Exploring Mechanisms Insurers Employ to Set Premiums and Maximize Profitability

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Abstract

The insurance industry is a very complex segment of the macroeconomy. An explain will be given as to how these companies are able to maintain and maximize their profits, allowing them to remain in business. A key area in this process is the setting of premiums. This activity draws from many areas of the business model. This paper will start with a birds-eye view and telescope in, starting with standard business practices and ending with specific undertakings of insurance companies. Companies must keep adequate liquid funds. This is done mainly through forecasting cash outflows and investing their assets under management. Within the realm of investing, insurance companies also use hedging techniques to maintain a healthy cashflow.

Keywords: *Insurance, Premiums, Profitability*

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Introduction

Insurers, like all companies, aim to maximize their profitability. The highest revenue they can attain in correspondence with the lowest expenses yields the highest profit margin. All companies seek to increase this margin in both the short and long term, and many have implemented precise plans and methodologies for attaining this maximum profit. This is obviously never an easy goal for a company as no company would go out of business if its profit remained high. The mechanisms insurers use to maximize profitability will be the focus. This paper will give an overview of the key areas insurers spotlight in attempts to continually attain the highest margins possible. It will be more conceptually based and less mathematically rigorous, though some mathematical principles must be covered.

A company must remain competitive and offer premiums at a comparable price to others in order to maintain business. Life insurance companies need assets under management to yield a profit from investment returns. This is where a company must balance the economic law of supply and demand with its need for assets and revenue. Using basic knowledge of economics, it is clear that if a life insurance company sets premiums too low, they will have a plethora of insured (Gründl, Gal & Dong, 2016). While this may seem advantageous on the surface, it could very well come back to harm the company in the long run. If the premiums are too low, the company will not have the liquid assets under management to pay claims from the mass of customers. This could cause the company to default on the claims and lose its assets and perhaps its reputation. On the flip side, if the company sets premiums too high, fewer individuals would pay to be insured, instead opting for a competitor. This may drive the company’s expenses over their revenue, putting them in “the red”. From this simple illustration, companies that set their premiums either too high or too low run into serious issues. Thus, it is of utmost importance for companies to set their premium prices adequately to maintain a net income with stable cash flow.

Macroeconomic Activities

Investing Strategy

Many companies are involved in some type of asset investing. While this may be a small segment of operations for some companies or even some industries, it plays a major role in the insurance industry. One of the primary actions of an insurance company is investing the assets under management which are accumulated initially from premiums paid by the insured. Once the assets are invested, the company continues to gain interest on the investment. The two main modes of revenue for an insurance company are premiums and interest earned on investments, so increasing the rate of return on investments subsequently increases revenue – a desirable result.

Though there are many different investment approaches and opportunities such as directly investing in the stock market, mutual funds, bonds, real estate, etc., “the basic theoretical model for investment management is the Markowitz portfolio selection model. The Markowitz model assumes that risk-averse investors only care about the expected return and risk of their portfolio investment” (Gründl, Gal & Dong, 2016, p. 8). The Markowitz model for investment management suggests increasing expected return while decreasing variance (risk). Markowitz (1952) himself wrote, “the investor does (or should) consider expected return a desirable thing *and* variance of return an undesirable thing. This rule has many sound points, both as a maxim for, and hypothesis about, investment behavior” (p. 77). Clearly, if two investment opportunities have the same expected value yet one has much larger variance, the investment option with a lower variance is more desirable. With a lower variance, predicting growth becomes much easier and more accurate. Under the Markowitz model, the investor has the option of fixing the expected return while attempting to minimize the variability or fixing the variability while attempting to maximize the expected return.

While Markowitz defended the principle of diversification, he also emphasized that diversifying does not negate all the variance. This was a common misconception derived from the law of large numbers. People thought that if they diversified their accounts, the expected value of the yield would be very similar to the actual value of the yield. The assumption was that if one investment “went south”, others would perform extraordinarily, thus counteracting the bad investment. Markowitz (1952) wrote, “this presumption, that the law of large numbers applies to a portfolio of securities, cannot be accepted. The returns from securities are too intercorrelated. Diversification cannot eliminate all variance” (p. 79). In other words, investments, even when diversified, have trends that are too similar to be deemed independent, so as one investment dips, the others will tend to follow. While this is not necessarily the case, it often is. With this in mind, when one investment dips, it is not a safe assumption to say that other investments will offset the bad investment as those investments may dip as well.

Insurance companies invest heavily in U.S. bonds because they are one of the safest forms of investment. This means that their returns have some of the lowest volatility, and they provide the option to select risk amount. For instance, “the U.S. insurance industry’s investment portfolio asset mix included approximately $234 billion in book/adjusted carrying value (BACV) of U.S. government bonds as of year-end 2013” (NAIC, 2014). Amongst insurance companies, life insurance companies account for the most investing activity with more than 60% of the insurance industry’s assets (NAIC, 2014). This means that life insurance companies invest heavily in U.S. bonds as a major part of their investment strategy.

Forecasting

Forecasting is used for investing purposes, but also has other applications in the realm of insurance and the macroeconomy. The strength of the macroeconomy, indicated by multiple benchmarks, has immediate and serious effects on different sectors within it, though the focus shall be upon the insurance industry. Because of the impact on the productivity and profitability of companies from the national and world economy, many have sought to find indicators of trends and cycles in these macroeconomies. In doing so, companies are able to gain a better understanding of what will likely happen in the future to the companies within the larger economic sphere. Thus, the need for accurate forecasting methodologies is imperative to the sustained growth of all kinds of companies.

Some of the different methods used in forecasting will be explored to give a working framework for understanding how insurance companies can capitalize on forecasting to enhance their business operations.

Methods

Many different methods have been developed as predictive models which are used for forecasting purposes. These methods include moving averages models, exponential smoothing models, regression models, time series decomposition models, and Bayesian models. Each of these have their advantages and disadvantages.

The most common patterns that forecasters attempt to predict are seasonality, cycles, and trends. Seasonality refers to the fluctuations in a variable that are repeated on a seasonal basis. This could be quarterly, monthly, weekly, etc. as long as it is within a year. Cycles, very often business related, are fluctuations in the variable that coincide with different movements in other related fields. For example, there are patterns in the national economy that effect many different sectors which are classified as cycles since they are not on a seasonal basis, yet they still ebb and flow relatively smoothly. Trends refers to the overall long-term movement of the variable, so if there is an upward trend, the graph depicting the variable versus time would show pseudo-steady increases. If there is a downward trend, the graph would show the opposite. The overall slope of the graph would also be useful in determining how significant of a trend exists. In a given dataset, it must be determined whether such patterns exist, and how to forecast the data with that information. Thus, such methods mentioned above have been formulated to deal with such patterns in a forecasting context.

The first of these methods, moving averages modeling, is a very simple type of model that takes previous *n* entries, adds them together, and then divides by *n*. This gives the forecast for the next period. This method is straightforward and does not require much computing power or expertise. However, it is not ideal for forecasting data when there are seasonal or cyclical patterns in the data. Thus, other methods have been produced to handle such challenges.

Exponential smoothing models have a little more exactness while approaching forecasting in much the same way as moving averages. This method uses the past forecasted value and compares it to the same period's actual value. Then it takes a constant, α, such that $0 < \alpha < 1$, and uses past actual data combined with forecasted data to give a new forecast. This method gives more weight to more recent data points and less weight to data points further in the past. It is given by this equation, where F_t is the forecasted value at time t and X_t is the actual value at time t:

$$F_{t+1} = \alpha X_t + (1 - \alpha)\,\alpha X_{t-1} + (1 - \alpha)^2\,\alpha X_{t-2} + \ldots + (1 - \alpha)^n\,\alpha X_{t-n} + (1 - \alpha)^{n+1} F_{t-n} \qquad (1)$$

where n is the last period that should be included in the forecast (Keating & Wilson, 2019, p. 100). Since $0 < \alpha < 1$, it is clear that the terms of equation (1) have less weight as they get further from time t. This is different from the moving averages model because it assigns different weights to the past values where the moving averages assigns the same weight to each past value.

There are other more complex versions of exponential smoothing that have been developed such as Holt's exponential smoothing and Winter's exponential smoothing. However, these will not be addressed in detail. These more advanced models can adequately forecast data with patterns such as seasonality and cycles.

Regression models have a wide variety of uses and can work well to forecast many types of data and are very helpful in determining significant independent variables for forecasting their dependent counterparts. Regression seeks to find a relationship between the independent and dependent variables and generate an equation relating them numerically. This paper will briefly discuss linear regression and not digress into other forms of regression models. Simple regression has just one independent variable used to forecast the dependent while multiple regression, as the name suggests, uses multiple independent variables to forecast the dependent variable. An equation of a multiple regression model follows this formula:

$$Y = a + b_1X_1 + b_2X_2 + \ldots \qquad (2)$$

where a is the value of the dependent variable with all the independent variables equaling zero (the intercept), the X_i's are the independent variables, and the b_i's are the coefficients associated with the independent variables. This gives a model that uses independent variables to predict the dependent one. One challenge is to first estimate where the independent variables will be before being able to estimate the dependent one as the dependent, by definition, depends on the independent variables.

A time series decomposition model in its multiplicative form can be displayed by this fairly simple equation:

$$Y = T * C * S * I \tag{3}$$

where T stands for the trend in the data, C stands for the cyclical factor in the data, S stands for the seasonality in the data, and I stands for the irregularity of the data (Keating & Wilson, 2019, p. 305). This model then, accounts for all the patterns in a dataset. The theory behind how the innerworkings of this model will be left for the reader to investigate.

Another model is the Bayesian model for forecasting. The mathematical rigor relating to this is too vast to include in detail, but this forecasting methodology stems from Bayes' Theorem used to calculate the probability of events happening based on given information, or conditional probabilities.

A look into the insurance industry's use of these can be investigated along with the correlation between the insurance industry and the national and global economies. Many other modeling techniques are closely related to one of these models or directly based on them.

Since many business decisions depend on the future outlook of the company's financial state, forecasting is an important tool. A Bayesian model of workers compensation insurance claims will now be the focus.

Insurance companies have an obligation to pay the claims which are required of them under contract. To do this, they establish a loss reserve to pay such claims. A pivotal operation is then to forecast the number and size of claims that an insurance company will be required to pay, so there can be adequate funds in the loss reserve but not more than necessary. There is much uncertainty associated with this task as Zhang et al. (2011) stated:

> This uncertainty has two fundamental sources. First, because claims are reported at random times, there is uncertainty about the number of claims for which the insurer will ultimately be liable. Second, there is uncertainty about the ultimate size of the insurer's liability even after the claim has been reported." (p. 637)

Because of this, it is the forecasters duty to predict the aggregate loss claims amount for the purpose of deciding the amount that can be invested and not kept to pay claims.

There is much rigorous mathematical development that is given to creating and testing models for loss reserves. Zhang et al. (2011) developed a Bayesian non-linear model for forecasting this crucial information for insurance companies. They applied their model to workers compensation insurance claims and completed model checking and diagnostics to find the value of their model. Their model had first-order auto-regressive feature, so to check heteroscedasticity, they standardized the residuals and plot them against the mean. This shows that the graph clearly has randomized residuals that follow no clear pattern and are centered on zero on the y-axis signifying a constant variance and an accurate predicted mean.

This model is a Bayesian non-linear regressive model that has been shown to accurately fit prior data (in-sample modeling) and can be extrapolated to forecast future data (out-of-sample modeling). This is just one example of a forecasting method used in the insurance industry to predict claim frequency and size, combined as aggregate claim liability costs.

While the models have become more rigorous in the 21st century, the same principles still applied previously when the more recent developments had not been made. For example, Cummins and Griepentrog (1985) tested the accuracy of using an ARIMA model versus a simple exponential trend model to forecast paid claim costs. They also tested another model they called an econometric model and deduce that, “a net gain in accuracy could be achieved by adopting econometric models.” (Cummins and Griepentrog, 1985, p.203). It is clear then that such modeling has been utilized and repeatedly verified to accurately forecast insurance paid claims, resulting in informed, more profitable decision-making. Forecasting within an insurance company is not the only job to be done though.

Macroeconomic

Another crucial facet for insurance firms to be mindful of is the macroeconomic patterns and how those affect the individual firms. Clearly, in some regard, the macroeconomy functions as a unit, or in the aggregate. This means that as the economy declines, the subdivisions in the economy will tend to do the same. This is certainly not a hard and fast rule, as many things such as new technologies disintegrate certain markets and make others flourish. Also, as the aggregate economy falls, markets such as precious metals begin to grow (i.e. countercyclical industries). Thus, the aggregate economy is not perfectly correlated to industry specific markets, but there is clearly much relation.

As an interesting note, it will be shown that the insurance industry is correlated to the banking industry, since the banking industry has much to contribute to the failure of the macroeconomy. Kerstin and Pick (2011) used a regression model to estimate the time until default for the banking and insurance industries. They ran many different variables and used Bayesian Information Criteria (BIC) to determine what variables were significant and independent of each other. Using this method, they found that “from the economic variables the indicators for financial openness and capital accounts openness, the inflation rate, and the unemployment rate, are include most often.” (Kerstin & Pick, 2011, p. 813). This means that probabilistically, the above regressors were included in the regression most often, signifying they are the most significant. And these regressors were included both for banking failure and insurance defaults, further signifying that these events have a high correlation.

Investigating the macroeconomy and how it affects the insurance industry directly without looking through the banking industry will be the next focus. Since insurance companies rely heavily on high interest rates and other aspects not directly associated with the insurance field, it is reasonable to infer some correlation between the macroeconomy and the insurance industry. Dorofti and Jakubik (2015) used “cross-country European aggregate data” (p. 56) to find the link between the insurance field and the macroeconomy. Their findings “suggest that low interest rates, along with limited economic growth, poor equity market performance and high inflation [have] a negative impact on insurance profitability.” (Dorofti & Jakubik, 2015, p. 56). It is certainly reasonable that the interest rate would have a positive relationship to insurance profitability. There would be a larger profit margin on investments yielding a higher overall profit level. The other factors are also fairly clear in their correlation. Thus, this study by Dorofti and Jakubik has shown that “despite, some further research needs to done, this study clearly points out the sensitivities of insurers to the macroeconomic environment.” (2015, p. 69).

Forecasting, as has been adequately displayed, has important uses in the insurance industry. There are many different models that can suffice for different purposes in the forecasting world, and each has its strengths and weaknesses. A form of the regression model is the most common choice for forecasting within the insurance field and has led to findings that the macroeconomy correlates highly with the insurance industry. Thus, insurance industries must use forecasting to determine their investment choices and other related business decisions to maximize their profitability.

Life Insurance Specific

Mortality Tables

Mortality tables are exceedingly useful in the actuarial practice. Actuaries rely on mortality tables to create and justify their predictive analytics and give suggestions on premium pricing. Before diving into the inner workings of these tables, a brief description of them is necessary. In short, a mortality table, also known as a life table or actuarial table, gives the probability that a person in a certain demographic dies before his/her next birthday. This is used to create models for predicting the future probability of death at any given age. Such a model can be used to find the life expectancy at any age along with the variability in these numbers.

Finding the mean (average) age of death of a certain population is exceptionally useful; however, variability in the actual age of death creates a problem. Suppose the company insures 20 people whose average age is 76 years old, but the standard deviation is 10 years. Now assuming the company insures its clients on the basis of their life expectancy, it would not be unreasonable for half or more of the 20 clients to die at age 72 or younger. With these numbers, the probability of such an event occurring is 4.77% (found using a z score of 4/10, finding the probability associated with a z score from a normal distribution, and entering this probability for each trail into a binomial CDF). This incident occurring would be problematic for the insurer, as claims would have to be paid and revenue from premiums would be greatly reduced because those dead are no longer paying premiums.

One incredibly useful law for all insurance companies is the law of large numbers. This law states that “the larger the number of exposures, the more accurate any predictions of losses will be to actual losses.” (Leimberg et al., 2018, p. 41). Stated generally, the more trails fitted to a certain model, the closer the variability in the sample will be to the population variance.

$$S^2 = \frac{\sum(x-\bar{x})^2}{n-1} \tag{4}$$

This is denoted mathematically by the sample variance equation which is stated in equation (4). It is well known that the sample variance is a consistent estimator of σ^2, the population variance. This means that as the number of trails in the sample approaches infinity, the sample variance approaches the population variance.

This has application to both the mortality tables and the insurance companies themselves. For the insurance company, this is useful since the more clients that are insured, the more the life expectancy from the mortality table will coincide with the observed life expectancy. This greatly reduces the chance a company will have to pay a much higher percent of claims than expected. Looking back at the previous example of the insurance company insuring 20 people, if this number is bumped to say 200 instead, the probability that half or more of the clients die at age 72 or younger is .00023%. This number converges to zero quickly as the number of insured increases. For the mortality tables, as more data is taken on death rates of certain demographics, the death rate of the sample will give a more and more accurate representation of the population death rates. The law of large numbers is crucial in actuarial assumptions.

By using different demographics of people (ex: men vs. women, smokers vs. nonsmokers, etc.), it is possible to compare life expectancies across demographics. This allows for understanding which subgroups are expected to die earlier, which has direct effects on premium rates. Especially under a term life plan, a life insurance company that insures an individual that has a lower life expectancy than average (higher probability of dying) will consequently have a greater likelihood of paying out a claim. To offset the higher probability of paying a claim, the insurance company must charge more in its premium. Conversely, an individual with a higher life expectancy will most likely be required to pay less in premiums.

As a brief side note, mortality tables are typically created using information from governmental censuses. Figure 1 gives an example of a mortality table.

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Worktable 23R. Death rates by 10-year age groups: United States and each state, 2007

[Rates per 100,000 population in specified group. Rates are based on populations estimated as of July 1, 2007]

Area	All ages 1/	Under 1 year 2/	1-4 years	5-14 years	15-24 years	25-34 years	35-44 years	45-54 years	55-64 years	65-74 years	75-84 years	85 years and over
United States	803.6	684.5	28.6	15.3	79.9	104.9	184.4	420.9	877.7	2,011.3	5,011.6	12,946.5
Alabama	1,009.0	1,027.4	30.9	19.6	129.1	157.9	281.8	592.3	1,179.7	2,544.2	5,790.2	13,954.3
Alaska	506.7	643.5	34.9	28.9	120.4	119.1	226.4	423.0	796.1	2,008.6	5,107.0	11,754.3
Arizona	718.7	685.0	29.1	17.4	102.4	113.6	193.7	419.5	829.7	1,786.8	4,234.7	10,996.7
Arkansas	994.5	775.6	45.5	21.3	113.5	140.2	273.2	585.9	1,108.0	2,382.5	5,583.6	13,146.7
California	639.4	528.4	23.6	13.1	67.3	82.0	148.1	369.8	779.0	1,725.8	4,508.6	11,940.1
Colorado	616.9	597.4	21.3	13.5	68.4	84.5	159.8	352.1	685.4	1,696.0	4,712.5	13,662.7
Connecticut	818.1	660.5	23.0	8.1	57.8	83.9	135.8	326.4	706.6	1,688.0	4,746.0	13,611.6
Delaware	847.3	751.3	17.1	7.2	76.0	100.4	232.6	467.5	852.4	2,011.3	5,053.8	13,050.3
District of Columbia	881.9	1,465.4	31.8	27.7	98.5	142.0	296.9	734.0	1,217.6	2,357.0	4,630.8	12,326.7
Florida	921.0	705.8	34.4	16.0	100.6	127.8	208.0	478.1	888.2	1,808.8	4,155.6	10,258.1
Georgia	715.9	797.8	32.4	16.0	94.2	117.0	204.7	469.6	982.4	2,276.3	5,467.0	14,056.7
Hawaii	739.8	589.6	30.5	16.1	59.4	77.0	157.1	388.0	742.7	1,613.7	3,626.8	10,622.1
Idaho	721.8	694.0	32.9	17.2	88.2	90.1	157.3	351.3	722.6	1,830.6	5,002.2	13,865.4
Illinois	782.0	665.8	26.5	15.4	73.6	89.8	165.0	393.9	867.1	2,003.1	5,183.5	13,168.6
Indiana	851.0	772.9	30.1	17.8	83.1	115.3	194.1	434.4	930.6	2,253.2	5,346.3	13,407.8
Iowa	911.0	566.1	27.5	15.2	59.3	79.9	154.5	346.7	784.4	1,888.0	4,899.1	13,109.2
Kansas	882.2	823.7	24.4	16.4	77.7	94.6	173.2	417.7	830.1	2,014.3	5,348.7	13,832.5
Kentucky	945.2	716.5	28.7	18.7	93.8	141.0	252.1	510.5	1,081.8	2,528.6	5,853.1	13,988.6
Louisiana	930.9	993.2	42.6	23.4	126.0	203.3	304.0	579.5	1,164.5	2,496.9	5,703.8	13,676.7
Maine	948.4	639.1	24.6	13.0	81.7	102.3	141.9	346.0	828.3	2,011.4	5,275.7	14,502.1
Maryland	778.8	800.7	32.1	16.6	89.2	119.5	209.3	425.4	847.9	1,967.8	5,089.9	13,881.5
Massachusetts	820.4	508.1	15.6	10.6	56.4	74.7	145.7	326.5	737.8	1,817.0	4,877.3	13,499.4
Michigan	861.0	782.1	23.9	15.3	74.6	108.0	183.1	446.5	908.3	2,088.3	5,390.2	14,049.3
Minnesota	714.5	561.5	23.1	12.0	58.8	65.2	123.9	285.8	666.6	1,644.9	4,588.3	13,056.0
Mississippi	968.0	1,027.9	52.9	26.7	122.5	166.4	310.6	637.3	1,247.0	2,635.0	5,815.6	13,051.5
Missouri	921.4	759.9	34.9	17.5	93.4	114.6	204.4	462.8	926.4	2,240.4	5,495.5	13,694.8
Montana	900.3	639.4	36.4	16.0	108.2	114.2	192.7	426.1	799.5	1,971.6	4,986.3	13,976.1
Nebraska	860.1	681.3	26.2	18.0	70.4	75.8	170.7	344.3	749.8	1,979.7	5,027.2	13,829.1
Nevada	728.4	640.3	35.1	17.1	101.9	117.0	218.3	482.8	969.3	2,239.9	5,231.9	12,414.6
New Hampshire	783.0	533.0	18.1	13.3	61.7	74.0	139.1	306.7	728.4	1,919.8	4,987.6	14,197.4
New Jersey	802.0	536.9	20.5	12.5	62.5	88.4	153.4	344.6	753.3	1,819.5	5,048.2	13,511.8
New Mexico	785.9	638.7	41.8	15.7	111.5	155.9	256.7	489.0	891.7	1,930.0	4,697.6	11,756.9
New York	765.3	572.1	22.4	12.7	51.1	76.6	152.3	357.7	776.1	1,779.9	4,612.8	12,432.5
North Carolina	839.3	847.0	28.4	18.5	89.0	119.3	206.9	468.8	960.5	2,285.1	5,448.8	13,801.7
North Dakota	869.3	787.9	28.5	14.4	77.4	110.3	146.8	336.0	688.2	1,837.0	4,408.7	12,320.9
Ohio	929.1	783.5	32.3	12.5	76.4	109.4	191.3	447.4	954.7	2,243.3	5,557.3	14,115.8
Oklahoma	996.1	844.6	40.9	23.6	98.5	142.7	253.6	579.3	1,144.5	2,540.3	6,021.2	13,742.6
Oregon	838.0	576.9	24.0	14.0	66.4	84.5	162.4	405.2	803.9	1,926.5	5,053.6	13,984.2
Pennsylvania	1,006.2	783.9	28.2	14.6	80.1	115.3	179.9	405.9	900.6	2,064.5	5,306.4	13,670.2
Rhode Island	919.1	731.7	10.2	8.6	45.4	80.1	153.7	357.6	821.3	1,868.8	5,122.3	14,375.9
South Carolina	894.8	895.3	39.8	19.2	105.1	143.0	256.9	547.6	1,092.8	2,323.3	5,296.0	12,669.1

Figure 1. *"Sample Mortality Table"*.

"Note". This is a mortality table from the CDC in 2007 showing death rate (CDC/NCHS, 2007).

While the amount of data gathered from such a census is vast, there are some issues with higher age death rates. With a certain population, finding the death rate in the first year of life would give an exact probability; however, as the age is given a death rate, the number of data points for each age are reduced. This leads to the higher ages of a mortality table being worse estimates of the aggregate. This issue spreads across all life table estimates and arises when the “data [is] sparsest [at] the highest ages, where population numbers are small.” (Dodd et al., 2018, p. 1). Mortality tables are not an exact science, and thus require actuaries to perform statistical modeling for predicting future death rates.

Mortality tables play an important role in the life insurance industry, as they are the framework for actuarial models that are used in the price determination of premiums. It is vital for mortality tables to be assembled using a large sample size, so the data is not grossly skewed or inaccurate. The law of large numbers is useful in understanding why this principle is so crucial.

Hedging

Hedging is used by a plethora of corporations to protect against exposure to risk. The use of hedging techniques within the life insurance industry will be focused upon here. Since the industry is very diverse, there are many products and services provided, leading to many variations in the hedging strategy employed. Such strategies are different for annuities as opposed to life insurance contracts and there are variations even within these product lines. Annuity benefit hedging, life insurance reserving, life insurance risk internalization, and natural hedging techniques will be discussed.

In its simplest form, hedging is used to counter cash flows. When a company has risk, typically downside risk, it can hedge the position or asset, so the cash flows from the position and the hedge are opposite. This prevents the company from having unexpected cash outflows that are too large to manage. Hedging also allows a corporation to free up cash for desirable investments (Altuntas et al., 2017). Downside risk is mitigated from this process of hedging as it acts similar to insurance by lessening the catastrophic downside risk with the substitution of a known loss. Thus, hedging a position costs a firm; there is a price for security. If a firm perfectly hedges a risk, the cost of the hedge will perfectly counter the gains from the risk. Thus, the risk will be perfectly neutralized, and the firm will see no increased profit. Therefore, in order to turn a profit, a firm must take some risk. This is done by not hedging everything or not perfectly hedging the risks. Risk is then mitigated without being removed completely. Perfectly hedging a risk is accomplished by entering into a long position in an asset that is perfectly negatively correlated to the risky asset. It can also be accomplished by entering into a short position in an asset that is perfectly positively correlated to the risky asset. In both cases, when the risky asset does well, the hedge does poorly, and when the risky asset does poorly, the hedge does well.

Annuities

Life insurance companies use hedging methods in two main fields: annuities and life insurance policies. The first to be discussed is annuities and the related guarantees that come with them. Annuities are simply a fixed cash flow stream (i.e., an income). Money is put into an account which grows for a set time before that money is taken out and paid back to the annuitant in equal installments. There are two main products within the annuity market: fixed and variable annuities. Fixed annuities grow the account balance at a rate specified by the provider, and it is the provider's responsibility to cover that accumulation. Variable annuities, as the name suggests, grow at a variable rate which is often tied to a market index. These are riskier than fixed annuities, but they yield higher returns because of the increased risk.

Most special guarantees are associated with variable annuities, but not all are limited as some can be applied to fixed annuities as well. The different guarantees that are going to be discussed in this paper are Guaranteed Minimum Death Benefit (GMDB), Guaranteed Minimum Accumulation Benefit (GMAB), Guaranteed Minimum Withdrawal Benefit (GMWB), and Guaranteed Minimum Income Benefit (GMIB). Each of these can be hedged using different derivative options in the capital market (Feng & Yi, 2019). These guarantees get complicated and often resemble exotic options, but this paper will stay mostly with simple market options.

A GMDB within an annuity is a guarantee that a party designated by the contract holder will receive a minimum amount should the contract holder die before funds are withdrawn from the account. There are two main types of GMDB: return of principal and earnings enhanced death benefit. These can be applied to fixed and variable annuities, but they are seen more often connected to variable annuities.

With a return of principal GMDB, the designated party receives the greater of the value of the principal and the current value of the annuity. The value of the death benefit can thus increase above the original principal added into the annuity, but it cannot dip below that value. This is represented as a put option, a type of market derivative that allows the option holder to sell the underlying at the strike price, since the beneficiary has the option to receive the funds from the original principal (strike price) but can choose the current value if that is higher (McDonald, 2013). From the perspective of the issuer of the annuity, the return of principal GMDB is represented as a short position in a put option, so it would be hedged with a long position in a put option. When the GMDB is activated, it is as if the long position in the put option is being exercised. Thus, the issuer is required to pay. The issuer would then exercise his/her put option to receive a similar payment as the one that is being paid out. This way, the issuer can neutralize the cash flows from the benefit and not have to worry about the potential losses that could occur.

Another type of GMDB is the earnings enhanced death benefit. This benefit pays the beneficiary a percentage of the increase in accumulated value. If the account principal was $400,000, the principal accumulated 50% of its value, and the earnings enhanced death benefit was 40%, the value to the beneficiary in the case of the policyholder dying would be $400,000*0.50*0.40 = $80,000. This is because the interest accumulated is half the principal or $200,000 and the beneficiary gets 40% of that increase from the guarantee or $80,000. If, however, the account value drops below the principal amount, no benefit is given. This is represented as a call option, a type of market derivative that allows the option holder to buy the underlying at the strike price, since the beneficiary essentially has the option to buy the policy at the principal (strike price) and sell it right away while keeping a percentage of the increase (McDonald, 2013). If the current value is lower than the principal, however, the beneficiary is left with the current value. This aligns with the standard call option. From the issuer's perspective, he/she has a short position in the call option. This is due to payment being required should the option be exercised. With a similar argument to the one for return on principal GMDB, a long position in an equivalent call option is required to hedge the earnings enhanced death benefit.

If a more complex version of a GMDB is used where both the return on principal and the earnings enhanced death benefit are employed, then a chooser option must be employed. The issuer must then take a long position in an equivalent chooser option to neutralize the cash flows.

The next benefit type is the Guaranteed Minimum Accumulation Benefit (GMAB). This type of benefit has some similarities to a return on principal GMDB, but it is certainly different.

It guarantees a minimum account value after a certain period. Unlike the GMDB, the GMAB is contingent on the annuitant surviving to the end of the guarantee period. Then if the account value is higher than the guarantee, it is no exercised and the account value remains. If, however, the account value drops below the guarantee, it is exercised, and the account value increases to the guaranteed amount. This annuity benefit is only applicable within variable annuities as they have the potential to drop in price as opposed to fixed annuities which have a set return. This benefit protects against the downside risk of market drops associated with the variable annuity. It, like the return of principal GMDB, is represented by a put option. Similarly, from the issuer's position, it would require a long position in a put option to hedge against this guarantee.

If a more complex version of a GMAB is employed where the account value minimum can be locked in multiple times, this is represented by a lookback option, a form of exotic option. Thus, the issuer would need a long position in an equivalent lookback option to counter the cash flows.

The next type of guarantee is the Guaranteed Minimum Withdrawal Benefit (GMWB). When exercised, this benefit allows the annuitant to annually withdraw a certain percentage of the current value of the annuity until the original principal amount is returned to the annuitant. This guarantee is available within fixed and variable annuities. As an example, suppose an annuitant bought an annuity with an original principal of $400,000 and after 20 years the value increased to $600,000 with a GMWB of 5%. Then the annuitant would be able to exercise the GMWB to receive $30,000 annually until the original $400,000 is recovered, or a little over 13 years. This is represented by a put option, as the GMWB can be recovered even if the account value drops below the guaranteed amount. Thus, it is hedged from the issuer's perspective with a long position in a put option.

The last common form of an annuity guarantee is the Guaranteed Minimum Income Benefit (GMIB). This benefit guarantees a minimum income stream when annuitizing. This is different from a GMWB since the GMIB is not withdrawing funds against the account value but is just a set minimum income stream. A GMWB is exercised during the accumulation phase of the annuity while the GMIB is exercised in the payout period of the annuity. GMIB's are mostly used in variable annuities to counter the risk from the market and offer more security. Often times, when an annuity is annuitized, a GMIB will guarantee a minimum payment based on the original investment compounded at some set interest rate. This actually creates a maximum function with the guaranteed amount and the market value of the annuity. If the market value would give a larger payment than the guarantee, payment is based on the market increase. If, on the other hand, the guarantee would provide a larger payment, for example, if the variable annuity does not perform well, payment would be made based on the guarantee. This type of guarantee would be represented by a put option as well. However, since it would be guaranteed before payments are made and it is a stream of income, many put options expiring at different dates may be the best representation of the guarantee. Thus, the issuer must take a long position in the different put options corresponding to the payments in order to hedge the GMIB and reduce the risk of market downturns.

Life Insurance Products

Life insurance products are guarantees that when the insured dies, payment will be made to the designated party. This is very similar to a GMDB, but it is not connected to an annuity. It is easier to see a GMDB as a life insurance addition to an annuity product. Life insurance products have their risk mitigated in ways different than annuities. Life insurance companies employ the principle of the law of large numbers to minimize catastrophic risk. This is done by accumulating many policies, so that when one policy requires a payout, it is covered by the other policies collecting premiums. Another way is through hedging with pure endowments (Bayraktar & Young, 2007). Yet another method for this is through delta-gamma hedging (Luciano et al., 2012) which is beyond the scope of this paper. These, however, do not always diversify all risk. There are periods where death in populations is not uniform, leading to increases or decreases in life insurance payouts. These are systematic risks that must be approached using other methods (Dahl & Møller, 2006). These systematic risks pose a serious threat to insurers and must be dealt with adequately.

Before addressing systematic risks, however, the concept of reserves must first be established. Insurance companies model risks associated with their policies using advanced actuarial techniques to generate an estimate on the required payouts. This is then used to generate an amount that the insurer must keep reserved for the payout claims to ensure that claims can be paid as they arise. An insurer needs to keep adequate cash to pay claims so that customers continue suing their products. They do not want excess cash though as that cash could have been allocated to investments earning interest instead of sitting in reserves. This method of reserving internalizes the risk of the insurance by not hedging the risk. The insurance provider is simply estimating an amount needed and keeping that amount on hand.

Another form of managing the risk of life insurance policies is through a process called natural hedging. This method is accomplished different ways, but the concept remains the same for the different approaches. It uses other products to hedge against the risk of life insurance instead of derivatives and other methods. Natural hedging is used to mitigate exposure from systematic risk of life insurance which arises when the deaths in a population cease being random and there are aggregate changes. As an example, if a life insurer writes policies for clients assuming an average mortality age of 80, but over the next 30 years, when those individuals would be reaching this age, the average age of mortality has increased to 85, the cash flows for paying claims will not be what is expected. In this situation, that may be a favorable move in mortality rate, though it can change in the opposite direction as well. Having a mortality table stay consistent for 30 years is highly unlikely (Wang et al., 2013). This is systematic exposure since adding additional policies will not reduce the risk. A way to naturally hedge against an increasing or decreasing age of mortality is to offer products that are inversely affected by these changes in death rate.

One such example of a product that is inversely correlated with life insurance products is an annuity that pays the annuitant until he or she dies. Clearly, for the annuity, the longer the annuitant is alive the worse it is for the issuer since more payments are required. When an individual with a life insurance policy lives longer, that is beneficial to the provider. This is because the provider can use the freed cash for investment purposes instead of paying a claim immediately (Li & Haberman, 2015). Thus, annuities and life insurance policies are inversely correlated and can be used to hedge each other.

There are other products that can act as a natural hedge to life insurance, but annuities are the clearest, most obvious product with this natural hedging capability. This concept of natural hedging encourages many life insurance providers to also offer an annuity product line. Providing both services even gives a firm a competitive advantage (Cox & Lin, 2013). Though this is an efficient hedging strategy to employ because it is, as the name suggests, natural, it is not without its concerns. Adding another product line means another set of risks being introduced along with more opportunities for mispricing risks and other human error. There is also the need to determine how much of each product is needed to efficiently minimize risk

(Wang et al., 2009). That is too specific for the purposes of this paper.

Annuities and life insurance products are highly used in the developed world, and both come with inherent risk. Insurance companies are willing to take on that risk for a corresponding return. Hedging is a method used by such firms to mitigate the risk that they take on, and it is achieved in multiple ways. Hedging annuities and life insurance products can be done in tandem through natural hedging. This is useful because simply having both products protects the insurers from much of the risk inherent in the products themselves. For the specific product lines, annuity guarantees are often hedged through market derivates. These guarantees are the Guaranteed Minimum Death Benefit (GMDB), Guaranteed Minimum Accumulation Benefit (GMAB), Guaranteed Minimum Withdrawal Benefit (GMWB), and Guaranteed Minimum Income Benefit (GMIB). They are hedged using a variety of call and put options, along with some exotic options. Life insurance products are often hedged internally using the concept of the law of large numbers and through reserving techniques. When risk is not random though, these techniques show to be inadequate, and other hedging techniques, such as natural hedging, are needed.

Future Research

To give an in-depth analysis of the mathematics behind predictive analytics and hedging risks properly is beyond the scope of this paper. There are, however, many benefits in understanding how insurance companies use numerical methods in their determination of valuation of policies and what the premium prices must be to receive positive returns.

Conclusion

Many of the impacts that directly and indirectly influence the process of setting premiums on products for insurance companies have been addressed above. These factors include investing assets under management, forecasting activities, mortality tables, and hedging.

Each of these has its own use to the insurance company. Investing in a diversified portfolio to maintain a risk averse position is used to increase revenue for an insurance company, which is crucial to the goal of maximizing profitability. Forecasting the macroeconomy and things such as death rate help the insurance firm accurately estimate future revenue and cash outflows, which is necessary in the field of reserving. For life insurance companies, having a mortality table that will accurately predict the death rate of the insured is important to estimate cash outflows as well. When these predictions are erroneous, however, hedging is essential for insurance company to be engaged in to negate the excess cash outflows from errors in predictions. Each of these factors work together to help firms maximize their profits.

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