THE DIFFERENCE IN GEOMETRY ACHIEVEMENT BETWEEN STUDENTS WHO UTILIZE LFM STRATEGIES AND THOSE WHO DO NOT: A NONEQUIVALENT CONTROL-GROUP PRETEST/POSTTEST STUDY

by

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Liberty University

A Dissertation Presented in Partial Fulfillment

Of the Requirements for the Degree

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ABSTRACT

The purpose of this quantitative, quasi-experimental nonequivalent control-group pretest/posttest study was to identify whether there is a difference in Geometry achievement between students who utilize Learning for Mastery (LFM) strategies compared to those who do not at the high school level. This study provided insight into instructional and assessment strategies that may increase student achievement, understanding, and retention. Seventy-three high school mathematics students enrolled in Geometry courses, grades nine through twelve, from a suburban high school in Northwestern New Jersey were assessed in this study. Students were given a pretest, participated in daily classwork and instructional strategies for ten weeks, and given a posttest. Data were analyzed using the analysis of covariance and descriptive statistics. The study showed that, while there was an improvement in the experimental group's mathematical achievement, there was no statistically significant difference in student achievement in mathematics between the control and experimental groups (p = .120). The researcher discusses the implications of the results and calls for additional research into the effects of LFM on student achievement in high school mathematics.

Keywords: mathematics, mathematics education, mastery learning, achievement, secondary education

Dedication

To my husband who challenged me to a competition of who could earn the highest score on our grad school entrance exam and continues to push me to achieve new goals – Thank you for always having my back! I love you.

Acknowledgments

First, I would like to acknowledge my team who worked with me throughout this journey: Dr. Steven McDonald and Dr. Amy Jones. They have been so helpful in providing me feedback and challenging me to push my ideas further from start to finish.

I also cannot thank my colleague enough for allowing this study to take place in her classroom. Her openness to the idea helped me to further develop my study.

Lastly, to all of my family and friends, thank you for checking in on me throughout this entire process and for being the first ones to say that they will see me at graduation. I will see you there.

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List of Abbreviations

Learning for Mastery (LFM)

Mastery-Based Testing (MBT)

Personalized System of Instruction (PSI)

New Jersey Student Learning Assessments-Mathematics (NJSLA-M)

Partnership for Assessment of Readiness for College and Careers (PARCC)

New Jersey Student Learning Standards (NJSLS)

Analysis of Covariance (ANCOVA)

CHAPTER ONE: INTRODUCTION

Overview

The purpose of this quantitative, quasi-experimental nonequivalent control-group pretest/posttest study was to determine if there is a statistically significant difference in Geometry achievement between students who utilize Learning for Mastery (LFM) strategies and those who rely on traditional strategies in high school mathematics. Chapter One provides a background for the topics of LFM, student achievement in mathematics, conceptual understanding, and retention. The background includes an overview of the theoretical framework for this study. The problem statement examines the score of the recent literature on this topic. The purpose of this study is followed by the significance of the current study and the research questions. The chapter closes with a list of key terms and their definitions.

Background

While achievement in mathematics correlates to opportunities that are important in life (e.g., employment or social and psychological well-being), less than half of students in the United States are proficient in mathematics (Begeny, et al., 2020). Mathematics perpetually builds on itself. Yet, mathematics content standards and standardized testing often have teachers rushing through the curriculum, skimming the surface rather than making sure students have mastered the content before moving on. A different way of teaching and assessing must be established to raise the achievement level in mathematics across the country. Mastery-based learning and testing is an instructional strategy that helps students master content that was not understood the first time it was taught (Farrell & Marsh, 2016). In this method, students can retake assessments after additional instruction emphasizing essential concepts and reducing test anxiety (Collins et al., 2019). Mastery-based instruction allows students to learn from their mistakes to reach understanding (Linhart, 2019). Bloom's Taxonomy and growth mindset support this instructional method.

Historical Overview

Mathematics teaches skills like scientific inquiry to deductive reasoning. In ancient times, mathematicians were high priests because they could explain the world around them using mathematics (Mastin, 2020). Today, mathematics education provides a place for students to develop critical thinking, problem-solving, and deductive reasoning skills that they will use throughout their lives (Ayalon & Even, 2010). However, the depth of knowledge taught in secondary mathematics education has been argued going back to 1915 when William Kilpatrick determined that Algebra and Geometry were being taught to too many students. Kilpatrick believed that only top students should have the privilege of learning these concepts because they are not functional skills for most people (Klein, 2002). As decades passed with arguments on what was and was not important in mathematics education, the 1983 report, *A Nation at Risk*, found education was not pushing students to achieve higher, college remedial mathematics course enrollment increased by 72%, and complaints from business and military leaders about the billions of dollars that needed to be spent on remedial education programs for their employees (Klein, 2002). Following this, emphasis was put on mathematics education by creating standards for schools and teachers to reach and follow in their curriculum. Standards have continued to change since then, but poor mathematics achievement still exists across the nation.

Benjamin Bloom proposed mastery learning in 1968, suggesting that teachers should use assessments as a learning tool to provide feedback and corrective action for students who may not have performed well (Guskey, 2005). Assessments should be used formatively instead of at a unit end. The goal here is to inform students of what they have mastered versus what skills they still need to work towards. Bloom proposed that students who have reached mastery of all skills on the original assessment should engage in enrichment activities while students who have not reached mastery should participate in corrective action and reassessment prior to moving on to the next unit (Guskey, 2005). In 1987, Slavin described three adaptations of Bloom's original mastery learning plan: Personalized System of Instruction (PSI), continuous progress, and LFM. All three forms of mastery learning require supplemental materials for corrective action for students who do not originally master the content followed by reassessment. The difference is how this corrective action is implemented and what the rest of the class is doing while it is happening (Slavin, 1987). Mastery learning allows students to see their learning progress and how the process can affect their academic achievement (Bloom, 1971). Learning from their mistakes is also an important growing opportunity for students because it allows their brains to form new connections. LFM is set up so that students can learn from their mistakes providing them with the opportunity for more brain growth. LFM turns the mistake mindset from one that suggests failure to one that is praised and suggests learning achievement (Boaler, 2013).

Society-at-Large

Mathematics plays an important role in education and society. Students need to be presented with the structures taught in mathematics to become strong problem-solvers and critical thinkers (Sriraman & Lesh, 2007). As students move into higher levels of mathematics education, their math anxiety increases as their motivation to learn math decreases. This poses a problem for society because mathematics education is an integral part of students becoming successful, contributing citizens (Teske, 2010). Therefore, educators must begin to provide the necessary supports needed to change students' mathematical mindsets.

Middle and high school grades mathematics achievement has been called into question. The mathematics performance in the United States has been below average among other countries in recent years, but the gap in achievement comes largely within the United States itself. The gap in performance exists between White and higher socioeconomic students and their minority and lower socioeconomic counterparts (Slavin et al., 2009). This is a concern for many reasons. One of those being that the mathematics content understood and retained in secondary education predicts a nation’s economic potential (Slavin et al., 2009). However, policymakers often do not give new teaching interventions and programs the time that they need to be deemed a success or a failure (Stigler & Hiebert, 1999). Therefore, instructional strategies must be reconsidered and be given the time to increase mathematics achievement in all students across the United States.

Theoretical Background

Bloom’s Taxonomy and Dweck’s theory of mindset emphasized the importance of this study because they speak to how students understand and learn new information. Bloom’s (1956) Taxonomy uses a hierarchal model to categorize learning objectives, while Dweck’s (2016) theory of mindset discusses how a student’s mindset affects their learning.

Bloom’s cognitive theory encompasses learning goals that require students to demonstrate conceptual understanding of content and analyze what they have learned (Murphy, 2007). The taxonomy is organized in the order of basic learning goals to advanced learning goals (Bloom, 1956), accentuating the need for critical thinking, understanding, and retention as well as the process it takes to get there (Murphy, 2007). Learning objectives require an understanding of facts and principles that students then interpret into other ideas. In Bloom’s taxonomy, these learning objectives are categorized in his hierarchal model in the levels knowledge, comprehension, application, analysis, synthesis, and evaluation—where knowledge is the most basic learning objective and evaluation is the most advanced (Murphy, 2007). Working through Bloom's Taxonomy helps students to accomplish content mastery (Furst, 1954) because students must master each level of the hierarchy before moving on to the next (Bloom, 1956).

Dweck's mindset theory originated as a social-cognitive approach to motivation (Dweck & Leggett, 1988). This theory expands upon motivation goals through fixed and growth mindsets. Dweck and Leggett (1988) discuss that individuals either have a helpless response to obstacles or a mastery-oriented response to obstacles. Those who have helpless responses have a fixed mindset and performance goals while those with a mastery-oriented response have a growth mindset and learning goals (Dweck & Leggett, 1988). The difference between the two is simple: those who have a fixed mindset believe they have set abilities, while those who have a growth mindset believe they can change their abilities with learning. However, it is possible for individuals to change their mindset (Dweck, 2016). Once an individual has a growth mindset, she is more likely to take on challenges, make mistakes, and learn from both to acquire more knowledge and intelligence (Dweck & Leggett, 1988). Helping students to achieve a growth mindset can result in goal achievement and academic success (Haimovitz & Dweck, 2017).

Bloom's Taxonomy describes the learning process that students need to take to achieve mastery, while Dweck's mindset theory describes the proper mindset needed to get there. In mathematics, students often get lost in a skill while the class moves on. Bloom and Dweck's theories suggest that mastery learning could lead to increased academic success, understanding, and retention. Teachers need to praise the process behind student academic performance to build a growth mindset (Haimovitz & Dweck, 2017) as students work their way through the learning goals of Bloom's Taxonomy.

Problem Statement

Student achievement in mathematics has been below average for several years yet learning and understanding mathematics is an important aspect in developing people that can problem solve, think critically, and contribute to society (Begeny, et al., 2020). Mathematics also builds on itself. Therefore, when students do not understand prior concepts, they struggle to learn new ones. This study will look at how incorporating Learning for Mastery (LFM) may increase student overall academic achievement in high school Geometry. The research suggests that there is a gap in the literature around LFM and its effects on student achievement in high school mathematics courses (Ellis, 2019; Linhart, 2019).

The concept of reteaching and retesting is often found in Individualized Education Programs (IEPs) as a method to help classified students learn and understand topics that they may not have performed as well on previously. Mastery-learning techniques have been shown to benefit students with learning disabilities in understanding mathematics (Marita & Hord, 2017). Linhart (2019) asserts that allowing students to take just one reassessment can help students to understand the benefits of correcting mistakes and revisiting their prior thoughts. However, there is still a need for research to determine if this test strategy increases student achievement, conceptual understanding, and retention at the secondary level (Linhart, 2019). Many studies surrounding this idea cannot infer causality because they are correlational. Therefore, a study should be at least quasi-experimental to determine if the teaching and testing strategy LFM affects student mathematics achievement in Geometry (Mouratidis, et al., 2018). As previously stated, there is a gap in the literature surrounding the research that has been conducted to determine if implementing LFM in a high school Geometry classroom is able to increase student mathematics achievement. The problem is that the literature has not fully addressed how LFM and reassessment affects student mathematics achievment in high school Geometry courses (Begeny, et al., 2020; Ellis, 2019; Linhart, 2019; Mouratidis, et al., 2018).

Purpose Statement

The purpose of this quantitative, quasi-experimental nonequivalent control-group pretest/posttest study was to identify whether LFM increases student mathematics achievement in the high school mathematics setting. One high school mathematics teacher incorporated LFM as part of her instruction throughout the school year in a high school Geometry class while some students utilized the strategy and others did not. The teacher gave instruction to the whole class at the same pace. Students were given a pretest at the start of the intervention and a posttest at the end of the intervention.

The independent variable was the instructional method LFM involving two groups: the experimental group who utilized LFM and the control group who did not. The dependent variable was high school Geometry student achievement. The covariate was the use of pretest scores. Students were placed into groups based on their teacher. The teacher implemented the independent variable strategy of LFM. Students had the opportunity to choose whether to utilize the intervention of LFM strategies throughout the study or not. Student achievement was measured using student scores on an assessment made from a bank of state assessment questions. Results on this assessment were compared between students who utilized LFM strategies throughout the study and those who did not to determine if there was a significant difference in student achievement in Geometry.

Significance of the Study

The study of mathematics is different than other subjects in that each topic builds upon previous information and foundational skills year after year. When students miss skills throughout their mathematical careers, they fall behind on new material. A solution to this problem could be implementing mastery learning. Pierrakos (2017) suggests that mastery learning can help students learn how to solve complex problems and encourage them to have grit and perserverance. Therefore, reteaching and retesting in high school general education mathematics could help the students who underperform on summative assessments properly learn the current skill as to not fall behind in future skills. This study was unique in that it specifically worked with high school mathematics students in a general education classroom to determine best practices for helping students learn and understand mathematics while being conscious of the time spent by both students and teachers in the process. Regardless of the results of this study, there is reason to expand the study to other schools and school districts.

Implementing LFM can also help to reduce test anxiety. If students do not show mastery on the original assessment, they are given the opportunity to receive corrective action in order to achieve mastery on a reassessment. Collins et al. (2019) found that implementing LFM and Mastery-Based Testing (MBT) techniques helps students reevaluate their study habits in order to decrease test anxiety. In fact, MBT is used in several state and federal licensing exams (Linhart, 2019), allowing candidates to take the assessment more than once to learn from mistakes and reach mastery.

This research study wase important because looked at the effects of LFM specific to the secondary mathematics general education classroom where it is not normally seen. This study was significant for teachers because it suggests that changing the traditional methods of instruction and assessment to LFM may increase student achievement in high school mathematics while also being feasible for teachers to implement. Regardless of the results of this study. using LFM with more advanced math concepts may enhance student achievement in

Geometry and implementing these strategies on a regular basis could help increase math performance across the country and grade-levels.

Research Question

As discussed in the problem and purpose statements, LFM helps to improve achievement levels among college-aged students and in different content areas. However, little to no research has been conducted in a high school mathematics classroom (Begeny, et al., 2020; Ellis, 2019; Linhart, 2019; Mouratidis, et al., 2018). For this study, ninth, tenth, eleventh, and twelfth-grade students in Geometry courses were the participants. Teacher instruction focused on using LFM to attempt to increase student achievement levels. Therefore, this quasi-experimental nonequivalent control-group study looked to find the answers to the following questions:

RQ: Is there a difference in *Geometry achievement scores* between students who *participate in LFM strategies and those who do not* when controlling for pretest scores?

Definitions

The following terms are pertinent to the study:

1. *Learning for Mastery (LFM)* – LFM is when instruction is given to the whole class at the same pace, and students are assessed at the end of the unit. Students who do not meet the standard for mastery take corrective action on their own time to master content before reassessment to determine if mastery was achieved (Slavin, 1987).
2. *Mastery-Based Testing (MBT)* – MBT is a form of testing that uses a grading technique that determines student mastery of individual standards (Lewis, 2019).
3. *Mathematics Achievement* - Mathematics Achievement is the expertise shown by a student in mathematics (Pandey, 2017).

4. *Retention* – Retention is the ability to recall information learned previously at a later date (Meconi, 1967).
5. *Understanding* – Understanding in mathematics is grasping concepts, operations, and relations while knowing what symbols, diagrams, and procedures mean (Kilpatrick & Swafford, 2002).

CHAPTER TWO: LITERATURE REVIEW

Overview

A systematic review of the literature was conducted to explore the problem of retention and conceptual understanding in high school mathematics and the role that mastery learning plays across other disciplines. This chapter will present a review of the current literature related to the topic of study. In the first section, the theory of mindset and Bloom's taxonomy will be discussed, followed by a synthesis of recent literature regarding mathematics education, assessment, and motivation to succeed in learning. Lastly, the literature surrounding mastery learning, the theory behind it, practice, and mastery testing will be addressed. In the end, a gap in the literature will be identified, presenting a viable need for the current study.

Theoretical Framework

This research study uses the theory of mindset and Bloom's Taxonomy as theoretical frameworks. Dweck's (2016) theory of mindset discusses how growth and fixed mindsets affect student learning. Bloom's (1956) Taxonomy categorizes learning objectives using a hierarchal model, with basic knowledge and comprehension being the most important at the base and more advanced understanding at the top. This is crucial to mathematical understanding and retention because students must understand prior knowledge before moving on to more advanced skills.

Theory of Mindset

Carol Dweck's theory of mindset stemmed from her 1988 work with Ellen Leggett. In this article, Dweck and Leggett (1988) discuss a social-cognitive approach to motivation theory as a precursor to growth versus fixed mindset. The original theory examines helpless response and mastery-oriented response to determine how students are motivated to learn best. The authors found that individuals who have performance goals tend to have a helpless response—

this is when an individual avoids challenges and has decreased performance when presented with obstacles—and individuals with who have learning goals tend to have a mastery-oriented response—this is when an individual seeks out challenges and learns from failure (Dweck & Leggett, 1988). In Dweck's (2016) theory of mindset, she expands upon these goals and responses by assigning mindsets to them. She posits that people either have a fixed mindset or a growth mindset. People who have a fixed mindset believe that their abilities are set, whereas people who have a growth mindset believe that their abilities can be changed through learning (Dweck, 2016). Those with a fixed mindset avoid and self-sabotage when faced with a difficult task, give up when faced with unfavorable feedback, and become complacent when faced with favorable feedback. When their ability is praised, people with a fixed mindset have their belief that talent and ability are unchangeable reinforced. Examples of high achievement make people with a fixed mindset feel threatened and their successes undervalued. Students with fixed mindsets feel that the purpose of learning activities is to show their ability in the content. People with growth mindsets feel the opposite way. Growth mindset people respond to challenges with an eagerness to learn, feedback of any kind looking for ways to improve, praising strategies and effort, examples of high achievement with inspiration to succeed, and learning activities as the opportunity to improve their abilities (Campbell et al., 2020).

In education, when students have a fixed mindset, they believe that they are born with a certain amount of intelligence, and it will not change. But, when students have a growth mindset, they believe that their intelligence is pliable and can change (Kapasi & Pei, 2021). This theory can offer an explanation to the underachievement in science, technology, engineering, and mathematics because students with a fixed mindset avoid challenges which reduces leaning in an effort to cover up any lack of understanding so that they may still be perceived as ‘smart’ especially if they used to excel in these content areas previously (Campbell et al., 2020).

Dweck (2016) suggests that individuals are not stuck in one mindset for their entire lives. Individuals can change their beliefs about risk and effort and what causes success and failure (Dweck, 2016) by using interventions that are used to promote growth mindsets in students (Kapasi & Pei, 2021). In education, achieving a growth mindset can be helped by teachers and parents praising the process that led to student academic success rather than the success itself (Haimovitz & Dweck, 2017). It is important to consider a student’s mindset because it can influence that student’s behaviors and psyche, like reaction to failure, level of effort put forward, expectations of success, and level of persistence. All of these factors impact academic achievement (Kapasi & Pei, 2021). In examining the process of goal setting (performance and learning goals), goal operating (helpless and mastery-oriented responses), and goal monitoring (negative emotions and expectations), a meta-analysis by Burnette et al. (2013) concluded that mindset matters in goal achievement.

Haimovitz and Dweck (2017) examine how mindset affects student achievement. They found that students with a growth mindset are more likely to be successful in academics than students with a fixed mindset because growth mindset students believe that they can continue to grow and increase their own abilities (Haimovitz & Dweck, 2017). Students with a growth mindset see challenges as an opportunity to improve and learn. They enjoy the process of learning and have an increased motivation to do just that (Kapasi & Pei, 2021). In contrast, fixed mindset students believe whatever they were born with is as much as they will ever be able to accomplish (Haimovitz & Dweck, 2017). While teacher praise itself does not necessarily affect student achievement (Brophy, 1981), the praise of the process of learning does (Haimovitz &

Dweck, 2017). For teacher praise to be effective, it must be in consequence to a student's academic performance, be specific, and be sincere (Brophy, 1981). Student beliefs about their own intelligence affect motivation and academic achievement. Elementary psychological interventions can affect mindset to increase academic achievement without making institutional changes (Haimovitz et al., 2011). Haimovitz et al. (2011) conclude that "encouraging a malleable mindset may help to sustain children's intrinsic motivation, thereby enhancing both academic success and life-long learning" (p. 751). School psychologists are in a good position to encourage and promote growth mindset in vulnerable student populations like those with learning difficulties and mental health concerns. These populations tend to be at-risk for low academic achievement. Research has shown that promoting growth mindsets in these students can have an important impact on student achievement (Kapasi & Pei, 2021).

Bloom's Taxonomy

Bloom's (1956) Taxonomy is used as a cognitive theory while playing a role in emotional and physical learning. It discusses different levels of cognitive ability while denoting what types of learning objective require a higher level of cognitive ability and which do not. The higher the level required of the learning objective, the deeper the learning and understanding of the content (Adams, 2015). Its aim is to classify educational goals and objectives using cognitive behaviors (Ullah et al., 2020). The original model is comprised of six learning objectives—each level designed to be more elevated than the last (Prasad, 2021). These learning objectives include knowledge, comprehension, application, analysis, synthesis, and evaluation, with knowledge being the most basic learning objective and evaluation being the most advanced (Bloom, 1956). However, a later revision of the model renamed some of the levels and changed the order of others. This revision names the levels as remember, understand, apply, analyze, evaluate, and create. The purpose of this revision was to place more emphasis on the synthesis of the skill or content learned and less emphasis on evaluation (Adams, 2015). Educators use these levels as verbs to describe their student expectations for critically thinking and understanding for each learning objective as well as to write test questions that assess higher-order thinking skills (Stanny, 2016). Assessments need to access each of the six levels of Bloom's Taxonomy in order to assess both student retention and their ability to be thinkers (Prasad, 2021).

Murphy (2007) describes these learning goals as cognitive operations that require students to demonstrate a depth of knowledge and understanding of the content to critically analyze what they have learned. The taxonomy was created to determine the most effective ways for students to learn and for teachers to meet these needs rather than rely on a textbook or universal curriculum. The idea is that students need to master each level of a learning goal in the model before moving on to the next using activities, formative assessments, and enrichments. This theory originated from Benjamin Bloom, an educational psychologist who formed a committee to identify a system of cognitive functioning. The committee began to work on creating a list of educational objectives. They organized the objectives in order from basic learning goals to advanced learning goals, arguing that the basics must be mastered before educators can expect students to complete the advanced goals (Bloom, 1956). Murphy's (2007) review of Bloom's Taxonomy is a theory that emphasizes the need for critical thinking and the process involved to get there. This is why educators use Bloom's Taxonomy as guide when writing student learning objectives and describing their content goals for students (Stanny, 2016).

Students need to accomplish a certain amount of content mastery (Furst, 1954), and working through Bloom's Taxonomy will help them accomplish that. Students can only apply what they know, which calls for retention (Horrocks, 1946). By moving through Bloom's

Taxonomy, students must first learn the content through observations, listing, locating, and naming (Bloom, 1956). In the knowledge level, students define each standard in the content (Ullah et al., 2020) and questions are asked specifically to test whether students have an understanding of the standard (Prasad, 2021). Then, students will move up the pyramid into comprehension, where they are asked to make sense of the information that has been presented (Bloom, 1956). In this level, students can explain each standard in the content (Ullah et al., 2020) and are asked to interpret the standards they have learned (Prasad, 2021). Third, students move into the application process, where they are asked to apply what they have learned in new but similar ways to how the content was originally presented (Bloom, 1956). In the application level, students apply each standard in context (Ullah et al., 2020) and questions are asked to make an argument for why students can apply the stand in such a context (Prasad, 2021).

On the top half of the pyramid, students move into the analysis level, where they are asked to dissect what they have learned and explore new relationships (Bloom, 1956). In this level, students breakdown the content by using several standards at the same time (Ullah et al., 2020) and are required to make observations on where the standards can be used in a different context (Prasad, 2021). Then, students move into the synthesis level, where they are asked to use what they have previously learned to create something new (Bloom, 1956). In the synthesis level, students create their own problems using the standards (Ullah et al., 2020) and are asked questions about the content like design, build, and change (Prasad, 2021). The most advanced level of the pyramid, evaluation, asks students to examine what they have learned and draw conclusions (Bloom, 1956). In the final level, students judge different problem criteria based on the standards they have learned (Ullah et al., 2020) and evaluate data to draw a conclusion about the standard in practice (Prasad, 2021). These levels must be considered in student learning and transfer of learning (Horrocks, 1946).

Bloom's Taxonomy describes the learning objectives and goals that students should complete as they master new content. Student learning objectives serve a purpose in education. Their main goal is to describe the knowledge and skills that students must master by the completion of a course. They also help design and align curriculum with learning activities and assessments (Stanny, 2016). However, often in mathematics, students get "stuck" on a topic while the rest of the class moves on. The reason for this is often that students lack the background knowledge that they should have had before entering their current course (Ullah et al., 2020). Implementing a form of mastery learning through reteaching and reassessing will hopefully allow students to master new content through each of the learning goals in Bloom's taxonomy. The goal is that each student will make it to the top of the pyramid and master the most advanced learning goal of evaluation. Bloom's Taxonomy pushes educators to think of learning objectives in the form of the verb heading each of the six levels—what students can do with the learning objective—and pushes educators to develop higher-order thinking questions which lead to a deeper understanding of the content (Adams, 2015).

Related Literature

Mathematics Education

Mathematics itself is described by structures. Students need to be exposed to these structures as early as possible to develop strong problem-solving techniques and critical thinking skills (Sriraman & Lesh, 2007). Mathematics learning plays an important role in expanding the deductive reasoning skills in students that can be applied in all aspects of their lives (Ayalon & Even, 2010). In fact, seven out of ten adults feel that a better mathematics education would lead to a more competitive economic advantage. Therefore, a majority of adults believe that students should complete at least geometry and algebra in high school and about a third of those adults believe high school students should be completing trigonometry and calculus, as well (Lucas & Fugitt, 2009). However, as students mature in their mathematics education, their motivation to learn math decreases as their math anxiety increases. Mathematics education must turn around because it is an essential aspect of how students learn to be successful citizens (Teske, 2010). Math teaches students general argumentative skills and systematic aspects of reasoning (Ayalon & Even, 2010). It is important to switch the mathematical mindset of students and teachers' beliefs to make mathematics learning possible for all students for those reasons.

Achievement in mathematics education has also been in question over the last several years, specifically in the middle and high school grades. Several people believe that the mathematics content learned and understood in secondary education is a major predictor of the economic potential of a nation (Slavin et al., 2009), but many high school students do not have the mathematics skills needed to be successful on college entrance exams or in a high-skill career (Stone et al., 2008). Almost one third of college freshmen in the United States are not prepared for college-level math even though high school graduation requirements have increased (Long et al., 2009). While students' mathematics performance in the United States has been below average in recent years, it is improving. However, the problem in secondary mathematics achievement is within the United States itself. Specifically, there is a huge gap between the performance of White and higher socioeconomic students and minority and lower socioeconomic students (Slavin et al., 2009). Something must be done to improve mathematics education to increase the achievement level for all students across the United States.

The 2005 National Assessment of Educational Progress results found that 37% of high school seniors performed below a basic mathematics levels and 23% of high school seniors performed at or above the proficient level in mathematics on the exam (Stone et al., 2008). Studies have shown that a lack of interest in mathematics is a contributing factor to low student achievement in mathematics. Often the disinterest is a product of students finding the subject difficult, feeling a lack of support during instruction, or finding the content boring. While there has been an increase in the number of students who are enrolled in advanced high school mathematics courses, only about one third of high school students in 2004 graduated having taken precalculus or calculus and about 22% of college students need to take a remedial mathematics course. Because mathematic ability portrays a growing role in careers in today's technological world (Stone et al., 2008), it is important for educators to do make a change in how mathematics is taught at the secondary level.

The first step to increasing mathematics performance is to be sure that prospective mathematics teachers understand the content that they are teaching. Secondary mathematics teachers need to understand the fundamentals of mathematics as well as what comes before and after each topic in the curriculum (Sriraman & Lesh, 2007). Once educators understand how secondary mathematics content connects, they can begin trying different instructional strategies to help reach all their students. Some of these strategies include cooperative learning, metacognitive instruction, individualized instruction, mastery learning, and comprehensive school reform (Slavin et al., 2009). The National Council of Teachers of Mathematics (NCTM) started to reform mathematics instruction in 1989 to emphasize experiences and exploration to solve complex problems rather than mimic a set of rules (Battista, 1994). More instructional strategies must be considered to help improve secondary mathematics instruction in the United States. Some research has also shown that taking more mathematics courses and more advanced mathematics courses increases student mathematics performance on standardized tests, high school graduation rate, and college admission (Long et al., 2009).

Mathematical Mindset

Using Dweck's mindset theory as a framework, the proper mindset is an extremely important factor in learning, specifically in mathematics. Students with a growth mindset learn more effectively because they desire a challenge and can learn from failure. Students with a fixed mindset are less likely to work outside their comfort zone, thus rarely pushing themselves beyond their current abilities (Boaler, 2013). Dweck's (2016) research found that about 40 percent of United States students have a growth mindset, 40 percent have a fixed mindset, and 20 percent have a combination of the two. Other studies have shown that students in middle school with a growth mindset have greater brain plasticity, which allows for greater potential in developing their intelligence while closing the mathematic achievement gap between the genders and races (Boaler, 2013). The myth that some people are "math people" and others are not is just that. Neuroscience research has shown that mathematics is a subject and skill that is learned through practice and effort. Studies have shown that all students have the ability to grow brain pathways that allow for mathematics learning to occur. It is important to promote growth mindsets in mathematics because fixed mindsets lesson a student's ability to build these mathematics pathways in the brain (Anderson et al., 2018).

Given this knowledge, teachers need to be aware of how they handle the mistakes made in their classrooms. Mistakes are an important part of the learning process, although students do not see them that way (Boaler, 2013). Teacher praise is an important part of the process, but how teachers give that praise is even more important. Teacher praise must extend beyond the simple support of a correct response (Brophy, 1981). Teachers should praise the process of learning from mistakes and failures, which should help to instill a growth mindset in all students (Boaler, 2013). Applying this idea in a mathematics classroom should help students understand that their mistakes are okay and change their mathematical mindset leading to smaller achievement gaps and increased performance in mathematics.

Teacher Beliefs

In the United States, the teaching profession works like a machine. Other countries do not necessarily share that belief. Many teachers in the United States see learning terms and practicing skills in mathematics as not exciting and try to mask those with other distractions (Stigler & Hiebert, 1999). Studies in mathematics education have shown that mathematics teachers' beliefs about teaching and learning in a math classroom are just some of the resources that educators use while teaching (Clark, et al., 2014). Educators' own beliefs about understanding and using mathematics can affect their students' beliefs. Educators must be confident in their mathematic abilities in order to help their students see themselves as capable in learning mathematics. Unfortunately, teachers (often at the elementary level) do not see mathematics as having a purpose, being fun, or being understandable (Chapman & Mitchell, 2018). It is just as important for teachers to have a mathematical mindset as it is for the students. Teachers must embrace the idea that mathematics should inspire growth and promote thinking and learning—especially at the elementary level. Some research has found that these educators teach mathematics with fear themselves because they were told at some point in their own schooling that they are unable to "do mathematics" (Chapman & Mitchell, 2018). Other studies have shown that teacher beliefs may be affected by Cognitively Guided Instruction (CGI). Schoen and LaVenia (2019) found that teacher beliefs fall markedly into one of four categories: Cognitive Constructivist, Direct Transmissionist, Facts First, and Fixed Instructional Plan.

Teachers who have Facts First and Fixed Instructional Plan beliefs are under the impression that they should stick close to the scope and sequence of mathematics textbooks. The authors identify that while many teachers still believe that they should tell students how to complete math problems, research has shown the opposite (Schoen & LaVenia, 2019). Ultimately, a teacher's mindset impacts the learning tasks that are offered, how class discussion is organized, assessments, and how she responds to mistakes (Chapman & Mitchell, 2018).

While teachers in the United States see individual student differences as a challenge in running their classrooms, other countries view these differences another way. Specifically, Japanese teachers believe that individual student differences in a class are valuable because they allow teachers to incorporate previous student responses to problems and questions as learning experiences in the classroom (Stigler & Hiebert, 1999). Clark et al. (2014) researched the relationship between teacher characteristics teacher and beliefs about mathematics teaching and learning. Their study found that upper-elementary teachers with strong mathematical content knowledge believe that students should struggle with problems before intervening. In contrast, teachers of the same age level with weaker mathematical content knowledge feel less confident in this because they are less likely to be able to help students through the struggle. These authors suggest that engaging students in challenging mathematics at the upper-elementary levels prepare students for learning challenging mathematics in the future (Clark et al., 2014). However, this belief is not the same at the middle school age level. Because of the fast curriculum pace, state testing, and tracking, middle school mathematics teachers tend to believe that it is not practical for students to struggle and that lower-tracked students cannot perform as well as their higher-tracked counterparts (Clark et al., 2014). Little research has been found on the beliefs of high school mathematics teachers.

Achievement

Students in the United States have been performing below mathematics achievement standards for decades (Teske, 2010). The achievement gaps exist among different subgroups of students, as well (Guskey, 2007). This is a problem because mathematical skills and thinking are used in becoming productive members of society, and they are continuing to get more difficult throughout K-12 education (Teske, 2010). In fact, achievement gaps among different groups of students have been a concern to political and educational leaders for year. In the 1960s, President Johnson homed in on the inequities of achievement in education among students from economically advantaged backgrounds versus those who were not. Politics attempted to close these gaps with the Economic Opportunity Act of 1964, the Elementary and Secondary Education Act of 1965, Title I programs, and No Child Left Behind of 2001 (Guskey, 2007). Ma and Kishor (1997) suggest the middle school grades have the largest impact on helping students develop a positive attitude towards mathematics leading to a higher level of mathematics achievement. Additional research shows that greater mathematical understanding leads to more positive attitudes towards mathematics; therefore, increasing student understanding should lead to greater student achievement (Ma & Kishor, 1997). To attain greater levels of student achievement in mathematics, students must master the unit of ten, fractions, and abstract concepts. However, students tend to struggle the most with fractions and algebraic concepts (Teske, 2010). This suggests the need for different strategies to help students master these three concepts to increase student mathematics achievement. Benjamin Bloom recommended closing the gap in achievement by reducing the differences in student learning outcomes. He saw that giving students the same instruction and time to learn resulted in a variation in how much students actually learned (Guskey, 2007).

Strategies & Practice

The success of mathematics education is built from effective teaching strategies and practices. There are various strategies to choose from, and there is no end-all-be-all of strategies. A popular teaching strategy, especially in an unsuccessful lesson or if students are still not understanding, is reteaching the lesson. However, this is only beneficial if reflection is used in a non-superficial way to revamp the lesson to be sure that more students have gained an understanding than the time before (Ganesh & Matteson, 2010). Farrell and Marsh (2016) researched how data can affect the reflection and reteaching process. They found that teachers respond to assessment data in the following ways: reteaching in a similar mode of instruction and retesting; using small group instruction; asking students to reflect on their scores; providing extra help; and changing the mode of instruction (Farrell & Marsh, 2016). Other teaching strategies have been shown to be beneficial for students both with and without learning disabilities in learning mathematics. These strategies include problem-based learning, systematic instruction, visual representations, cognitive processing (Marita & Hord, 2017), and proficiency-based learning (Fergus & Smith, 2022). The goal in implementing any combination of these teaching strategies and practices is to provide all students with the ability to gain access to an understanding of all mathematics at the high school level.

Often the above-mentioned strategies fall into the following teaching styles: cooperative learning, metacognitive strategy instruction, individualized instruction, mastery learning, and comprehensive school reform. Cooperative learning incorporates students working in groups to master content but being assessed individually. Metacognitive strategy instruction involves students working in small groups and asking themselves questions out loud to comprehend and make connections between material. Individualized instruction has students work through content at their own pace while meeting goals established by the teacher (Slavin et al., 2009). Mastery learning is a form of reteaching and retesting used to help students master content that was already taught but not understood the first time (Farrell & Marsh, 2016). This approach is meant to help all students master class objectives and standards prior to moving on to the next (Slavin et al., 2009). Proficiency-based learning falls under the guidelines of mastery learning. This form of instruction sets clearly defined learning objectives, uses a grading system that communicates how much content a student has progressed through rather than how much content a student knows are one particular time, and measures student progress based on the objectives completed instead of as a comparison to other students. Like mastery learning, students are able to reassess to demonstrate their learning and can do so through a variety of methods (Fergus & Smith, 2022). Comprehensive school reform is a larger, more complicated picture to paint because it does not happen on the classroom level. This requires the school to implement programs to help with mathematical learning and understanding and looks different from school to school (Slavin et al., 2009). For this study, mastery learning will be used to determine if it helps students learn and retain mathematics.

Studies using preservice teachers have found that their ability to reflect, receive feedback, and Notice are important aspects to them applying the most effective teaching strategy for any given situation. One study found that preservice teachers who reflected on peer lessons were able to see a change in their pedagogical content knowledge and skills. They were able to adjust pacing, timing, teaching strategies, and lesson sequencing from reflecting on previous lessons (Ganesh & Matteson, 2010). This idea falls under the Japanese lesson design of lesson study, where teachers reflect upon previous lessons and then repeat them. The idea of Noticing is also important for preservice teachers to grasp because it is the ability to recognize student thinking and learning. Amador, Carter, and Hudson (2016) found that when preservice mathematics teachers are extremely focused and engaged in Noticing, they are able to pinpoint details about students' procedures in solving mathematics problems. Giving effective and appropriate feedback is one of the most powerful ways to increase student achievement. It can be given verbally or in writing and is most effective when it points out what needs to be done moving forward and helps students understand how much of the learning goal they have met (Fergus & Smith, 2022). Reaching preservice teachers in their methods programs about ways they can grow to use different strategies to help all students can help increase mathematics achievement for all students.

Assessment

Throughout education, there have been several different types of assessments used to measure student academic success and understanding. Some assessments suit the learning styles and personalities of some students better than others. Different assessment types include formative, summative, oral, written, and performance. While these are just a few, educators have been looking at assessments to measure student learning and have been trying to find the best ways to do that for centuries (Bennett, 2011).

Formative vs. Summative Assessment

Two major types of assessment in secondary education are formative and summative assessments. Formative assessments are a focus in education because they are supposed to measure where students are in the knowledge base of the curriculum being taught. Formative assessments provide student feedback throughout each stage of the learning process, what students still need to know. Summative assessments measure what students have achieved at the end of a unit or course (Bennett, 2011).

Summative assessments can provide students with a valuable learning experience because of the preparation process. These types of assessments can strengthen the representation of material needed to be retrieved for tests as well as increase rates of content knowledge retention. Summative assessments can also help teachers to identify which students need follow-up instruction and reassessment while identifying which teaching strategies are most effective (Bennett, 2011). Some researchers argue that summative assessments are too dominant in education and that there should be more assessments surrounding assisting learning and student feedback. Most importantly, though, summative assessments need to emphasize the skills, knowledge, and attitude that are recognized as most important by educators, districts, and standards (Black & William, 1998).

Formative assessments can let teachers know how instruction should be changed in the future while showing teachers what their students know. These assessments can inform teachers of progress in student achievement (Bennett, 2011). Formative assessments do not need to be tests but can also be as simple as an exit card or survey. However, they do need to provide information that can be used by students and teachers to determine what steps need to be taken next in the learning process. Researchers suggest that the effectiveness of formative assessments is dependent on the content of the feedback and the learning opportunities they provide. Changes to formative assessments must be made so that they do not provide superficial feedback or promote memorization (Black & William, 1998).

Classroom Practices & Strategies

There are several different ways that summative and formative assessments can be implemented in the classroom. Studies have shown that while there is no statistically significant difference between standards mastered by male and female students, they have shown that male and female students perform differently based on the type of assessment given (Lewis, 2019).

Oral assessments are the oldest form of assessment and require students to give spoken responses to questions from the teacher. However, this form of assessment is rarely used lately, even though it does have several academic advantages. Oral assessments develop student communication skills, are more authentic because they provide students experience in interviewing and defending their ideas, and are more inclusive to different races, genders, and learning abilities (Huxham et al., 2012). For example, women tend to underperform on traditional, in-class, written assessments but excel in a traditional setting verbally answering questions (Lewis, 2019). Students with dyslexia also prefer oral assessments because they do not need to focus on whether their writing is legible. Oral assessments encourage critical thinking and prevent plagiarism. Studies have shown that these types of assessments also help relieve test anxiety and suit some learning styles and personalities better than others (Huxham et al., 2012).

Written assessments tend to be preferred more by educators but less by students. For educators, written assessments take less time to implement. There is also a smaller chance of bias and greater reliability with a written assessment. Oral assessments are not anonymous and can bring out different unintended biases. Written assessments also tend to lend themselves to assessing more complex and abstract ideas (Huxham et al., 2012). While there have been some studies involving giving a traditional in-class assessment followed by an untraditional oral reassessment, little has been researched to show how reassessments improve grades using a similar type of assessment as the original.

Regardless of whether the assessment is oral or written, standards-based grading can be used to help decrease mathematics anxiety, especially at the middle school level. Being able to take a reassessment has been found to relieve this anxiety. One of the most distinguishing characteristics of proficiency-based learning and grading is the ability for students to take feedback from one assessment, learn from it, and reapply it to a new assessment. In traditional assessment, teachers instruct students, assess learning, and assign a grade based on how much of the learning can be regurgitated. In proficiency-based grading, students learn from teachers, practice, receive feedback, assess, and reflect (Fergus & Smith, 2022). While research has been conducted in the middle school mathematics setting using proficiency-based grading and assessment, little research has been done in the high school mathematics setting.

Motivation

Another topic in education that is discussed by educators includes student motivation. Motivation is the process of initiating and sustaining goal-directed activities. Goal orientation theory explains motivation in terms of mastery goals and performance goals. Mastery goals stem from a growth mindset and stimulate learning, whereas performance goals stem from a fixed mindset and stimulate fear of failure (Cook & Artino, 2016). In this theory of motivation, individuals either seek to document their abilities and not discredit them (performance goals) or to increase their abilities and master new skills (mastery goals) (Elliott & Dweck, 1988). Because learning, motivation, and identity are closely related, educators find themselves trying to integrate different cognitive theories, like achievement motivation, in their classrooms (Summers, 2006).

Performance goals promote a fixed mindset and motivate students to maintain the status quo. Some research has shown that students with performance goals build a helpless response to failing. These students ask themselves if their ability is adequate (Elliott & Dweck, 1988). Mastery goals promote a growth mindset and motivate students to take risks and learn from their failures. Some research has shown that students with mastery goals will increase their abilities over time and react to obstacles with a mastery-oriented response. These students ask themselves how they can best master the task at hand (Elliott & Dweck, 1988). Students who have mastery goals are more likely to be motivated to learn new and more challenging tasks.

Achievement

Using student motivation to determine and increase student achievement is an important aspect to consider in education. Teachers can observe student motivation in their student's behaviors and engagement. Achievement motivation is defined by a student's persistence to pursue learning activities, engagement in learning activities, and performance on learning activities. Student motivation can determine how diligently students work on tasks and if they will pursue challenges (Dweck & Elliott, 1983). Achievement motivation changes as students grow and mature physically, cognitively, and emotionally. Social norms affect how students perceive school and acceptance from their peers. Student values may change both positively and negatively from what they previously were, depending on what their goals were and what they are in the present moment. Studies have shown that students who are intrinsically motivated are more likely to be higher achievers than those who are extrinsically motivated (Dweck & Elliott, 1983). Determining how to increase intrinsic motivation in students should lead to an increase in academic achievement.

Data

There is a push to use data to help motivate students to learn and achieve. Some researchers argue that data will encourage students to put in the extra effort. Others feel that data will help students understand their own strengths and weaknesses and how to improve. Students with performance goals pay greater attention to their grades and achievement. They are more likely to compare themselves with their peers (Farrell et al., 2015). This can lead to negative future outcomes if students do not feel like they can measure up to their peers. However, students with mastery goals are more concerned with developing new skills and increasing their understanding. They are more likely to use data to increase their effort because they believe that greater effort leads to greater success (Farrell et al., 2015).

Teachers tend to promote performance goals when using data to make decisions for students. This can foster fixed mindsets in their students. Instead, teachers can create growth mindsets by using data to focus on individual improvement and to reward the process rather than the grades themselves (Farrell et al., 2015). Using data in this way can help students to change their mindsets and focus on mastery goals. Therefore, increasing motivation and achievement.

Mastery Learning

In traditional education formats, teachers expect a normal distribution in their grades, with a third of their students learning the content well, a third learning the content sufficiently, and a third just getting by or failing (Bloom, 1971). This format can stifle teacher aspirations and student desire for academic success and increased learning. Instead, if educational formats shifted to the use of mastery learning, teachers could utilize prior understanding, factual knowledge, and self-monitoring to help students learn (Donovan & Bransford, 2006). Over the last several years, there has been an influx of support for mastery learning. In this method of instruction, students must prove that they understand a certain level of proficiency before they can move on in the content (Lineberry et al., 2015). Mastery learning requires the teacher to determine a standard level of performance that represents mastery of the content, assess frequently, and provide corrective instruction for students who may not have mastered the concept the first time (Slavin, 1987). Although, the success of any educational intervention is affected by how well an educator understands how learning happens (Campbell et al., 2020).

Mastery learning increases intrinsic motivation for learning because it teaches students that they can learn the material and reach their goals. In mastery learning, all students have the opportunity to earn an A because they are graded based on their mastery of the content standards rather than compared to their peers. (Bloom, 1971). Mastery learning achieves this by allowing the learning time between students to vary (Lineberry et al., 2015). Slavin (1987) describes three main types of mastery learning. These include Personalized System of Instruction (PSI), continuous progress, and Learning for Mastery (LFM). In PSI, students work on self-instructional materials to learn the content while teachers give supplemental lectures to guide learning. Unit objectives and assessments are established at the start of the course, and students can take the assessments as many times as they need to earn a passing score. In continuous progress, students work on units at their own pace and are provided with activities to help correct their learning if they are unable to meet the standards the first time being assessed. In LFM, the teacher gives instruction to the whole class at the same pace. Students are assessed at the end of each unit. If students do not meet the standard for mastery, they take corrective action on their own time in the form of tutoring or small group instruction. Then students may take a reassessment to determine if they have achieved mastery (Slavin, 1987). Regardless of the type of mastery learning, formative assessments are used throughout the learning process to help students pace their learning and understand what they need to focus on more. Using formative assessments appropriately allows students to master each learning standard before moving on to the next. When students show mastery on these assessments, they know that their learning approach and study habits are strong enough to lead them to success. Mastery learning also promotes lifelong learning. By allowing students to see their progress and how the process affects the outcome, students can apply these habits throughout their lives (Bloom, 1971).

Theory

Learning and understanding prior knowledge are essential aspects of mathematics education. Failure to do so impacts a student's ability to learn future concepts. Therefore, the theory behind mastery learning is relatively simple: instruction should be created to ensure that virtually all students learn each skill and standard in a hierarchical order so that students have the prerequisite knowledge needed so they may learn the next skill. Mastery learning theory posits that instructional time and resources should allow all students to achieve mastery of content (Slavin, 1987). Benjamin Bloom stated that when there is little variety in instructional time and method among different student, there is typically larger variation in how much each student learns. Some students require more instructional time and different instructional methods than others in order to appropriately learn material (Guskey, 2007). The idea of mastery learning is not new in education. Throughout the 1920s and 1930s, educators required students to show their mastery of each lesson before they could move on to new material. The two most influential types of mastery learning to the theory are PSI and LFM because they require teachers to create many short units and students to take formative assessments at the end of each one (Kulick et al., 1990). In particular, studies support that LFM has a positive impact on exam scores, retention, grades, and attitudes towards learning, particularly in mathematics and science (Guskey & Pigott, 1988).

Mastery learning theorists argue that instructional time should vary per student, and the standard of achievement should be constant in education rather than the instructional time being the constant and achievement the variable as it is in traditional instruction. When instruction varies according to student need, each student has the opportunity to master the concept (Guskey, 2007; Kulick et al., 1990). However, this idea can create large gaps in the content covered between higher-achieving students or quicker learners and lower-achieving students or slower learners (Slavin, 1987). Although, Bloom suggests that the separation in the time needed to master concepts only occurs at the beginning of a course. Once students master the fundamentals, they do not need to spend as much time mastering future concepts (Kulick et al., 1990). Bloom suggested that this would help increase student achievement among diverse groups of students and that the instructional variation and differentiation were essential to learning (Guskey, 2007).

Practice

The teacher's contribution to student learning is undervalued in mastery learning and its research. Often, this is due to teacher pushback on implementing new methodologies and strategies because they are unsure of how they work. The idea of having students move forward at their own pace also sounds like a bit of a logistical nightmare on the grading end of the scale (Ellis, 2019; Linhart, 2019). If teachers understand what and how it is being done when implementing mastery learning, they are more likely to implement the strategy. However, mastery learning procedures do contribute to increased student academic achievement (Martinez & Martinez, 1988). If implemented appropriately, mastery learning can support the development of students, give them the ability to solve complex problems, and apply what they learn to real life. Mastery learning encourages students to have grit, persevere, and collaborate (Pierrakos, 2017). The purpose of mastery learning is to measure academic growth in each student instead of individual differences in their abilities (Zimmerman & Dibenedetto, 2008), so it is important to investigate practices that lead to the successful implementation of mastery learning. Individual tutoring is one practice that research has shown to increase student achievement by two standard deviations over students receiving traditional instruction. While mastery learning can be part of the solution to closing acheivement gaps, it does have practical problems in application (Pelkola et al., 2018)—like taking too much time for some students to learn the material (Slavin, 1987).

Mastery learning often presents itself in practice as reteaching and reassessing content. It requires teachers to support students who did not master the originally assessed concepts through post-instructional teaching strategies (Bellert, 2015). Mastery learning is a concrete way to implement competency-based education frameworks at a variety of age levels. It can also help educators be more accountable and effective in teaching their content well the first time (Lineberry et al., 2015). The reteaching strategy is a second chance for students to learn the concepts and teachers to refine their instructional strategies. Reteaching is most effective after implementing a formative assessment to learn what students understand and what still needs to be mastered. Effective reteaching uses both direct and strategic instruction; deliberate and monitored practice; and reassessment (Bellert, 2015). In reassessment, it is a good idea to use specifications grading. In this type of grading, teachers determine if a student's work meets required specifications to master a concept by assigning a pass/fail grade to that question. Incorporating specification grading upholds a high academic standard, reflects student learning outcomes, motivates students to learn and excel, discourages cheating, reduces student stress, makes students feel responsible for their grades, gives students usable feedback, makes expectations clear, and fosters higher-order thinking and development (Nilson, 2015).

Formative assessments are an important practice in mastery learning. They allow for teachers to adapt their instruction to meet the needs of students who did not originally master the concept at hand. Incorporating these practices gives both student and teacher timely feedback about progress and the opportunity to ensure each student reaches concept mastery (Zimmerman & Dibenedetto, 2008). In LFM, assessing students regularly with formative assessments is key and students should be able to correctly answer atleast 90% of problems on the test in order to demonstrate mastery. When a student is unable to meet that standard, that student requires further instruction before assessing again (Pelkola et al., 2018). Mastery learning has been shown to increase both student achievement and motivation (Martinez & Martinez, 1988), but more work needs to be done surrounding secondary mathematics education.

Mastery Testing

Traditional exams are high-stakes and tend to cause test anxiety. Because of this, many educators have called the effectiveness and fairness of high-stakes exams into question. The test anxiety, in particular, is caused by students not necessarily learning at the same pace and, therefore, are not ready to be tested at the same time (Harsy & Hoofnagle, 2020). Suppose students do not show mastery of content on a particular assessment, that grade follows them throughout the course—even if the material is eventually understood. The National Center for Educational Achievement determined that there are two characteristics that the highest performing schools have in common: the configuration of curricular needs for students in order to introduce, develop, and master content and assessments that must be mastered at each level as a prerequisite of moving onto the next level (Harsy & Hoofnagle, 2020). Test-taking is an integral part of mathematics courses as it helps students and teachers assess learning. However, how these tests are structured has a great impact on the study habits of students. Mastery-Based Testing (MBT) has been explored in some mathematics classrooms to do just that. MBT assesses concepts in small chunks allowing students to focus on a few challenging peaks rather than an entire landscape (Collins et al., 2019). MBT is not a new concept. It is used in several licensing exams, such as medical and pilot exams (Linhart, 2019). MBT uses standards-based grading, a grading technique that determines student mastery of individual standards, to ensure that students eventually master all standards in a course (Lewis, 2019). In this method of assessing, students only receive full credit for a problem if they have demonstrated mastery of the standard being assessed—there is no partial credit. The point of MBT is to increase understanding of material taught using a growth-mindset approach and, subsequently, helping to alleviate anxiety while learning mathematics (Harsy & Hoofnagle, 2020). Students may reassess standards until they are able to demonstrate mastery (Lewis, 2019). Doing this allows students to evaluate their content knowledge, persevere in problem-solving, and develop confidence in challenging themselves (Collins et al., 2019).

With MBT, educators must consider specific validity issues that do not occur in traditional, standard assessments. These four issues include defining what mastery means in terms of learner readiness to move on in the content, learner retention of prior knowledge, and complete content mastery of each unit; threats to validity because of retesting; needing reliable and specific measurement in determining mastery versus non-mastery; and consisting among different versions of the assessments. Retesting makes it challenging to analyze the reliability and internal structure of each content assessment (Lineberry et al., 2015). Another difficulty that LFM and MBT bring to the table is challenging task that the teacher has in organizing the work of her students who may be at all different stages of the learning and assessment process as well as different formative assessments that test the same content (Pelkola et al., 2018). Without changing the structure of the classroom, this could prove to be impractical with timing and classroom logistics (Harsy, 2020; Pelkola et al., 2018). Educators must also be sure to create questions that are direct and specific in assessing the current content standards, provide substantial feedback that let students know where they fall in the class academically, find opportunities to give time for students to reassess, and continue to show students that the process matters (Harsy, 2020). Some researchers and educators have found that creating organization around the process of retesting to make MBT easier to manage on a day-to-day basis (Mangum, 2020). Mangum (2020) suggests that using a rubric to compare traditional testing to MBT and creating more time in the day devoted to extra help, therefore creating more reassessment opportunities, are helpful ways in staying organized and committed to the MBT process.

MBT clearly outlines important course concepts, motivates students to develop a deeper conceptual understanding of content, encourages a growth mindset, and reduces test anxiety by allowing students to retake assessments after additional instruction (Collins et al., 2019). Providing students the opportunity to learn from their mistakes is an important aspect of MBT (Linhart, 2019). When Bloom first researched mastery learning, he concluded that the model includes four parts: defining mastery, planning for mastery, teaching for mastery, and grading for mastery. Regular, targeted assessments are needed in order to implement this model (Zimmerman & Dibenedetto, 2008). Harsy et al. (2020) found that students who claimed to have test anxiety at the beginning and middle of a semester eventually felt less anxious due to MBT. Once students get used to MBT, their anxiety drops. Students also felt less anxious before MBT exams than traditional exams (Harsy et al., 2020).

Motivated students believe that they can continue to improve and grow in learning through effort, reflecting a growth mindset (Collins et al., 2019). The assessment methods used by teachers affect their students’ attitudes (Harsy, 2020). Harsy and Hoofnagle (2020) found that students assessed with MBT felt their understanding of the content was better reflected and that students had higher final averages while spending less time studying on their own time. Any student can learn if they are given the time do so. This is particularly important because students vary in their ability to persevere and understanding instruction (Harsy & Hoofnagle, 2020). Traditional exams do not foster this way of thinking in all students. While high achieving fixed mindset students will perform well on these exams, low achieving fixed mindset students will have a difficult time accepting that they can do better in the future (Collins et al., 2019). To counteract fixed mindsets and encourage students to learn from their mistakes, educators must consider what they emphasize in the classroom. MBT is just one-way educators can lessen math anxiety, increase STEM retention, and nurture growth mindsets (Harsy, 2020). To foster a growth mindset in all students, assessment strategies must evolve. Allowing students to go over and correct their mistakes is an integral part of MBT and increases student motivation by encouraging students to participate in enough repetition to develop an understanding of the material (Linhart, 2019). MBT allows for a shift in mindset by both student and teacher.

Harsy and Hoofnagle (2020) researched how MBT affect undergraduate math students and their academic achievement. While they found no statistical difference in the average scores on the final mastery assessment, they did find a significant difference in the final average between students who were in the MBT group versus the traditional assessment group. They also found that MBT is best suited for the average student—not the hardest worker or the students who will likely not pass the class (Harsy & Hoofnagle, 2020).

Summary

Mathematics education has brought on questions about the best possible strategies to help students retain, conceptually understand, and achieve at the high school level. Past researchers have explored how student mathematical mindset and teacher beliefs affect these aspects in the classroom. Additionally, researchers have investigated how growth versus fixed mindset and Bloom's taxonomy play a role in achievement and student motivation to achieve. A growth mindset is defined as the internal belief that one can increase one's abilities through hard work and learning, whereas a fixed mindset is defined as the inner belief that one is born with a certain amount of skill that cannot be changed. Studies have shown that students with a growth mindset are more likely to achieve higher than students with fixed mindsets. Different assessment strategies and how they have been implemented in the classroom have also been discussed in previous research in affecting student achievement. However, little is known about how student mindset, teacher beliefs, and assessment strategies work best together in a high school mathematics classroom. A gap exists in the literature pertaining to how these coexist to help students retain and conceptually understand high school mathematics and, therefore, increase student achievement.

Some research suggests the use of mastery learning and mastery testing to help facilitate retention and conceptual understanding of content. However, this process and teaching strategy can be challenging to take on in a high school mathematics classroom, even though it is essential for students in mathematics to understand previous concepts prior to moving on to the next. By examining a variance of mastery learning, as well as the growth mindset of students, in the high school mathematics classroom, educators can better understand how to help future students retain and conceptually understand mathematics content. This, in turn, could also potentially apply across content areas.

CHAPTER THREE: METHODS

Overview

This was a quantitative study. This study used a quasi-experimental, nonequivalent control-group pretest/posttest design taking place in a high school in northwestern New Jersey. Descriptions of the instrument and procedures are provided. This study defines the independent variable as the learning strategy, the dependent variable as high school Geometry achievement scores, and the covariate as pretest scores. This study used ANCOVA and descriptive statistics to analyze the data.

Design

A quantitative, quasi-experimental, nonequivalent control-group pretest/posttest design was used. It is the most used quasi-experimental design for educational research. This type of design is executed much like the pretest-posttest experimental control-group design except participants in the research are not randomly assigned to the experimental and control groups. However, both groups take the same pretest and posttest. Both groups must be treated as similarly as possible apart from the treatment variable on the experimental group. Because there is no random assignment, this study is quasi-experimental. A nonexperimental control-group pretest/posttest design can include more than two groups and all groups can receive the treatment. Some design limitations include the threat to internal validity due to the group differences being from preexisting differences between the groups instead of the treatment effect (Gall et al., 2007).

This study lent itself to this type of research design because the goal was to determine the effect LFM has on student achievement in high school Geometry. Specifically, this research used a quasi-experimental, nonequivalent control-group design to identify how those important educational phenomena may change (Gall et al., 2007) with the teaching and reassessment technique of LFM. Previous research suggested that a nonequivalent control-group research design was best suited for this type of research. Demirel and Yilmaz (2019) used a nonequivalent groups pretest-posttest control group design to determine if students who play mind games show a difference in their perceived problem-solving skills and academic achievement compared to the control group in math and grammar courses. Denny et al. (2017) studied how a nonequivalent control-group design was used to determine the impact of how multiple intelligence teaching strategies may impact student engagement and learning while using questionnaires, scales, and assessments to gather data for analysis. Kang and Bae (2021) used a nonequivalent control-group pretest-posttest design to determine if a nursing student emotional competency promotion program is effective in strengthening the psychological well-being of patients.

The control and experimental groups had a different number of participants that fell naturally in their groups in Demirel and Yilmaz's (2019) study and in Denny et al.'s (2017) study. While the number of participants in each group in Kang and Bae's (2021) study were initially supposed to be different, they ended up being the same. These studies support the use of a quasi-experimental, nonequivalent control-group research design when determining if students utilizing LFM strategies creates a statistically significant difference in student acheivement, retention, and conceptual understanding compared to students who do not.

In this study, the independent variable was defined as the learning strategy. The learning strategy that was used as the treatment was LFM while the control group received traditional instructional strategies (Slavin, 1987). The dependent variable was defined as high school Geometry mathematics achievement scores. Mathematics achievement was defined as the expertise shown by a student in mathematics (Pandey, 2017). The covariate was defined as pretest scores. Pretest scores were defined by the scores students earned prior to receiving the treatment.

Research Question

RQ: Is there a difference in *Geometry achievement scores* between students who *participate in LFM strategies and those who do not* when controlling for pretest scores?

Hypothesis

The null hypothesis for this study is:

H_0: There is no significant difference in *Geometry achievement scores* between students who *participate in LFM strategies and those who do not* when controlling for pretest scores.

Participants and Setting

This section describes the population, participants, and setting of the study. The study took place in a medium-sized high school in rural northwestern New Jersey school district. The participants were selected using a convenience sample with a sample size of 73 students. Participants were selected from three in-person Geometry courses using ninth, tenth, eleventh, and twelfth grade students over the course of the first marking period.

Population

The participants for the study were drawn from a convenience sample of a high school student population located in northwestern New Jersey during the 2022-2023 school year. The school district resides in a rural area and encompasses middle-class families—median household income of $106,451—with moderately educated parents—44.4% of those living in the area hold a bachelor's degree or higher (Census Bureau, 2021). During the specified school year, the school had 1,255 students grades ninth through twelfth enrolled with an average class size of 22 students across all core subjects.

Participants

For this study, the number of participants sampled were 73 which meets the required minimum when assuming a medium effect size. According to Gall et al. (2007), 66 students is the required minimum for an ANCOVA when assuming a medium effect size with statistical power of .7 at the .05 alpha level. The sample came from the high school in the district. Within this school, students were selected from three Geometry classes. The major focus of these classes in the timeframe measured was teaching rigid motion transformations to advance students' geometric knowledge further and prepare them for college level math courses.

The sample consisted of 30 males and 43 females from ninth to twelfth grade in Geometry. Students ranged between 14 and 18 years of age. There were 23 students in Class A, 23 students in Class B, and 27 students in Class C. The students in Class A consisted of 11 males and 12 females: 12 Caucasian, 5 African American, and 6 Hispanic American. The students in Class B consisted of 8 males and 15 females: 14 Caucasian, 2 African American, 2 Asian American, and 5 Hispanic American. The students in Class C consisted of 11 males and 16 females: 19 Caucasian, 1 African American, 3 Asian American, and 4 Hispanic American.

Participants from the sample were able to choose to utilize LFM strategies (experimental group) or not (control group). The control group consisted of 39 participants and the experimental group consisted of 34 participants. The nonequivalent group size and the nonrandomized sample makes this study fall into the quasi-experimental, nonequivalent control-group pretest/posttest research design (Gall et al., 2007).

Setting

This study took place in one high school in three Geometry in-person mathematics classes. Students enrolled in these classes are on the middle track for high school mathematics difficulty and are taking these courses in preparation for college. The study took place over the course of the first marking period of the 2022-2023 school year. In this time, the classroom teacher implemented LFM through reassessment and MBT. Class A and C took place in Classroom #1. Class B took place in Classroom #2. Both classrooms had similar set ups with whiteboards, SmartBoards, class set of graphing calculators, and single seat desks. All mathematics classes were 75 minutes long running on block scheduling. All three classes took place at different times of the day: Class A 7:30-8:75am, Class B 11:30-12:45pm, and Class C 12:50-2:05pm.

Instrumentation

A quantitative, quasi-experimental, nonequivalent control-group pretest/posttest study was used to measure how the treatment, LFM, affected the dependent variable student achievement in high school Geometry. Achievement was measured through an assessment created from a bank of standardized Geometry questions from the New Jersey Student Learning Assessments – Mathematics (NJSLA-M) (See Appendix A for instrument). This instrument measures New Jersey students' grade or course level proficiency with regards to skills, knowledge, practices, and concepts that will be needed in college and career readiness. The assessment includes a variety of questions that assess reasoning and modeling at the appropriate grade level (New Jersey Department of Education, 2019). There is public access to the bank of questions (Pearson, 2020a; Pearson, 2020b).

The purpose of this instrument was to measure the pretest and posttest of high school Geometry acheivement scores. The state of New Jersey has committed to standards-based assessments for more than 40 years in order to provide the children of the state with the opportunity to be successful in society politically, economically, and socially. Students in the state of New Jersey have been required to pass a Mathematics and English/Language Arts skills assessment since 1981 (New Jersey Department of Education, 2016). In the 2014-2015 school year, New Jersey implemented a new standardized state test, the Partnership for Assessment of Readiness for College and Careers (PARCC), to measure higher-level thinking skills and to align with the Common Core State Standards for Mathematics and English/Language Arts. In 2016, the name of the assessment was changed to the New Jersey Student Learning Assessment to match the new name for the standards, New Jersey Student Learning Standards (NJSLS) (New Jersey Department of Education, 2016). The NJSLA-M was developed together by a team of experts at the New Jersey Department of Education and Pearson Education, Inc. It was developed by aligning NJSLS to questions that require higher-order thinking skills and measure a student’s ability to apply what they about a concept instead of regurgitate memorized facts by using mathematical reasoning and modeling. The NJSLA-M also offers teachers information on student progress, proficiency, and targeted support (New Jersey Department of Education, 2016).

The instrument has been used in numerous studies (e.g. Hashmi, 2021; Kitchin, 2020; Panetta, 2021). A team of experts from the US Department of Education peer reviewed the state assessment. The team determined that the assessment meets “adequate overall validity evidence...consistent with nationally recognized profession and technical testing standards” (Brogan, 2020, p. 20). The team of experts concluded that the State of New Jersey’s assessment measures the knowledge and skills that are specified in the NSLS (Brogan, 2020). The same team of experts from the US Department of Education peer reviewed the NJSLA for its reliability. They determined that there is “adequate reliability evidence for its assessments for the following measures of reliability for the State’s student population overall and each student group consistent with nationally recognized professional and technical testing standards”

(Brogan, 2020, p. 25). A stratified alpha was used to determine the reliability coefficient of the NJSLA. The stratified reliability coefficient for NJSLA-M Geometry is $\rho_{strata} = .90$ (New Meridian Corporation, 2020). Data from the *New Meridian Technical Report* of the 2018-2019 NJSLA determined that the "high reliability of the test scores implies that the test items within a domain are measuring a single construct" (New Meridian Corporation, 2020, p. 142). This is necessary in determining the validity of the assessment (New Meridian Corporation, 2020).

The instrument consisted of 12 questions from the NJSLA-M Question Bank (Pearson, 2020b). Participants were measured on a continuous, overall score where they could earn between 0 and 19 points. The assessment consisted of Type I and Type II questions. Type I questions ranged from 0 to 2 points—zero points indicated no mastery of that question, and two points indicated mastery. Type II questions ranged from 0 to 3 points—zero points indicated no mastery of that question, and three points indicated mastery. The combined possible score on the NJSLA-M Question Bank Assessment ranged from 0 to 19 points. A score of 0 points is the lower possible score, which indicated that the participant had the lowest high school Geometry achievement score, and a score of 19 points is the highest possible score, which indicated that the participant had the highest high school Geometry achievement score.

This assessment was administered in class on paper. The classroom teacher passed out the assessment to each student to be completed without a calculator. Participants were given 45 minutes to complete the assessment. The instrument was then be scored by the researcher and classroom teachers participating in the study. Prior to scoring, all scorers were trained on how to determine the number of points earned for each question. Questions were broken down into parts for scorers to understand how and when to take off points. This instrument was utilized for both a pretest and posttest.

The New Jersey Department of Education stated that the determination of approval is to be made at the district level. Permission to use this instrument was granted by the supervisor of the mathematics department in which the instrument would be used in. All questions available on the Question Bank are public information and are released the year after the previous year's test is completed. See Appendix C for permission to use the instrument.

Procedures

Prior to beginning the research study, IRB approval was be sought by submitting the information surrounding the study to CAYUSE IRB (see Appendix D). Because the research involves normal educational practices that are unlikely to impact the students' ability to learn negatively, the IRB found that the study was exempt from further review. District approval was also granted after sending a letter to the superintendent permission (see Appendix E and F). Then, participants were chosen from four Geometry courses in a northwestern New Jersey high school taught by one teacher. A pilot study was conducted in the fall semester of the 2021-2022 school year to determine if LFM was feasible to implement and to provide information on how to train the teacher implementing the instructional and assessment strategy. In this pilot, the teacher incorporated LFM as part of her instruction throughout the school year. The teacher gave instruction to the whole class at the same pace. Students were assessed at the end of each unit. In traditional LFM, only students who do not meet the predetermined standard for mastery take corrective action on their own time before reassessing (Slavin, 1987). In this study, however, any student who wishes to increase a grade on an assessment had the opportunity to take corrective action in the form of showing completed, correct homework assignments; assessment corrections; and supplemental assignments. Students had one week to communicate their intention to reassess from the time they receive their original assessment feedback using a

Google Form (see Appendix B) and three weeks to complete the three tasks. Once the tasks were completed, students were able to take a reassessment to determine if they have achieved mastery and increase their grade. At the end of the first marking period (10-week period), students took an assessment made from the NJSLA-M Question Bank to determine the level of student mathematic achievement in Geometry. All assessment scores were recorded in a spreadsheet broken down by student, assessment, and reassessment.

At the beginning of the study, all students took the same assessment made from the NJSLA-M Question Bank to determine initial achievement levels as a pretest. Next, LFM strategies (as described above) were implemented among all students, and they were able to choose if they wanted to utilize the LFM strategies or not. At the end of the study, all students took the same assessment made from the NJSLA-M Question Bank to measure student achievement levels in Geometry after the intervention, LFM, was implemented as a posttest.

At all stages of data collection, all information that can identify the participants was protected. Data was stored securely with only the researcher having access to the records on a password protected computer and a protected spreadsheet. Student names were replaced with numbers. The data was retained for a period of five years after the completion of this research study.

Data Analysis

This research study used one-way ANCOVA and descriptive statistics to analyze the data. The most appropriate statistic for the nonequivalent control-group design is ANCOVA because this method of data analysis tests the effects of the treatment without needing to match the data (Campbell & Stanley, 1963). ANCOVA makes compensated adjustments to the posttest means of the control and experimental groups reducing the effects of the initial group differences. Its purpose is to control for these differences before comparing the data. The strength of ANCOVA is that it controls internal validity (Campbell & Stanley, 1963; Gall et al., 2007) which is important because the nonequivalent control-group design threatens internal validity because differences on the posttest between the control and experimental groups could be a result of preexisting differences in the groups rather than the treatment. There also exists a threat to external validity because of the interaction between the pretest and the experimental treatment (Gall et al., 2007). The internal validity of this quasi-experimental, nonequivalent control-group pretest/posttest design was controlled by utilizing ANCOVA for data analysis. External validity was controlled by ensuring that the pretest has no bearing on the experimental treatment (Gall et al., 2007). In addition to ANCOVA, descriptive statistics, like mean and standard deviation, were used and reported for both the pretest and posttest (Gall et al., 2007).

Data screening included visual screening for missing and inaccurate entries as well as the creation of a Box and Whisker plot to find any outliers in the data for the control and experimental groups (Gall et al., 2007). ANCOVA also requires certain assumptions to be made, including normality, linearity, bivariate normal distribution, homogeneity of slopes, and equal variances (Campbell & Stanley, 1963; Gall et al., 2007). For the assumption of normality, Shapiro-Wilk was used. A series of scatter plots between the pretest variable and posttest variable, high school Geometry achievement scores, for the control and experimental groups were used to test the assumption of linearity and the assumption of bivariate normal distribution. In bivariate normal distribution, the researcher looked for a cigar shape. To test the assumption of homogeneity of slopes, the researcher looked for interactions within the data. Levene's Test of Equality of Error Variance was used to test for the assumption of equal variance (Gall et al., 2007). If the means of the control and experimental groups are too different, then the treatment cannot be said to have an effect (Campbell & Stanley, 1963).

An ANCOVA was appropriate for this study because there were two categorical groups for the independent variable: the experimental group who utilized LFM strategies, and the control group who did not. The dependent variable of high school Geometry achievement was a continuous variable (Gall et al., 2007). These statistics determined if there was a statistically significant difference between the mean scores of the dependent variable, student achievement in high school Geometry after the independent variable, participation in LFM strategies, was enacted as an intervention (Gall et al., 2007) for the null hypothesis $H_0 1$ when assuming a medium effect size with statistical power of .06 using partial eta squared at an alpha level of $\alpha = 0.05$. The mean and standard deviation of student achievement scores were calculated for the control and experimental groups for both the pretest and posttest. These results were analyzed using an ANCOVA. The assumption of homogeneity of regression were needed to determine if the means of the control and experimental groups were too different. If the means were too different, then it would not be possible to say if the treatment had an effect (Campbell & Stanley, 1963). Internal validity was controlled by utilizing ANCOVA and external validity was controlled by utilizing a pretest (Campbell & Stanley, 1963; Gall et al., 2007). The null hypothesis would be rejected at the 95% confidence level.

CHAPTER FOUR: FINDINGS

Overview

The purpose of this quantitative, a quasi-experimental, nonequivalent control-group pretest/posttest design research study was to understand the difference in posttest achievement scores, as measured by the assessment created from NJSLA-Math released items, among high school Geometry students participating in various learning strategies, when controlling for pretest achievement scores at a high school in northwestern New Jersey. Chapter four presents the results of this study. It states the research question, hypothesis, and the descriptive statistics. A one-way ANCOVA was used to test the hypothesis, which the researcher failed to reject. The results of the assumption tests and data analyses are outlined.

Research Question

The study used a quantitative, quasi-experimental, nonequivalent control-group pretest/posttest design to answer the question regarding student achievement in high school mathematics.

RQ: Is there a difference in *Geometry achievement scores* between students who *participate in LFM strategies and those who do not* when controlling for pretest scores?

Null Hypothesis

The researcher tested one hypothesis for the proposed research question to examine student achievement as it pertains to high school mathematics.

H_0: There is no significant difference in *Geometry achievement scores* between students who *participate in LFM strategies and those who do not* when controlling for pretest scores.

Descriptive Statistics

Descriptive statistics were secured on both the covariant, achievement pretest scores, and the dependent variable, achievement posttest scores, for each group. The sample consisted of 73 participants from one high school. Scores on the assessment created from the NJSLA-Math released items can range from 0 to 19. A high score of a 19 meant that the student met all assessed New Jersey mathematics learning standards while a low score of zero means that the student met no assessed New Jersey mathematics learning standards.

The overall pretest scores, which were used for the covariant, ranged from 0-8 for both the control group, students who did not utilize LFM strategies, and the experimental group, students who did utilize LFM strategies. The control group had a mean pretest score of 3.74 with a standard deviation of 1.89 while the experiment group had a mean pretest score of 3.79 with a standard deviation of 2.32. The median and mode of the control group was 4 (n = 12). The median and mode of the experimental group was 3 (n = 8). Descriptive statistics for the covariant can be found in Table 1. See Figure 1 and Figure 2 for graphs of the data distribution of covariant.

Table 1

Descriptive Statistics for Covariate (Pretest Score)

Group	*N*	Range	Min	Max	*M*	*SD*
Control Group (Traditional Strategies)	39	8	0	8	3.74	1.888
Experimental Group (LFM Strategies)	34	8	0	8	3.79	2.320

Figure 1

Distribution of Covariant (Control Group Pretest Score)

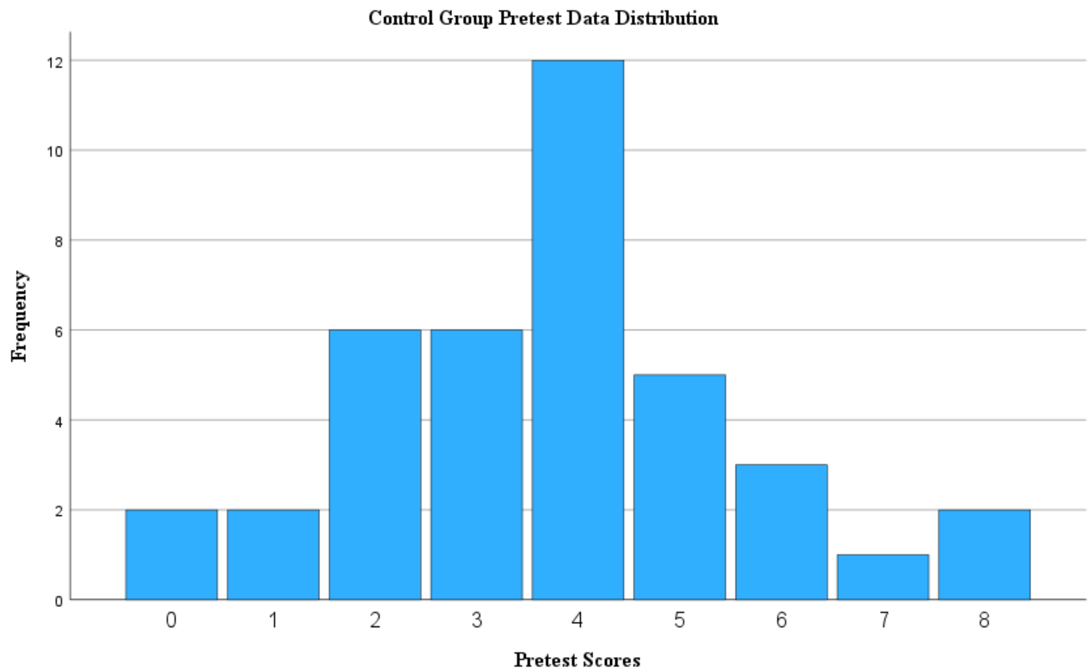
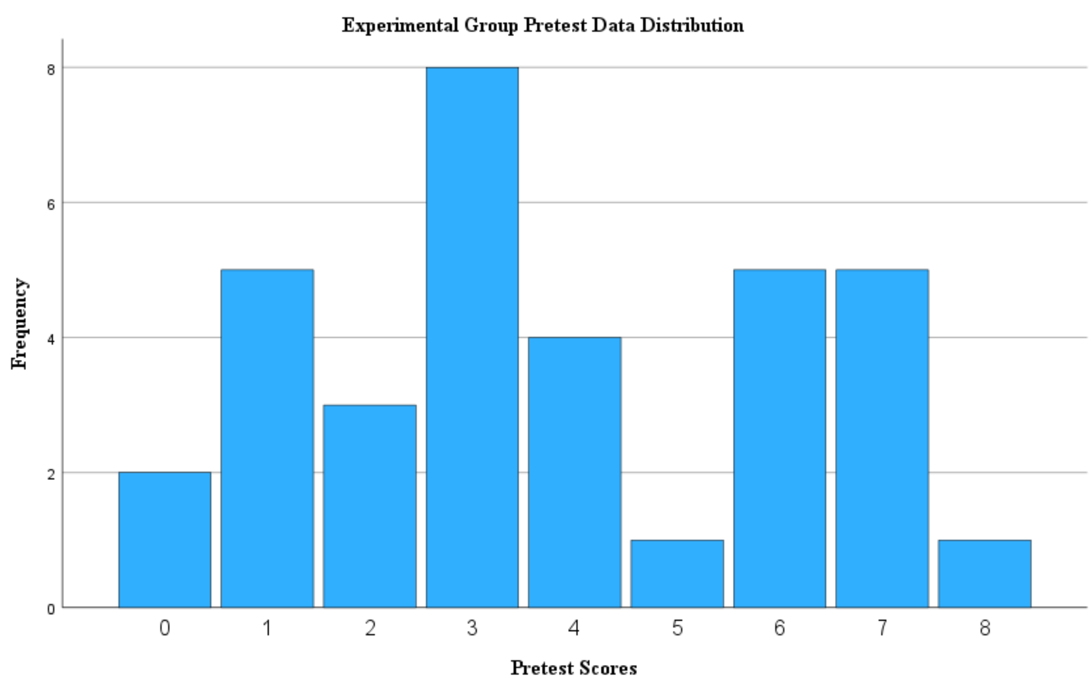


Figure 2

Distribution of Covariant (Experimental Group Pretest Score)



The overall achievement posttest scores, the dependent variable, ranged from 1-15 (1-15 in the control group and 2-13 in the experimental group). The control group had a mean of 7.72 and a standard deviation of 3.42. The experimental group had a mean of 7.76 and a standard deviation of 2.65. The control group had a mode of 10 ($n = 7$) and a median of 7.5. The experimental group had a mode of 9 ($n = 6$) and a median of 8. Descriptive statistics for the dependent variable can be found in Table 2. See Figure 3 and Figure 4 for graphs of the data distribution of the dependent variable.

Table 2

Descriptive Statistics for Dependent Variable (Posttest Score)

Group	*N*	Range	Min	Max	*M*	*SD*
Control Group (Traditional Strategies)	39	14	1	15	7.72	3.418
Experimental Group (LFM Strategies)	34	11	2	13	7.76	2.652

Figure 3

Distribution of Dependent Variable (Control Group Posttest Score)

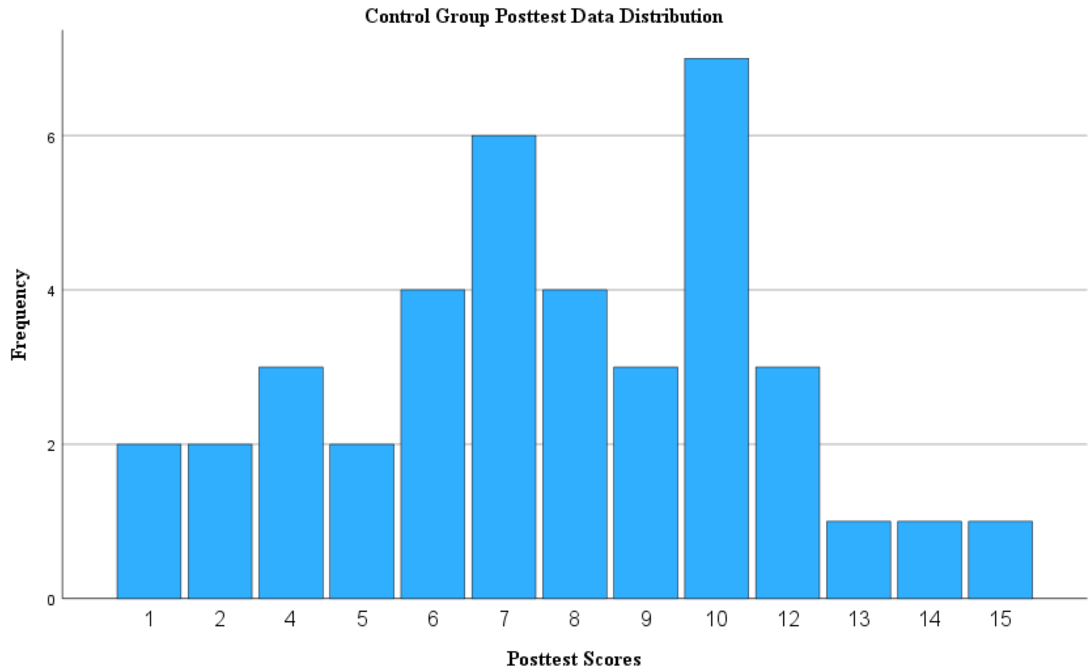
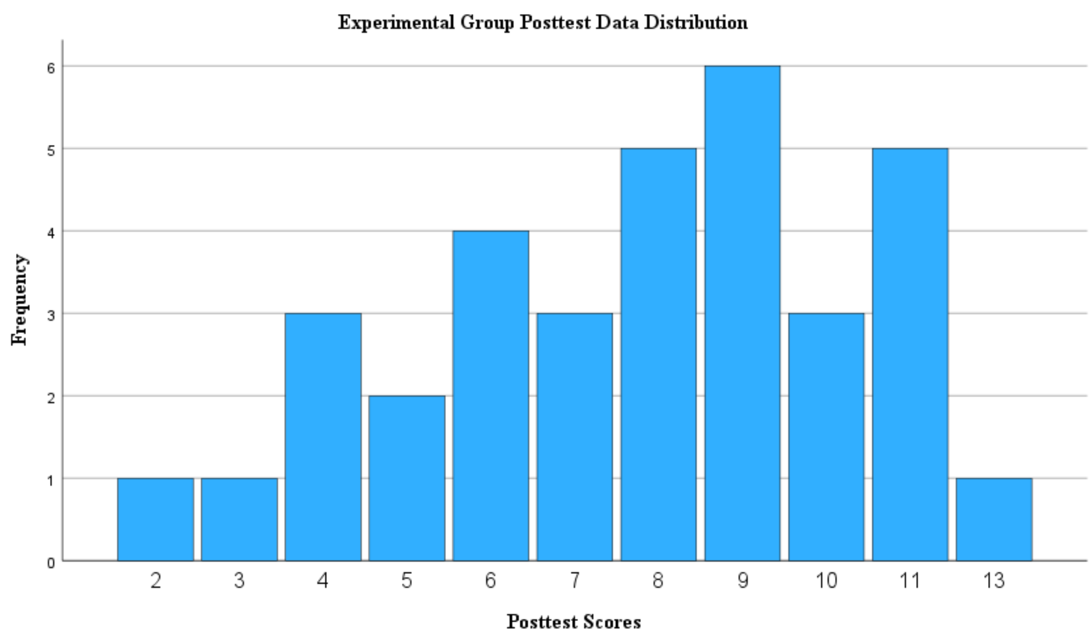


Figure 4

Distribution of Dependent Variable (Experimental Group Pretest Score)



Overall, 66 students showed an improvement from their pretest to their posttest; one showed a degradation; and six had no change. Both groups showed an improvement in their scores (3.98-point average increase in the control group and 3.97-point average increase in the experimental group). The control group showed the greatest average point increase.

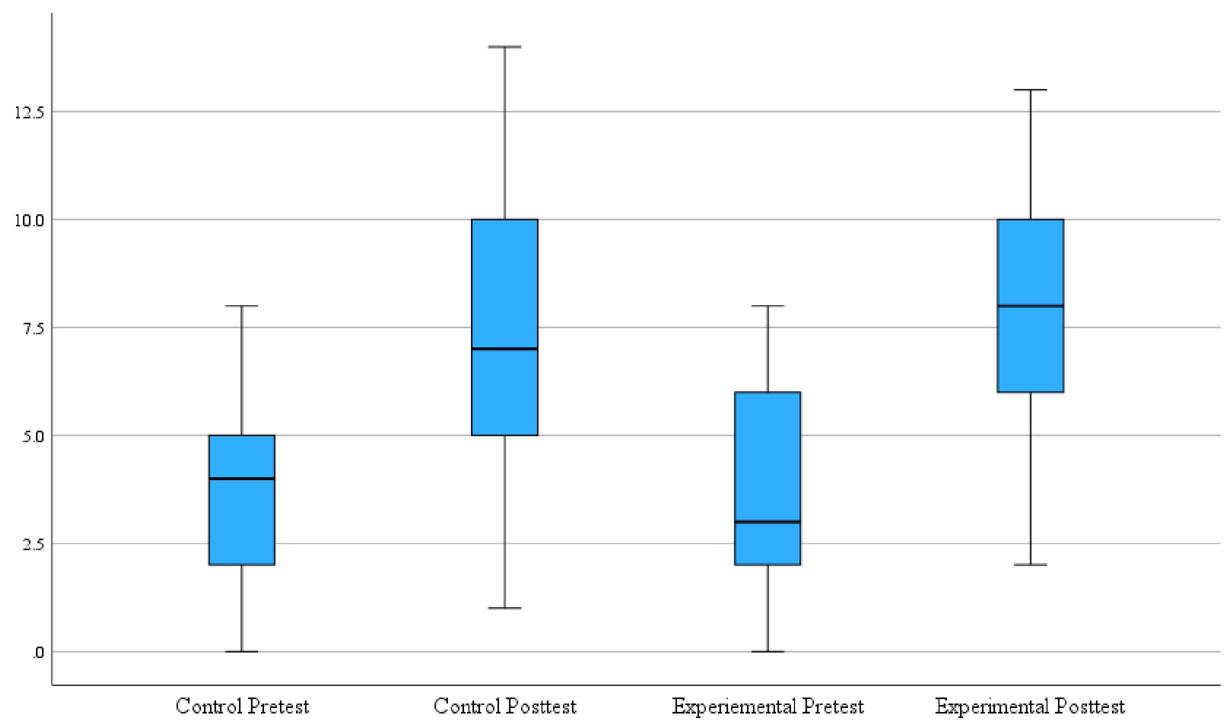
Results

Assumption Tests

The assumptions for a one-way ANCOVA were met. Independence of observations was maintained. Data screening was conducted on each group's dependent variable. The researcher sorted the data on each variable and checked for inconsistencies. No data errors or inconsistencies were detected. Box-and-whisker plots were used to identify outliers on the dependent variable. No outliers were identified. See Figure 5 for box-and-whisker plots.

Figure 5

Covariant and Dependent Variable Box-and-Whisker Plot



The *Shapiro-Wilk* test was used to check the assumption of normality since each group's population was under 50 participants. The assumption of normality was met for all groups. See Table 3 for Tests of Normality.

Table 3

Tests of Normality by Group

		Shapiro-Wilk		
		Statistic	*df*	Sig.
Control Group	Posttest Score	.978	39	.619
Experimental Group	Posttest Score	.969	34	.434

Scatter plots were used between the covariant, pretest scores, and the dependent variable, posttest scores, for each group to assess the assumptions of linearity, bivariate normal distribution, and the homogeneity of slopes. These assumptions were met for all groups. See Figure 6 and Figure 7 for the scatter plots.

Figure 6

Covariate to Dependent Variable Scatter Plot for the Control Group

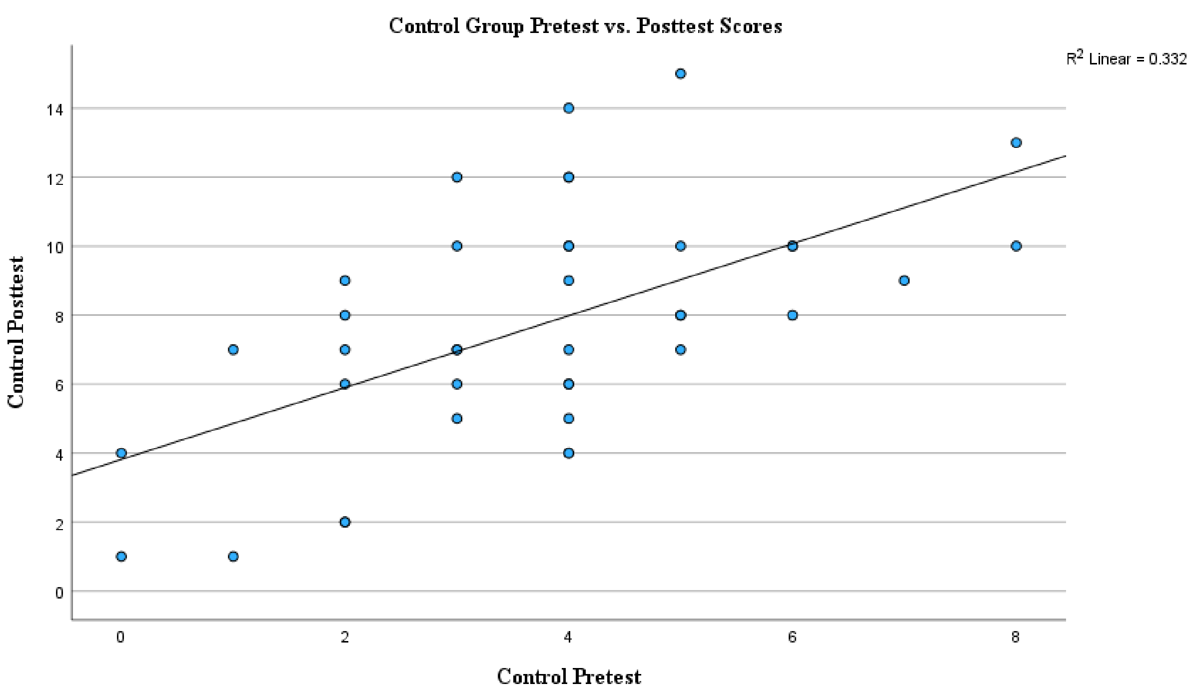
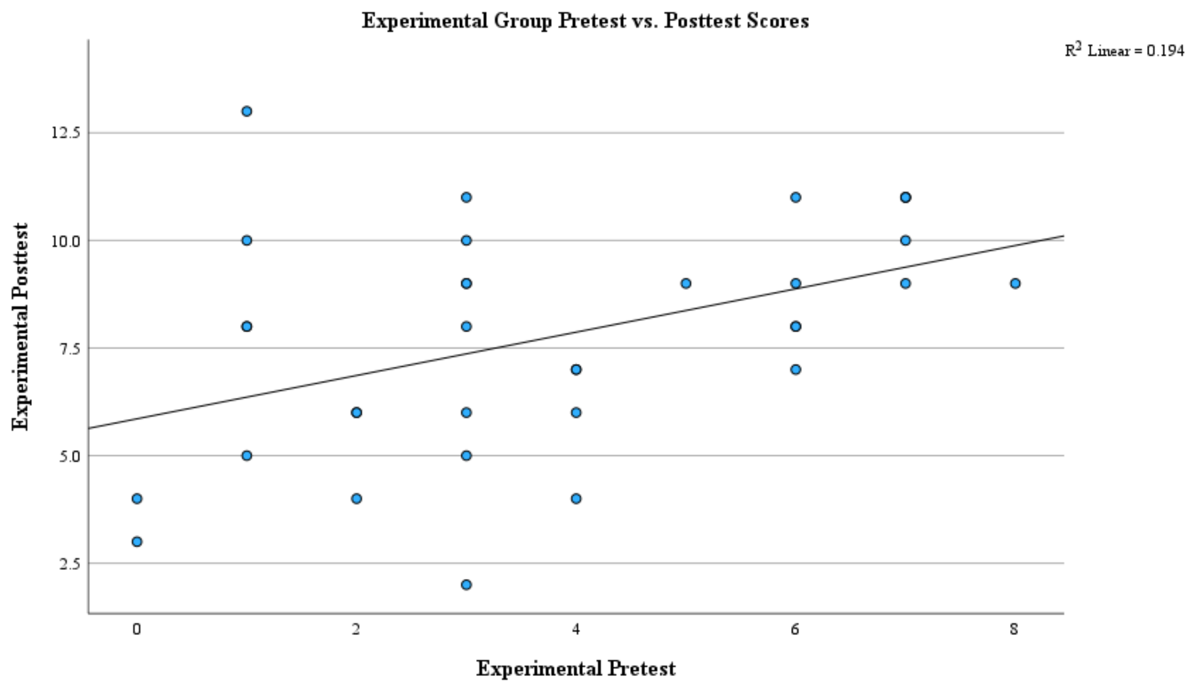


Figure 7

Covariate to Dependent Variable Scatter Plot for the Experimental Group



Levene's Test of Equality of Error Variance was used to test the assumption of homogeneity of variance. The assumption of homogeneity of variance was met where $p = .296$. See Table 4 for the results of *Levene's Test of Equality of Error Variances.*

Table 4

Levene's Test of Equality of Error Variances

Dependent Variable: Posttest Score

F	*df1*	*df2*	Sig.
1.108	1	71	.296

a. Design: Intercept + Pretest Score + Group

Hypothesis

An ANCOVA was run to determine if there was a significant difference in posttest achievement scores among high school Geometry students who utilized LFM strategies (experimental group) versus those who did not (control group). All statistical analysis and assumption tests were performed using IBM SPSS Statistics, a statistics software program. The independent variable was learning strategy, specifically LFM and traditional. The dependent variable was student achievement posttest scores, as measured by an assessment created from NJSLA-Math released items. This study utilized student achievement pretest scores from the same instrument as a covariant. The researcher failed to reject the null hypothesis at the 95% confidence level where $F(1, 65) = 2.476$, $p = .120$. An alpha level of .05 was used. The effect size was between small and medium: partial eta squared ($\eta^2_{\text{part}} = .044$). There was not a statistically significant difference in student achievement scores. See Table 5 for *Tests of Between-Subjects Effects*. Because the researcher failed to reject the null, no post hoc analysis was required.

Table 5

Tests of Between-Subjects Effects

Dependent Variable: Posttest Scores

Source	Type III *SS*	*df*	*MS*	*F*	Sig.	η^2_{part}
Corrected Model	192.171[a]	3	64.057	9.134	<.001	.284
Intercept	390.346	1	390.346	55.662	<.001	.447
Pretest Scores	183.550	1	183.550	26.174	<.001	.275
Group	17.365	1	17.365	2.476	.120	.044
Error	483.883	69	7.013			
Total	5049.000	73				
Corrected Total	676.055	72				

a. R Squared = .284 (Adjusted R Squared = .253)

CHAPTER FIVE: CONCLUSIONS

Overview

This chapter consists of the discussion of this research study, including relevant literature from other studies as it pertains to the hypothesis. In addition, this chapter offers the implications and limitations of this study, as well as recommendations for future research.

Discussion

The purpose of this research study was to identify whether LFM increases student mathematics achievement in the high school mathematics setting. Achievement in mathematics correlates to important life opportunities, like employment or social and psychological well-being, but less than half of the United States' students are proficient in mathematics (Begeny, et al., 2020). Because mathematics builds on itself, it is important to find a way for teachers to help students master the content so that they can retain it as students move into upper-level courses. Mastery-based learning and testing is an instructional strategy that is designed to do just that and help students master content that may not have been understood the first time (Farrell & Marsh, 2016). LFM is one of the mastery-based learning strategies and is intended to help students learn from their mistakes and promote a growth mindset (Linhart, 2019).

In the past, it was found that education was not pushing students to higher achievement in mathematics (Klein, 2002). This has been a concern of policymakers because it has been suggested that the mathematics content understood and retained in middle and high school predicts a nation's economic potential (Slavin et al., 2009). Therefore, strong mathematics achievement plays an important role in students becoming contributors to society (Teske, 2010).

The research question for this study was as follows: Is there a difference in Geometry achievement scores between students who participate in LFM strategies and those who do not when controlling for pretest scores?

The null hypothesis (H_0) stated there is no significant difference in Geometry achievement scores between students who participate in LFM strategies and those who do not when controlling for pretest scores. Results of this research study validated the null hypothesis because the experimental group, students who utilized LFM strategies, had a mean achievement posttest score of 7.76 and the control group, students who did not utilize LFM strategies, had a mean achievement posttest score of 7.72. The difference between the groups, as shown in Table 5, was not found to be statistically significant according to the one-way ANCOVA's test between-subjects effect at $F(1, 65) = 2.476$, $p = .120$; therefore, the researcher failed to reject the null hypothesis at a 95% confidence level.

Knowing which learning strategies are effective in increasing student achievement in mathematics is important because understanding mathematics is fundamental to developing people who can problem solve, think critically, and contribute to society (Begeny, et al., 2020). However, learning and understanding mathematics requires mastering the skills taught at each level. Many researchers report that LFM helps to achievement levels among college-aged students and across different content areas (Begeny, et al., 2020; Ellis, 2019; Linhart, 2019; Mouratidis, et al., 2018) since this strategy is meant to fill in the gaps that students have in mathematics.

Bloom's Taxonomy emphasizes how students understand and learn new information and discusses how learning goals create a template to student understanding and achievement (Murphy, 2007). Mastering the mathematical content taught in class increases student understanding while promoting a growth mindset (Dweck & Leggett, 1988). LFM provides students with the opportunity to learn missed or misunderstood content on their own time to work towards mastery (Slavin, 1987). A focus of this research study was to determine if LFM strategies do increase student mathematics achievement.

LFM strategies, reteaching, and retesting have been found to be a useful in special education as methods to help classified students learn and understand course content in mathematics (Marita & Hord, 2017). Other research has found that giving students the opportunity to retake even one assessment teaches them the benefits of correcting mistakes and revisiting prior ideas (Linhart, 2019). This research, however, showed that mathematics achievement did not vary in a statistically significant manner among high school Geometry students who participated in LFM strategies and those who participated in traditional instructional and assessment strategies.

Implications

The use of LFM strategies did result in students performing better on short-term, summative assessments. Students who did utilize LFM strategies performed better on their in-class reassessments. This learning strategy is effective in that it provides students with the ability to revisit content that has previously been covered to correct their misunderstandings (Donovan & Bransford, 2006). While correlational results in studies conducted by Begeny, et al. (2020), Ellis (2019), Linhart (2019), and Mouratidis, et al. (2018) infer that LFM strategies can increase student mathematics achievement at different grade levels and in different content areas, the results of this study suggest that more research needs to be conducted in high school mathematics to determine that stance. The results of this study as well as the summative assessment scores captured throughout the course of the study imply that while LFM may increase short-term achievement of individual assessments, it has little to no impact on student retention of the content to increase student mathematic achievement on cumulative assessments for the population of students in the given environment of the study. Further research, across different high school math content with different teachers in different schools, may still yield different results and should be considered.

It is also important to note that while the results were not statistically significant, students in this study who utilized LFM strategies did see improvement in their grades. Therefore, incorporating these strategies into the high school math classroom regularly may help to improve student grades. Academically, there appears to be no harm in trying it. The review of the literature also discussed math and testing anxiety. In a world where educators see increasingly more students with anxiety in the classroom, could LFM strategies help to reduce it by giving students a second chance to relearn the material before retesting? Could LFM strategies help to improve confidence in mathematics for high school students? Running the study for a longer period of time may provide opportunities for students to gain better understanding of the content and, in turn, increase their retention and student achievement in mathematics.

Furthermore, students in this study were able to decide if they wanted to be in the LFM experimental group or be in the control group. As a result, it is important to consider the type of student who may want to utilize these strategies to improve their grades and understanding of the material. Are students intrinsically motivated to utilize these strategies to improve their academic achievement? Or, are students extrinsically motivated by other factors, like parents, to improve their grades? Determining the motivation behind student involvement in the study could give more information on improving student achievement in high school mathematics. Taking all of this into consideration may help high school math teachers to utilize reassessment strategies in their classrooms to improve academic achievement in high school mathematics.

Limitations

The reader should consider a few limitations when applying and interpreting the results of this study. While many threats to internal validity were controlled for with the control group, pretest-posttest design (Gall et al., 2007), others need to be supplemented in the design of the study, for example instrumentation, participants, and length. The threat to internal validity in this study was self-selection bias. Self-selection bias may have limited the study because students could decide if they would be utilizing the LFM strategy, and it is possible that students have a specific quality that motivated them to choose to participate.

Limitations to external validity influenced the generalizability of the study. These limitations were caused by the participants, setting, and timeframe of the study. The majority of students in this research study were Caucasian and came from middle-class families attending the same high school in a rural, northwestern New Jersey town. A study including more students from different schools in different settings, like urban communities or economically disadvantaged areas, could result in different findings. A larger sample size of students could also lend itself to more generalization. This study lasted for ten weeks with a posttest at the end of the timeframe. A study spanning the whole school year assessing more NJSLS might produce different results. Performing the study in a different content of high school mathematics may yield different results, as well.

Recommendations for Future Research

Recommendations for future research include:

1. Continuing to explore the impact of LFM on student achievement in high school Geometry;
2. Extending the duration of the study;
3. Analyzing the impact of LFM on student achievement in other high school mathematics content areas;
4. Determining how LFM strategies impact retention of mathematics content;
5. Exploring how LFM strategies impact student achievement in high school Geometry in different school districts set in different environmental and economic settings.

Conclusion

With high school mathematics proficiency continuing to be a topic of mind among the population of the United States as well as its greater societal implications, research to increase student achievement in mathematics must be ongoing. This study continues to close the gap between student achievement in mathematics and the learning strategies that can be used to help increase it in the classroom. With the findings of this study being inconclusive, it is vital that this study act as a catalyst to encourage others to continue exploring the impact of LFM and other learning strategies on student high school mathematics achievement.

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APPENDIX A

Removed to comply with copyright.
Released Items:

CCSS.MATH.CONTENT.HSG.CO.A.3: M41170, VH017425
CCSS.MATH.CONTENT.HSG.CO.A.5: VF803321, M40060P, 2523-M42570, VF647255, VH096531, VH003640, VH233587
CCSS.MATH.CONTENT.HSG.CO.B.6: VF815346, VF904191, VF819867

Geometry Pretest/Posttest

Directions: Complete the questions below to the best of your ability. Show all work.

1.) [redacted]

2. [redacted]

3.) [redacted]

4.) [redacted]

5.) [redacted]

6.) [redacted]

7[redacted]

8.) [redacted]

9.) [redacted]

10.) [redacted]

11.[redacted]

12.) [redacted]

APPENDIX B

Assessment Retake

Fill out the form below within a week from receiving a test or quiz back. The form is timestamped. Any submissions beyond one week will NOT be accepted. You will need to know the name of the assessment (this can be found in the upper left hand corner of the assessment). You will also need to upload all homework covered on the assessment (if you are unsure what you need, ask me and I will let you know).

The name, email, and photo associated with your Google account will be recorded when you upload files and submit this form

* Required

Name: *

Your answer

Block: *

- Block 4
- Block 5

Name of assessment that you are retaking: *

Your answer

Completed homework from the assessed section:

Add file

APPENDIX C

Panico, Amanda [redacted]

NJSLA Sample Question Bank

3 messages

Panico, Amanda [redacted] Wed, Aug 17, 2022 at 2:48 PM
To: "Hilaman, Lara" [redacted]

Good afternoon Lara,

I am a teacher in New Jersey looking to gain permission to utilize the sample question bank from the NJSLA-M as a data analysis instrument in a pre- and post-test for students in my district. Please let me know if this is something that I will be able to use.

Thank you for your consideration,

Amanda Panico
Mathematics & Computer Science Teacher
Roxbury High School

Hilaman, Lara [redacted] Mon, Aug 22, 2022 at 10:14 AM
To: "Panico, Amanda" [redacted]

Good morning,

This determination is make the local school or district level. Please reach out to your direct supervisor regarding the use of available NJSLA resources for this purpose.

Sincerely,

Mrs. Lara Hilaman

She/Her/Hers

Education Program Development Specialist 2

Office of Assessments

[redacted]

New Jersey Department of Education

Work: [redacted]

Email: [redacted]

Web: https://www.nj.gov/education/

Panico, Amanda [redacted]

Permission to Use NJSLA Sample Question Bank

3 messages

Panico, Amanda [redacted] Mon, Aug 22, 2022 at 11:08 AM
To: Jeffrey Fiscina [redacted]

Good morning Jeff,

I reached out to the NJ DOE for permission to use the NJSLA Sample Question Bank for my pre/posttest. Their response was that the permission needs to be granted from you. Will you grant me permission to use the NJSLA Sample Question Bank for the pre/posttest for my research?

Thank you,

Amanda Panico
Mathematics & Computer Science Teacher
Roxbury High School

Fiscina, Jeffrey [redacted] Mon, Aug 22, 2022 at 1:39 PM
To: "Panico, Amanda" [redacted]

Hi Amanda. You have my permission, but not quite sure what else I need to do. Let me know if you need any help.

See you later this week!

[Quoted text hidden]

> [Quoted text hidden]
> Please be advised that this email and any documents accompanying this email may be subject to public access, except as specifically protected based on the laws governing confidentiality of information. If you have any questions regarding the confidentiality of your information, please contact the office of the Board Secretary for guidance.

--
Mr. Jeffrey Fiscina
Supervisor of Mathematics Grades 7-12
Supervisor of Business and Family & Consumer Science Grades 7-12
Roxbury School District
[redacted]

APPENDIX D

LIBERTY UNIVERSITY.
INSTITUTIONAL REVIEW BOARD

August 9, 2022

Amanda Nicol
Steven McDonald

Re: IRB Exemption - IRB-FY21-22-1228 The Difference in Geometry Achievement Between Students Who Utilize LFM Strategies and Those Who Do Not: A Nonequivalent Control-Group Pretest/Posttest Study

Dear Amanda Nicol, Steven McDonald,
The Liberty University Institutional Review Board (IRB) has reviewed your application in accordance with the Office for Human Research Protections (OHRP) and Food and Drug Administration (FDA) regulations and finds your study to be exempt from further IRB review. This means you may begin your research with the data safeguarding methods mentioned in your approved application, and no further IRB oversight is required.

Your study falls under the following exemption category, which identifies specific situations in which human participants research is exempt from the policy set forth in 45 CFR 46:104(d):

Category 1. Research, conducted in established or commonly accepted educational settings, that specifically involves normal educational practices that are not likely to adversely impact students' opportunity to learn required educational content or the assessment of educators who provide instruction. This includes most research on regular and special education instructional strategies, and research on the effectiveness of or the comparison among instructional techniques, curricula, or classroom management methods.

The final versions of your study documents can be found under the Attachments tab within the Submission Details section of your study on Cayuse IRB.
Please note that this exemption only applies to your current research application, and any modifications to your protocol must be reported to the Liberty University IRB for verification of continued exemption status. You may report these changes by completing a modification submission through your Cayuse IRB account.

If you have any questions about this exemption or need assistance in determining whether possible modifications to your protocol would change your exemption status, please email us at irb@liberty.edu.

Sincerely,
G. Michele Baker, MA, CIP
Administrative Chair of Institutional Research
Research Ethics Office

APPENDIX E

June 15, 2022

Dear Dr. Radulic and the Board of Education,

As a graduate student in the School of Education at Liberty University, I am conducting research as part of the requirements for a Doctor of Philosophy degree. The title of my research project is The Difference in Geometry Achievement Between Students Who Utilize LFM Strategies and Those Who Do Not: A Nonequivalent Control-Group Pretest/Posttest Study and the purpose of my research is to identify whether Learning for Mastery (LFM) increases student mathematics achievement in high school Geometry.

I am writing to request your permission to conduct my research at Roxbury High School. Participants will be asked to participate in a pretest comprised of released questions from the New Jersey Student Learning Assessment–Mathematics (NJSLA-M); contribute to class as usual by participating and completing assignments/assessments; and participate in a posttest comprised of released questions from the NJSLA-M. For participants who would like to participate in utilizing LFM, the following tasks must be completed within three weeks from receiving an assessment back: request the reassessment by filling out the Reassessment Request Google Form; show his or her teacher all completed homework from the section; complete quiz/test corrections in the instructional center with the help of a math teacher; complete a supplemental assignment to work towards understanding the assessed content; and complete the reassessment. Participants will be presented with informed consent information prior to participating. Taking part in this study is completely voluntary, and participants are welcome to discontinue participation at any time.

Thank you for considering my request. If you choose to grant permission, please provide a signed statement on official letterhead indicating your approval.

Sincerely,

[redacted]

Amanda Panico
Teacher of Mathematics
[redacted]

APPENDIX F

Panico, Amanda [redacted]

Doctoral Dissertation Permission Request

Radulic, Loretta [redacted] Wed, Jun 29, 2022 at 2:10 PM
To: "Panico, Amanda" [redacted]
Cc: Charles Seipp [redacted]

Good afternoon,

You have permission. Dr. Seipp and I will discuss it at an upcoming BOE Education Committee meeting. We are interested in hearing the results of your study.

Good luck!

Loretta

Loretta L. Radulic, Ed. D.

Superintendent

Roxbury Township School District
42 N. Hillside Ave.
Succasunna, NJ 07876

PHONE- [redacted] | **FAX-** [redacted]
WEB- www.roxbury.org | **FACEBOOK-** Roxbury Public Schools | **TWITTER-** [redacted]

Preparing the children of today for tomorrow...