of teachers who use manipulatives, representations, and numbers/symbols in abstract learning, all of which parallel Bruner's work. Through my research, I extended Bruner's theory by including the experiences of teachers who use the stages in his theory to teach mathematics. I documented and analyzed the lived experiences of teaching with this method. While research was conducted on the effectiveness of the CRA framework for students who struggle with mathematics (Agrawal & Morin, 2016; Bouck et al., 2018; Dubé & Robinson, 2018), I found no research that examined teachers' lived experiences teaching in this manner. Through this study, I confirmed and possibly extended the research that builds upon Bruner's learning theory (1966) by illuminating lived experiences of teachers who use CRA constructivist strategies. It also confirmed the importance of visual modeling with representations, which is how teachers instruct students in top-ranking Singapore (Urban et al., 2021). Results provided insights for assisting teachers, students, and researchers to grasp what does or does not help build mathematical understanding. Documenting, researching, and analyzing teachers' experiences extended the theory by including teachers' voices, which were not included in Bruner's learning theory.

Empirical

Through the results of my current study, I filled a gap in the research literature by providing educators and researchers an augmented and increased understanding of participants' lived experiences regarding math instruction when using CRA constructivist strategies. While students continue to have difficulty with math, only a few studies included students' perceptions of CRA strategies (Milton et al., 2019; Purwadi et al., 2019; Zhang et al., 2022), and a few examined teachers' perceptions of the CRA methods (Purwadi et al., 2019; Zhang et al., 2022). Findings from my study confirmed the work of Bouck et al. (2022) and supported the importance of fading teacher support using CRA with explicit instruction, modeling, and moving to independent student work.

Qualitative studies include participants' experiences in real-world settings (Cristancho et al., 2018). According to Kaur (2019), master teachers in Singapore were found to believe that the model method, which is a heuristic that utilizes pictorial representations to help solve mathematical problems, assisted students in problem-solving. Kaur's research was one study that examined teachers' opinions about CPA. The results of my study supported this research. My study added much needed qualitative research to the body of work on the CRA approach when teaching math and increased understanding of its use. My research extended the impact of constructivist CRA strategies in classrooms by examining teachers' lived experiences.

Practical

CRA is an evidence-based practice (Bouck et al., 2018). Results of my study illustrated teachers experiences in the classroom using with CRA strategies and revealed what teachers find meaningful when teaching math with this framework to a variety of learners and learning styles. The lived experiences of teachers who teach with CRA strategies impacted instructional practices and possibly student achievement. It provided insights into math teachers' planning and action steps and *how* and *why* they make certain teaching moves to help make math understandable to students. Through this research, I highlighted teachers' experiences teaching all students, including those struggling with mathematics and those identified as gifted in math. This study could encourage increased collaboration among teachers regarding instructional strategies and perhaps even lessen teacher anxiety about teaching math by encouraging increased professional learning in math. Professional learning in math reduces teacher anxiety (Artemenko et al., 2021). Saran and Gujarati (2013) examined positive changes in pre-teachers' negative thoughts about mathematics after participating in a class that taught mathematics using a concrete, pictorial, and abstract (CPA) method. Kutaka et al. (2018) suggested a need for more professional learning in mathematics for elementary teachers. Newton (2018) agreed; much more professional development is needed to address the needs of elementary math students and their teachers. Teachers need resources and professional learning to address the Standards of Mathematical Practices (Schweig et al., 2020). My research could lead to program reform and refinement and may provide ideas for professional learning that include CRA methods. Researching teachers' lived experiences teaching math highlighted this need. This novel study illuminated the lived experiences of elementary math teachers who use CRA strategies not found in research.

Research Questions

I used a main research question and several other sub-questions to examine the experiences of math teachers as they work with students implementing constructivist strategies. Qualitative research investigates participants' experiences in real-world settings (Cristancho et al., 2018). The open-ended research questions in this study allowed participants to describe their experiences in the classroom. Research questions that guided this study are below:

Central Research Question

How do elementary math teachers describe their lived experiences of implementing constructivist strategies in the classroom?

Sub Question One

How do elementary math teachers describe their experiences of using concrete manipulatives in the classroom?

Sub Question Two

How do elementary math teachers describe their experiences of using representations or models in the classroom?

Sub Question Three

How do elementary math teachers describe their experiences of using abstract mathematical concepts in the classroom?

Definitions

1. *Abstract Concepts* - Described as only using symbols and numbers in mathematics (Miller & Kaffar, 2011). This can be as simple as a math fact stated with numbers and symbols, such as 15+17=32.
2. *Concrete Representational Abstract* - A strategy, model, or framework consisting of three stages of instruction used to teach mathematics that begins with a concrete stage where hands-on manipulatives/objects are used while learning a novel concept (Bouck & Flanagan, 2010). It progresses to a representational stage where pictures can be used to represent the concrete objects in an immediate phase between concrete and the abstract (Peltier & Vannest, 2018). Finally, the last stage moves to the abstract level, where symbols and numbers are only used (Miller & Kaffar, 2011).
3. *Manipulatives* - Generally defined as objects that can be moved or manipulated to support instruction and learning during mathematics (Moyer, 2001). Manipulatives are tangible objects used for math instruction that can be seen, physically touched, or heard (Hurrell, 2018). The use of manipulatives assists students in literally seeing what they may not be able to visualize abstractly. Manipulatives are often used before students move to the representational stage, where students draw pictures to solve problems.
4. *Math Anxiety* - "Feelings of tension and anxiety that interfere with manipulating numbers and solving mathematical problems in a wide variety of ordinary life and academic situations" (Richardson & Suinn, 1972, p.551).

5. *Representations* - Include drawings, pictures, models, and symbols representing numbers, quantities, and concepts. They bridge understanding from the concrete manipulatives to the abstract (Peltier & Vannest, 2018).

Summary

Despite the use of various strategies and standards to teach mathematics, many elementary students still need help learning mathematics. The purpose of this transcendental phenomenological study was to describe elementary teachers' lived experiences of implementing constructivist strategies when teaching mathematics. Students should be able to solve math problems procedurally while conceptually understanding the math they perform and applying their new knowledge when solving real-world problems. Teachers should conceptually understand math for effective teaching and student mastery of current rigorous standards. The constructivist CRA framework was based on Bruner's learning theory (1966) and aligns with increased mathematical understanding for both students and teachers. The CRA framework has been shown to benefit students who struggle in math and supports teachers with limited math understanding, including those with math anxiety.

It was important that I investigated and further explored teachers' experiences using CRA methods to add to the research literature, as there is scant research on teachers' opinions and experiences of using CRA strategies when teaching. By listening to teachers' experiences, it was possible to tease out what they believed was important for math instruction to increase student learning and achievement. Focusing on teachers' lived experiences when instructing with constructivist CRA strategies highlighted areas in teaching that others could benefit from and revealed teaching practices that are not valuable. Through this study I gained a better understanding of teaching with constructivist strategies and highlighted other research areas that need further investigation.

CHAPTER TWO: LITERATURE REVIEW

Overview

This literature review contains the theoretical framework and a review of the literature on the constructivist theory the concrete, representational, and abstract (CRA) approach is based upon. The literature review covers CRA components, related literature on students and teachers who have experience with CRA strategies, and research on CRA constructivist strategies used in Tier I general classroom settings. It also illuminates how the CRA framework has been shown to help with Tier II interventions for students who struggle with mathematics and Tier III interventions for those with learning disabilities. The importance of teachers' beliefs, attitudes, and anxieties towards mathematics are also reviewed, as this information is pertinent when examining teachers' lived experiences while teaching mathematics to children. Finally, an examination of the literature also reveals Singapore's approach to mathematics and their teachers' experiences using constructivist strategies, as the entire country is trained to use the CRA framework with excellent success. The CRA/CPA approach has been used with success in Singapore and has also been used successfully for intervention instruction in America and around the world, which leads to the pondering of why America's students still struggle in mathematics and the research in this study. What are elementary math teachers' lived experiences of implementing constructivist strategies?

Theoretical Framework

In this section I described the theoretical framework that guided this phenomenological research study. Bruner's learning theory (1966) set the theoretical foundation upon which this study was grounded by describing elementary teachers' lived experiences teaching mathematics with constructivist strategies such as the CRA framework. Bruner's learning theory provided a similar model to learning as the CRA framework for teaching and learning mathematics. Bruner’s learning theory stemmed from constructivism, whereby learners gain new understandings through their experiences.

Learning Theory

Bruner was a psychologist who believed that children are active learners and construct their knowledge (Bruner, 1960). Bruner advocated children construct new knowledge through experiences by progressing through three modes, or representations, of learning, described as enactive representations (hands-on), iconic representations (images or pictures), and symbolic representations. In the enactive stage, learners use physical actions to learn by doing with physical movements and objects; this is then stored in the memory. In the iconic stage, learners begin to use images or pictures of their concrete experiences and knowledge. Imagery in the mind is then constructed once students have experiences with actual objects; students can then mentally visualize the concepts. After moving through the enactive and iconic stages, learners proceed to the last symbolic stage, where symbols replace their pictorial representations (Bruner, 1966). According to Bruner, as students learn, they continue to add to their schemata. He theorized that students *and* adults learn through the progression of enactive, iconic, and symbolic representations.

Bruner was clear not to rush the learner through the various stages of learning. He also believed that scaffolding or assisting children when learning new skills was helpful to make new learning accessible through simplification, motivation, and giving models to follow. This scaffolding offered support to children as they learn new skills. Bruner theorized that moving too fast or assuming students can visualize a concept without having had the experiences would not be helpful to the learner. Bruner (1966) argued that once a symbolic system in an individual is firmly in place, skipping the concrete and representational stages is feasible. However, it could be risky because the student may not have the necessary visual imagery available should there be difficulty with strictly symbolic problem-solving.

Bruner (1966) asserted that the ultimate goal is for the learner to get to the symbolic or abstract phase and not become or remain complacent in the enactive or iconic stages. He recognized that students must be savvy with their mathematical knowledge when in the symbolic stage, as required in higher math and other life experiences. Experiences with concrete objects and representations helped students with the imagery and visualization needed when faced with difficulty in the symbolic stage of a problem.

Application of Bruner's Learning Theory

Bruner's learning theory set the foundation for the CRA process and hands-on constructivist math practices many teachers use today in America and other countries. Singapore uses the same strategy as a nationally recognized educational method for mathematics instruction; it is a concrete, pictorial, and abstract (CPA) sequence (Kim, 2020). CRA follows the enactive, iconic, and symbolic stages Bruner's learning theory was built upon by using manipulatives, representations, and symbols while working through a math concept (Yakubova et al., 2020). CRA is an evidence-based practice and was shown to be successful with general education and special education students when taught by instructors who understand the process (Agrawal & Morin, 2016; Dubé & Robinson, 2018; Mahayukti et al., 2019). Because of the success of Bruner's constructivist strategies used in CRA, Bruner's learning theory was used when researching math teachers' experiences using CRA constructivist strategies instructing elementary students. Bruner's learning theory guided this study.

CRA/CPA

CRA parallels Bruner's (1966) theory about how learners use representations to comprehend new information. CRA has been referred to as a strategy, a method, a sequence, and a framework. Early research by Witzel (2005) defined CRA as a method of teaching mathematics that consists of first using concrete manipulatives, then representations or pictures, and in the last stage, using abstract numbers and symbols to solve problems. CRA is an instructional framework that allows teachers to instruct sequentially through three phases: manipulation of concrete objects, pictorial representations, and abstract numbers (Yakubova et al., 2020). Several researchers found the CRA framework was an evidenced-based approach for teaching students who need extra mathematical support and have conducted considerable research on this strategy and framework (Agrawal & Morin, 2016; Bouck et al., 2018). While this approach is most often referred to as CRA, some countries, such as Singapore, refer to it as the concrete, pictorial, and abstract approach (CPA). CRA and CPA can be used interchangeably, but for this study, the acronym CRA was utilized.

Concrete Stage

At the concrete stage in the CRA framework, students create internal meaning from the manipulation of concrete objects (Miller & Hudson, 2006). Concrete objects are often referred to as manipulatives and are used to support students in mathematical procedural and conceptual understanding (Peltier et al., 2020). Math manipulatives can be mass-produced materials designed for math or simple teacher-created materials (Bouck et al., 2018). Additionally, Hurrell (2018) defined concrete manipulatives as objects that can be seen, touched, and/or heard, such as structured materials created for mathematics, like Base 10 blocks, etc., or unstructured materials not specifically designed for mathematics, like buttons used as counters. Hurrell (2018) expanded the understanding of concrete objects further and described manipulatives as tangible, external representations of mathematical concepts that assist in creating an internal model.

Representational Stage

Early research by Witzel et al. (2001) defined the representational stage, sometimes called the pictorial stage, as a stage where drawings or pictures represent three-dimensional manipulatives. The representational stage effectively bridges the concrete to abstract mathematical concepts (Agrawal & Morin, 2016; Hurrell, 2018). Students who solve problems in the representational stage typically draw models of concrete representations. Flores and Milton (2020) suggested that actively creating pictures from manipulatives provides students with greater understanding and comprehension. Students then internalize those representations, and their meanings as the reliance on concrete objects fade, and they draw their own two-dimensional pictures (Gibbs et al., 2018).

Abstract Stage

In the final abstract stage, students use all that was learned from the concrete and representational stages and perform operations with numbers only (Milton et al., 2019). An example of the abstract is an equation such as 23+8=31, with no need for manipulatives or visual representations. By this final stage in the sequence, students no longer require the use of any materials or pictures because they used their constructed mental images to solve problems in the abstract form (Gibbs et al., 2018). While solving in the abstract stage is the goal, moving from the representational to the abstract is the most difficult aspect for students (Strickland & Maccini, 2013). This stage is difficult for students because they must generalize what was learned and use their conceptual understanding from previous stages to solidify conceptual understanding and fluently solve math problems (Hurrell, 2018).

Bruner’s learning theory utilized constructivist strategies through the progression of enactive representations (hands-on), iconic representations (images or pictures), and symbolic representations that guided this research study. Bruner’s learning theory also guided the research questions of this study by focusing on the stages of the theory in relation to teachers’ experiences using them when teaching math to elementary students. This novel study added to the existing research and literature by providing the qualitative experiences and voices of teachers who used these strategies when teaching math to elementary students.

Related Literature

The related literature section overviewed the constructivist strategy of the CRA framework, math achievement in America, possible reasons students struggle with mathematics, and related literature on teachers, students, and researchers who used CRA strategies in general education and intervention settings. Research pertaining to the importance of teachers’ mathematical knowledge, efficacy, and personal feelings regarding math and how the lack of knowledge, efficacy, and mathematical anxiety can negatively impact what takes place during mathematics lessons was also included. A focus on professional learning showed a positive impact on mathematics teaching experiences. Constructivist teaching and learning with the CRA/CPA framework is used successfully in other countries and was also covered. All of this set the foundation for understanding teachers’ experiences of using constructivist strategies to teach math in elementary classrooms.

Examining the lived experience of teachers who used the CRA framework or parts of the framework added to current research. Several studies highlighted the successful use of CRA for students learning mathematics (Bouck et al., 2018; Peltier & Vannest, 2018; Zhang et al., 2020). However, in most studies, researchers were the instructors (Flores, Hinton, & Meyer, 2020;

Kanellopoulou, 2020). There was little research that examined the experiences of teachers who use CRA or its components when teaching. This study provided additional support for Bruner's learning theory by examining teachers' lived experiences of implementing CRA strategies while teaching elementary mathematics. Further, it extended the existing literature by providing needed qualitative research on teachers' experiences with the CRA framework, of which there was little information.

Math Achievement in America

Despite the changes in math over the years and high per-pupil costs for instruction, America's math scores remained poor. It was estimated that 6% of students have a math disability (Powell et al., 2013), and roughly 14% had difficulty with basic math (UNESCO, 2017). According to the National Assessment of Educational Progress (NAEP, 2019) scores, only 41% of fourth-grade students and a mere 34% of eighth-grade students scored at or above proficiency in mathematics. Internationally, America ranked only 25th of 79 countries participating in the Programme for International Student Assessment (OECD, 2019).

Regardless of students' poor performance on assessments, mathematics remains a necessary skill for student and adult success; it is needed daily for everyday functioning, such as understanding elapsed time, comprehension of data, and monetary skills for financial decision making and even measurement (Soares et al., 2018). It is one of the essential subjects in both elementary and secondary schooling and prepares students for real-world problem-solving. The world continues to become more interconnected, and countries' labor forces compete for the same jobs, so knowledge of science, technology, engineering, and mathematics (STEM) plays a role in every country's competitiveness (Miao & Reynolds, 2017). Li and Schoenfeld (2019) argued that STEM education should shift instruction from what teachers should do to what math experiences will push students to become deep and powerful thinkers. Simply put, mathematics is important in America and the world.

Mathematical knowledge starts before formal schooling and can impact years of learning. Even mathematical talk at home between a mother and a child can influence a child's mathematical language before kindergarten (Gürgah Oğul & Aktaş Arnas, 2021). Basic mathematical knowledge of children entering kindergarten predicted math outcomes later in elementary school (Jordan et al., 2009). According to Khanolainen et al. (2020) and their longitudinal study that followed students from kindergarten to ninth-grade, parents' mathematical difficulties predicted lower performance in addition and subtraction fluency among their children. Starting school with lower math understanding set students up for difficulties in the subject on the first day of school, which continued in later years. Rittle-Johnson et al. (2017) found that students from lower socio-economic backgrounds entered school with lower math knowledge than those from higher socio-economical households. However, not all students who struggle with math came from a lower socio-economic status (SES). Soares et al. (2018) posited, some students are low performers due to a variety of variables, such as ineffective instruction and environmental reasons. Students who struggle with mathematics are often described as having *math difficulties*.

Unlike students who generally struggle with mathematics are those who have a math disability. A disability in mathematics is defined as having difficulties in math that have lasted over half a year, skillsets that are considerably below that of their same-aged peers, and shortfalls that interfere with functioning, all of which are confirmed by standardized and clinical assessments (American Psychiatric Association, 2013). Mathematics disability, also called dyscalculia, involves dysfunction of parts of the brain used for math and is considered a neurodevelopmental disorder (Soares et al., 2018). Students with dyscalculia demonstrate difficulties with basic numerical processing when young, and this struggle with mathematics typically continues through schooling (Wilkey et al., 2020).

Mathematics has been a challenge for many students and for many years despite multiple efforts from America's educational system. In 2010, the United States created new standards designed to improve student learning and math achievement and required students to use multiple strategies to solve problems (Common Core State Standards Initiative [CCSSI], 2010). While rigorous, adding the CCSS did not come with teacher training or preparation. Local school districts scrambled to understand the new standards and had little to no time to prepare teachers to instruct according to new standards properly. Despite new standards, students continue to struggle with math (Loveless, 2020). This was supported by Liu and Jacobson (2022). Only 22% of 214 fourth-grade students could accurately order three fractions with proper justifications, even after years of learning with CCSS. Visual strategies were used more than symbolic or verbal strategies. However, even though the number line strategy was required in the third-grade CCSS, only 5% of the 214 students used it to justify their answers. Only three of the 41 students who chose to procedurally cross-multiply got the correct answer, which caused Liu and Jacobson to question using an abstract shortcut to teach fraction comparison.

Levels of Support Needed Through MTSS

To meet the needs of students who struggled academically and/or behaviorally, including those who have difficulty with mathematics, a multi-tiered system of support (MTSS) was put in place in many schools across the country. In some districts, it took the place of Response to Intervention (RtI). Schools often used RtI models to identify academic and behavioral deficits early so that interventions can be provided based on the intensity of the need (Fletcher &

Vaughn, 2009). According to Nelson et al. (2020), MTSS was designed to provide increased levels of teacher interventions depending on student data and specific needs. The goal was to catch students up. Much progress has been made toward developing systematic methods to support students in America by adopting a multi-tiered system of support MTSS models (Berkeley et al., 2020). Students who struggle in mathematics often receive intervention support in the context of small-group instruction in addition to their core instruction (Nelson et al., 2020). Students are typically progress-monitored to measure student improvement and intervention effectiveness.

Tier I was created for all students and offers quality, evidence-based instruction aligned to standards and taught by a general education teacher in their classrooms (Flores, Hinton, & Meyer, 2020). Should students need help finding adequate success in particular academic or behavioral areas, they receive Tier II services. Tier II services require supplemental, differentiated evidenced-based intervention support by an educator (Flores, Hinton, & Meyer, 2020). Depending on the school, sometimes, the teacher is a specialist with additional training in the subject, and remediation differentiates the instruction based on student needs. Tier III is the most intensive intervention designed for students who significantly and persistently struggle in a subject or behavioral area (National Center on Intensive Intervention, 2018). Students serviced in Tier III may have disabilities that fall under a range of labels and often require support from special education services and an Individualized Educational Plan (IEP).

Researchers are interested in finding evidence-based teaching methods and interventions to assist students in becoming more math competent (Codding et al., 2020). According to Codding et al., using evidence-based intervention strategies such as modeling, visual representations, corrective feedback, guided practice, explicit practice, and flashcard drills improved second-grade whole number knowledge. When motivational strategies were included, increased benefits were evidenced. Early math interventions provided in preschool, kindergarten, and first grades showed promise for students at risk and those with disabilities (Nelson & McMaster, 2019). Students who received interventions in early numeracy content showed more growth than peers who did not receive comparable interventions. The CRA framework is one method that has been shown to benefit student learning, especially for those who struggle with mathematics, and has been successfully used in many interventions for school-aged students (Bouck et al., 2018; Flores et al., 2018).

CRA as an Evidenced Based Practice for Tiers I, II, and III

The CRA framework is a method of teaching mathematics that has received a considerable amount of empirical research and has shown positive benefits for students with or without math disabilities (Agrawal & Morin, 2016; Dubé & Robinson, 2018; Mahayukti et al., 2019); in Tiers I, II, and III. Peltier and Vannest (2018) argued that, unlike published curriculums or programs, the CRA framework could be utilized for all math concepts and age groups and help students with various disabilities. Bouck et al. (2018) found CRA to be an effective, evidence-based method for teaching students with disabilities in mathematics, specifically with computation problems.

Teachers often use CRA strategies to help children construct mathematical understanding because CRA stresses conceptual understanding; it bridges the gap between more difficult abstract concepts by first using manipulatives and drawn models and fading these visual supports until students can solve the problems with only numbers and symbols (Flores et al., 2014). Learning mathematical concepts through representations bolsters conceptual understanding and mathematical thinking (Flores & Hinton, 2022). According to Yakubova et al. (2020), students first engage with actual objects and physically perform the math, then draw representations of the concrete concept from the first stage, and move to the last sequence that uses abstract numbers - all while making connections through each stage. The ultimate goal of the CRA sequential progression is to enable students to understand the skill they are learning and why it works (Yakubova et al.). The CRA framework is not a program; it is a sequence that moves from one phase to the next and can be used with any math text, curriculum, or skill (Peltier & Vannest, 2018).

Exact parameters for when to move to the next stage in the sequence or framework are lacking in research; however, Bouck et al. (2018) posited that mastery is demonstrated in each phase when students can show independence in each stage with 80% success. Hurrell (2018) did not consider a percentage of success for demonstrating mastery; rather, he suggested that educators look for signs of conceptual understanding through student manipulatives and representations. Regardless of what is defined as mastery, students should feel secure enough to move to the next level of support with understanding.

While CRA is considered a sequence to many, Peltier and Vannest (2018) argued that CRA is not a sequence but a framework. They suggested it was important for teachers not to teach each stage in isolation until competence for fear students would not generalize their new knowledge from concrete and representational stages to the abstract. Peltier and Vannest also stressed the importance of teachers' introducing the concrete *along with* the abstract for students to see the connections in both stages. In agreement, Yakubova et al. (2020) highlighted the importance of showing connections between all three phases. Highlighting connections among all three stages helped to illustrate the integration of the stages and solidified understanding.

CRA strategies are beneficial for students who find specific concepts challenging, as several stages are involved in helping with mathematical understanding. According to Jones and Tiller (2017), it is vital that all students be able to access the curriculum and enter math problems in a variety of ways. For educators, the flexibility of using CRA strategies provide multiple entry points to math problems; this is important as no two students learn in the same manner. Simply moving through the concrete, representational, and abstract stages or moving back to a stage when further understanding and clarification are required offers natural differentiation for students when it is needed to support learning. As an evidence-based practice, CRA is appropriate for all students in Tier I general education settings for instruction, as well as in Tier II and Tier III interventions (Dubé & Robinson, 2018).

Teaching With CRA

Manipulatives are important to be used as a single strategy, but they are also beneficial when used in the CRA framework (Bouck & Park, 2018; Bouck et al., 2018). Manipulatives are not only helpful for general education students learning mathematics but also for those who find math difficult. Manipulatives provide students access to math problems. Early research from Witzel et al. (2001) described that hands-on involvement with math concepts evened the playing field and accessibility for students. From their research, Bouck et al. (2018) recommended that instructors intentionally add manipulatives to their instructional practices for students who struggle in mathematics. Peltier et al. (2020) found that manipulatives effectively increased the mathematical achievement of both at-risk students and those with a learning disability.

Much consideration must go into using manipulatives. Hurrell (2018) posited that concrete objects should be chosen with care and used to trigger student thinking; manipulatives should assist students in displaying their thinking. Intentionality is also key, and incorporating concrete objects can be complex. Using manipulatives is more complicated than it might appear, and teachers must understand why and how to use them. Sometimes some teachers use manipulatives in ways that do not explicitly show the mathematical concepts trying to be conveyed; therefore, strong pedagogy for using manipulatives is required (Newton, 2018). Flores and Hinton (2022) suggested to reference manipulatives as math tools when teaching, as this may encourage their proper use and importance in the classroom. The purpose of using manipulatives goes beyond representing math concepts; it should help connect students' math understanding across all skills moving them forward in understanding and should be done with careful teacher support (Coles & Sinclair, 2019).

In the second stage of CRA, representations or pictures help build the conceptual understanding needed for a richer understanding of the concept or skill (Gibbs et al., 2018) and can also be in the form of a number line (Flores & Hinton, 2022). Research conducted by Zulfakri et al. (2019) supported the use of CRA to teach geometry by increasing the use of representational skills. In their study, students improved their representational abilities and problem-solving compared to those taught conventionally without CRA strategies. The use of representations helped build conceptual understanding and is needed for today's math standards.

The benefits of using representations were evidenced in a recent study conducted in Sweden, where researchers focused on an intervention that involved collective reasoning and number sense with kindergarten students (Sterner et al., 2020). Researchers trained the teacher to use the CRA method to provide interventions to a group of kindergarten students. In this study, the teacher used concrete objects and representations in the form of pictures and drawings, along with guided reasoning opportunities to teach number sense. Participants' number sense increased, and that effect lasted nine months later and into the next school year. Using concrete objects and representations with reasoning resulted in meaningful learning for young students (Sterner et al., 2020). The use of modeling with representations to problem-solve is also taught in Singapore for problem-solving; it helps students make abstract problems more concrete (Carter et al., 2002).

In the last abstract stage, students use prior knowledge from the first two stages to solve math problems with equations, symbols, and numbers (Milton et al., 2019). Manipulatives and representations are no longer needed because students use the visualizations they constructed to solve abstract problems (Gibbs et al., 2018). Abstractly manipulating mathematical symbols was used to solve problems and show understanding of the task (Africa et al., 2020). In contrast to moving through three stages, Coles and Sinclair (2019) argued that students can simultaneously engage with the concrete and the abstract, and students need not start with the concrete. Coles and Sinclair posited that some students have difficulty moving from the concrete to the abstract. This can be a difficult stage for students because they must use their conceptual understanding from previous stages and generalize to solve problems fluently (Hurrell, 2018).

Concreteness Fading

Concreteness fading is utilized to teach abstract concepts through the presentation and movement between concrete, pictorial representations, and the abstract as levels of concreteness diminish (Suh, 2019). Concreteness fading has been utilized in learning, but Fyfe and Nathan (2018) sought to explain it further and provide more clarity in its process and procedures. They posited that concreteness fading is a three-step process that starts with a concrete representation and gradually fades into the next representation, linking the representations together on a continuum. Physical and visual support fades as students understand the concept taught. They suggested that physical manipulatives be used first, which parroted Bruner's (1966) original theory. Fyfe and Nathan (2018) hypothesized that students would remember the first concrete representation more readily and that CRA stages should be taught separately for students to see that all representations refer to the same thing and that fading through each stage is more effective for students as they can reference previous stages. In contrast, Bruner (1963) originally argued that the order of the three stages could be altered for different purposes.

Similar to the above researchers, Kim (2020) stressed that a key component of concreteness fading is using instructional *strategies* to connect the concrete, representational, and abstract. Concreteness fading assists students by providing a variety of visual representations of abstract concepts and also helps students make connections among representations as they moved to the abstract (Kim, 2020). Despite the many benefits of using concreteness fading, Suh et al. (2020) found that the most common mistake in CRA concreteness fading was the lack of teachers' explicitly explaining the connections between the different representations and argued for the need of a common design and understanding of concreteness fading. Research by Donovan and Fyfe (2022) supported the importance of connecting manipulatives in the form of Base ten blocks to numerical place value. Students in kindergarten, first, and second grades who understood the connection between numbers and Base ten blocks had higher posttest scores than those who showed no understanding and knowledge of those connections.

Explicit Instruction

Effective execution of the CRA framework should also use explicit instruction (Agrawal & Morin, 2016). When teaching with explicit instruction, teachers use interactive modeling of the concept or skill to guide students through more examples until students can work on the problem independently (Gibbs et al., 2018). Explicit instruction should also use precise, succinct, and consistent language to assist students in practicing skills and giving feedback (Gunn et al., 2021). Luevano and Collins (2020) argued, explicit instruction should provide students with clear expectations, appropriate pace, heuristics for each student, and ample feedback.

Research by Bouck et al. (2018) found that manipulatives in mathematics are typically paired with teachers' explicit instruction. Hurrell (2018) also argued that the transparency between math manipulatives and explicit teaching links the relationship between the concrete and the abstract. Peltier and Vannest (2018) suggested that concrete manipulatives and drawn representations could not teach an abstract mathematical concept; the instructor needs to model and teach through explicit instruction, helping students relate the concrete representations to the abstract. The use of manipulatives alone cannot teach a concept (Hurrell, 2018). Without instruction, objects and pictures have no real meaning to a student facing a novel skill. Nor can relying on textbooks to teach concepts either, as teachers must have a firm understanding of the connection between the topics in the texts and mathematical pedagogies to assist student understanding (Newton, 2018). Knowledgeable teachers teach with explicit instruction and for understanding.

Suh et al. (2020) examined the framework and design of concreteness fading and found that explicit instruction helped connect the concrete with representations, as noted previously; however, they also found some implementations that showed no significant student learning. In the experiments that showed no growth, instructors did not use explicit instruction with concreteness fading. Studies, where the researchers did not explicitly explain the connections between representations, showed no significant learning improvement in students (Arawjo et al., 2017; Suh et al., 2020).

CRA and Tier II Interventions

Tier II intervention math services are offered in many schools across the country for students who find mathematics a challenge; these students need more assistance beyond what their general education teacher can offer. According to Fuchs et al. (2012), Tier II interventions take place in small groups and use specific targeted teaching strategies. Typically, Tier II intervention students are not identified with a math disability. Intervention services provide students with all abilities to have equal access to a problem, and the use of manipulatives helps even the playing field (Jones & Tiller, 2017). Interventions for Tier II math students should focus on conceptual understanding (Flores, Hinton & Meyer, 2020). While there are studies that show promise for CRA effectiveness for Tier II interventions, there was a significant lack of teachers who provide the interventions (Flores, Hinton, & Meyer, 2020; Flores et al., 2018).

Fraction Interventions

The importance of understanding fractions cannot be stressed enough, as it is crucial for algebra in later years (Siegler et al., 2012); however, many students have difficulty with fractions (NAEP, 2019). Students need a conceptual understanding of fractions for middle, high school, and college (Stohlmann et al., 2020). Flores et al. (2018) argued that in order for students to understand rational numbers, representations should be included, not simply procedural knowledge, and research by Zhang et al. (2022) supported that. Students must fully grasp the conceptual understanding of rational numbers for success, and using representations is an important method to secure that understanding; otherwise, procedures will have no meaning, Zhang et al. As a result of student difficulty with rational numbers, researchers argued for the need of interventions that promote rational number understanding (Kärki et al., 2022).

Roesslein and Codding (2019) examined fraction interventions used to support kindergarten through sixth-grade students who struggled with mathematics. They examined the focus of the interventions and instructional components and how effectively the fraction interventions increased student performance. Roesslein and Codding found that conceptual and procedural focus on learning fractions was important and should be used to increase fraction understanding in interventions; however, more research was needed to show an explicit connection of both conceptual and procedural understanding. According to Peltier et al. (2020), CRA supports conceptual and procedural learning. Milton et al. (2019) argued that conceptual understanding continues while procedural knowledge and fluency increase.

There have been several studies conducted to research the effectiveness of using the CRA/CPA framework to teach fractions (Flores, Hinton, & Meyer, 2020; Flores et al., 2018; Purwadi et al., 2019), and these methods showed promise of effectiveness with students needing Tier II interventions. Flores, Hinton, and Meyer (2020) conducted a study with Tier II students who struggled with fraction and decimal concepts and were identified through the MTSS process. They found that a CRA intervention consisting of area and length models, manipulatives, modeling, teacher guidance, and practice helped students better understand area and length models than in a typical ‘business as usual’ intervention, according to pre and posttests from both groups. This multiple strategy approach supported findings from Roesslein and Codding (2019). Of note, in the study by Flores, Hinton, and Meyer (2020), researchers conducted and taught the interventions themselves, and they did not fully investigate what materials were used in the “business as usual” intervention taught by general education teachers. Flores, Hinton, and Meyer (2020) described that further research was needed, as some limitations may need to be examined further for generalizability. Despite the limitations in their study, they argued that the results demonstrated that CRA interventions for Tier II students may have potential, and further research should persist in refining the implementation to prevent further student failure.

Flores et al. (2018) investigated the effectiveness of the CRA framework with fifth-grade students learning fractions. This was a follow-up to their previous study on CRA fraction interventions. They found the CRA framework was effective for fifth-grade students who received Tier II intervention 20 minutes a day for five weeks in a small-group setting. Surprisingly, student participants never used manipulatives before this intervention; they only witnessed their teacher use them. Students had much more practice and feedback with this intervention than in the general education setting, and the explicit instruction offered through the CRA framework was effective. However, there was no comparison group involved in this study, and the researchers were the instructors once again. Flores, Hinton, and Taylor suggested that more research be conducted on math interventions that teach mastery of fractions. Nowhere in the studies conducted by Flores et al. (2018) and Flores, Hinton, and Meyer (2020) did current math teachers teach CRA interventions. Researchers functioned as instructors, which does not occur in a natural school environment. In contrast, according to Peltier et al. (2020) and their research, teachers with the proper training were as effective as researchers who are experts in implementing interventions.

Purwadi et al. (2019) conducted a mixed-method study investigating whether the CPA strategy positively affected students' conceptual understanding and their mathematical representation when working with fractions. The study examined concepts supported by the National Council of Teachers of Mathematics (NCTM, 2020), such as defining and distinguishing what fractions are, using them for a correct purpose, creating and utilizing models to assist with problem-solving, and interpreting a variety of math phenomena. Third-grade students in a village in Padangbulia, Bali, demonstrated a richer understanding of fractions and could verbalize their understanding of fractions, and fraction concepts, and could explain how to solve fraction problems. Additionally, they were also more engaged in the lesson. According to Purwadi et al., students were able to express their understanding and agreed that using CPA strategies helped them understand fractions.

Problem-Solving Interventions

Mathematical problem-solving is exceptionally important to learn as it corresponds with daily problems children may face in life and was required to solve by using strategies (Swanson et al., 2020). An integral factor when learning to problem solve is the ability to generalize mathematical skills to real, authentic experiences (Spooner et al., 2019). Problem-solving can be complex and automatic, and memorized knowledge does not always help when problem-solving. Performing math includes different ways to solve problems (Ergen, 2020). CRA strategies have also been shown to assist Tier II students when problem-solving and offer a different method than typical problem-solving strategies. In contrast to studying keywords for problem-solving, Flores et al. (2016) conducted a study with Tier II intermediate elementary students that incorporated the CRA framework, schema-based instruction, and a problem-solving plan. Students used the CRA framework as they worked through each step of the plan. Results indicated that practice with concrete manipulatives and representations helped students with cognitive processes in the abstract stage with an overall strong effect. In this study, the researcher was again the instructor. In agreement, Ergen (2020) posited that non-routine math problems can be solved with drawing strategies.

Flores et al. (2016) agreed that general education teachers should conduct instruction within an intervention setting and with teacher professional development. Teacher professional development for problem-solving is important to note. According to Kaskens et al. (2020), students faced great complexity when problem-solving, and teachers may not fully realize what is required to teach it to students, especially with more advanced mathematical problems.

CRA and Tier III Interventions

Just as Tier II students found success in mathematics by using the CRA framework, Tier III students have too. Tier III interventions are typically provided for those who significantly struggle and need the most intensive interventions in school. Students with mathematic disabilities need much more concentrated and personalized intervention supports to assist in their increase of mathematical improvement (Zhang et al., 2020). Several studies have highlighted the success of using the CRA framework with students who have mathematics disabilities (Bouck & Park, 2018; Bouck et al., 2018; Zhang et al., 2022).

Functional Mathematics

Counting numbers and understanding quantities are important skills established in young children. Research by Nelson and McMaster (2019) suggested that students who received early numeracy interventions made increased math gains compared to those who did not and this included those with and without disabilities. In a qualitative case study that supported these findings, Kanellopoulou (2020) used CRA methods to teach a kindergarten student with an intellectual disability counting skills and found that this method helped the participant learn to count, visualize, and quantify numbers one through 10. While the researcher conducted the instruction, the special education teacher who worked with and knew the student well, was also interviewed. This educator was surprised by the student’s ability to remember how to count to 10 and felt that her student showed great improvement due to the intervention. As measured on a kindergarten curriculum-based assessment, post-tests showed gains in all areas, demonstrating improved understanding.

Adding to the literature exploring the benefits of CRA strategies for students with math disabilities, Bouck et al. (2017) investigated how the CRA framework assisted middle school students with intellectual disabilities in functional mathematics. They found that by using the CRA approach as an intervention, all student participants were able to solve a real-world problem by correctly making change. Interestingly, Bouck et al. moved students through the different phases after students were able to solve 80% of the problems in each stage independently. This was the only study found that included students with intellectual disabilities that also required a percentage correct demonstrated in one stage before moving to the next CRA stage. In this study, lessons were taught by a secondary special education teacher and the researcher.

Conceptual Understanding of Basic Multiplication and Division

Milton et al. (2019) illuminated the positive effects of using CRA instruction to teach multiplication, division, and conceptual understanding to students who have math disabilities and other health impairments (OHI). In this mixed-methods study, teachers were given professional development by the researchers who modeled CRA methods. Teachers were required to demonstrate their newly learned CRA instructional strategies for multiplication, division, and the operations' inverse relationships before teaching the CRA intervention. Training teachers *how* to teach aligns with findings from Holm and Kajander (2020), who posited that math courses alone may not be enough to prove helpful for teaching math; courses that specifically focus on teaching may be more helpful.

In the study conducted by Milton et al. (2019), students were taught basic multiplication and division and asked how they solved the problems. After CRA instruction, there was evidence that students understood the functional relationship between multiplication and division facts, and students could also explain it with words and pictures, demonstrating conceptual understanding. This was one of only a few studies found where researchers used a qualitative component to examine students' experiences and understandings of multiplication and division. Of particular note, it is important to highlight that teachers who taught using CRA strategies were instructed on *how* to conduct the CRA intervention, and research did not start until teachers demonstrated mastery. Criteria such as these were not found in other studies.

Fractions

Many general education students struggle with fractions, even with only estimating fraction sums requiring no written computation (Braithwaite et al., 2018). It is especially true that students with math disabilities find understanding fraction concepts difficult (Bouck et al., 2020; Tian & Siegler, 2017). Recently, CRA instruction has improved fractional understanding in students with math disabilities (Hinton & Flores, 2019). This includes CRA taught as a three-step process and as in integrated intervention with multiple steps explicitly taught at the same time (Morano et al., 2020). The struggle with fractions and CRA benefits is not only evidenced in America. New research conducted in China also suggested that CRA math instruction can improve fraction competence in students with math disabilities and included the opinions of the participants and their teachers (Zhang et al., 2022).

Zhang et al. (2022) focused on the effects of CRA interventions for students with math disabilities while comparing and ordering fractions taught by two researchers with special education expertise for 10 weeks. Contrary to how CRA is typically taught, students were introduced to all three components of CRA during each lesson. All four students showed improvement in their conceptual and procedural understanding of fractions and maintained it two weeks after the intervention. Additionally, three of the four student participants noted they enjoyed the interventions, and, via questionnaires, the students' teachers agreed that students made improvements in learning. This was one of only a few studies that examined student opinions about the interventions and asked the participants' teachers about their students' improvement. This study was also in contrast to most CRA research because all three CRA components were used during each lesson, rather than progressing through CRA stages over time and with demonstrated competency before moving to the next sequence.

Interventions and Core Instruction

Quality math programming should certainly be offered to those who struggle with mathematics and students in general education Tier I settings. Tier II and Tier III interventions were helpful for those who have difficulty with mathematics, as evidenced in the above research. However, recent research revealed that math interventions and programs may not always align with core classroom instruction (Nelson et al., 2020). In addition, interventions varied in instructional strategies, vocabulary, and cover less topics than Tier I instruction (Fuchs et al., 2016). This could cause even more confusion to students who struggle with mathematics because students with math challenges found generalizing math skills difficult (Nelson et al., 2020).

Nelson et al. (2020) were the first to examine the alignment between core math instruction in general education classrooms and math instruction provided in intervention programs and settings. Although several mathematical practices were aligned, vocabulary and instructional approaches taught by teachers varied between both settings. Explicit instruction, concrete and representational models, feedback, and aligned vocabulary are principal for math intervention and general education instruction. This alignment was vital. Nelson et al. posited that providing core instruction and intervention that reinforces generalizing mathematical skills using concrete and pictorial representations is needed for students. This is what CRA constructivist strategies provide.

Examining research on CRA and its impact on a variety of learners is important. However, quality teaching strategies are only as effective as the math teachers who use them for instruction. According to Kilpatrick (2020), students learned the majority of their mathematics in school, and teachers and classmates influenced their mathematical cognition and problem-solving. Teachers are key when providing the best instruction according to students' needs. Teachers' knowledge influences their teaching practices and beliefs about math, learning, and instruction (Yang et al., 2020). According to Kaskens et al. (2020), teachers' knowledge in mathematics also positively impacted students' ability to problem-solve. Examining research on teachers' mathematical beliefs, attitudes, knowledge, and even anxieties is needed to understand current math instruction, training, and classroom environments, as all of the above impact teachers' lived experiences teaching constructivist strategies.

Teachers' Mathematical Beliefs, Attitudes, Knowledge, and Anxiety

Kutaka et al. (2017) suggested, quality math teachers must have solid mathematical and pedagogical knowledge, constructivist beliefs about teaching math, and good attitudes toward learning math. Teachers must understand the math they teach and make specific parts of particular content visible to students so they can digest and readily learn it (Ball et al., 2008). Miao and Reynolds (2017) posited that learning mathematics could be improved through what teachers directly do in the classroom. Some teachers struggle with self-efficacy and math anxiety. Therefore, a deeper look at how teachers feel about mathematics, what they know and can do in that content area, and possible anxieties about mathematics was in order.

Teachers' Mathematical Beliefs and Attitudes

Early research conducted by Raymond (1997) argued that math content-related beliefs in elementary school teachers mainly stem from past experiences when they learned math. Teacher practice is tied closely to beliefs about math content rather than how to teach it and the pedagogy behind it. Furthermore, teachers' beliefs about subject content, pedagogical understanding, and classroom environment affect instructional practice (Siswono et al., 2019). In addition to teachers' past mathematical experiences, Bandura (1993) found that teachers' beliefs about their math abilities impact the types of math challenges offered to students and the strategies used for instruction. Additionally, many teachers believe mathematics is simply a subject for "math people" or those who have a gift in mathematics (Chestnut et al., 2018), which only adds to the angst for those who struggle with mathematics. Contrary to that, efficacious mathematics teachers tend to push pupils with more challenging work and set more rigorous goals (Bandura, 1977, 1993).

Teachers' attitudes towards math were originally researched years ago and defined as a general liking or disliking of mathematics, a proneness to become involved or not involved with mathematics, the feeling that one is either good or bad at it, and whether they feel mathematics is useful or not (Neale, 1969). Attitudes toward math can cause positive or negative feelings about the subject, and negative feelings can even turn into math anxiety. Unfortunately, these poor feelings and anxieties about math could easily spill into the classroom when teaching and negatively impact student learning (Schaeffer et al., 2021). This is important to note because, according to Holm and Kajander (2020), students' conceptions about mathematics were formed by their previous teachers.

Teachers Mathematical Knowledge

Teachers' knowledge impact quality teaching and beliefs (Siswono et al., 2019), and the importance of teacher knowledge in mathematics cannot be underscored enough. Strong mathematical content knowledge is mandatory for teachers to instruct for deep understanding (Tan et al., 2021). Elementary teachers' mathematical content knowledge impact students' understanding of math (Masingila & Olanoff, 2022). According to research by Copur-Gencturk and Tolar (2022), fourth and fifth-grade teachers who had strong mathematical content knowledge also had strong pedagogical content knowledge. Even with early elementary mathematics, educators need a thorough understanding of why concepts and procedures mathematically make sense because more difficult concepts needed in later years are built upon basic skills (Alex & Roberts, 2019). For example, equal sharing is a precursor to fractions. Teachers' pedagogical content knowledge in mathematics is vital and is linked to effective teaching and student achievement (Ball et al., 2008).

Nations around the world measured student achievement through a variety of assessments such as PISA and Trends in International Mathematics and Science Study (TIMMS); however, with the age of teacher accountability, many states in America developed their own student achievement accountability tied to teachers of record. In Ohio, student growth is measured through a value-added system that measures the growth students show year after year on state assessments according to the expected growth of the group (Ohio Department of Education, 2022). These data are directly tied to the teachers who teach each student; therefore, it is imperative that teachers have the knowledge base to teach math as it impacts student learning and achievement.

Contrary to measuring teacher knowledge and effectiveness through assessment results, Stipek and Chiatovich (2017) sought to examine teachers' observed practices in their longitudinal study. They measured the effects of high-quality instruction and class climate of third-grade teachers on student engagement and academic achievement in reading and math. All students were socio-economically disadvantaged; some had challenges with math and reading, while others were considerably high-performers in those subjects. After having a high-quality third-grade teacher, as indicated on observation sub-scales, students with previous poor academic skills predicted better test score achievement in math and reading than their peers who started with higher academic skills. Kaskens et al. (2020) agreed that teachers' mathematical knowledge for *how* to instruct mathematics to students is extremely important and predicted growth in fourth-grade students' problem-solving in the Netherlands. High-quality teaching matters.

Teachers' mathematical knowledge matters; however, it is not the only skill needed to teach mathematics. Mathematical topics in kindergarten through grade 12 may seem relatively easy; however, they require deep understanding to teach them well (Newton, 2018). This was evident in a Canadian study of prospective mathematics teachers with undergraduate mathematics degrees preparing to teach students in grades fourth through eighth or up to tenth grade with further math courses (Holm & Kajander, 2020). Even though all nine participants had strong mathematical backgrounds, researchers found no evidence that their mathematical knowledge adequately assisted them to *teach* conceptual understanding to elementary mathematics. Most participants agreed that procedures were important to know in order to solve math problems, and most believed it was also important to be able to explain why and how to solve problems. Interestingly, participants could not demonstrate or explain how to multiply across a decimal, multiply fractions, or perform integer subtraction. Participants mainly explained how to solve problems by using procedures only. However, after taking a math methods course, participants improved their conceptual understanding. Most were finally able to explain *how* to solve problems, not just the procedures used to get the answers, Holm and Kajander.

The lack of mathematical, conceptual knowledge and understanding needed to teach students was again noted in a study by Ramsingh (2020). Three first and second-grade South African teachers were asked how they would teach basic word problems; one such problem involved division that included sharing 20 crayons. They were required to demonstrate representationally how they would teach the concept to students. All three teachers talked about using concrete objects, drawings, and symbols to solve the problem. All knew the answer; however, not one could use the representations created to solve the problem. More troubling, their representations caused them confusion.

The significance of teachers' mathematical knowledge was also highlighted by Miao and Reynolds (2017). They compared the effectiveness of teaching mathematics to nine and 10-year-old students from similar socio-economically equivalent schools in England and China. Miao and Reynolds recommended several suggestions for improving instruction, including increasing teachers' mathematical and pedagogical knowledge and moving them to become specialists, increasing lesson time for whole-class interactions, and improving teacher training. They also suggested decreasing whole-class lectures and partial class interactions and reducing group and individual work time. Researchers inadvertently discovered that the test instrument used in the study, TIMMS 2003, proved too easy for Chinese students and resulted in a ceiling effect;

therefore, learning gains could not be measured. Of particular note, primary teachers in England were generalists, whereas Chinese primary teachers were specialists in mathematics. Teachers' mathematical knowledge matters.

With increased focus on early mathematics instruction and interventions, preschool teachers also need solid mathematics content knowledge. Only 9% preschool and pre-service preschool teachers in the United States could correctly determine the errors made in a preschool game that included a counting component with concrete objects (Li, 2021). Errors included count-out errors, one-to-one correspondence, and principle of cardinality errors; and only 29% knew how to assist students with their errors. Most participants lacked the precise and consistent vocabulary needed to teach mathematics to this age group effectively. However, it was not known how many of the participants had mathematics pedagogy classes. Regardless, the need for teacher mathematics training to increase mathematical teaching knowledge was evident even in a preschool setting.

Teachers' Mathematical Anxieties

While math anxiety in students has been researched for years, teacher math anxiety is gaining further study recently. Math anxiety was defined by Richardson and Suinn (1972) as feelings of anxiety or tension that interfere with solving real-life math problems in a variety of contexts. Further, math anxiety was defined as apprehension and fear regarding math-related situations (Barroso et al., 2021; Ramirez et al., 2018). Unfortunately, elementary math teachers had higher levels of math anxiety than others in different work environments (Malinsky et al., 2006). There could be several reasons for this.

According to Snyder et al (2019), elementary teachers do not often take numerous math classes in college or obtain math degrees. Knowing that elementary teachers are not required to take as many math classes, it is also possible that those with lower math achievement and/or math anxiety may self-select elementary teaching as a profession. Additionally, 89% of elementary school teachers are female (Snyder et al., 2019). Scholars agreed that more females were inclined to have greater math anxiety than males (Hart & Ganley, 2018). This may explain the higher levels of math anxiety in elementary teaching than in other occupations. Adding to this research, Artemenko et al. (2021) examined the anxiety levels of in-service and pre-service elementary teachers in Germany and Belgium. They found that female elementary teachers had more mathematics anxiety than females in other areas of study, and roughly one-fourth had extremely high math anxiety.

The anxiety and negative attitudes towards math that some teachers possess have unfortunately been shown to negatively impact math achievement in elementary students, both males and females (Schaeffer et al., 2021). This added to earlier research by Beilock et al. (2010) that suggested teachers' math anxiety negatively impacted the mathematical achievement of females. Beilock et al. (2010) and Stoehr (2017) asserted that teachers who have math anxiety display negative attitudes towards math. Displaying negative attitudes about math inadvertently fostered environments that convey those beliefs to children learning mathematics and negatively impact student math achievement (Artemenko et al., 2021). This was also supported in a large-scale national sample study of mathematics teachers and ninth-grade students that suggested teachers with math anxiety were associated with lower student mathematics achievement (Ramirez et al., 2018). Should teachers fear mathematics, they may avoid it or teach it without full understanding.

Gresham (2018) noted that even teachers recognize that their own fear and anxiety toward math can impact students. One teacher in Gresham's study clearly articulated that while teaching a concept, she started crying in front of students because she did not understand it and recognized that seeing her frustration caused students a lasting impression. Another teacher in the same study commented that, although she was taught good teaching strategies to teach math, she despised the actual mathematics and felt her math anxiety "screaming at her" (p. 98).

It is important to note that despite elementary teachers having more anxiety than others, not all were impacted by it and demonstrated negative attitudes when teaching mathematics in the classroom Artemenko et al. (2021). According to Artemenko et al., additional math training appeared to lower anxiety levels. The results of their research showed that elementary teachers in Germany and Belgium who were trained mathematics specialists did not show heightened levels of math anxiety. The increase in math training may be one reason these educators lacked anxiety.

Teachers' general math anxiety shifted and decreased with quality professional development (Battista, 1986; Kutaka et al., 2017). Instructing teachers by filling some gaps in their own mathematical understanding to a certain extent provided intervention to teachers (Ganley et al., 2019). According to Tunç et al. (2020), when educators were instructed how to use concrete models both as learners and teachers, it increased teachers' and pre-service teachers' efficacy. Increased conceptual understanding of mathematics builds teacher self-efficacy and improved teacher instruction (Looney et al., 2017). The more teachers understand the math, the better their instruction, which may lead to improved student learning.

Pre-service Teachers' Mathematical Anxieties and Knowledge

Pre-service teachers with mathematical anxieties tend to believe math is hard, lack confidence in the subject, and avoid it (Gómez Escobar & Fernández, 2018). This was supported by research conducted by Park and Flores (2021); their study showed that pre-service teachers reported high anxiety about mathematics and felt less confident in mathematics and science.

While pre-service teachers felt less confident in both subjects, they felt less anxiety and more positive feelings teaching science than mathematics. These feelings of anxiety negatively impacted pre-service teachers' attitudes and happiness when teaching those two subjects. According to Marbán et al. (2021), mathematical anxiety significantly affects pre-service teachers' enjoyment of mathematics.

Anxious pre-service teachers may not have the amount of math knowledge needed or may not have obtained the required mathematics pedagogical skillsets needed to teach it (Novak & Tassell, 2017). Boote and Boote (2018) found that many pre-service teachers have poor conceptual knowledge and rely heavily on procedures. Some elementary teachers also lack conceptual understanding needed to teach as well (Ramsingh, 2020). However, Keazer and Phaiah (2022) posited that using procedural knowledge flexibly to increase fluency means that procedural knowledge works with learners' conceptual understanding. Both procedural knowledge and conceptual understanding are important for mathematical learning and are needed for current math standards.

In a mixed-methods study, Aydin and Özgeldi (2019) gave prospective elementary teachers released PISA math questions that were originally given to 15-year-old students who took the 2012 PISA. The math questions consisted of contextual, conceptual, procedural knowledge questions, and a combination of the three with a large emphasis on procedural knowledge and techniques. Prospective teachers struggled the most with the combination questions that required contextual, conceptual, and procedural knowledge. The qualitative data showed that most could not give mathematical explanations for conceptual knowledge questions. They also lacked the ability to express mathematical arguments on questions that required contextual knowledge. Prospective teachers struggled slightly less with understanding math and calculations, and most used memorized algorithms to get their answers, Aydın & Özgeldi. Pre-service teachers' knowledge and anxiety can be improved. Research by Gresham and Burleigh (2019) showed that by modeling and using multiple instructional strategies in math methods classes, pre-service teachers noted their anxiety decreased.

Importance of Mathematics Professional Development

Despite the angst felt by some teachers regarding mathematics and the lack of math content knowledge of some, there is hope for educators who do not like mathematics. This means there is hope for increasing student mathematical understanding. To improve students' mathematical understanding, educators must also be offered opportunities to learn as it is teachers who determine how math is taught (Newton, 2018). Most elementary teachers were taught to be generalists who teach all subjects, however, special training for elementary math teachers was a recent change in teacher preparation (Myers et al., 2020; Schubring & Karp, 2020). To address the cognitively challenging Standards of Mathematical Practices (SMP) in the new CCSS, teachers needed professional development and proper resources (Schweig et al., 2020).

There is building research that quality professional development improved elementary teachers' pedagogical knowledge, attitudes, and beliefs; this was supported by a five-year longitudinal study conducted by Kutaka et al. (2018). The study focused on changes in elementary teachers' knowledge, attitudes, and beliefs regarding the teaching and learning of mathematics through professional development called *Primarily Math*. *Primarily Math* is an 18-credit hour graduate-level professional development program designed for teachers to become elementary mathematics specialists. The program taught conceptual understanding and multiple strategies, including constructivist hands-on learning, for teachers to use in classrooms (Center for Science, Mathematics, and Computer Education, University of Nebraska Lincoln, 2018). Kutaka et al. (2018) argued that improved focus on mathematics courses and instructional courses aimed to foster increased understanding and how to respond to elementary students' mathematical reasoning and thinking helped increase teachers' mathematical knowledge base, attitudes, and beliefs.

Stohlmann et al. (2020) added to the research on professional development and suggested effective ways to increase teachers' conceptual understanding when solving fraction division problems. Professional development that focused on teachers' abilities to create and solve real-world fraction division word problems by incorporating pictures and representations helped improve teachers' understanding. When teachers' pictorial understanding increased, it helped with their conceptual understanding of dividing fractions. However, 11 out of 63 teachers were still challenged with using the correct reference unit with representations to solve fraction word problems even after professional development.

Those who design teacher preparation programs looked at ways to determine how prepared their teacher candidates are before receiving their teaching degrees through assessments, thus increasing accountability (Swars Auslander et al., 2020). However, math knowledge may not be enough to help with conceptual understanding. According to Holm and Kajander (2020), even undergraduate teaching candidates with mathematics degrees benefitted from math methods courses as it can help develop conceptual understanding. In a study conducted by Gresham and Burleigh (2019), all 12 elementary pre-service teachers noted that hands-on strategies they experienced in a math methods class were not the way they were taught math in elementary school. However, pre-service teachers found these constructivist strategies helped their understanding of solving problems in multiple ways.

As evidenced in another study, the use of manipulatives in a math methods course for prospective elementary math teachers helped them understand procedures they learned as children, confirming the benefits of hands-on learning even with pre-service teachers Holm and Kajander (2020). When provided enough learning and practice, pre-service teachers can increase their self-efficacy in mathematics when preparing lessons (Gonzalez-DeHass et al., 2022). Brinkmann (2019) posited that to increase pre-service efficacy pre-service teachers need content specific methods courses, data-driven intervention lesson planning, conceptual and procedural knowledge integration, math connections with real world examples, and encouragement to increase positive attitudes in math. Teaching these courses to pre-service teachers demanded knowledgeable college instructors; roughly 50% of college math instructors have math degrees and teach math content courses to prospective teachers (Masingila & Olanoff, 2022). However, 74.7% of content method instructors in four-year institutions that offered doctoral programs had no elementary teaching experience and may not be fully prepared to cultivate the mathematical knowledge prospective teachers need to teach (Masingila & Olanoff).

Specific math training is important for those who teach children math. According to Urban et al. (2021), mathematics teachers need specific training for visual modeling expertise. However, it is important to note that even with training and resources, some teachers reverted to using more traditional teaching methods. Urban et al. found that some teachers prefer strategies that are more helpful to adults, such as heavily relying on verbal explanations despite training to use visual models. Their findings suggested that elementary teachers use visual models inconsistently. Additionally, according to Blazar et al. (2020), providing training on using CCSS-aligned textbooks did not result in teachers' using them when teaching. After a median of three days of teacher training in six states using new CCSS-aligned textbooks, only 25% of the schools had teachers who used them for almost all of their math lessons. Most teachers used their own materials and strategies to teach. Furthermore, there was little evidence of differences in achievement growth in the elementary schools that used CCSS-aligned textbooks versus schools that did not (Blazar et al.). Training to use visual models and textbook resources did not change the majority of teachers' teaching methods in these studies.

There is research that some training can change and improve teacher instruction. Looney et al. (2017) highlighted negative to positive changes in pre-service teachers' beliefs about math after an instructional math course; strategies learned increased teacher understanding. Professional development that included constructivist strategies, including bar modeling found in the CRA framework, paid off for Singapore (Urban et al., 2021). This frontloaded approach to empowering and educating teachers resulted in better scores than most countries in the world. The professional development offered in Singapore includes the concrete, pictorial, and abstract approach (CPA), also called CRA.

Math Achievement in High Performing Singapore

While it is important to investigate the research and benefits of the CRA framework and teachers' beliefs, attitudes, and anxieties, it is also worth examining a country that scores well on international math tests compared to America. Singapore ranks as one of the highest mathematically performing countries in the world and is second behind China on the Programme of International Student Assessment (OECD, 2019). The Ministry of Education in Singapore recognized that the key to its country's success was directly tied to its educational system. They invested heavily in teacher recruitment and training (Ministry of Education, Singapore, 2015). Singapore constructed its educational system from the ground up and with the coordination of primary and secondary schools. Singapore uses the concrete, pictorial, and abstract approach to teach mathematics; Bruner's enactive-iconic-symbolic theory was the foundation of the CPA approach for mathematics as it is for the CRA framework in America (Kaur, 2019).

Singapore endorses CPA, and it is utilized for all of its mathematics instruction nationwide; it is represented in math books and taught to pre-service teachers who are in training to teach mathematics (Leong et al., 2015). Much of Singapore's mathematics teaching requires students to visualize abstract relationships (Kaur, 2019). One of the models Singapore teachers use to represent the abstract is the model method. Singapore uses a spiral curriculum that moves from concrete objects that represent numbers to rectangle models, and then abstract symbols (Ng, 2022). The model method requires students to use rectangular bars to represent a whole and/or parts of a whole depending on the problem (Kaur, 2019). Rectangle pictorial representations stand for numbers and can be used to solve arithmetic problems in early years of schooling and later used to represent variables or unknown numbers (Ng, 2022). Slightly different than simply drawing pictures, Ng and Lee (2009) explained that the bar models represent quantities and their connections between quantities. These models assist students in creating visualizations for abstract problems and relationships.

In Kaur's (2019) study of the perceptions of expert primary teachers and the model method, all five teachers interviewed felt that using models was very helpful in assisting students to visualize relationships in addition, fractions, ratios, and percentages when solving word problems. This study consisted of a very small sample size and did not consider the views of novice teachers; however, it was one of the very few studies found that examined teacher perceptions about using pictorial models or representations. In Kaur's study, master teachers were clear that the model method helped students; they also noted that the representations did not have to be the model method; any representation could be used according to students' preferences or understandings.

Teachers in America choose how to teach math and use many different mathematical strategies to teach concepts according to state standards (Schweig et al., 2020). Some strategies include using cognitively challenging problems, using various representations and math tools, problem-solving, justifying, connecting, applying math moves, and using students' understandings to steer further instruction (Myers et al., 2020). Examining elementary teachers' lived experiences with constructivist strategies, specifically, CRA, may help educators learn more about its impact on teachers and students.

Summary

Teachers in America have a history of using multiple approaches to teach mathematics; however, students still struggle. There are many reasons why students are challenged with math, and research shows that their struggle is a multifaceted problem. New rigorous standards demand that teachers provide evidence-based instructional practices that focus on building conceptual understanding, not just procedural knowledge for all students. Some elementary teachers may lack mathematical and pedagogical knowledge and the ability to demonstrate conceptually how to solve math problems without overly depending on procedures. Additionally, some elementary teachers have angst and anxiety with mathematics; when teaching it to students, all of which impacts teaching and learning.

Benefits of the constructivist CRA framework as an evidence-based method for all learners in Tier I, Tier II, and Tier III abound. The CRA framework helps students make sense of the math they perform by allowing students to visualize problems first with concrete manipulatives, than with pictures or representations on paper, and then finally with abstract symbols and numbers. Students can move along this scaffolding of support when they need more or less support. This model parallels Bruner's learning theory (1966), which was the theoretical framework of this study. Researchers who used the CRA framework in their studies most often utilized researchers as the instructors to teach students, not teachers. Teachers' voices using constructivist CRA strategies needed to be heard.

Little was known about the experiences and opinions of teachers who teach elementary mathematics, and far less was known about teachers' experiences using the CRA framework during instruction. The CPA framework is used successfully in Singapore; it is taught to their teachers and pre-service teachers and represented in their textbooks. Their entire country is familiar with its concepts and has found outstanding success. In America, procedural teaching has been the norm for decades and has been hard to dismantle. This research study illuminated teachers' lived experiences of implementing constructivist strategies such as the CRA framework and its components. Listening to the experiences and voices of teachers helped identify strengths and weaknesses in teaching and learning with the CRA framework and helped identify the next steps needed for educators to improve their mathematical knowledge and beliefs. It also highlighted areas needed for professional learning that may eventually be used to improve mathematical understanding in teachers and possibly even impact student academic achievement in students.

CHAPTER THREE: METHODS

Overview

The purpose of this transcendental phenomenological study is to describe elementary teachers' lived experiences of implementing constructivist strategies, specifically the concrete, representational, and abstract (CRA) approach when teaching mathematics in a midwestern school district. In this chapter, I cover the research design based on the work of Moustakas (1994), the setting, the rationale for the setting, and participant recruitment. I describe my positionality as the researcher, the interpretative framework, the philosophical assumptions, and my role as the human instrument. I articulate the procedures for the study, beginning with site permission, IRB approval, participant recruitment, and consent. I also explain procedures for interviews, focus groups, and journal entries. I include a description of how data are collected and analyzed, and I describe how ethical considerations are honored, including an explanation of how research data remains safe and secure and confidentiality is maintained. I conclude with a chapter summary.

Research Design

In qualitative research, researchers study how people or groups of individuals create and construct meaning and use the research to understand participants' experiences (Patton, 2015). Husserl is the founder of qualitative methodology (van Manen, 2014). He considered all knowledge to be created through experiences (Moustakas, 1994). Researchers use a qualitative methodology to explore a problem and construct a detailed understanding of a common phenomenon shared by participants (Creswell & Guetterman, 2019). Participants and researchers bring a varied background of experiences and meanings from those experiences to qualitative studies (Adu, 2019). Qualitative research is used to understand the complexities of an issue by listening to the voices and experiences of others (Creswell & Poth, 2018). I selected a qualitative methodology for my study to gain insight into the nature of the experiences of elementary math teachers using constructivist CRA strategies. Using a qualitative research method requires the researcher to play a key role by gathering data through open-ended questions through interviews, focus groups, and through participants' words (Creswell & Poth, 2018). As the researcher, I had the opportunity to hear participants' voices in a variety of ways: individually, in small focus groups, and through their personal reflection in journals. This qualitative method shed light on their lived experiences. As Adu (2019) posited, qualitative data must include information about contexts, such as participants' characteristics, history, and location, because it helps the audience deeply understand the research themes. Qualitative research was the best fit for this study to understand how participants make meaning of their experiences in their teaching because listening to, documenting, and analyzing their experiences helped me to understand the complexities of teaching while using constructivist strategies.

The focus of a phenomenological research design is to express a common meaning from the lived experiences of multiple persons and their shared common phenomenon (Creswell & Poth, 2018). Gaining a deep understanding of how humans make sense of lived experiences and converting those experiences into consciousness is the core of phenomenology (Patton, 2015). According to van Manen (1990), descriptions of a shared phenomenon can be narrowed to the universal essence or meaning. I used a phenomenological design for my study to focus on participants' shared experiences while teaching mathematics with constructivist strategies to find the universal essence of that experience. Adu (2019) described that researchers using a phenomenological design recruit those who experience the same phenomenon and interview them with open-ended questions. Interviews, focus groups, and participants' journals provided me with the thick and rich data needed for research to determine the meaning of my participants' shared experiences.

"Meaning is at the heart of transcendental phenomenology" (Moustakas, 1994, p. 57); it focuses on the overall or completeness of an experience. Moustakas stated that transcendental phenomenology requires an approach free of preconceived beliefs and biases, and researchers should examine the investigated phenomenon with a new lens or a fresh take. Patton (2015) suggested that there is an assumption of essence or an underlying meaning of a phenomenon experienced by groups. Those common experiences can be uncovered through the collection and analysis of those experiences. Moustakas (1994) described four phases of the transcendental phenomenological approach: epoché, reduction, imaginative variation, and synthesizing meaning. Transcendental phenomenology was appropriate for my research because I researched the experiences of elementary teachers who used constructivist strategies to teach math with a fresh perspective open to perceived experiences. Moustakas stated that utilizing the epoché will allow complete focus on what is being observed and heard. I set aside all of my presuppositions and viewed participants' experiences with a new lens and a fresh perspective to accurately collect descriptions and experiences of teachers using a constructivist approach to teach. This shed light on the phenomenon. This phenomenological study helped provide a rich description of lived experiences, which was the purpose of this study and was missing in research.

Moustakas (1994) suggested that transcendental phenomenology must begin with a good research question that stems from deep interest and must have social and personal importance. I was passionately interested in why students struggle in mathematics despite years of increased focus and more rigorous standards. I investigated what teachers experienced when teaching math with constructivist strategies such as the CRA framework. Transcendental phenomenology allowed me to conduct personal interviews, focus groups, and journal entries that illuminated the descriptions and rich experiences of the participants. I learned about participants' experiences and beliefs regarding this teaching framework through their responses to interviews, focus questions, and journal entries. Using a qualitative method of research and a transcendental phenomenological design disclosed the essence of their shared experience and gave insight into their experiences with constructivist math methodologies.

Research Questions

Central Research Question

How do elementary math teachers describe their lived experiences of implementing constructivist strategies in the classroom?

Sub Question One

How do elementary math teachers describe their experiences of using concrete manipulatives in the classroom?

Sub Question Two

How do elementary math teachers describe their experiences of using representations or models in the classroom?

Sub Question Three

How do elementary math teachers describe their experiences of using abstract mathematical concepts in the classroom?

Setting and Participants

Selecting a setting and participants is important, as the results of the research depend on both. According to Creswell and Guetterman (2019), care should be taken to select an appropriate setting and participants that will allow an understanding of the phenomenon. The setting of my study took place in elementary schools within a midwestern suburban school district in the United States of America. All participants were elementary teachers, and/or math specialists, and/or academic coaches representing kindergarten through fifth grades experiencing the same phenomenon.

Setting

The setting of this study took place in several elementary schools (kindergarten through grade five) in Montgomery City Schools District (MCS); MCS is a pseudonym. MCS is located in the midwestern United States and is a public suburban school district just outside a large metropolitan city. MCS contains two large high schools, an alternative high school, a high school academy for the most at-risk youth, and a special needs preschool. It also contains four middle schools and 11 elementary schools. Three of the elementary schools are Title One buildings that receive federal funding due to the size of their free and reduced population (MCS, 2022).

District data from the Ohio Department of Education (2020) showed the demographics, in grades preschool through 12, 67.9% of the district's population was White, African Americans represented 9.1%, Hispanics also represented 9.1%, 5% were Asian Americans, and students who identified as multi-racial represented 8.8% of the district's total population. Additionally, 14.8% of the student population was identified as having special education services requiring an Individualized Education Plan (IEP); students who qualified as English Learners (EL) represented 5% of the district population, and this population was growing rapidly. MCS instructed roughly 11,000 students; and like many suburban schools, there were pockets of affluence and pockets of poverty. Close to 23% of students were Economically Disadvantaged (ED) and qualified for free- or reduced-lunch status (Ohio Department of Education). ED students represented an even higher percentage of the population in the Title One elementary buildings at 36%, 51%, and 33%, according to the Ohio Department of Education.

This suburban school district operates under an educational hierarchy with a superintendent, assistant superintendent, chief academic officer, and secondary and elementary directors of education who oversaw school principals. The district also employs curriculum specialists and special education directors. Mental health specialists, full-time counselors, and a diversity, equity, and inclusion director adequately meet all students' physical, academic, and emotional needs and focus on students' social-emotional learning (MCS, 2022). The setting for this study represented a growing suburban school district in the state, and the study could be easily replicated in other suburbs with similar demographics obtained through departments of education across midwestern United States.

Patton (2015) described the importance of spending extensive time with participants in the field for qualitative studies. I used MCS as the district of choice for this study, as its location and demographics could be replicated. It was a good mix of students from many socio-economic levels and was ideal for me to spend ample time with participants collecting data from interviews and focus groups. In this setting, I did not use participants from the elementary school where I was in a supervisory or evaluative role. A neighboring district just a mile away agreed to allow me access and permission to conduct my research were I not able to use MCS as my research site.

Participants

After obtaining IRB approval (see Appendix C), I used purposeful sampling to recruit participants. Purposeful sampling is a method of sampling that pulls from the most information-rich participants with experiences and understandings that could support issues in the study

(Patton, 2015). Out of a sample pool of approximately 160 math teachers from kindergarten through fifth grades, including intervention specialists, special education teachers, and elementary academic coaches, I used 10 participants for my study. I used a gatekeeper, the elementary director of education, to forward my recruitment letter via email (see Appendix D). With the superintendent's and the elementary principals' permission, I also promulgated hard copies of the recruitment letter at staff meetings. When I was not available for a staff meeting, the principal distributed the hard copy of the recruitment letter (see Appendix E). This hard copy of the recruitment letter served as a second reminder for recruiting potential participants. All participants worked at MCS and used constructivist CRA strategies to participate in this research study.

While Patton (2015) suggested no specific number is required for sample size in qualitative inquiry, he stressed that in-depth and enlightening material from a small number of participants could have great value. My sample pool included approximately 160 general education teachers, over 25 special education and Title One teachers, and six instructional coaches who often assisted with classroom instruction. Patton posited the quality and richness of the information collected combined with valid, meaningful data and analysis is key. Participants included variances in gender, ethnicity, age, experience teaching, and educational background. Participants met the requirement of teaching mathematics using CRA constructivist strategies to students from kindergarten through fifth grade. Depending on the school, some teachers only taught math, math and an additional subject, all subjects for that grade level including math, or taught math as an academic coach. Interested volunteers responded to me via email or phone within a two-week deadline that was previously communicated in all correspondence. The number of participants for this study was 10.

Researcher Positionality

The motivation for my research was based on my personal experiences and observations as an elementary educator, a reading and math intervention teacher, an academic coach, and an elementary school principal. I entered teaching as a lover of the language arts, history, and science; I felt most comfortable with those subjects. Mathematics was simply a subject I was required to take; it was not my passion. My indifference towards math changed when the new Common Core State Standards (CCSS) spread from coast to coast in the United States in 2010. I was put in charge of leading a large elementary school of 1,500 students in math intervention and professional development.

This journey in math led me to learn a great deal about mathematics, how best to teach it to elementary students, and what was needed for teacher professional development. The new standards caused me to look at math differently. I had to deeply understand what the standards meant and what students were expected to know and do. Additionally, I had to examine what was expected from teachers to teach according to new standards. I learned the days of learning math through procedures only were gone; it was simply not enough. Students must now understand procedures and concepts *and* apply that learning to challenging real-world problems (CCSSI, 2010). The new standards had a greater emphasis on higher-order cognition (Porter et al., 2011). The shift in standards required more from teachers; they, too, had to understand procedures, concepts, and applications in mathematics and teach those skills to students. In my quest to improve my mathematics instruction, I learned about natural differentiation by using the CRA method, also known as the concrete, pictorial, abstract (CPA) approach. I also learned how math was taught in Singapore and used those methods to learn, teach, and coach. It completely changed my thinking and instruction by improving my interventions and increasing students' learning and academic achievement. As an academic coach, I recognized many teachers did not understand the rigor of the new standards or how to teach conceptual understanding because they were taught procedurally. There was a disconnect. I also noted an increase in teacher conceptual understanding when coached to use more manipulatives and pictures/models with their students. The increase in teachers' conceptual understanding fascinated me.

As a result of my experiences, I wanted to learn about the experiences of other teachers who use constructivist strategies in the CRA framework which included using concrete manipulatives, representations, pictures, and/or abstract equations. I believed giving voice to teachers' thoughts and shared experiences could impact preparation for pre-service teachers, professional development for current teachers, and research for scholars. Hearing from the teachers who instructed students daily added new insights to the field of mathematics.

Interpretive Framework

The interpretive framework, or the lens through which I conducted this study, was from a social constructivist paradigm. Social constructivist theory is a learner-centered theory where knowledge is constructed in a social environment, and learning continues from a student's current intellectual state to a higher level (Vygotsky, 1978). Vygotsky described the Zone of Proximal Development (ZPD) as the space in learning between where a student can problem-solve independently and the next potential level of learning with guidance from the teacher. Knowledge is constructed through experiences and depends on the learners' environments (Morchid, 2020). Learning occurs within a social construct (Vygotsky, 1986) and social interaction assists learners and their prior knowledge (Orru et al., 2018). It is best utilized when teachers engage students with real-life situations (Morchid, 2020). I believe that students and adults learn from experiences and their environments in life. Their learning is constructed through interactions, language, encounters with others, and through all of life’s activities. Knowledge constructed is based on the understanding of those experiences.

Philosophical Assumptions

Researchers of a phenomenological study must examine the philosophy adopted (Neubauer et al., 2019). In phenomenology, it is recognized that researchers bring with them worldviews and biases (Patton, 2015). I believed it was important that I shared my physiological assumptions, values, and beliefs as it would assist participants in understanding how I view the world, specifically in the areas of teaching and learning. In this transcendental phenomenological study, I recognized I came with my own philosophical assumptions: ontological, epistemological, and axiological.

Ontological Assumption

Ontological assumptions are based on the multiple realities from participants’ different experiences (Creswell & Poth, 2018). While I, as the researcher, had my own realities, others had different realities based on their life experiences. I completely understood and respected that others had different realities. Moustakas (1994) suggested explaining how participants look at their experiences in different ways is important. I used several methods for data collection to capture participants’ experiences and realities, such as interviews, focus group interviews, and participants’ journal entries. According to the experiences of the participants in this study, I anticipated that participants would demonstrate subjective realities, and common themes would emerge in the data and the findings in this study, which they did.

Epistemological Assumption

With epistemological assumptions, researchers try to situate themselves as close to the participants as possible to collect information from their subjective experiences which helps to see how knowledge is known (Creswell & Poth, 2018). Many believe that knowledge is derived from experiences that are all subjective. I was deeply interested in the participants' views and how they interpreted meaning through human interaction. The researcher is the research instrument in a transcendental phenomenological study and must become as close as possible to the participants and the settings where they live their experiences (Patton, 2015). I spent time with all participants, listened to their experiences through interviews and focus groups, and examined participants' journal entries to extract meaning.

Axiological Assumption

Researchers with axiological assumptions recognize their position and biases and the value of information collected from participants; however, they are free to make those biases known to participants (Creswell & Poth, 2018). As the researcher, I recognized I had my own worldview and biases based on my life experiences as a teacher, academic coach, and administrator, and I brought them to this study. As an educator, I believed putting the needs of students first is paramount and that educators should teach according to the vast array of students' needs as best practice. I also believed that teacher reflection on teaching practices and student learning was very important and can improve instruction. I shared those personal and educational biases with participants. Creswell and Poth suggested being biased-free and bracketing out subjectivity during data collection and analysis. As the human instrument in this transcendental phenomenological research study, it was my job to listen to the responses of the participants and set aside my own biases as I learned from participants' experiences and made meaning from the data collected. Remaining in the epoché is vital to understanding a phenomenon (Moustakas, 1994). I desired to learn more about what teachers notice and experience as they taught students with constructivist strategies in a bias free manner.

Researcher's Role

My previous experiences as a teacher, math intervention teacher, academic coach, and administrator offered me a unique perspective as the human instrument in this qualitative study. Patton (2015) posited that qualitative research is personal in nature and unlike quantitative research, the researcher is the primary research instrument. Being the researcher as an instrument, I had the main role and responsibility to gather data and used my skillsets to observe, question, analyze, and communicate results from this study. Qualitative researchers use multiple methods to gather and organize data, work back and forth with all data to determine main themes and patterns to extract meaning (Creswell & Poth, 2018). Carefully listening, documenting, and recording interviews, focus groups, and reading participant's journal entries were the first steps in data collection and, by doing so, tapped into teachers' experiences, thoughts, and feelings needed for qualitative research. I reread and journaled while I found themes and analyzed data to understand the deep meaning participants attributed to their experiences. I took great effort to reflect on all data from interviews, focus groups, and participants' journals which helped me with meaning-making discovery.

It was mandatory that I bracketed out my experiences as much as possible to enable me to look at the phenomenon with a novel perspective and used the epoché process defined by Moustakas (1994). Creswell and Poth (2018) found while bracketing is never perfect, it helped researchers examine their own experiences with the same phenomenon before investigating the experiences of participants. As van Manen (1990, 2014) described, the researcher deciphers and interprets data with assumptions from personal experiences. Knowing this, I was mindful of my biases regarding quality math instruction and how teachers could best help students master concepts. I also needed to set aside my assumptions about planning and implementing lessons and focused on collecting data and using personal reflection. Engaging in the epoché must be done continually (Moustakas, 1994). Using reflective journaling throughout my research helped me remain bias free and remain in the epoché.

If teachers thought I was conducting research with a supervisory or evaluative slant, it could have been detrimental to the study. Therefore, I was very open and honest about my role as a researcher who desired to illuminate and understand participants' lived experiences when teaching math. I emphasized this when I recruited participants and throughout the research process; this helped build quality rapport and trust. Any perceived or possible conflict of interest were non-existent by not permitting those under my direct supervision, authority, or evaluative responsibility to participate. All participants were educators who do not have a personal relationship with me. Creswell and Poth (2018) warned of the possibility for the researcher to side with participants. I remained mindful to allow participants' experiences and opinions to speak through data. Eddles-Hirsch (2015) suggested that the phenomenon explored is not measured or characterized through the typical view of reality; rather, it searched for how participants made sense of their experiences in their everyday world.

Procedures

Steps necessary to conduct this research study included IRB approval through Liberty University for research involving human subjects, obtaining documentation of permission to perform the study and site approval from the school superintendent of MCS, and securing participants through recruitment via emails, invitation at meetings, obtaining consent to participate, recording data, and debriefing with participants.

Permissions

The first step in this study was to secure site approval from the MCS superintendent, which I completed. A permission letter was sent to the superintendent of MCS asking for permission to conduct the study with specifics regarding the research to be conducted and a description of the voluntary participants needed (see Appendix A). Permission to conduct the study was granted (see Appendix B). After that, I obtained Institutional Review Board (IRB) approval from Liberty University (see Appendix C). After securing IRB approval, I focused on recruiting participants for this study.

Recruitment Plan

Once the site and IRB approvals were obtained, I adhered to the following steps for recruitment: purposeful sampling, participant recruitment, and garnering participant consent before gathering data. Purposeful sampling was used to deliberately sample individuals or a group who are in the best position to offer information about the proposed research (Creswell & Poth, 2018). Purposeful sampling was used to find participants with rich information on the study's topic that can be deeply studied (Patton, 2015). I used purposeful sampling of math teachers (kindergarten through fifth grades), academic coaches, and math specialists who taught elementary students in MCS. I gathered this information from the district website. These individuals used constructivist instructional strategies, such as concrete manipulatives, representations/models, and/or abstract equations for teaching mathematics. This offered information-rich experiences. I approached the superintendent of MCS, described the purpose of my qualitative study, and asked for the superintendent's permission for either he or the director of elementary education to be my gatekeeper. The director of elementary education forwarded my recruitment letter/email to my sample pool of qualifying teachers who used one or more of the CRA strategies (see Appendix D). Additionally, with permission from building principals, I reiterated the same information in the recruitment email at building-level meetings by redistributing a hard copy of my recruitment email as a recruitment follow-up. The recruitment follow-up gave details of the study and asked for participants (see Appendix E). For meetings I could not attend, the principal promulgated the email to staff.

From the forwarded recruitment emails and face-to-face informational meetings reiterating the information in the recruitment email at all elementary schools in the district, I received enough responses via email or phone from interested parties to create a list of 10 participants for the study. I distributed a consent letter to all potential participants with details of what would be expected from the participants, including the time required, benefits, risks, confidentiality, and privacy considerations (see Appendix F). Potential participants were informed about the voluntary nature of the study, and all participants were given pseudonyms for my use only to maintain confidentiality. Informed consent was mandatory for all participants. Participants first completed a questionnaire to gather their demographic data (see Appendix G). This information was important for my study as it helped to verify participant credibility, described the participants, and noted their educational background and experiences that may have impacted their teaching methods. New teachers in the profession may have had different experiences and training than those who have taught for decades.

Data Collection Plan

I began to collect data after I received approval from the IRB, the participating school district, and after receiving informed consent from all participants. Using multiple sources of data from different qualitative methods and at different times was needed to provide triangulation (Patton, 2015). I conducted interviews, focus groups, and collected journal entries from all participants. To understand the experiences of the phenomenon in research, the voices and actions of participants must be heard, collected, documented, reduced, and synthesized to get to the essence of the experience (Patton, 2015). Qualitative inquiry demands accurate data collection (Patton, 2015) and must be collected in an ethical manner (Creswell & Poth, 2018). Data in my study was collected with ethical integrity and in a variety of methods through interviews, focus groups, and participant's journal entries.

Individual Interviews Data Collection Approach

Rubin and Rubin (2012) stated that an interview is founded on conversation. Interviews flowed like conversations and elicited participants to open up and express themselves freely so I saw their experiences through their viewpoints. Moustakas (1994) suggested that interviews be an interactive process allowing individual experiences of the phenomenon to be understood. He suggested broad, open-ended questions elicit thorough accounts of experiences and allow deep and detailed descriptions of participants' experiences. The goal of interviews is to identify how participants see their world (Patton, 2015). In addition, interview questions are often sub-questions to the main research question (Creswell & Poth, 2018). Semi-structured interview questions stemmed from the research questions, participants' experiences teaching math, and their use of the CRA framework (see Appendix H). Follow-up questions were asked to assist with understanding the proper intent and context of participants' answers (Dahlberg & Dahlberg, 2019). I asked for clarification about any answers to my questions that were confusing or had the possibility of multiple meanings. It was important I fully understood participants' answers and participants' intent in order to code them correctly.

Quality interviews are dependent on the interviewer (Patton, 2015). Interviews established rapport and trust between me and the participants. I conducted one-on-one interviews with 10 participants with semi-structured open-ended questions in comfortable locations and times that worked with their schedules; all were via Zoom. In order for participants to feel safe disclosing their experiences during interviews, they should be conducted in a familiar location (Patton, 2015). All interviews in my study occurred in locations determined by the participants to ensure maximum comfort and accessibility. Zoom can be used as a useful tool for qualitative data collection as it is convenient, easy to use, offers security features, and data management options (Archibald et al., 2019). Each participant was interviewed once, which took less than an hour. Interviews provided the first chance to collect data pertaining to teachers' descriptions of mathematical constructivist experiences, which included CRA strategies. Participating in individual interviews gave participants the freedom to talk about their experiences teaching the different parts of the CRA framework without the input or biases of others. This allowed me to note their individual experiences, beliefs, and views before colleagues joined in the discussion during focus groups.

Interviews only occured between the individual participant and me. I made use of the epoché to help set aside any preconceived beliefs or biases (Moustakas, 1994). I purposefully set aside my own biases before each interview and journaled when needed. I took notes during interviews, noting facial expressions and emotions emitted by the participants along with my impressions and thoughts. I also took notes afterward during transcription reading and rereading. I electronically recorded all interviews with my Apple iPhone and through Otter.ai. I personally checked electronic transcript accuracy against my personal audio recordings and promulgated individual transcripts to participants for member-checking. All transcripts were electronically transcribed through Otter.ai.

Individual Interview Questions

1. Please state your name, job title, job expectations, and introduce yourself. CRQ
2. Why did you choose to become a math teacher for elementary-aged students? CRQ

3. Describe your experiences when writing a math lesson. SQ1
4. If I walked into your classroom, what would I see when you introduce a new concept to your math students? SQ1
5. Describe a time when you responded to a student who did not understand a new skill or concept. SQ1
6. What are your experiences when a child already understands the concept you are teaching and may have the skill mastered? SQ1, 2, 3
7. Describe experiences you have had with using concrete manipulatives when teaching math. SQ1
8. How do you teach conceptual understanding to your math students? SQ1, 2
9. Tell me what it looks like and sounds like when you use representations and modeling for math instruction. SQ2
10. Describe your experiences with teaching abstract concepts to students. SQ3
11. Why do you use constructivist strategies (CRA) when teaching mathematics? SQ1
12. You have shared a lot with me during this interview; thank you. What more would you like to add regarding your experiences with the CRA framework and its components?

Questions were designed to pull from participants' experiences with teaching mathematics. Question one was an experience and behavior question that was asked to start the interview with a simple introduction and to build rapport (Patton, 2015). Moustakas (1994) suggested creating a relaxed and trusting environment for interviews. This question offered participants the chance to describe their roles and observe what each participant does during their day. Question two was an opinion and a feeling question. Opinion and feeling questions aim to stir the participants' emotions and highlight their feelings about their beliefs (Patton, 2015). With this question I sought to get participants talking about their profession and to uncover if they considered themselves math teachers because most teachers teach all subjects at the elementary level.

Question three was a knowledge question regarding participants' math lessons and showed how or why participants included any constructivist strategies when they initially planned for their math lessons or why they did not include those strategies. Knowledge questions are based on what participants know (Patton, 2015). Question four was a sensory and experience question that offered teachers the opportunity to describe what was seen, heard, manipulated, and experienced. This allowed teachers to go deeper than answering a simple question about lesson introductions. Participants needed opportunities to describe what was seen, heard, manipulated, and experienced when they taught math.

Questions five and six were knowledge questions focusing on participants' knowledge base and understanding of how students learned and comprehended the material. Knowledge questions focus on what participants know (Patton, 2015). I asked questions to uncover participants' experiences when faced with students challenged with a concept or math problem and what participants did with students who previously mastered a concept. According to research conducted by Mudaly and Naidoo (2015), master teachers used scaffolding in the form of the CRA approach when teaching. Mudaly and Naidoo also found that when master teachers noticed students struggling with a particular math concept or skill, they automatically turned to concrete examples and switched to representations or pictures to help with student understanding.

Questions seven through 11 referenced participants' experiences. Experiences could also be considered sensory questions (Patton, 2015). Experience questions elicited responses that described what participants did, heard, and saw when they taught. The variety of participants' experiences illuminated the use of CRA strategies when teaching. Thames and Ball (2010) found teachers who are weaker in math are also not as flexible in their mathematical understanding and use more of a procedural emphasis to provide instruction that is more repetitive, not as rigorous, or as interactive. Heck et al. (2008) found that the more hours of professional development teachers received in math, the more hands-on activities and investigative practices occurred in classrooms.

Lastly, question 12 allowed participants to illustrate their experiences in an open forum. This question elicited useful, different data and provided me with a better understanding of their experiences with the CRA framework. After talking about math practices, participants had other information to add at the end of the interview. As an administrator, I found that a final open-ended question at the end of job interviews provokes responses that reveal new understandings and information that would not be heard in the interview process. The same was true for these interviews as well. The interview questions elicited responses that allowed me to hear what participants did, heard, and observed when teaching mathematics.

Individual Interview Data Analysis Plan

Data analysis in transcendental phenomenology is a process that consists of several parts: epoché, reduction, imaginative variation, and synthesis of texture and structure (Moustakas, 1994). I used these steps to analyze data from interviews, focus groups, and journal entries. Moustakas adhered to a routine process for analyzing data. I followed an analysis method created by Moustakas (1994) to analyze data from interviews. I used Moustakas's modified method of analysis of phenomenological data to analyze all data from interviews, focus groups, and observations to be sure I followed a specific process and remained consistent when analyzing data. According to Moustakas (1994), researchers should describe and bracket out personal experiences to avoid bringing biases to the study; bracketing out biases is referred to as the epoché.

The second stage in phenomenological analysis is reduction (Moustakas, 1994). This process includes extracting the most important statements from the data, and researchers can begin to create themes that can then be categorized and grouped based on similarities (Adu, 2019). Textural descriptions are developed from these themes. Moustakas (1994) stated that a textural-structural description will emerge to reflect the meaning of the phenomenon and experience. The third stage, imaginative variation, involves developing interpretations of the possible connections among the themes and narrowing it to one that best represents data (Moustakas). Participants' experiences create textural descriptions that become broader universal textural descriptions and themes to structural descriptions (Moustakas). Lastly, synthesizing meanings will form the essence of the experience of the participants (Adu, 2019).

Before any data were analyzed, I described my experiences with the CRA framework as a teacher, a former academic coach, and an administrator and bracketed that out to avoid my biases; I did this through journaling to remain in the epoché. I used speech-to-text electronic transcriptions through Otter.ai for interviews and cross-checked them with my electronic recordings**.** After data collection through interviews, I gave participants a copy of their transcripts for member checking to ensure it was accurate. I reviewed data from participants' in-depth individual interviews and my notes during and after the interviews. Moustakas (1994) encouraged member checking to ensure accuracy.

To ensure I conducted this study according to Moustakas's (1994) routine process, I spent hours reading and rereading transcripts from participants' in-depth experiences, descriptions, and ponderings. I made notes in the margins when needed. It was important that I examined all data, allowing me to examine the phenomenon from various perspectives. I reviewed relevant statements and any expressions that were not repeated to be the horizons. I used full transcriptions from every participant in interviews, I first needed to compile any relevant statements about the phenomenon or experience and listed them all. Moustakas referred to this as horizontalization.

Moustakas (1994) stressed that all statements or expressions must be examined for two conditions; they must be essential and required for understanding the experience and possible to abstract. If these conditions are not met, or the expressions are duplicated, they must be eliminated. According to Moustakas, the core themes are clustered and checked against the record of every participant while asking two critical questions. Are the themes expressed in the transcription? If not expressed, are they similar? If they are not expressed or similar, then they must be extracted. From the core themes, the individual textural descriptions, or what each participant said, are used. Individual textural descriptions highlighted participants' words and stated common themes of their lived experiences.

I manually coded by highlighting sections of the transcripts to look for words and phrases that I noticed were repeated in interviews. When I was satisfied with found themes, I found sub-themes manually by color coding on printed transcripts. I included all participants' interview data (transcripts, notes, reflections), and worked through every sentence examining codes, themes, and sub-themes for organizing data. Moustakas (1994) speculated that coding begins when data organization begins.

Focus Groups Data Collection Approach

Focus groups may benefit research, especially when the participants interact, dialogue, and discuss a shared phenomenon, as they may illuminate different perspectives (Patton, 2015). In focus groups, researchers have an opportunity to note interactions and insights among participants that may not be readily expressed in one-on-one interviews (Creswell & Poth,

2018). Educators often dialogue in staff meetings and professional development throughout the workday. Being around their colleagues in focus groups helped participants feel comfortable and shared experiences that spurred rich discussions. My focus group questions were open-ended, encouraged discussion and dialogue between participants, and reflected the research question and sub-questions.

Two focus groups of three to five participants each were conducted with open-ended questions that spurred conversation and dialogue about teaching math with a constructivist focus (see Appendix I). Participants for each group were randomly selected and placed in one of the two groups, dependent on participant availability. Focus group questions stemmed from my central research question. Focus group participants felt encouraged to answer questions and bounced ideas off one another and propelled conversation. Follow-up questions were asked. Follow-up questions are important to determine what participants mean and allow more information about the statements in context (Dahlberg & Dahlberg, 2019). I asked follow-up questions when there was any ambiguity regarding participants' answers. All focus groups were conducted via Zoom for participants' comfortability and availability and transcribed electronically with Otter.ai. I manually checked those transcripts with my own electronic recordings. This ensured accuracy and heled me understand the experiences of the participants. Moustakas (1994) and Patton (2015) stressed the importance of in-depth interviews and observations. I took notes regarding what was said and notated any facial expressions and hand gestures during interviews and focus groups for analysis.

Focus Group Questions

1. Please introduce yourself to your fellow teaching colleagues to get to know each other if you do not already. Please state your name, your job title, and introduce yourself. CRQ
2. Tell me about your experiences when you noticed something "click" in the mind of a student or group of students when teaching mathematics. What helped the students' understanding? SQ1, 2, 3
3. What are your experiences when you have tried to help students master new skills and concepts in math? SQ1, 2, 3
4. When teaching math, describe your experiences when a student needed reteaching. SQ 1
5. Tell me about your experiences with constructivist learning, such as the concrete (hands-on manipulatives), representational (pictures and drawings), and abstract (symbols and numbers) strategies when teaching math. SQ1, 2, 3
6. What else would you like me to know regarding your experiences when using the CRA framework? SQ 1, 2, 3

Patton (2015) advocated that focus groups are interviews with the exception that participants hear others' responses and can add to their original statements. Starting focus groups with nonconfrontational questions that were easy to answer helped participants feel welcome and safe in focus groups, just as in the interviews. In question one, I wanted to help break the ice and help participants recognize commonalities in one another. Question two allowed for an open opportunity for positive answers that set participants at ease as they shared and collaborated in a social setting. Patton (2015) described focus groups as a safe place for people to talk. From my experiences as a teacher and administrator, teachers enjoy sharing and hearing positive teaching and learning experiences and often share challenges; participants did this readily.

Questions three and four were knowledge and experience questions. By asking knowledge and experience questions, I provided the opportunity for participants to share their knowledge base and experiences which highlighted similar and diverse perspectives and encouraged deeper mutual understanding. Discussing their experiences stimulated memories and other experiences of teaching that illuminated strategies used when teaching math. Mudaly and Naidoo (2015) found that master teachers used CRA strategies along with scaffolding through the support of teacher guidance. I was interested in learning if any of the participants referred to using this type of strategy when introducing a novel skill or for students who are challenged.

Question five required participants to verbalize their experiences teaching and learning with CRA strategies. Participants described rich and descriptive information. Finally, question six, allowed participants to articulate what they were thinking at the end of all previous questions and comments and allowed them to add to and enrich data. Verbalization, sharing experiences, and articulating what participants think can help increase the richness of data (Patton, 2015). Hearing how colleagues used the CRA framework encouraged others to share experiences they previously did not mention.

Focus Group Data Analysis Plan

I used the structured outline by Moustakas (1994) to analyze focus group data, just as I did in one-on-one interviews. After I took all electronically transcribed and confirmed data from focus groups and allowing participants to review their transcripts for member-checking, I manually coded transcripts, notes, and reflections. I suspended my biases as Moustakas suggested for engaging in the epoché and organized codes through reflective journaling.

Horizons were identified, repetitive comments were extracted, and themes began to emerge and solidify. I clustered teachers' descriptions into themes to help create textural and structural descriptions. As in interviews, I needed to create sub-themes by color coding on paper copies of transcripts. I used this information in Chapter Four just as I did with interview data.

Journal Prompts Data Collection Approach

Journal writing assists individuals to make sense of one's experiences (Thompson, 2011). Journal prompts are considered to be a researcher-generated document (Merriam & Tisdell, 2015). Journaling was quite different than verbalizing with another person or with a group of people. Unlike interviews and focus groups, journaling was a private internal process. The process of journaling helps express what one may find challenging to verbalize (Sheather, 2019). Journaling offered time and space for participants to reflect. Cousins and Giraldez-Hayes (2022) conducted a study where participants journaled during coaching. They found that the process of journal writing empowered individuals to participate in deep personal exploration where they felt safe and judgment free. Reflective journaling about a teaching experience using CRA strategies offered new, personal, and rich insights into participants' experiences teaching.

It is important in qualitative research to witness participants in their environments and see firsthand what participants are experiencing as it is discovery positioned (Patton, 2015). To saturate myself in their experiences, participant journals were used (see Appendix J). I had the opportunity to hear, see, and feel the experiences of the participants through their own words by reading their journals. Reading their journals helped me to better understand their experiences through the eyes of the participants. I used participants' entries from all journals as the third source of data for my study.

I asked participants to write about a lived experience using constructivist CRA strategies to teach mathematics to students. Writing about a personal experience can help researchers to gain access to participants' experiences (van Manen, 1997). I asked participants to write their journal entries electronically on their personal Google Drive account and share those with me to maintain privacy. For those who did not have a personal Google Drive account, they used another program such as Microsoft Word and sent their entry via email. I took all journal entries and placed them in an electronic file on my personal computer. All entries remained confidential, stored on my password protected computer, and backed up on an external hard drive that remained in a locked office. Pseudonyms were also used for journals. I anticipated at least three sets of data from interviews, focus groups, and journal entries from 10 participants. I kept a personal reflective journal throughout the study to help me set aside biases.

Journals

1. Please reflect on an experience using constructivist strategies such as concrete, representational, and abstract (CRA) methods when teaching math and write about that experience. Please consider using as many details as possible to help fully describe your experience. Your entry must be at least one paragraph in length. SQ1

This prompt was based on an experience question. Experience questions allow participants to share what they experienced (Patton, 2015). This included what was observed, heard, said, materials used, actions, feelings, thoughts, etc.

Journal Entry Collection

I used all participants' entries for coding and reflective analysis. These entries and my reflections were used for further analysis. It was one thing to hear about teaching experiences; it was another to read deep reflections in participants' journals. Before I began, I was reflective and noted any biases in my own reflective journal to remain bias free and better understand participants' words, actions, and experiences.

When examining participants' journals, I kept three sets of organized electronic notes on the following: raw data (participants' journals), interpretations, and personal data and reflections. Journaling helps individuals to reflect and think deeply in a safe environment and manner (Cousins & Giraldez-Hayes, 2022). Offering participants the opportunity to write about an experience teaching with CRA strategies allowed a deeper reflection on what happens in their classrooms when using those strategies and may offer new data not found in interviews and focus groups. It was imperative that participants described their experiences in rich detail. Examining participants' experiences of the phenomenon through their journaling helped me garner the data needed to investigate the specifics of their shared phenomenon.

Journal Data Analysis Plan

I followed the same analysis plan I did with interviews and focus group data. Moustakas (1994) called for researchers to participate in the epoché, reduction, and imaginative variation, which leads to synthesizing texture and structure. I noted my own biases before reviewing participants' written data. I used reduction by closely examining written journal entries several times, memoing as I read. I reviewed all coding to ensure accuracy, found sub-themes, and color coded them manually. Saldaña (2016) stated that initial coding is merely a beginning, and initial codes are provisional. Codes can be categorized and clustered, and themes will naturally emerge (Adu, 2019). Imaginative variation is used to identify emerging themes to create textural descriptions (Moustakas, 1994). I used detailed notes and reflections from participants' experiences as notated in journals to construct textual descriptions of the experiences the teachers described. I noted the structural explanations of the phenomenon through personal intuition and reflection to find structural themes from the descriptions that resulted from the reduction. I was extremely careful to suspend any of my opinions, assumptions, and judgments before, during, and after data collection. I noted them in my own reflective journal to remain free from bias and in a state of epoché.

Data Synthesis

I aimed to understand the meaning of participants' lived experiences as the research instrument in this phenomenological study. Interpreting data goes beyond the raw descriptive data; the goal of interpreting data is to look for patterns in participants' responses and make sense of it (Patton, 2015). When data begin to overlap, and characteristics of categories emerge, understanding of the phenomenon increases (Morse, 2015). According to Moustakas (1994), it includes epoché, phenomenological reduction (bracketing, horizontalization, clustering themes, and textural descriptions), moves to imaginative variation and structural descriptions, and finally, the synthesis of those descriptions. I followed Moustakas's steps to synthesize data in my study.

To analyze and synthesize phenomenological data, researchers should first strive to look at the phenomenon without bias and with as much openness as possible by recognizing their own opinions and experiences of the said phenomenon; it is referred to as epoché (Moustakas, 1994). Epoché is the initial step in the reduction procedure and is required in this type of research, as it involves the setting aside of the researcher's ideas, beliefs, and theories of the phenomenon or experience and embracing a new view or lens to see what is before the researcher. It offers the space to take in the new experiences in an unbiased manner. According to Neubauer et al. (2019), the researcher is to stand aside; no subjectivity can enter the process to inform the descriptions generated by the participants. This freedom from bias is difficult for researchers and must be consciously practiced for true openness of what is seen and heard without judgment.

Transcendental phenomenology also employs the use of reduction. The reduction stems from participants' experiences, and each experience is perceived and described in its entirety with rich detail (Moustakas, 1994). The descriptions, statements, and observations of an experience create textural descriptions referred to as "the what" (Moustakas, 1994). Observing and noticing themes and reviewing data over and over is a reflective activity and is needed to understand the phenomenon. The more perspectives, the more examination, the more correction, and the clearer the reflection becomes (Moustakas, 1994). After looking at the themes from various perspectives, constructing imaginative reflections is the next step.

Imaginative variation looks at the phenomenon from various experiences (Moustakas, 1994). The goal is to pull out the structural explanations of the phenomenon by allowing intuition and reflection to find structural themes from the descriptions that resulted from reduction. Imaginative variation uses textural descriptions to create structural descriptions; these structural descriptions are sometimes referred to as "the how" that describes "the what" of the phenomenon (Moustakas, 1994). Finally, through the synthesis of the textural and structural descriptions, or the "what and how" of the phenomenon, the essence of the experience can be explained.

I was incredibly detailed in my descriptive analysis and kept data *in situ* from interviews, focus groups, and participants' journals by following the process outlined by Moustakas (1994) above. I collected all data according to protocols designed for all interviews, focus groups, and participants' journals. Then I coded and organized the data from interviews, focus groups, and journals separately. I examined those preliminary codes from interviews, focus groups, and journal entries while I documented and organized emerging themes. Creswell and Poth (2018) suggested rereading, reviewing, and analyzing data for qualitative inquiry. I read and reread, reviewed all data and continued to do this throughout the analysis process. Patton (2015) called for three sets of data needed for triangulation. I continually looked for significant statements, made notes in the margins of the triangulated data from interviews, focus groups, and journals as I also noted the horizons.

Moustakas (1994) suggested using the textural descriptions from all data and integrating them to create universal textural descriptions from participants' experiences. I synthesized all preliminary codes and themes from all three data sources into structural descriptions to generate a greater comprehension and understanding of the experience of teaching math with CRA constructivist methods manually. I pulled from all of the teachers' textural descriptions from all three data sources, sorted them, and merged common textural descriptions manually and with the use of Excel to keep data organized to find the universal textural descriptions of the overall phenomenon. This is the "what" of the experience. To see the "how" of the experience, I used imaginative variation to understand the phenomenon's essence. I examined all data to look for structural qualities through phenomenological reduction that revealed the meaning of how teachers experienced the studied phenomenon of teaching math with constructivist strategies.

Patton (2015) called for examining all data and working back and forth among all data and the researcher's perspectives to take all the evidence and make sense of it. After I organized and analyzed all structural and universal themes, continually working back and forth, the meaning and essence of the phenomenon emerged. Qualitative research findings and results were incorporated into a full and rich description of the experiences and phenomena of the participants (Creswell & Poth, 2018). Through manual synthesis of all data and using my intuition, I integrated universal and structural descriptions of the participants' experiences with the phenomenon to find the essence of the experience. I was then able to share my findings and the limitations of the study.

Trustworthiness

Trustworthiness is simply a requirement in qualitative research; quality research and conclusions are only as good as they are valid. Validity is based on trustworthiness, external reviews, and triangulation (Creswell & Poth, 2018). To increase trustworthiness, Patton (2015) suggested researchers consistently look inward to remain bias free through reflexivity. I practiced reflexivity by journaling and bracketing out my personal biases and used triangulation by incorporating interviews, focus groups, and journal entries in this study. Lincoln and Guba (1985) recommended using four concepts to assess trustworthiness when conducting qualitative research: credibility, dependability, conformability, and transferability. Credibility, dependability, transferability, and confirmability must be evident to build trustworthiness in a qualitative study. Building positive rapport with participants is a solid foundation for rich and robust data collection. I built empathy with the participants through consistent and caring interactions to understand the participants and their experiences as much as possible.

Credibility

Credibility plays a key role in creating trustworthiness. Credibility is defined as the researcher’s ability to accurately describe the truth in the research findings (Lincoln & Guba, 1985). It also requires accurate descriptions of participants’ views and cannot aid in any vested interests of researchers (Patton, 2015). As the human instrument, it was not my place to prove a viewpoint or sway data to my own beliefs or biases. According to Lincoln and Guba (1985), prolonged engagement through systematic, extensive time spent in the field is required for credibility. I spent hours with participants in interviews, focus groups, and by reading and reviewing experiences described in journals to ensure credibility. Triangulation of data from different sources, member checks after interviews, representation of findings for verification, peer debriefing, journaling, and analyzing negative cases help maintain credibility in qualitative research (Patton, 2015). I used data from interviews, focus groups, and participants' journals for data triangulation and invited feedback through member checking in interviews. According to Lincoln and Guba (1985), member checking is critical for credibility. Additionally, I bracketed out all of my own biases and presuppositions to ensure credibility throughout all portions of this study.

Transferability

Transferability in research is the ability of readers to generalize or transfer the similarity of the current study to another case (Lincoln & Guba, 1985). Rich, thick data that is extremely descriptive of the phenomenon and robust accounts of all data collection experiences help provide transferability. I cannot prove the findings in this study. Findings should apply to other contexts, times, or even populations (Lincoln & Guba). I spent hours with participants and examined their experiences through their journal writing, collecting rich details. This data provided in-depth and detailed information from teachers' experiences, and data collected from interviews and focus groups added credibility. I included positive, neutral, and negative comments, quotes, and reflections from all data sources to offer the study transparency, authenticity, and credibility.

Dependability

Dependability focuses on the researcher's responsibility to document the entire inquiry process with accuracy and order and is also considered reliable (Lincoln & Guba, 1985). Dependability in a study should show consistent findings and be replicated by following detailed procedures and descriptions (Lincoln & Guba). I consistently preserved and accurately recorded all data. Careful documentation was done through electronically recorded interviews and focus group facilitation and was done through recorded interviews via Otter. ai and through an electronic recording devise. As I read their journals, I took copious detailed notes on my reflections. Member checking is the process of having one or more than one participant to check the accuracy of the report or account (Creswell & Guetterman, 2019). I used member checking with participants to strengthen dependability. I had participants check their transcripts for accuracy.

Confirmability

Lincoln and Guba (1985) stressed that confirmability confirms the findings of a study are based on participants' responses and not the researcher's presumptions or biases and are also considered reliable. Confirmability connects actual data to the researcher's interpretations and findings (Patton, 2015). I used peer and expert reviews of data to ensure confirmability in this study. It is important to note that participants were allowed to receive draft findings for clarification. Patton suggested that participants review drafts to determine areas of clarity, focus, and possible confusion; this allows reflexive triangulation.

Ethical Considerations

Ethical considerations are paramount in qualitative research; they must be considered and applied throughout the research process (Creswell & Poth, 2018). Participants of my study required a measured level of ethical considerations and actions that began with informed consent after IRB and site approval. I obtained informed consent from all adults in the study, and I honored the privacy and confidentiality of participants. Confidentiality of names and responses through passwords and pseudonyms must be used throughout the research (Patton, 2015). I kept all names and pseudonyms of my participants, and data associated with each, confidential and secure. In addition, all of my notes, electronic files, and memos are password protected; I kept paper files and notes in a secure locked location and will retain data for three years, after which time I will delete my electronic files and shred any hard copies.

Creswell and Poth (2018) indicated the study's details and purpose should be clearly shared with participants, and the voluntary nature of their participation must also be understood. I communicated all of this information to participants before their giving consent and agreeing to contribute to this study. I conducted interviews with participants' convenience in mind and where participants felt most comfortable. I reported all perspectives, regardless of findings, and participants received a copy of the information they provided. All local laws and regulations were followed. The potential benefits from my study included collaboration with other educators, learning from one another, and the study's findings. I communicated with participants about the potential risks associated with the study and that they were no greater than those faced in everyday life.

Summary

In this chapter, I described the methods used in this qualitative study that explored teachers' experiences of implementing constructivist strategies when teaching elementary mathematics. I used transcendental phenomenology to illuminate the experiences of participants and examined their shared common phenomenon. Data collection consisted of three sources: interviews, focus groups, and participant journal entries. I followed Moustakas's modified method of analysis and adhered to the process: epoché, reduction, imaginative variation, and synthesis of texture and structure (Moustakas, 1994). I honored the study's validity through the use of credibility, transferability, dependability, and confirmability, establishing good rapport between me and the participants and exhibiting ethical integrity throughout the study. By using a structured analysis method, I ensured all themes were examined, and the core themes surfaced with a unified statement of the meaning and essence of participants' experiences. As the human instrument, it was my job to not hinder hearing participants' experiences by having my own agenda or conceived conclusion. The methods mentioned in this chapter reflected the integrity and truth of participants' experiences while allowing reflection and new meaning.

CHAPTER FOUR: FINDINGS

Overview

The purpose of this transcendental phenomenological study is to describe elementary teachers' lived experiences of implementing constructivist strategies, specifically the concrete, representational, and abstract approach (CRA) when teaching mathematics in a midwestern school district by asking, What are elementary teachers' lived experiences implementing constructivist strategies? A phenomenological study allows researchers to examine participants' shared experiences. The central research question that guides this phenomenological study is the following: How do elementary math teachers describe their lived experiences of implementing constructivist strategies in the classroom? This chapter includes the descriptions of 10 purposefully criterion selected participants, data from participants in the form of narrative themes and sub-themes created through transcendental phenomenological reduction, data from outliers, and responses from research questions. Chapter 4 concludes with a summary.

Participants

All 10 participants in this research study were from the same midwestern public school district, and purposeful criterion sampling was utilized for participant selection. Participants were full-time educators who taught mathematics and worked with students at the elementary school level; grade levels ranged from kindergarten through fifth. Participants were sent a recruitment email from a gatekeeper, the director of elementary education, and follow-up hard copies were also shared at staff meetings at all 11 elementary schools. A total of 10 participants responded, and all 10 took part in individual interviews, seven turned in journals, and seven participated in focus groups. All participants remained in the study for its entirety. Pseudonyms were given to responding participants and to their corresponding schools to protect personal and locational confidentiality. (See Table 1).

Table 1

Teacher Participants

Teacher participants*	Gender	Years of teaching	Highest degree	Content area(s)	Grade level	School*
Aimee	F	18-22	Masters	Math	5th	Bridgeton Elementary
Ingrid	F	12-17	Masters	Math specialist	2nd-5th	Taylor Elementary
Sara	F	12-17	Masters	All content areas	1st	Bridgeton Elementary
Nancy	F	17	Masters	Math/science	4th	South Elementary
Erika	F	41	Masters	Math/science	3rd	Washington Elementary
Elise	F	5	Bachelors	Intervention specialist cross-categorical All content areas	Kdg-2nd	Bridgeton Elementary
Sabrina	F	23	Masters	Math specialist	2nd-5th	Westfall Elementary
Wendy	F	23	Masters	Math specialist	1st -5th	Birch Run Elementary
Brad	M	7	Masters	Math gifted teacher	3rd-5th	Lake Ridge Elementary
Tia	F	11	Masters	Academic coach	K-5	Bridgeton and Ellis Elementary

* Pseudonyms

Aimee

Aimee indicated during her interview she taught math for 18-22 years in Montgomery City Schools (MCS), and last year she taught fifth grade mathematics at Bridgeton Elementary. Aimee has her master's degree. When asked why she wanted to become a math teacher, Aimee responded in her interview, "I became a math teacher, but not by choice." The math position was open, she interviewed, was offered the job, and took it. Aimee reported, "I struggled tremendously in math in school, and I think part of the reason why was because my family moved in middle school." She was placed in the gifted program in her new school because she was in a similar program in her previous school. However, she explained, "The rigor was different, everything was hard for me, and I cried every day. Every day I hated it." Thankfully, her father was an engineer and helped her. Aimee stated, "Finally, in college, it all started clicking. My background motivated me to become the math teacher I am today, who teaches for understanding."

Ingrid

Ingrid taught in MCS for 23 years. Ingrid expressed in her interview that she was a Title One math specialist at Taylor Elementary, where she taught Tier II interventions to students who struggle with mathematics but have not been identified with a math disability. Ingrid has her master's degree and her intervention specialist license, so she can teach students with disabilities from kindergarten through twelfth grade. Ingrid originally served students as a math aide in a different district and was encouraged by her son's first-grade teacher to become a teacher. Ingrid's assisting and teaching experiences have been exclusively with mathematics, first as an aide then as a math specialist. In Ingrid's words, "I help kids gain confidence in math as well as social emotional learning skills and just being able to handle life better." Ingrid also taught math to high school students in the summers who needed credit recovery.

Sara

Sara taught between 12-17 years in MCS and shared in her interview she was a first-grade teacher at Bridgeton Elementary, where she taught all subjects. Sara has her master's degree. When asked why she wanted to become a math teacher, Sara replied, "I love little kids, and I love how their minds work. And I get to start building that solid foundation for math." She stated she enjoys teaching first grade because, "in kindergarten, they are learning how to do school, but in first grade, they know how to do school."

Nancy

Nancy explained in her interview that she taught math exclusively for 17 years in multiple states and districts. She taught three years out of state, seven years in another district in-state, and is in her seventh-year teaching fourth grade in MCS at South Elementary. When asked about herself, Nancy explained, "I was always a good student, had good teachers, and math had always come easy to me. I thought I might want to continue that and help replicate that for other students." Nancy enjoys the challenge math provides and declared, "Math and science were in my wheelhouse." She does not want to teach anything other than math and science. Nancy learned to teach math using the concrete, pictorial, and abstract (CPA) approach in her undergraduate education and used the CPA framework every year teaching mathematics. Nancy was the only participant who received this training in undergraduate education.

Elise

Elise said in her interview she taught for five years in MCS as an intervention specialist for a cross-categorial unit to students in kindergarten through second grade in Bridgeton

Elementary School. Elise taught math, English language arts, and social emotional learning skills to her students with two paraprofessionals who assisted in her classroom. Students had a variety of different special education identifications and learning needs. Elise has her bachelor's degree; she also taught summer school to students with special needs. According to Elise,

> I like to work with students who have high needs and need more support than typical students, and that kind of falls in line with math in that the hands-on approach, hands-on manipulatives. Those types of activities work best for students who need a concrete approach and need high levels of support as well.

Erika

During Erika's interview she shared a wealth of experience from her 41 years of teaching. She has been an intervention specialist for reading and taught various grades in several schools. Last year she taught mathematics to third-grade students at Washington Elementary School. Her students switched teachers for instruction, so Erika taught two third-grade math classes. Erika has her master's degree, and she loves teaching math. Erika was not always a fan of mathematics. Erika expressed, "I remember coming home, sitting on the kitchen counter with my dad working on math homework and crying because I didn't understand it." Erika did not solve the problems the way the teacher taught them at school, and her work was marked incorrect. Erika taught with a different mindset and explained, "I always say to the kids that I don't want any tears in math unless they're laughter. And so, the expectation is that we're growing in our math understanding and how to make kids love math again."

Sabrina

Sabrina expressed in her interview that she taught elementary classes for 23 years. Last year she served as a math specialist who taught math Tier II interventions to students in second through fifth grades who found math a challenge. Sabrina has her master's degree and was pleased that she understood math instruction through two perspectives, as a classroom teacher and as a specialist who taught intervention. When asked about herself, Sabrina explained that math did not come easy for her, and she had to work hard at it. As a teacher, she gravitated to the kids who needed that extra support. Sabrina explained, "There are some kids who need a lot of support, and I understood those kids."

Wendy

Wendy indicated in her interview that she taught for 23 years in MCS; 13 of those years were in general education classrooms in grades four, five, and six before she became a math specialist. Last year Wendy was a math specialist and taught math interventions to Tier II students in first through fifth grades. She also taught one reading intervention class last year. Wendy has her master's degree. Her goal was to close the gap between where students are currently at in their mathematical understanding and where they should be and increase learning beyond a year's worth of growth. According to Wendy, "I want to close that gap so that it doesn't get bigger or turn into a special education situation that doesn't need it." She also stated, "The biggest battle is first overcoming the mindset that math is horrible; it's too hard."

Brad

Brad explained in his interview that he taught in MCS for 5 years in elementary schools and 2 years in another state where he taught algebra. He taught mathematics exclusively for 7 years total. The last few years he taught students identified as gifted in math in grades third through fifth. He also taught in various primary classrooms introducing students to additional thinking skills and math lessons. Brad has his master's degree and is licensed to teach math in grades fourth through ninth as well as high school through twelfth grade. Brad enjoyed the demeanor of elementary students and decided to stay at the elementary level. According to Brad, “I had exposure to the gifted program in Montgomery Schools and knew I kind of wanted to go that route.”

Tia

Tia shared in her interview she taught in MCS for 10 years as a fifth-grade math teacher, and last year was her first year in an academic coaching position for a total of 11 years teaching and learning. She has her master’s degree and is licensed to teach math in grades fourth through ninth. Tia reported, “I was never good at math until I had a really good teacher in high school.” Tia became a math teacher so that she could make an impact on her students when math became more complex in the upper elementary and middle school grades. As a coach, she served students and teachers in multiple elementary grades in two elementary buildings. In this role, she taught and modeled lessons to students and teachers, helped improve their teaching craft, and assisted teachers in implementing new expectations in the classroom.

Results

The focus of this phenomenological study was to describe elementary teachers’ lived experiences of implementing constructivist strategies, specifically the concrete, representational, and abstract approach (CRA) when teaching mathematics in a midwestern school district. This study was guided by a central research question and three sub-questions. Data were collected through individual interviews and focus groups via Zoom, and journals all of which offered a variety of rich information that was later used for analysis. Data collection and data analysis included using epoché, phenomenological reduction, imaginative variation, and synthesis of textural and structural descriptions. I followed this process throughout all data collection and analysis.

Before I began to read and evaluate data, it was important that I remained ready to take in the phenomenon without personal bias. In order to remain in the epoché during data collection and analysis, I used a reflective journal so that I could set aside my personal presumptions. After transcription of interviews through Otter.ai, I listened to my secondary recordings and compared the transcripts to the separate electronic recordings to confirm interview and focus group transcripts were correct. I made corrections to any words that were transcribed in error.

I then read and reread transcripts from interviews, focus groups, and journals multiple times for accuracy and deep understanding. I began to notice patterns in the data. After reviewing and analyzing data from interviews, focus groups, and journals I created initial codes that represented the patterns and meanings I noticed. I continued to look for patterns and meaning from data and I combined codes that helped to represent the relationships and meanings. I also used horizontalization, and gave all statements equal value, and removed repeated statements.

After initial codes and open codes were developed, data were examined again for possible textural meanings and codes were manually grouped into initial themes. Important and meaningful themes were reviewed and revised. In vivo quotes that exemplified the themes emerged and were grouped together. I reviewed and revised all themes. Four themes and nine subthemes developed. Themes arose from my engagement with the data as I pursued an answer to each research question. Themes expressed the meanings of the participants and emerged through my interpretations. Themes and subthemes are outlined in Table 2.

Table 2

Themes and Subthemes for Triangulated Data

Theme	Subthemes
Mathematical understanding with concrete objects	Explain thinking Show understanding Visualizing
Mathematical concepts with representations	Teacher modeling Student modeling
Providing inquiry with abstract problems	Productive struggle Manipulative use
Recognizing needs with abstract concepts	Student needs for learning Teacher needs for teaching

Mathematical Understanding with Concrete Objects

The first theme identified through data analysis of participants' experiences using constructivist strategies when teaching math was mathematical understanding with concrete objects. Three distinct subthemes emerged from the first theme and were the following: explain thinking, show understanding, and visualizing. From interviews, focus groups, and through their journals, all participants expressed the importance of knowing their students' level of understanding of skills and math concepts when teaching. Participants wanted students to share their thinking verbally and through their actions with manipulatives, representations, and numbers to ensure their conceptual understanding, and ultimately visualizing it in their minds. Teachers were insistent that students gave evidence of how they arrived at their answers to ensure they understood the skill taught. Erika stated, "I want to *make sure* they understand the math first of all rather than just announcing they understand it, right?"

Explain Thinking

The first subtheme identified from the theme mathematical understanding with concrete objects was explain thinking. Teachers and students benefitted from students explaining their thinking to their teacher and others when completing math problems. Every participant explained that they expect students to be able to talk about their math work as they solve problems and work on concepts. These explanations helped teachers know where students were in their mathematical understanding, and it also helped other students' understanding. Sara commented, "It's one thing to tell me the answer is four. But it's another to be able to tell me how you got there." Explanation of student thinking was expected in the classroom, small groups, and one-on-one with the teacher.

Participants agreed that teachers should teach for students' conceptual understanding. It was important to teachers that students understood what they were learning and doing in math, how and why it works, and be able to explain it. Sabrina, who works with Tier II math intervention students, noted, "They'll sit there, and they'll not understand. Right? So, in here, I push them to talk." Teachers stressed they needed to know if students understood their answers or not and more importantly, why. Wendy agreed with the importance of students talking through their work, "Oh, you got the right answer, but that still doesn't prove to me you understand the rules on how to do it, but not that you understand the concept." Sabrina added,

> I find students can often look ready to move on to the representational and abstract math problems, but they are not ready. The process of teaching back to me and peers has really made my students accountable and aware of their level of learning.

Teachers emphasized the usefulness of knowing their students were ready for the next step in math so teachers would know how to proceed with their next steps in teaching. Talking about how students solved problems helped teachers gauge student understanding.

Participants noted that fellow classmates also benefitted from listening to student conversations about how problems were solved. Sara explained, “They’re listening to their classmates, and they may really tune in then.” She also lets students who are really struggling with a problem call a friend for help but refuses to let the friend tell them the answer. Sara stated, “They can give ideas of strategies to use which benefits the understanding of all.” Sabrina concurred, “Sharing out what you think might be different from what somebody else thinks, but that’s why it’s so important to share. Because how you solve the problem may be an a-ah moment for someone else.” Teachers experienced the benefits of providing students opportunities to explain their thinking as they solved problems, not only for their own teaching purposes but also for the conceptual understanding of other students.

Show Understanding

The second subtheme found from mathematical understanding with concrete objects was show understanding. Teachers garnered knowledge of students’ conceptual understanding by observing student actions with concrete manipulatives and representations such as pictures and/or models. All participants agreed that it helped them to *see* where students were in their understanding, so they knew who actually understood the problem, or not, and how to meet student needs. Aimee explained that she liked to see what students know. Aimee stated, “Show me what it looks like, then I will trust you understand.” Sabrina agreed that she wants to visibly see student understanding, “If I see he’s not getting the right answer, well, show me how you got that. And then he’ll pull out the blocks. I want you to talk out loud. Tell me what your brain was doing." Tia experienced something similar. Tia said, "I ask, what happened in your head? So, trying to make them stop doing just the trick that they had learned but pulling it back to what was happening in their brain and putting it on paper."

Brad and Aimee explained they make sure students show her their understanding, so they know students have the skill secured for future progression in math. Brad stated, "Just because they have the procedural understanding doesn't mean they conceptually understand the topic. If they don't have the conceptual understanding they will get to a point where they're going to struggle more than they would otherwise." Aimee agreed, "If they think they have the skill or concept mastered, I try to make sure that they truly understand what the concept is and that they have not just memorized it." Participants agreed that students should show what they know, but not just to the teacher. They also invited students to show their understanding by sharing their pictures and drawings with the class. Erika described,

> I have the tendency to pull up my sleeve and say, "Oh, you just gave me teacher goose bumps. Share that again. Would you mind coming up to the board or would you write on that chart what you just did?"

Visualizing

For the third subtheme, participants experienced and expressed the importance of visualization when using constructivist strategies. Teachers perceived that without the ability for students to understand math concepts through manipulatives and/or representations, students would not be able to visualize concepts in their minds. Teachers agreed that visualization could be difficult if students did not comprehend what was happening and what that looked like in a concrete and/or representational way first. Wendy concurred, "Because with math, I think unless you can picture it in your mind it is very hard to understand." Aimee also had a similar experience but with fractions, “If they don’t fully understand what a fraction is, it’s really hard for them to visualize. When I got to fraction multiplication, they were like, wait a minute. I’m looking for pieces of pieces?” Teachers agreed that students had to be able to visualize it in their heads to draw it and so they stressed that when teaching math. Tia shared what she often asks students,

> What if you were to close your eyes? What would it look like in your brain? Okay, well, I’d have five groups of two or two groups of five. Well, then draw that. Going back to picturing either in your brain or using manipulatives really helps with conceptualization.

It was important for teachers to know that students were able to visualize what the problem asked and demanded, and they taught for it.

Mathematical Concepts with Representations

The second theme identified from teachers’ experiences using constructivist strategies was mathematical concepts with representations. Two sub-themes emerged and were teacher modeling and student modeling. Through interviews, focus groups, and journals, modeling when teaching math was utilized by all 10 participants and included modeling with manipulatives, drawing representations and/or pictures, and solving equations. Students also modeled how they solved math problems when teaching and sharing with peers. Sara confirmed, “I always tell my students that good mathematicians can get the answer, but great mathematicians can tell me how. And that’s when I feel good because I have something to show them and teach them.” Teachers and students experienced the benefits of modeling in their classrooms.

Teacher Modeling

Participants described their experiences with teacher modeling while using constructivist strategies. Teachers found that students needed to see models created by the teacher to help them move from the concrete, representational, and abstract to grasp concepts. Teachers explained how the practice of modeling with constructivist strategies demonstrates to students how someone else can learn a skill or solve a problem. Modeling by teachers was differentiated based on student's needs and ability levels. Elise spoke about introducing new concepts to her students with special needs. Elise stated, "You would definitely see the 'I do, we do, you do' approach; that's kind of what we use at the beginning. And then you would see the modeling. For that, 'I do' exactly how to solve the problem." Sara parroted this and included her internal dialogue in her process of modeling to her first-grade students, "I do a lot of modeling of my internal thinking and how I thought about solving the problem. I also teach with 'I do, we do, you do' and model that."

Teachers pointed out that they used modeling when teaching math with objects, representations, and equations and then let students try it on their own. Sabrina described her math teaching, "You would see me talking a little right in front of my kids either showing them, modeling for them what they're learning, or I might be using my document camera." She explained further that she modeled different ways to make 10 and encouraged students to look at what their classmates were doing. Elise also described,

> For a learner that learns in a concrete way, being able to *show* them how to solve a problem in a concrete way is going to make a lot more sense than only showing abstract or showing it on a board or in that type of way.

Participants also modeled what students did in lessons and when solving rich tasks so that all could visually see what students described. Erika stated, "I take visual notes on chart paper. They start to see the connection between their drawings and an equation."

Student Modeling

Teachers described the purpose and benefits of student modeling in their math classes. Sabrina explained what she told her students, "This is math. And this is to help our brains make sense of it, so we can move through the problem. So, draw a quick pic. What might it look like?" Participants expressed that while students might be able to use procedures to get correct answers for problems many had no idea what they did, why they got their answers, and what the answer should represent. Teachers stressed the importance of student modeling as it gave them important information about what students know and can do. Aimee expressed, "I want them to be able to show me what fifteen-sevenths looks like. Those models are so important for that concept of what it looks like moving forward."

When students did not know how to model mathematics, participants found that helpful and informative as well because it informed teachers of next steps needed when teaching. Aimee explained,

> We were talking about changing mixed numbers into improper fractions, and I wanted them to draw what a mixed number looked like. What does that mean, and what does three and one-half look like if you draw it? Students did not know.

This discovery showed Aimee that her students needed more time and experience with concrete models and pictorial representations. After the student modeled it correctly, she asked the student, "How did that make you feel when you can explain to me what it looks like as a model and show it and talk about the pieces?" All participants agreed that teachers needed to provide opportunities for students to model mathematics as it assisted their teaching and student learning.

Participants allowed for and experienced the benefits of students constructing their own understanding when modeling. Aimee explained that she does not give her students a lot of information when having students model with representations to solve problems. Aimee stated,

> I really let them discover. They all wanted to use circles for fractions, and I said go right ahead. No, those aren't equal parts. I wanted them to discover on their own that a circle was not a good idea. So now you're gonna use a rectangle. Now show me with a rectangular fractional model. We do a lot of trial and error.

Teaching experiences and in vivo quotes from participants' interviews, focus groups, and journals were grouped together and revolved around the theme mathematical concepts with representations. Representations were discussed as pictures and models to help with student understanding; teachers and students used modeling more than any other form of representation.

Providing Inquiry With Abstract Problems

The third theme identified was providing inquiry with abstract problems and the two sub-themes were productive struggle, and manipulative use. In interviews, focus groups, and in journals, teachers stressed that they believed in providing exploration and inquiry opportunities for students when teaching with constructivist strategies, especially to assist students with abstract concepts. Tia described why she liked using an inquiry approach. Tia explained, "Building kids from a place of exploration first allows them to come from not just going straight into procedural." Wendy agreed, "It's not just that these are the rules, and you will do it. It's just with everything, understanding that why and teaching kids to ask and question."

Productive Struggle

Participants valued inquiry and exploration because it helped students construct their own learning and solidified understanding from the concrete, representations, and the abstract. When teaching mathematics, teachers experienced the benefits of providing inquiry-based opportunities to enable students to construct their own understanding. Aimee shared,

> Through discovery learning, my students started to put together the procedure to multiply fractions by using their models and by noticing patterns. As we continued to work, they used their models to check their understanding with the abstract thinking they were demonstrating.

Participants shared that they encouraged their students to explore, think, struggle, and learn from mistakes. Teachers experienced benefits in teaching and learning by allowing students to productively struggle with different types of problems and concepts before offering suggestions. They explained why that process was beneficial. Aimee shared that she thought of things differently than she did when she started teaching. Aimee said, “I think, what do I want them to struggle with? What conceptions do I want them to develop? I don’t teach by showing, showing, showing. I like them to kind of investigate a lot.” Wendy described a process she used when students were given only representational puzzles on an online program used at her school, “We’ll walk through a puzzle, kind of talking through it to see if we can figure it out. And I don’t even tell them what the topic is.” Participants wanted students to learn from their mistakes and expressed that learning from mistakes was important. Sara explained, “So I don’t just go around the room calling on kids because I want the right answer because what I want them to learn is that we can learn from our mistakes.”

Manipulative Use

Through participants’ experiences, teachers asserted that having and using manipulatives was crucial when teaching abstract concepts with student understanding, from basic number sense to more advanced math. All participants used manipulatives and mentioned their importance. Elise shared, “For students with disabilities, concrete manipulatives are so important. I was always trying to figure out how to pull in concrete manipulatives to work with their needs. My students wouldn’t be able to do concepts to mastery without them.” Elise stated, “The hands-on learning approach, hands-on manipulatives, those type of activities work best for students who need a concrete approach and need higher level of supports as well.”

Math intervention specialists reported using manipulatives daily and had many manipulatives at their disposal. Sabrina stated, “I have every math manipulative known to man, yes. So, I never feel like I have a lack of resources, which is not usual. There are many teachers who are not in the same boat as me.” Math specialists described a new kind of manipulative they were using during interventions. The blocks are already connected and color-coded to represent numbers one through nine. Wendy described the benefits of using this new manipulative in the Math-U-See program she used for interventions, “That visual of really being able to see truly what the numbers are and being able to visualize what subtraction is and what it looks like–it’s not just counting backwards.” Sabrina concurred, “Kids don’t have to count out nine individual ones cubes; they can pull out a nine block. Saves so much time; it makes so much sense. I wish our classroom teachers all had those for kids.”

Participants recognized the benefits of starting with concrete manipulatives and proceeding through the CRA framework. Brad stated, “I think it just helps students create their own understanding of the concept that they are learning. And before long you see that starting off at a really foundational concrete level of understanding–this concept does build quickly.” Teachers reported using manipulatives as much as possible due to their effectiveness. Nancy was taught how to use CRA strategies in her undergraduate schooling and recognized its benefits for teaching and learning. Nancy stated, “It is the only way I teach because it works.”

However, participants stressed that not all teachers have the manipulatives they need or the amount they need for the entire class. Most believed more manipulatives were needed in classrooms. Access to materials impacted how Nancy planned lessons. Nancy explained,

> I try my best to start with concrete and then get to the pictorial, representational, and then move to into the abstract with just numbers and equations. Of course, you know, I have to look at materials that I have at my disposal – manipulatives, anything hands-on. I have to account for that.

Nancy also described that sometimes concrete manipulatives may confuse some students, especially if they have learned a skill strictly through memorization and do not understand the concept. Nancy stated, "I've had some kids that just know their multiplication facts, but they don't really understand what it is. I have to really push them to understand what multiplication is."

Recognizing Needs with Abstract Concepts

The fourth theme identified from data analysis was recognizing needs with abstract concepts. As with the other themes, participants' in vivo quotes, explanations, experiences, and observations they shared during interviews, focus groups, and journals were coded and grouped into themes. Teachers were clear that students required specific needs to access abstract problems and teachers were adamant that they were the ones to provide those diverse opportunities in the classroom. Two subthemes that stemmed from the main theme were student needs when learning math and teacher needs when teaching math with constructivist strategies. While teaching with constructivist strategies, participants expressed there were a variety of strategies and actions that students needed for mathematical learning, as not every student learns the same way. Nancy stated, "Not every strategy will work for every student." Teachers shared that a variety of strategies and presentations were needed so that all students could get started on novel problems. Participants also expressed that they needed to provide multiple strategies for maximized learning for all styles of learners. All participants agreed that teachers must have a variety of math tools on hand and take the time to use them. Erika expressed, “I think that you need to have a variety of tools that are available to you because we all learn so differently. You can’t just do it one way in my opinion. You have to have a variety.”

Student Needs for Learning

All participants agreed that not all students learn the same way, so students need multiple entry points and access to problems to build upon prior learning to learn new skills. Sabrina explained, “I’m thinking lots of different kids need to hear it in different ways.” Access to abstract problems included constructivist methods such as using manipulatives or representations. Teachers did not want passive students when faced with a novel or difficult task. Sabrina had students use their blocks when they worked on a particular math skill and explained why. Sabrina said, “Everybody has an entry point to the problem. So, nobody’s sitting and not knowing what to do, how to begin, looking around at their neighbors.” Tia also experienced this and expressed that students need a starting point, “Even if they had not been taught how to do a particular skill, students can build on what they do know.” It was important to teachers that students used tools to help them with abstract problems.

Brad described his experiences teaching while using multiple pictures or representations to reach all learners. Brad explained, “There are other times other representations might be an easier way to go about solving a tough problem. Having all of those tools in your toolkit really lets you dive into problems and work through them.” Sara agreed about the variety of student needs,

> A lot of them just look when you are doing something really abstract. I think that you need to find everyone's avenue. How they would learn to choose best is not how I would learn best. If I was showing my thinking, I would draw pictures.

Teachers were clear that students needed to know that there were multiple ways to solve problems. Erika stated, "There's not just one way and so our brains, we all think differently and isn't that great?"

Teacher Needs When Teaching

Through their experiences teaching with constructivist strategies, teachers expressed the need to slow down and take the time to teach constructivist strategies. Teachers expressed the need to provide activities and classrooms that were conducive to learning with manipulatives and representations. Teachers mentioned the importance of offering flexibility between CRA stages when instructing students to solve abstract equations. Participant Wendy explained,

> I just think you gotta slow down to do it. Yes, we have content to cover, but I do think using different manipulatives when they actually have to conceptualize and visualize seems to be what gets skipped. But in the end, the slower you go, you end up making further progress.

Sabrina agreed, "I pulled out the blocks. Okay, well, look at what you have and tell me what number is that? So just really pulling back, slowing down, and talking through the steps. I have to do that a lot for my kids." Aimee also concurred, "We have to stop just pushing and pushing and pushing to get through everything. I'd rather make sure they're strong in 10 things rather than somewhat knowledgeable in 20." Erika had similar thoughts and expressed, "I want them to take their time. Slow and steady wins the race. Mathematicians typically are not fast. It isn't about speed, you know? I want them to fill their thinking papers. I call scrap paper thinking paper."

Teachers explained that after using tools and pictures to teach for student understanding, then students were ready to move on to the abstract. Teachers wanted students to get to the point of mastery so they could solve problems with understanding *on their own*. Nancy explained that multiplication can be hard for students, but once students understood what it represented and what it was then efficiency was the goal. Nancy described,

> I spend a lot of time in the first few months of the year having them understand what arrays are and 'groups of' for them to understand what arrays are. But by the end of the year, we need to let that go. That's not an efficient way.

Teachers also recognized that the way they organized their classrooms and lessons impacted student learning. These steps helped to foster opportunities for students to find multiple ways to solve problems with manipulatives and representations and were important for student learning. Sara explained, "I love using manipulatives, and the area that I have some struggles with is just having the right amount of manipulatives." Sara utilized small-group rotations to solve this problem. Sara said, "In order to kind of counterbalance that lack of materials, I do rotations, like a group of seven or six. It allows me to get manipulatives in everyone's hands." Erika used a different organization structure in her classroom. Erika had students leave their manipulatives, pictures, and drawings on their desks and conducted gallery walks so all students could see what others were seeing and thinking. During the walk, Erika asked, "How many vertices do you see? How many angles? In the real world, where would you see right angles in the real world?" Participants were creative in offering time and opportunities in classrooms for students to solve problems using manipulatives and pictures. Nancy expressed, "It's worth teaching that way. It takes the extra step in planning and making sure that you have materials supplies." Teachers' quotes focused on recognizing students' and teachers' needs when teaching mathematics.

Outlier Data and Findings

The outlier in this study was the use of concrete manipulatives for more than the use of the CRA framework and mathematics. A teacher explained manipulatives were also beneficial for her students' occupational therapy goals. Elise discussed the additional benefits of using manipulatives that had nothing to do with mathematics. This participant taught students with special needs. Her students had a range of learning needs she addressed all year long, including fine motor skills. Her mother was an occupational therapist, which impacted some of this participant's teaching. Elise stated, "She was always throwing manipulatives in front of me and saying that was the most important thing for learning." Through her teaching experiences, Elise found an additional benefit from using concrete objects other than helping students with mathematics. She found that manipulating concrete objects in math also helped students with the fine motor skills they needed to acquire. Elise stated,

> Most of my students require support with occupational therapy (OT), and so manipulatives are such a big part of OT, as well as fine motor support. So being able to work with them to access both math, but also fine motor at that time is really helpful.

No other participants discussed other benefits of using concrete manipulatives outside of mathematics in this study. Elise used a more concrete approach in many of the subjects she taught, possibly due to the needs of her students.

Research Question Responses

This transcendental phenomenological study was guided by one central research question and three sub questions. My research questions were designed to reveal and describe elementary teachers' lived experiences of implementing constructivist strategies, specifically CRA when teaching mathematics. Data analysis of participants' responses to the research questions and three sub questions lead to the identification of four themes (a) mathematical understanding with concrete objects, (b) mathematical concepts with representations, (c) providing inquiry with abstract problems, and (d) recognizing needs with abstract concepts. Responses to the central research question and sub questions are below.

Central Research Question

How do elementary math teachers describe their lived experiences of implementing constructivist strategies in the classroom? Elementary math teachers described their experiences of implementing constructivist strategies from a vast array of time teaching in general education classrooms, small-group interventions, reflection on their teaching practices, and their insights from those experiences. Through data analysis, four themes answered the central research question: (a) mathematical understanding with concrete objects, (b) mathematical concepts with representations, (c) providing inquiry with abstract problems, and (d) recognizing needs with abstract concepts. Teachers experienced benefits of using constructivist CRA strategies for teaching and student learning. Teachers experienced the importance of using constructivist strategies as a framework to reach all students during their mathematics instruction.

Sub Question One

How do elementary math teachers describe their experiences of using concrete manipulatives in the classroom? Sub question one was used to extrapolate teachers' experiences and perceptions using constructivist strategies with concrete manipulatives in their classrooms. One primary theme emerged from data analysis. The first primary theme (a) mathematical understanding with concrete objects, and three subthemes (b) explain thinking, (c) show understanding, and (d) visualization. Teachers used concrete manipulatives to help students make sense of the math that was taught and to create a visual of the math to assist with teaching and student understanding. See Table 3 for the open codes, themes, and subthemes regarding sub-research question one.

Table 3

Open Codes, Themes and Subthemes in Relation to Sub-Research Question One

Open Codes	Occurrence of open codes across all data points	Theme	Subthemes
Make sense	123	Mathematical understanding with concrete objects	Explain thinking
Seeing math	81		Show thinking
Talking	41		Visualization

Teachers' experiences with concrete manipulatives aligned with mathematical understanding with concrete objects. Using this strategy to teach helped teachers observe if students really understood the math they performed and assisted teachers in determining next steps when teaching. Teachers expressed the benefits and paramount importance of providing and using concrete manipulatives when teaching mathematics with constructivist strategies. Participants' experiences pointed to the value of starting with the concrete to assist and solidify student understanding. Teachers used concrete manipulatives to teach concepts and expected students to demonstrate their understanding with objects to ensure they understood. Aimee explained,

> If I just jumped right in and started teaching them things they're not ready for, that they don't have the conceptual understanding of, they're not gonna learn. They're just going through the motions. So, if you truly want kids to understand math, you have to start with the basics, which is your concrete.

Regarding constructivist strategies, Wendy stated, "The biggest thing is kids are way more successful when you take them through that concrete, the conceptual, and then move to the abstract."

Sub Question Two

How do elementary math teachers describe their experiences of using representations or models in the classroom? Sub question two was utilized to discover teachers' experiences using constructivist strategies with representations, pictures, and/or models when teaching math. One primary theme, (a) mathematical concepts with representations, and two subthemes (b) teacher modeling and (c) student modeling, emerged from data analysis. Teachers used representations to help make math they taught visible through modeling and used models and pictures to bridge the concrete and the abstract. See Table 4 for codes, themes, and subthemes in connection with sub-research question two.

Table 4

Open Codes, Themes, and Subthemes in Relation to Sub-Research Question Two

Open Codes	Occurrence of Open Codes Across All Data Points	Theme	Subthemes
Doing Math	130	Mathematical Concepts with Representations	Teacher Modeling
Using Pictures	54		Student Modeling
Scaffolding	12		

Teachers' experiences with representations aligned with mathematical concepts with representations. Teachers felt that representations using models, pictures, and drawings were a significant link between the concrete and the abstract and it was a necessary step for understanding how to complete abstract problems. Teachers used modeling to teach concepts and expected students to use modeling when learning and performing math tasks. Teachers also had students model back to teachers and classmates to solidify concepts and share visually through representations of how the problem was solved. Teachers used representations to scaffold their teaching between the concrete and the abstract through modeling. When talking about teaching students to use representations for multiplication, Tia stated,

There's more than one way to model a problem. Going back to using a picture, drawing three groups of five, or using an area model–connecting that to partial quotients so they're able to see the building blocks of how we get to a standard algorithm. Brad explained, "We kind of really focus on, here are some different picture representations. Which one do we think represents three-fourths and why? And they'll argue back and forth until we come to a consensus." Participant Aimee stated, "I don't let them move on to that more abstract thinking and using just numbers for a while. We are constantly drawing, drawing, drawing. Show me your thinking." Teachers found success when teaching concepts through modeling and was an important segue to teaching abstract concepts.

Sub Question Three

How do elementary math teachers describe their experiences of using abstract mathematical concepts in the classroom? Sub question three was designed to explore teachers' experiences using constructivist strategies when teaching abstract concepts. Two primary themes surfaced. The first primary theme was (a) providing inquiry with abstract problems and two subthemes (b) productive struggle and (c) manipulative use, emerged from data analysis. See Table 5 for codes, themes, and subthemes in relation to sub-research question three.

Table 5

Open Codes, Themes, and Subthemes in Relation to Sub-Research Question Three

Open codes	Occurrence of open codes across all data points	Theme	Subtheme
Investigate	18	Providing Inquiry with Abstract Problems	Productive Struggle
Try	28		Manipulative Use
Reteaching	15		
Rich Tasks	12		
Concrete Work	95		

Teachers' experiences with abstract mathematical concepts aligned with providing inquiry with abstract problems. When discussing the abstract stage, Aimee stated, "That's the hardest part." All participants agreed that using prior experiences with manipulatives and representations should occur before jumping to the abstract immediately. Ideally, teachers wanted students to have the knowledge and conceptual understanding of the math concept before going to the abstract phase. However, they also provided opportunities to help guide students with abstract concepts by connecting it to situations in life that were identifiable and experienced in students' lives.

To make the abstract less of a challenge, teachers posited that providing a rich task or a real-world problem was helpful. Erika explained, "We do more associating with things that we know and comparing it–and trying to make sense of it in the real world." Teachers encouraged students to try and solve abstract problems using what they know and allowed them to wrestle with rich tasks. Wendy described her experiences with teaching abstract concepts, "I think as a teacher that can be very frustrating, but I mean, at least with our kids being younger, the more we can take it back to something conceptual that they can visualize, and they could see." If students struggled with abstract problems, teachers commented they would move back to the concrete. Regarding the abstract, Erika reported, "If they're really having trouble grasping something, we can break it down into concrete again first and then kind of rebuild it."

All teachers explained that the starting point for abstract problems and equations was to begin with concrete manipulatives. Aimee insisted, "You have to start with the basics, which is your concrete." In response to teaching abstract concepts, Sabrina summed it up, "Don't jump right into that–ever."

The second primary theme (a) recognizing needs with abstract concepts and two subthemes (b) student needs for learning and (c) teachers' needs for teaching, surfaced from data analysis. See Table 6 for the open codes, themes, and subthemes regarding sub-research question three.

Table 6

Open Codes, Themes, and Subthemes in Relation to Sub-Research Question Three

Open Codes	Occurrence of Open Codes Across All Data Points	Theme	Subtheme
Showing Access	41	Recognizing Needs with Abstract Concepts	Student Needs for Learning
Connections	19		Teacher Needs for Teaching
Patterns	13		

Teachers' experiences with abstract mathematical concepts aligned with recognizing needs with abstract concepts. This included students' and teachers' needs. All participants agreed that students do not learn in the same manner. Therefore, students need multiple ways to enter problems and repeated practice until mastery. Teachers expressed that students needed multiple representations, manipulatives, and time until they could grasp novel abstract concepts and ideas. Sabrina used concrete models until students understood adding two-digit numbers with regrouping and explained, “It was eye-opening to see the different levels of learning.” Elise used counting bears, cubes, and connecting cubes to count on and count back. She found that students needed multiple methods to learn. Elise described, “Currently, the strategy that seems to work well for my student is the combination of a number line and counting bears to provide visual and hands-on support.”

Participants also agreed that teachers needed to provide the right activities and create classrooms with the materials, pace, and time for learning with CRA strategies. Nancy taught with CRA strategies for 17 years and worked with fourth- and fifth-grade students who were identified with learning disabilities and attention difficulties. Nancy knew she had to provide the right CRA strategies to reach these students. After multiple attempts, first building equivalent fractions, then coloring equivalent fractions, students could finally solve the abstract problems independently. Nancy shared, “By the end of these lessons, the students were able to find equivalent fractions on their own using solely numerical representations.” These experiences and quotes when teaching abstract concepts pointed to participants’ recognizing the needs of teachers and students when teaching mathematics.

Summary

Outlined in this chapter were the findings from this transcendental phenomenological study regarding teachers’ lived experiences of implementing constructivist strategies when teaching mathematics. The findings reflected the experiences of 10 participants who taught elementary mathematics with constructivist strategies. Four themes, nine subthemes, one outlier, answers to a central research question, and three sub questions emerged from data analysis. The four themes were as follows: (a) mathematical understanding with concrete objects, (b) mathematical concepts with representations, (c) providing inquiry with abstract problems, and (d) recognizing needs with abstract concepts. In vivo quotes were used that exemplified the themes. Results from individual interviews, focus groups, and journal reflections revealed that teachers found benefits from using constructivist CRA strategies for teaching students conceptual understanding. Participants' experiences demonstrated that constructivist strategies were an important and central part of their mathematics instruction. One participant found that improving fine motor skills was an additional benefit of using concrete manipulatives.

CHAPTER FIVE: CONCLUSION

Overview

The purpose of this transcendental phenomenological study is to describe elementary teachers' lived experiences of implementing constructivist strategies, specifically the concrete, representational, and abstract approach (CRA) when teaching mathematics in a midwestern school district. Chapter five includes five discussion subsections: (a) interpretation of findings, (b) implications for policy and practice, (c) theoretical and methodological implications, (d) limitations and delimitations, and (e) recommendations for future research. This chapter concludes with a summary.

Discussion

This study examined teachers' lived experiences using constructivist strategies when teaching mathematics. Ten participants provided the shared lived experience through three triangulated data sources, which led to the creation of four themes: (a) mathematical understanding with concrete objects, (b) mathematical concepts with representations, (c) providing inquiry with abstract problems, and (d) recognizing needs with abstract concepts. This section outlined the results of the study's findings connecting the themes and supporting the interpretations of the findings with theoretical and empirical literature and participants' experiences. This discussion covered five major subsections, including the interpretation of findings, implications for policy or practice, theoretical and empirical implications, limitations and delimitations, and recommendations for future research.

Interpretation of Findings

This section recapitulated the thematic findings of the study followed by an interpretation of the findings. Teachers' shared experiences resulted from teaching in classrooms, instructing in small groups, one-on-one teaching sessions with students, and through their observations and perceptions using constructivist strategies. Teachers found benefits for teaching and student learning when they used CRA constructivist strategies to teach math. They were able to scaffold their instruction using the CRA framework for their students. Teachers experienced instructing mathematical understanding with concrete objects, mathematical concepts with representations, providing inquiry with abstract problems, and recognizing needs with abstract concepts. Several interpretations emerged from the four themes: teachers' use of natural differentiation for students' needs, confidence in teaching mathematics, and teachers' determination to ensure learning.

Summary of Thematic Findings

Four themes arose through data analysis in this phenomenological study and include (a) mathematical understanding with concrete objects, (b) mathematical concepts with representations, (c) providing inquiry with abstract problems, and (d) recognizing needs with abstract concepts. These themes corresponded to the theoretical framework of the study. The first theme, mathematical understanding of concrete objects, consisted of three subthemes: explain thinking, show understanding, and visualization. Teachers experienced teaching for conceptual understanding and expected students to demonstrate their understanding verbally, with manipulatives, and through visualization. The theme of mathematical concepts with representations consisted of two subthemes teacher modeling and student modeling. Teachers experienced teaching concepts through modeling and encouraged students to use modeling when working math problems. The third theme, providing inquiry with abstract problems, consisted of two subthemes, productive struggle and manipulative use. Teachers experienced using exploration and manipulatives to help teach abstract problems. The last theme, recognizing needs with abstract concepts, consisted of two subthemes, students' needs for learning and teachers' needs for teaching. Teachers experienced the variety of students' needs to solve abstract problems and teachers' needs to provide what helped students succeed.

Confidence in Teaching Mathematics. Teachers experienced confidence and self-efficacy in mathematics and in their instructional abilities to teach math to students when using CRA constructivist strategies. Teachers' beliefs about their own math capabilities influenced math challenges they provide to students (Bandura, 1993). All teachers in this study expressed they enjoy mathematics and teaching it to students. Nancy exclaimed, "I've always loved the challenge of math." Quality math teachers demonstrated positive feelings about math and the knowledge of how to teach it with constructivist strategies (Kutaka et al., 2018).

Teachers shared they knew their students, math as a subject, and how to teach it with the CRA framework. They were confident they could find where the breakdown in student understanding occurred because they understood conceptual and procedural knowledge themselves and taught it daily to students. Yang et al. (2020) suggested that teacher knowledge impacted their teaching practices. Teachers in this study espoused confidence and believed they could help students learn by using CRA strategies because of their past successful experiences working with students and using manipulatives, models, and equations. Increased self-efficacy in pre-service teachers is connected to their mathematical conceptual and procedural knowledge integration (Brinkmann, 2019).

Teachers experienced self-efficacy through their explanations and in their actions while teaching and planning to teach. Regarding CRA strategies, Nancy stated, "I see it work." Teachers claimed CRA strategies benefitted students' learning regardless of their learning needs. Brad asserted, "I think it's beneficial for all students, really beneficial." This supported Zhang et al. (2020) who posited that CRA is an effective method for all students to learn math. Teachers experienced the benefits when teaching with constructivist strategies by using concrete manipulatives, representations, and abstract equations.

Teachers also expressed confidence in teaching with constructivist strategies and desired to intervene early. Teachers were confident in providing interventions and believed they needed to intervene immediately and early in their students' educational journeys, so they did not fall further behind. Wendy explained, "The sooner we get to kids, the better." Early math interventions may help primary students in preschool through second grade (Nelson & McMaster, 2019).

Teachers' experiences and beliefs in learning math and teaching math influenced how they taught students. Teachers described their personal experiences learning math when they were in school and put themselves in one of two self-described groups. One group described they were good at mathematics when younger, felt comfortable with math and wanted the same enjoyable experience for their students. Nancy loved school when she was younger and always had good experiences. According to Nancy, she wanted to "replicate that for other students."

The other group previously disliked math as students and believed they were not good at it. Several shared memories of frustration and sadness because they could not understand it. Aimee explained, "I felt stupid because I couldn't understand it, and it was because of the way it was taught to me." Even though many teachers said they were not good at math when younger, they overcame their distaste for the subject. This happened by either having a quality math teacher who helped them understand or they had another adult in their life who helped them understand. Tia explained things were difficult "until I had a really good teacher in high school." Both Erika and Aimee stated they learned math from their fathers. These teachers changed their perceptions and feelings about math because someone in their lives helped them understand it. Conceptual understanding was what teachers needed. Teachers' beliefs regarding subject content, pedagogical understanding, and class environment impacted instructional practice (Siswono et al., 2019).

Regardless of their background, all teachers in this study wanted to be sure that they taught with methods that promoted student understanding. They recognized math is not a subject most students enjoy, and they wanted to change that for students. Erika desired "to make kids love math again." Confidence in teaching math was apparent in all teachers in this study. Teachers with more teaching experience and math training were likelier to have higher self-efficacy (Alshehri & Youssef, 2022). Teachers worked hard to spread their own confidence in math to their students. Erika explained she "holds their hands as they go to the abstract, so they don't get anxious."

Although this study used participants from the elementary level, many teachers had more undergraduate math classes than typical for elementary licensed teachers. Six of the teachers were licensed to teach fourth through ninth grades, and two were licensed to teach mathematics even at the high school level. Math licensure at the upper elementary levels and beyond requires more mathematics in undergraduate schooling. The higher mathematical knowledge of teachers, the more confidence in teaching (Alshehri & Youssef, 2022).

Natural Differentiation for Students' Needs. Teachers used natural differentiation to meet students' needs through CRA constructivist methods for all levels of student understanding, including students in general education and those with special educational needs. They recognized that the CRA framework offered a natural method to differentiate and scaffold for students. Nancy stated, "I structure my class a lot with more differentiation from the beginning because I know my students." Teachers experienced the benefits and flexibility of using CRA methods to reach their students. Sara explained she had to "show them different ways of solving." Teachers used concrete objects and models to support student learning. Teachers used CRA methods because it helped them to differentiate their teaching until students understood the task and could adequately explain and show their thinking while performing mathematics. Bouck et al. (2022) found that using CRA strategies while fading teacher support, modeling, and using explicit instruction helped students move to independent work. Teachers expressed the need to move forward through the framework but only at the pace needed for each student and back up to a more visual or concrete example. Moving forward and backward along the CRA framework ensured teachers could catch students at the point of their misunderstanding. If students struggled, teachers took a few steps back. Regarding this, Wendy stated, "working back to where they do understand and slowing down, really building on that, and then moving forward." Teachers agreed that differentiation with CRA strategies met students where they were in their mathematical understanding.

Teachers perceived that using the CRA framework offered flexibility and was important when teaching to meet individual students' needs with differentiation. Teachers recognized a variety of student needs when teaching, so they remained flexible in their instructional approach. Wendy expressed, "It's just a lot of flexibility and movement." Suh (2019) posited that movement between the concrete and representational is utilized in concreteness fading when teaching abstract concepts. Teachers looked for ways they could make math accessible to all, and using manipulatives assisted with that. Elise agreed that using concrete manipulatives was "the way to access math." Manipulatives have been found helpful for all types of learners, provide accessibility, and evens the playing field (Witzel et al., 2001). Peltier et al. (2020) found that manipulatives supported students' procedural and conceptual understanding. Wendy expressed frustration “when manipulatives are taken away from kids in the classroom too early” because she saw the benefits for students who needed that scaffolding and differentiation.

Representations students created were helpful in making connections with manipulatives, pictures, equations, and understanding. Tia explained, “It’s really challenging for them to connect their model to the numbers.” Teachers found that teaching those connections with representations was vital. Sabrina stated, “When I’m modeling with pictures, I’m trying to make that connection for kids.” Flores and Hinton (2022) found that representations were important in mathematics instruction because as students move to the abstract, they can access visual pictures they previously created. Brad explained, “Creating representational models with their hands helps them significantly.” Understanding and explaining mathematical relationships confirmed previous work by Milton et al. (2019).

Determination to Ensure Learning. Teachers in this study expressed steadfast determination to ensure students understood math concepts before moving on to the next skill. Ingrid explained, “You have to go slow to go fast.” Teachers were adamant that students showed they could complete math problems and skills before moving on to new skills and independent work. Most teachers experienced working with students in small groups or one-on-one if students needed assistance and ensured students proved their conceptual understanding beyond simply getting the correct answers. Sara stated, “I am pretty stubborn; I’ll have them stay until they show me they can do it independently.” Teachers stressed that demonstrating their thinking and modeling their thought processes to students as they moved between CRA stages helped students make connections. Flores and Hinton (2022) posited that learning math through representations strengthened mathematical thinking and conceptual understanding. Teachers made requests to students such as, “Show me, make sure, prove to me, explain, draw it, etc.” and pushed students to demonstrate they understood and saw connections between concepts and CRA stages. Teachers reported students obeyed their requests and tried hard in trusting learning environments. Teachers agreed that teaching for conceptual understanding was essential for student learning because mathematical understanding builds upon itself. Teachers in this study stressed the importance of conceptual understanding in math and were determined to teach that. Suh et al. (2020) suggested that when connections between representations were not present in teaching, students did not improve (Suh et al., 2020).

Teachers were determined to keep students engaged and participating in math class and were resolved to include talking about math and modeling consistently. Sara described her goal was to “always get them to be mathematicians that can explain their thinking.” Wendy confirmed, “The more they’re actually teaching me or driving the lesson, the more successful they are.” Nancy agreed and encouraged her students to talk about math in class. She wanted to “let their voices be heard.” Teachers were insistent and agreed that students must be able to visualize math to understand it, which was one reason they used manipulatives and models when teaching. Prior research from Kaur (2019) confirmed that teachers believe modeling assists students’ visualization capabilities.

Confident math teachers taught using the CRA framework for conceptual understanding and expected students to demonstrate understanding in a variety of ways: verbally, with objects and pictures, and through visualization. Teachers used modeling and exploration through CRA strategies to teach math problems. Teachers used CRA to differentiate their teaching and were determined that all students understood math skills before moving on to the next skill.

Implications for Policy and Practice

The findings in this phenomenological study generated several suggestions for policy and practice for school districts, school leaders, and those who work with students. Recommendations support general education and special education teachers, educational assistants, academic coaches, math intervention specialists, curriculum coordinators, administrators, and district coordinators. Implications also impact public and/or private school teaching practices. This study yielded implications for policy and practice that warrant further investigation and offer insights that assist professional development, curriculum development, teacher instruction, and student learning.

Implications for Policy

The findings from this study generated several policy implications for districts and universities. Findings from this study illuminated that teachers with mathematical conceptual understanding and teaching methodologies that utilized the CRA framework were successful with teaching students mathematical understanding. Districts should consider increasing the number of teachers with strong math conceptual knowledge and methods for implementing CRA strategies in their workforce. Newton (2018) suggested that increased conceptual understanding was needed for math teachers. When teachers increased their pictorial understanding when dividing fractions, it improved their conceptual understanding (Stohlmann et al., 2020). District and administrative leaders should screen candidates well during the hiring process and seek to hire those with strong backgrounds in mathematics and math methods. Districts should look to hire teacher candidates who are knowledgeable in mathematics and their own conceptual understanding. Holm and Kajander (2020) suggested that mathematical knowledge needed to be combined with math methods teaching to increase teacher conceptual understanding.

District leaders should consider working with local universities and colleges and communicate the need for quality mathematics pre-service education in their teaching programs. Training that includes constructivist strategies, such as the CRA/CPA framework, has been successful for math teachers and students in Singapore (Urban et al., 2021) and should be included in pre-service training. This would build the mathematical capacity of new elementary school teachers. Creating this partnership would provide a win-win opportunity for both universities and school districts as more highly qualified math teachers with a CRA skillset would be ready for teaching opportunities in local districts.

School districts should adopt a district-wide professional development plan for current teachers to increase mathematical understanding and provide training with the CRA framework, its components, and how to implement those strategies. Professional development using CRA methods also increased teachers' knowledge of mathematics so that educators could build their conceptual understanding (Stohlmann et al., 2020). Therefore, it would behoove school districts to invest in continued learning for their professional workforce. What happens in classrooms depends on classroom teachers and specialists who teach students.

Results from nationwide testing showed a deficit in elementary mathematics; only 41% of fourth graders scored proficient or above on the National Assessment of Educational Progress (NAEP, 2019). New standards did not improve math assessment scores (Loveless, 2020). Teachers need quality professional development and the proper resources to meet the demands of rigor in the Common Core State Standards (Schweig et al., 2020). Professional development with concrete models increased self-efficacy and the knowledge of how and why models work as learners and teachers (Tunç et al., 2020). Increased professional development that includes evidenced-based CRA constructivist strategies may empower teacher confidence in teaching and bolster student achievement and should be considered by districts.

School districts should also consider providing specific training for math specialists as they work with students requiring intervention support in Tier II and Tier III services identified through MTSS. Students who receive Tier II or III support need supplemental evidenced-based interventions and more intensive instruction (Hinton & Flores, 2022). Teachers who work with students who struggle with mathematics should have a strong base of knowledge in both mathematics, how best to teach it; their school district should provide this training. High-caliber teachers require mathematical and pedological understanding, constructivist knowledge, and positive attitudes about mathematics. (Kaskens et al., 2020).

Implications for Practice

This research study led to several specific suggestions for school leaders to consider, such as offering local in-house professional development to staff and providing an increased supply of manipulatives. Implications impact administrators, academic coaches, intervention specialists, teachers, and students. In addition to district-level professional learning, teachers would benefit from learning from resident experts in their own schools who already teach with or have been trained to use CRA strategies. The addition of teacher collaboration could increase the understanding and use of CRA methods when teaching. Administrators should consider providing in-house professional development from math specialists and/or academic coaches. Elementary schools with full-time math coach specialists had higher fourth-grade math achievement than schools that did not (Harbour & Saclarides, 2020). Coaching and training could take place during the school day, after-school meetings, or professional development days.

It could save schools money and participants' time, and having a math mentor in the school could provide internal support that is always present for teachers trying CRA instruction.

At the building level, principals should budget for and purchase resources that support constructivist strategies, such as various concrete manipulatives, models that can be constructed and destructed, and other visual supports that make mathematical concepts visible. Schweig et al. (2020) found that math instructors needed materials and professional development to teach to the Standards of Mathematical Practices. Manipulatives help build conceptual understanding of abstract equations (Hinton & Flores, 2022). Increasing teacher and student access to manipulatives could help level the playing field so that all students can access mathematics in ways they can understand. Peltier et al. (2020) found that using manipulatives improved math achievement in students with learning disabilities and those at risk.

Theoretical and Empirical Implications

This phenomenological study examined teachers' lived experiences of implementing constructivist strategies when teaching mathematics. Participants reflected upon their observations, actions, and perceptions while teaching math with constructivist strategies such as the CRA framework. The theoretical and empirical implications of this study are outlined below.

Theoretical

The theoretical framework that guided this phenomenological study was Bruner's learning theory (1966). Bruner theorized that children learn best when constructing their knowledge through their experiences (Bruner, 1960). Continued research of children's cognitive stages of development showed that they moved through three modes of thinking: enactive action-based (hands-on), iconic image-based (pictures or images), and symbolic (abstract) language-based learning (Bruner, 1966). CRA follows the enactive, iconic, and symbolic stages. Bruner's learning theory builds upon using manipulatives, representations, and symbols while working through math concepts (Yakubova et al., 2020).

This study confirmed Bruner's learning theory through participants shared lived experiences teaching math with constructivist strategies. Teachers experienced success using CRA to help teach students' conceptual understanding of mathematics. They found the CRA framework helped their teaching and positively impacted students' learning when they used CRA strategies. Teachers believed this learning framework built students' conceptual understanding and allowed them to construct their understanding through hands-on, concrete methods, representations, and abstract problems. Teachers experienced using the progression of stages in CRA and found that flexibly moving back and forth between the stages was helpful for students to construct understanding. Constructivist teaching shifts instruction from a rigid, structured, teacher-controlled sequence and pace to a more student-centered, participative teaching and learning style (Bature & Atweh, 2020).

Teachers' experiences using concrete objects confirmed Bruner's theory of the importance of using them for student learning. Teachers described that manipulatives helped make math visible and provide entry points into difficult problems for students. This echoed early research that manipulatives provide access to problems (Witzel et al., 2001). Teachers experienced the need for using a variety of concrete manipulatives and yearned for more, so their entire class benefited, not just small groups of students. Manipulatives increase the mathematical achievement of at-risk students and those with a learning disability (Peltier et al., 2020).

This study validated Bruner's theory about the advantages of using representations and pictures for learning. Teachers highlighted mathematical connections made between objects, models and pictures, and abstract concepts when they taught. Teachers and students utilized modeling to make these connections visible. Guiding students through multiple models helped them eventually solve problems independently (Gibbs et al., 2018). Teachers also affirmed that referencing concrete objects helped students create their own images and pictures. Teachers were emphatic that pictures and models assisted student visualization and required students to explain those connections verbally while doing the problems. Pictures alone did not teach. Teachers were determined to be sure students understood the math they worked on as they completed problems. Learning through representations encouraged conceptual understanding and mathematical thinking (Flores & Hinton, 2022).

This study confirmed Bruner's theory of using the abstract stage as the final point in the learning progression. Teachers experienced fading concrete and pictorial supports when students were ready to move to independent work. However, teachers in this study wanted to ensure that students comprehended how they got their answers and why and demanded explanations before they knew students had obtained mastery and were ready to move to the abstract.

Teachers experienced the need to slow down and meet students where they were in their learning. They desired to teach to the edge of what students could do by themselves and what they could do with support from a teacher. This study validated Bruner's learning theory because teachers also used a scaffolding approach when teaching with CRA methods. Teaching to the point between what students can do independently and with some assistance aligned with Bruner's learning theory and confirmed Vygotsky's Zone of Proximal Development (Vygotsky, 1978).

Empirical

This qualitative phenomenological study about the CRA framework differed from most previous studies because the vast amount of research on CRA was quantitative in design. Most research examined the effectiveness of the CRA framework for students who struggled with mathematics (Agrawal & Morin, 2016; Bouck et al., 2018; Dubé & Robinson, 2018). This qualitative study included teachers' shared experiences when teaching using CRA constructivist strategies. Only a few studies explored teacher perceptions of CRA and its components (Purwadi et al., 2019; Zhang et al., 2022).

This study investigated teachers' experiences teaching mathematics and utilized those experiences to make meaning of their shared phenomenon. Teachers stressed the importance of teaching students the conceptual understanding of the math they performed and not simply the procedures they used to get the answers. Teachers wanted students to grasp the concepts well enough that they could explain *how* they got the answers. The goal of the CRA framework is to enable students to understand the skill they are learning and why it works (Yakubova et al., 2020). Findings from this study confirmed those previous findings.

This study also highlighted the importance of teachers pulling support away when students were ready for the next independent step when they solved math problems. Teachers moved along the CRA framework when needed, adding and/or subtracting concrete models and pictures when needed. This study confirmed the research of Bouck et al. (2022), supporting the importance of fading teacher support using CRA with explicit instruction, modeling, and moving to independent student work. This study extended the research by examining how and why teachers used the CRA framework and included the voices of teachers not found in most CRA studies.

Teachers were confident when using CRA strategies to teach because they saw it worked for their students, and all teachers expressed it was the approach they used when teaching math. This study affirmed previous research that the CRA framework is a successful method for student learning (Bouck et al., 2018; Peltier & Vannest, 2018; Zhang et al., 2020). Teachers in this study represented general education teachers from first through fifth grades. They taught intervention to students who struggled with math, students who needed special education classes, and students identified as gifted in mathematics. All teachers experienced success teaching with CRA strategies. This supported previous research that CRA methods benefited students with or without math disabilities (Agrawal & Morin, 2016; Dubé & Robinson, 2018; Mahayukti et al., 2019); in Tiers I, II, and III.

Another difference in this study from most research studies on CRA was that general education, intervention, and special education teachers taught their own students CRA strategies. Researchers did not do the teaching. In many CRA studies, researchers were the instructors (Flores, Hinton, & Meyer, 2020; Kanellopoulou, 2020). In addition, previous studies using students' teachers required teachers to have prior training on implementing CRA strategies (Milton et al., 2019; Sterner et al., 2020). Teachers in this study did not have additional training in CRA practices prior to the research.

This study illuminated everyday teaching experiences with the CRA framework and highlighted the voices of teachers who used them. This study extended Bruner's work by utilizing and examining teachers' experiences who used CRA methods to teach. The CRA framework aligned to Bruner's learning theory of enactive, iconic, and abstract methods for learning and added a qualitative focus to the body of research pertaining to Bruner's learning theory. Prior to this study there was little research that examined the lived experiences of teachers who used constructivist strategies and the CRA framework.

Limitations and Delimitations

Limitations are expected in research. Limitations are weaknesses or problems with studies that are noticed by the researcher (Creswell & Guetterman, 2019). Limitations in this study consisted of the setting by only using one school district and a small sample size. This study was conducted in a suburban school district of approximately 11,000 students. While there were other districts in proximity, being available for potential individual interviews and focus groups would have been impossible for me to personally achieve. I was a full-time employee and my work hours during and after school made my commuting and personal availability limited.

The second limitation was the sample size. The sample size of this study was small. Even with a large sample pool, I had 10 participants in this study that responded. All 10 participants stayed in the study for its entirety; however, the small sample size may not be generalizable to other settings and participants in kindergarten through fifth grade. In addition, the small sample size only contained teachers who self-identified as those who enjoyed teaching mathematics. It would have been interesting to find participants who did not enjoy teaching mathematics as much.

Delimitations are common in research and set the confines of the study. Delimitations were needed for this qualitative study and included my choice of using a qualitative design to acquire the shared experiences of elementary teachers who teach math with constructivist strategies, as there is scant qualitative research on this topic. Knowing my own background in the field of education, I chose to use a transcendental phenomenological design so that it forced me to set my own biases aside to focus on the experiences of others. This study was designed to investigate participants' experiences; a quantitative design would not have produced the data I needed for a shared lived experience and phenomenon. I chose not to conduct a case study because I wanted to obtain more teachers' experiences and voices, and I also chose not to conduct an ethnographic study as that focused on culture.

This study set limits on who participated in this qualitative study. This study focused only on kindergarten through fifth-grade math teachers who used CRA constructivist strategies. Participants also had to work in Montgomery City Schools. Purposeful criterion sampling was used to recruit participants and to ensure they met the confines of the study.

Recommendations for Future Research

In this phenomenological study, I described elementary teachers' lived experiences of implementing constructivist strategies, specifically the CRA approach, when teaching math in a midwestern school district. A small sample size of 10 participants from the same school district participated in this study. However, there was a good variety of participants: general education teachers from first through fifth grades, math specialists, a special education teacher, a teacher for gifted math students, and an academic coach. My recommendations for future research include additional qualitative studies on teachers' perspectives using the CRA framework. I suggest there be broader recruitment of participants from multiple sociographic and geographic areas. Teacher training, experiences, and materials vary greatly among rural, suburban, and inner-city schools; research conducted in all of these environments would add to research. More districts and schools should be included in future CRA qualitative studies, as that would increase the sample size and number of participants. Another recommendation I suggest is that middle school and high school teachers who use CRA strategies should be included in future studies. In addition, I recommend future research should contain observations of teachers teaching mathematics using CRA strategies. Participants may not realize the strategies they use during their teaching, and a non-participant observer may capture what happens in the most authentic environment by acquiring phrases, actions, and interactions of participants. Doing so may add new information to the field for researchers and educators.

Conclusion

The purpose of this transcendental phenomenological study was to describe elementary teachers' lived experiences of implementing constructivist strategies, specifically the CRA approach when teaching math in a midwestern school district. The theoretical framework that guided this study was Bruner's learning theory (1966). This theory was used to create a central research question and three sub-research questions. Data used in this study included teacher personal interviews, focus groups, and journals and were used to answer all research questions. Participants were selected purposefully, and all agreed to participate in this study.

Four themes and nine sub-themes emerged from data analysis. Themes included: mathematical understanding with concrete objects, mathematical concepts with representations, providing inquiry with abstract problems, and recognizing needs with abstract concepts. Sub-themes comprised of explain thinking, show understanding, visualize, students' needs for learning, teacher needs for teaching, productive struggle, manipulative use, teacher modeling, and student modeling.

This study revealed that teachers found success using CRA to help teach students conceptual understanding of mathematics. Math teachers with a strong knowledge base of understanding mathematics and how to teach it using CRA differentiated strategies were confident in their abilities to reach all students. They knew how to help a variety of learners and how to differentiate through the CRA framework to meet students' needs. Teachers also recognized the importance of using manipulatives, models, and access to various tools for teaching and student learning. Teachers used CRA components in their teaching that paralleled the three components of Bruner's learning theory: enactive, iconic, and symbolic. This study confirmed Bruner's learning theory and extended the body of work with a qualitative focus.

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Appendix A

Permission to Conduct Study

Jenny Wielinski
Doctoral Student, Liberty University

January 10, 2023

Dr. [redacted]
Superintendent
[redacted] City Schools
[redacted] OH [redacted]

Dear Dr. [redacted]:

As a graduate student in the School of Education at Liberty University, I am conducting research as part of the requirements for a doctoral degree in educational leadership. The title of my research project is Elementary Teachers' Experiences of Implementing Constructivist Math Strategies: A Qualitative Study, and the purpose of my research is to illuminate teachers' lived experiences of implementing constructivist strategies, such as the concrete, representational, and abstract (CRA) approach when teaching elementary mathematics through a phenomenological study.

I am writing to request your permission to conduct my research in the [redacted] City School district and utilize your membership list to recruit participants for my research. I would like to ask permission to contact members of your district organization to invite them to participate in my research study.

Participants will be asked to contact me to schedule an informational meeting. Participants will be presented with informed consent information prior to participating. Taking part in this study is completely voluntary, and participants are welcome to discontinue participation at any time.

Thank you for considering my request. If you choose to grant permission, please provide a signed statement on official letterhead indicating your approval.

Sincerely,

Jenny Wielinski
Doctoral Student, Liberty University
[redacted]

Appendix B

Permission Response Form

[redacted]

[redacted]

January 25, 2023

Jenny Wielinski, Doctoral Student at Liberty University, has my approval to conduct research in the [redacted] and recruit members of our organization to assist with her research study.

Please contact me at [redacted] with questions.

Respectfully,

[redacted]

Superintendent

Appendix C

IRB Approval Letter

LIBERTY UNIVERSITY.
INSTITUTIONAL REVIEW BOARD

April 4, 2023

Jenny Wielinski
Ellen Ziegler

Re: IRB Exemption - IRB-FY22-23-1123 Elementary Teachers' Experiences of Implementing Constructivist Math Strategies

Dear Jenny Wielinski, Ellen Ziegler,

The Liberty University Institutional Review Board (IRB) has reviewed your application in accordance with the Office for Human Research Protections (OHRP) and Food and Drug Administration (FDA) regulations and finds your study to be exempt from further IRB review. This means you may begin your research with the data safeguarding methods mentioned in your approved application, and no further IRB oversight is required.

Your study falls under the following exemption category, which identifies specific situations in which human participants research is exempt from the policy set forth in 45 CFR 46:104(d):

Category 2.(iii). Research that only includes interactions involving educational tests (cognitive, diagnostic, aptitude, achievement), survey procedures, interview procedures, or observation of public behavior (including visual or auditory recording) if at least one of the following criteria is met:
The information obtained is recorded by the investigator in such a manner that the identity of the human subjects can readily be ascertained, directly or through identifiers linked to the subjects, and an IRB conducts a limited IRB review to make the determination required by §46.111(a)(7).

Your stamped consent form(s) and final versions of your study documents can be found under the Attachments tab within the Submission Details section of your study on Cayuse IRB. Your stamped consent form(s) should be copied and used to gain the consent of your research participants. If you plan to provide your consent information electronically, the contents of the attached consent document(s) should be made available without alteration.

Please note that this exemption only applies to your current research application, and any modifications to your protocol must be reported to the Liberty University IRB for verification of continued exemption status. You may report these changes by completing a modification submission through your Cayuse IRB account.

If you have any questions about this exemption or need assistance in determining whether possible modifications to your protocol would change your exemption status, please email us at irb@liberty.edu.

Sincerely,
G. Michele Baker, MA, CIP
Administrative Chair of Institutional Research
Research Ethics Office

Appendix D

Recruitment Email

Dear Elementary Math Teachers, Intervention and Math Specialists, and Academic Coaches:

As a doctoral student in the School of Education at Liberty University, I am conducting research as part of the requirements for a doctoral degree in Educational Leadership. The purpose of this study is to describe teachers' experiences of implementing constructivist strategies such as using concrete manipulatives, representations/models, and/or abstract equations when teaching math to students in kindergarten through fifth grade. I am writing to invite eligible participants to join my study.

Participants must be elementary teachers, intervention specialists, math specialists, and/or academic coaches who teach math to students in kindergarten, first, second, third, fourth, and/or fifth grade in [redacted]. Participants must also use constructivist strategies by using concrete manipulatives, representations/models/pictures, and/or abstract equations (CRA) when teaching math. Participants, if willing, will be asked to fill out a brief questionnaire to gather basic demographic information, which should take about 1 minute to complete. Participants will be interviewed individually by the researcher once during the study for roughly 45 minutes. The interview will be audio- and video-recorded. Research participants will also be randomly selected to take part in one of 2 - 3 focus groups to discuss their experiences when teaching math with approximately 3-5 other participants. Focus groups may take 45 – 60 minutes to complete, will be conducted virtually via Zoom, and will be electronically recorded via Zoom and/or Otter.ai. Participants will also be asked to write a journal reflection of at least a paragraph on an experience teaching math using constructivist strategies such as concrete, representational, and abstract (CRA) methods. Completion time for the journal reflection may vary depending on participant's answers. Participants will receive a copy of their individual interview transcripts for member checking to ensure transcription accuracy and will return their approved transcripts to me through email within 4 days of receipt. Focus groups will not be member checked, but will be reviewed manually alongside the researcher's electronic recordings. Names and other identifying information will be requested as part of this study, but the information will remain confidential.

To participate, please contact me at [redacted] for more information and to set up a time for an interview.

A consent document is attached to this email and will be given to you at your next staff meeting roughly a week before the first interview. The consent document contains additional information about my research. If you choose to participate, you will need to sign the consent document and return it to me either via email or at the time of the interview before completing the demographic survey.

Sincerely,

Jenny Wielinski

Jenny Wielinski
Doctoral Student, Liberty University
[redacted]

Appendix E

Hard Copy of Recruitment Email Follow Up

Dear Elementary Math Teachers, Intervention and Math Specialists, and Academic Coaches:

As a doctoral student in the School of Education at Liberty University, I am conducting research as part of the requirements for a doctoral degree in Educational Leadership. The purpose of this study is to describe teachers' experiences of implementing constructivist strategies such as using concrete manipulatives, representations/models, and/or abstract equations when teaching math to students in kindergarten through fifth grade. I am writing to invite eligible participants to join my study.

Participants must be elementary teachers, intervention specialists, math specialists, and/or academic coaches who teach math to students in kindergarten, first, second, third, fourth, and/or fifth grade in [redacted]. Participants must also use constructivist strategies by using concrete manipulatives, representations/models/pictures, and/or abstract equations (CRA) when teaching math. Participants, if willing, will be asked to fill out a brief questionnaire to gather basic demographic information, which should take about 1 minute to complete. Participants will be interviewed individually by the researcher once during the study for roughly 45 minutes. The interview will be audio- and video-recorded. Research participants will also be randomly selected to take part in one of 2 - 3 focus groups to discuss their experiences when teaching math with approximately 3-5 other participants. Focus groups may take 45 – 60 minutes to complete, will be conducted virtually via Zoom, and will be electronically recorded via Zoom and/or Otter.ai. Participants will also be asked to write a journal reflection of at least a paragraph on an experience teaching math using constructivist strategies such as concrete, representational, and abstract (CRA) methods. Completion time for the journal reflection may vary depending on participant's answers. Participants will receive a copy of their individual interview transcripts for member checking to ensure transcription accuracy and will return their approved transcripts to me through email within 4 days of receipt. Focus groups will not be member checked, but will be reviewed manually alongside the researcher's electronic recordings. Names and other identifying information will be requested as part of this study, but the information will remain confidential.

To participate, please contact me at [redacted] for more information and to set up a time for an interview.

A consent document is attached to this email and will be given to you at your next staff meeting roughly a week before the first interview. The consent document contains additional information about my research. If you choose to participate, you will need to sign the consent document and return it to me either via email or at the time of the interview before completing the demographic survey.

Sincerely,

Jenny Wielinski

Jenny Wielinski
Doctoral Student, Liberty University
[redacted]

Appendix F

Consent Letter

Title of the Project: Elementary Teachers' Experiences of Implementing Constructivist Math Strategies: A Qualitative Study
Principal Investigator: Jenny Wielinski, Doctoral Student, School of Education, Liberty University

Invitation to be Part of a Research Study

You are invited to participate in a research study. To participate, you must be an elementary teacher, intervention specialist, math specialist, and/or academic coach that teaches mathematics in [redacted] to students in kindergarten, first, second, third, fourth, and/or fifth grade. Participants must use constructivist strategies by using concrete manipulatives, representations/models/pictures, and/or abstract equations (CRA) when teaching math. Taking part in this research project is voluntary.

Please take time to read this entire form and ask questions before deciding whether to take part in this research project.

What is the study about and why is it being done?

The purpose of this study is to describe elementary teachers' lived experiences of implementing constructivist strategies when teaching mathematics.

What will happen if you take part in this study?

If you agree to be in this study, I will ask you to participate in the following:

1. Complete a brief demographic questionnaire before participating in an interview. The questionnaire should take about 1 minute to complete.
2. Participate in an individual audio and video recorded interview with the researcher once during the study, for roughly 45 minutes. The interview will be electronically recorded.
3. Participate in one of 2 – 3 focus groups to discuss experiences teaching math with a constructivist focus with approximately 3-5 other participants. Focus groups may take 45-60 minutes, will be conducted via Zoom, will be audio and video recorded, and will be electronically transcribed via Zoom and/or Otter.ai.
4. Write a journal reflection on an experience teaching math using constructivist strategies such as concrete, representational, and abstract (CRA) methods. Time may vary depending on the participants' answers.
5. Review interview transcripts for accuracy with participants. Focus group transcripts will not be member checked but will be manually reviewed alongside the researcher's electronic recordings.

How could you or others benefit from this study?

Participants should not expect to receive a direct benefit from taking part in this study. However, direct benefits participants may receive because of taking part in this study include:

- Increased personal reflection on mathematical instructional practices.
- Collaboration time with other teachers who also teach math.
- Reflection on the impact of their instruction on student learning

Benefits to society include an increased understanding of teachers' experiences when teaching mathematics. This could possibly lead to mathematical instructional reform and/or professional learning opportunities for new and practicing teachers.

What risks might you experience from being in this study?

The risks involved in this study are minimal, which means they are equal to the risks you would encounter in everyday life.

How will personal information be protected?

The records of this study will be kept private. Published reports will not include any information that will make it possible to identify a subject. Research records will be stored securely, and only the researcher will have access to the records.

- Participant responses will be kept confidential through the use of pseudonyms.
- Interviews will be conducted in a location where others will not easily overhear the conversation.
- Data will be stored on a password-protected and locked personal computer in a locked office. After three years, all electronic records will be deleted and all hard copies will be shredded.
- Interviews and focus groups will be audio- and video-recorded and transcribed. Recordings will be stored on a password-protected and locked personal computer for three years and then erased. Only the researcher will have access to these recordings.
- Confidentiality cannot be guaranteed in focus group settings. While discouraged, other members of the focus group may share what was discussed with persons outside of the group.

Is study participation voluntary?

Participation in this study is voluntary. Your decision whether to participate will not affect your current or future relations with Liberty University or [redacted]. If you decide to participate, you are free to not answer any question or withdraw at any time without affecting those relationships.

What should you do if you decide to withdraw from the study?

If you choose to withdraw from the study, please contact the researcher at the email address/phone number included in the next paragraph. Should you choose to withdraw, data collected from you will be destroyed immediately and will not be included in this study. Focus group data will not be destroyed, but your contributions to the focus group will not be included in the study if you choose to withdraw.

Whom do you contact if you have questions or concerns about the study?

The researcher conducting this study is Jenny Wielinski. You may ask any questions you have now. If you have questions later, **you are encouraged** to contact her at [redacted] or at [redacted] You may also contact the researcher's faculty sponsor, Dr. Ellen Ziegler, at eziegler@liberty.edu.

Whom do you contact if you have questions about your rights as a research participant?
If you have any questions or concerns regarding this study and would like to talk to someone other than the researcher, **you are encouraged** to contact the IRB. Our physical address is Institutional Review Board, 1971 University Blvd., Green Hall Ste. 2845, Lynchburg, VA 24515; our phone number is 434-592-5530, and our email address is irb@liberty.edu.

Disclaimer: The Institutional Review Board (IRB) is tasked with ensuring that human subjects research will be conducted in an ethical manner as defined and required by federal regulations. The topics covered and viewpoints expressed or alluded to by student and faculty researchers are those of the researchers and do not necessarily reflect the official policies or positions of Liberty University.

Your Consent

By signing this document, you agree to be in this study. Make sure you understand what the study is about before you sign. You will be given a copy of this document for your records. The researcher will keep a copy with the study records. If you have any questions about the study after signing this document, you can contact the researcher using the information provided above.

I have read and understood the above information. I have asked questions and have received answers. I consent to participate in the study.

☐ The researcher has my permission to audio-record and video-record me as part of my participation in this study.

____
Printed Subject Name

____
Signature & Date

Appendix G

Questions for the Demographic Survey

1. **Gender**
   a. **Male**
   b. **Female**
   c. **Other**
   d. **Prefer not to report**

2. **Education – highest degree**
   a. **Bachelor's degree (e.g., BA, BS)**
   b. **Master's degree (e.g., MA, MS, MEd)**
   c. **Doctoral degree (e.g. Phd, EdD)**

3. **Years of teaching**
   a. **0-5 years**
   b. **6-11**
   c. **12-17**
   d. **18-22**
   e. **23 or more**

4. **Grade you currently teach**
   a. **Kindergarten**
   b. **Grade 1**
   c. **Grade 2**
   d. **Grade 3**
   e. **Grade 4**
   f. **Grade 5**
   g. **Multiple elementary grades**

5. **Subject(s) you currently teach**
   a. **Math**
   b. **Math and an additional subject**
   c. **All elementary subjects including math**
   d. **Elementary academic coach**
   e. **Intervention specialist**
   f. **Math specialist**

Appendix H

Interview Questions

Interview Questions:

Participant:	Role/Title:	Setting:	Date/Time:

Question:	Quotes:	Researcher's Reactions/Impressions
1. Please state your name, job title, job expectations, and introduce yourself. 2. Why did you choose to become a math teacher for elementary-aged students? 3. Describe your experiences when writing a math lesson. 4. If I walked into your classroom, what would I see when you		

introduce a new concept to your math students? 5. Describe a time when you responded to a student who did not understand a new skill or concept 6. What are your experiences when a child already understands the concept you are teaching and may have the skill mastered already? 7. Describe experiences you have with using concrete manipulatives when teaching math? 8. How do you teach conceptual		

understanding to your math students? 9. Tell me what it looks like and sounds like when you use representations and modeling for math instruction. 10. Describe your experiences with teaching abstract concepts to students. 11. Why do you use constructivist strategies (CRA) when teaching mathematics? 12. You have shared a lot with me during this interview; thank you. What more would you like to add regarding your		

experiences with the CRA framework and its components?		

Appendix I

Focus Group Questions

1. Please introduce yourself to your fellow teaching colleagues so you can get to know each other if you do not already. Please state your name, your job title, and introduce yourself.
2. Tell me about your experiences when you noticed something "click" in the mind of a student or group of students when teaching mathematics.
3. What are your experiences when you have tried to help students master new skills and concepts in math?
4. When teaching math, describe your experiences when a student needed reteaching.
5. Tell me about your experiences with constructivist learning such as the concrete (hands-on manipulatives), representational (pictures and drawings), and abstract (symbols and numbers) framework when teaching math.
6. What else would you like for me to know regarding your experiences when using the CRA framework?

Appendix J

Journal Prompt

Name:	Date:
Students' Grade(s):	Math Topic:
Please reflect on an experience using constructivist strategies such as concrete, representational, and abstract (CRA) methods when teaching math and write about that experience. Please consider using as many details as possible to help fully describe your experience. Your entry must be at least one paragraph in length.	