The McEliece Cryptosystem as a Solution to the Post-Quantum Cryptographic Problem

Isaac Hanna

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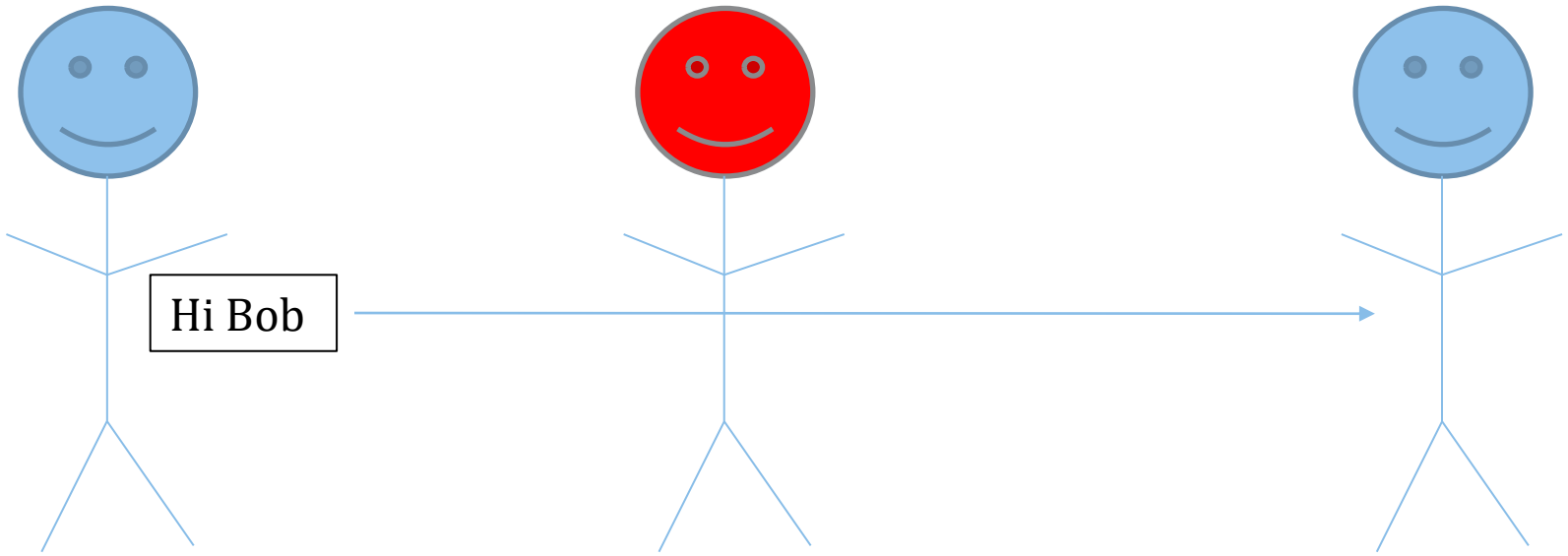
The Cryptographic Problem



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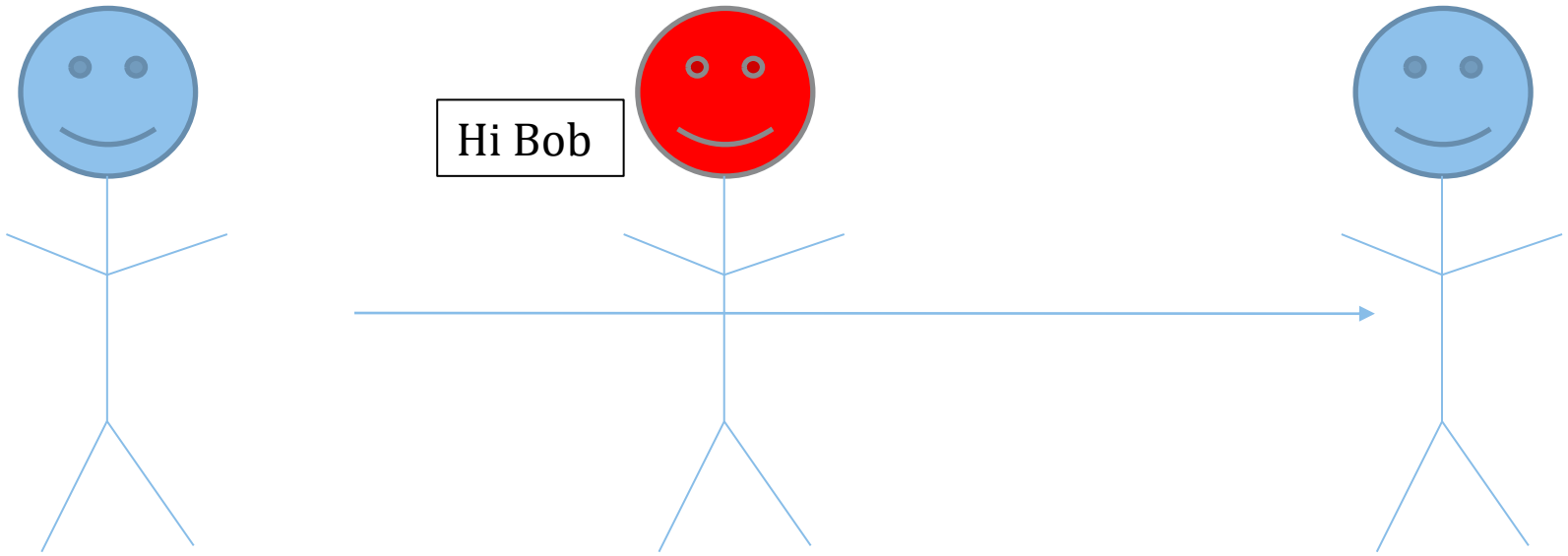

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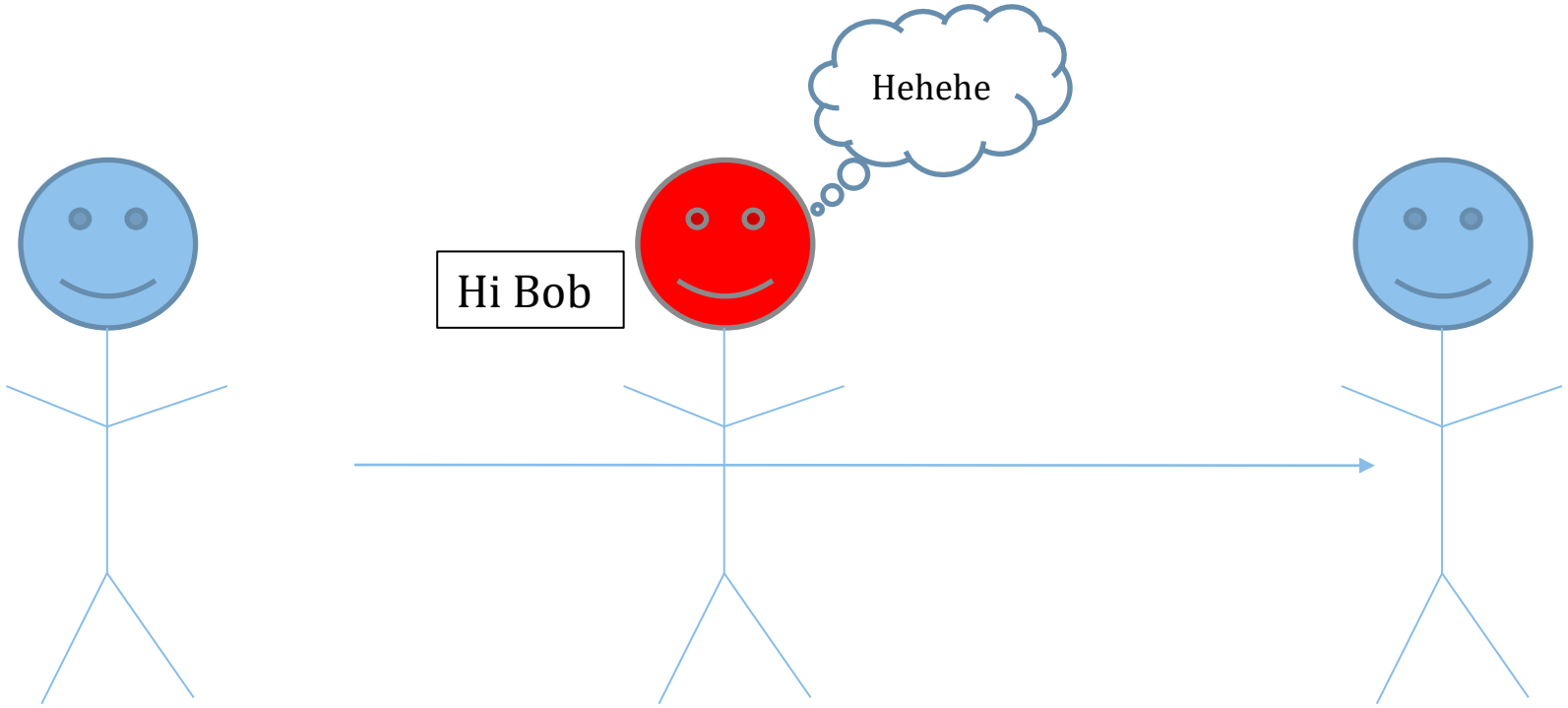

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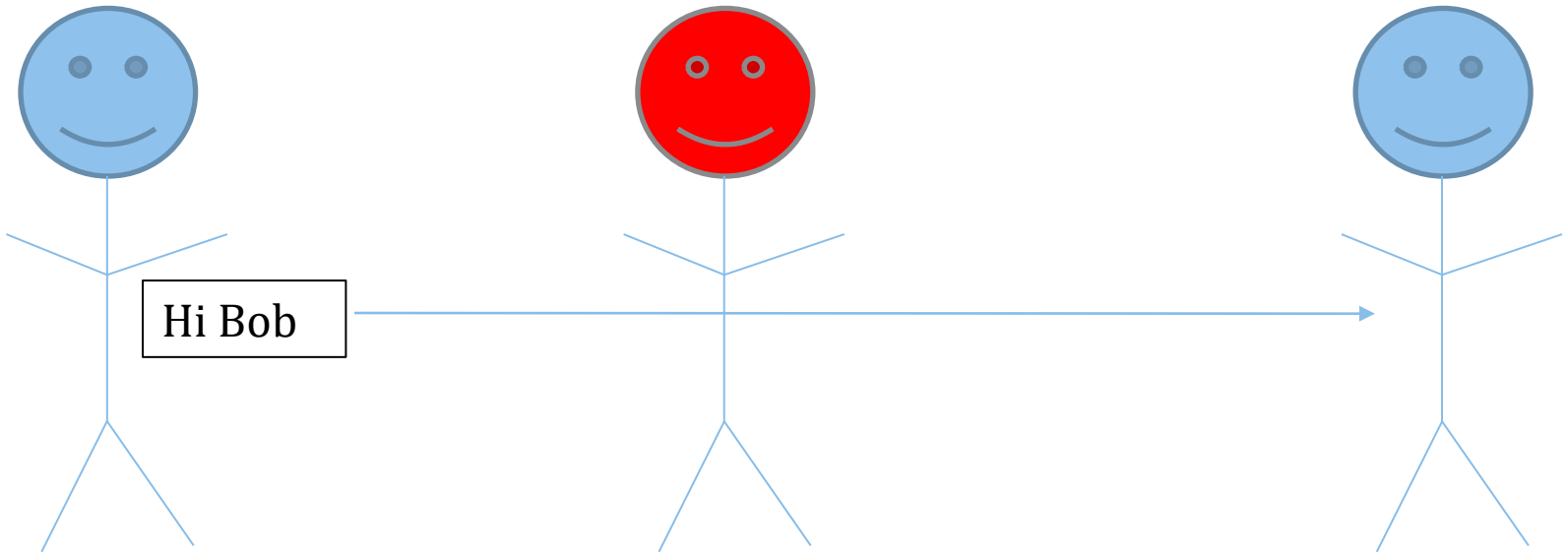

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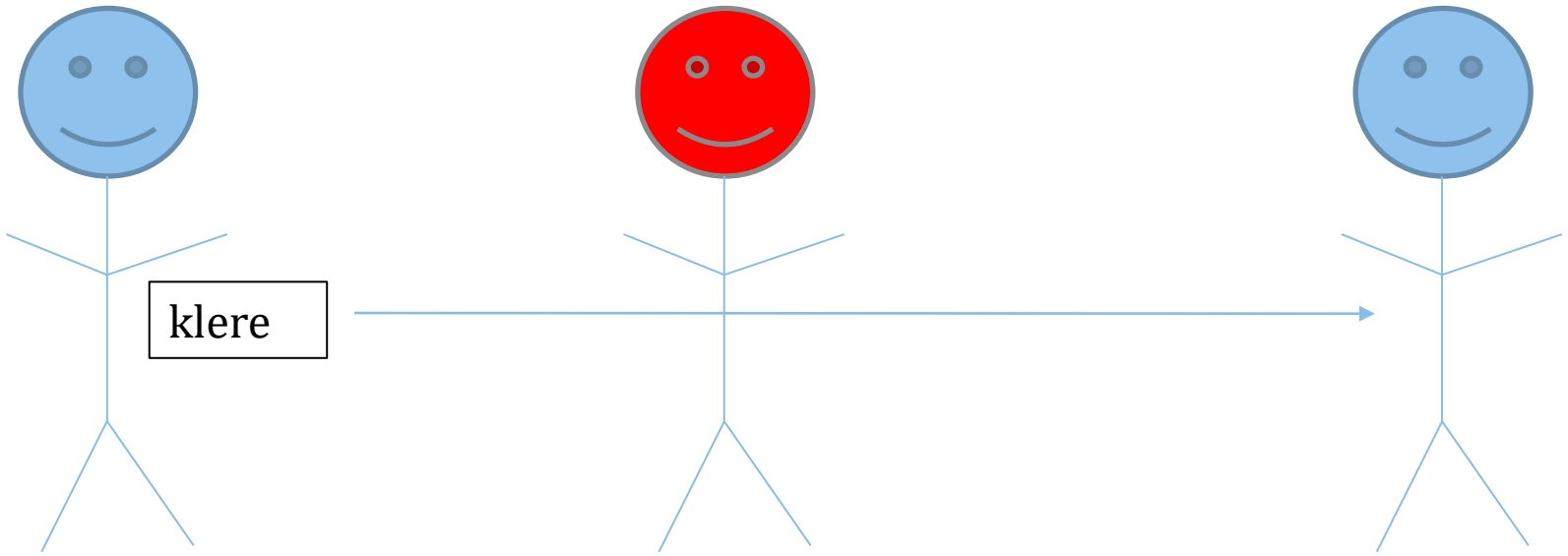

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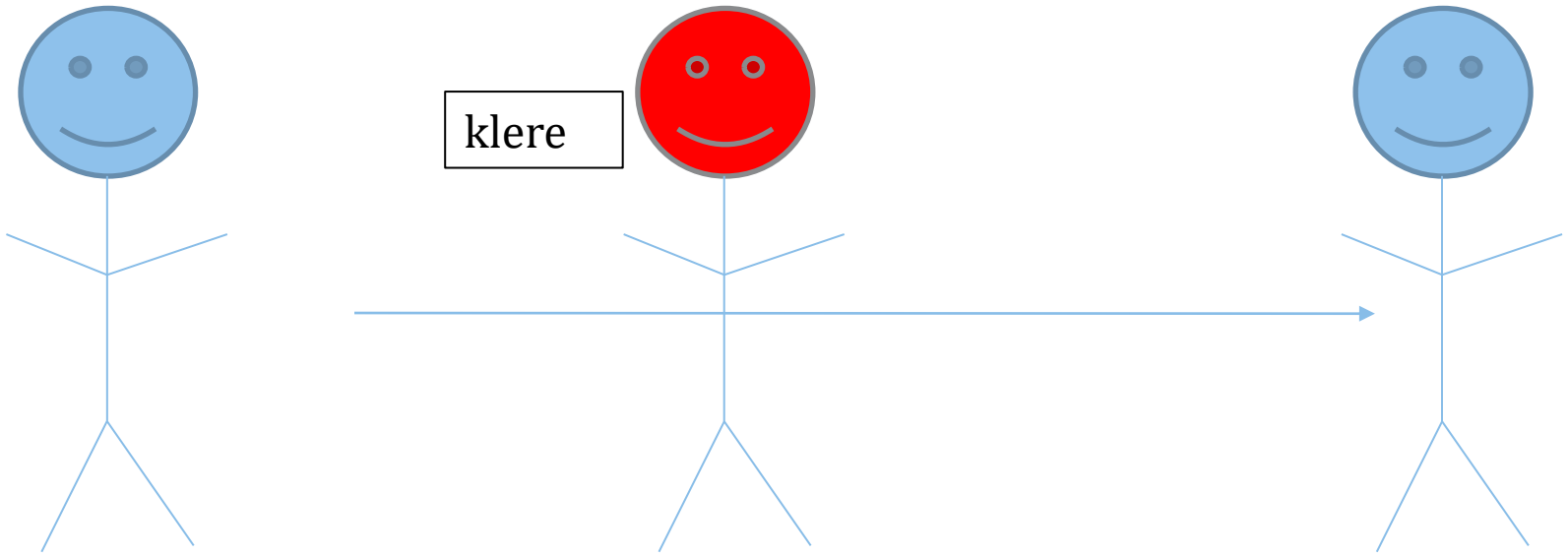

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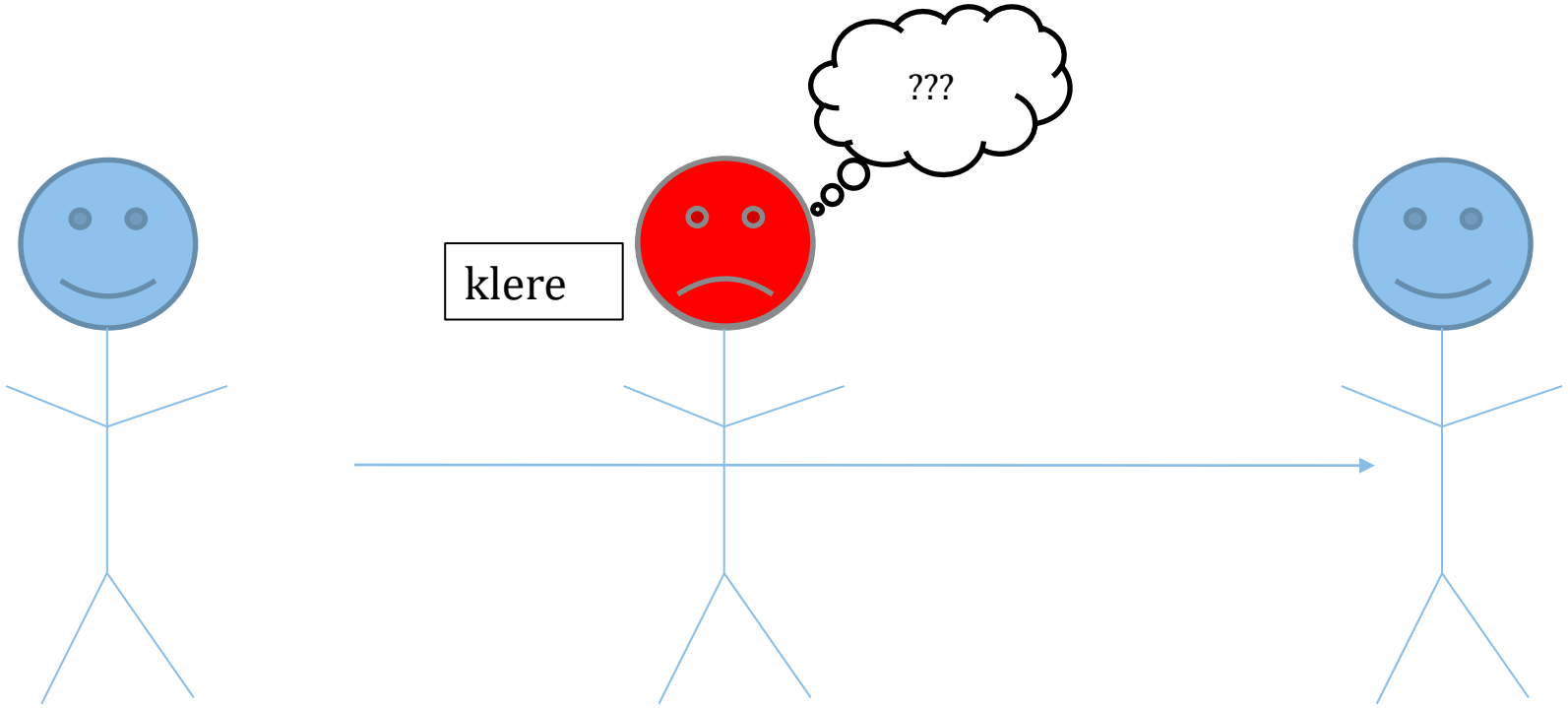

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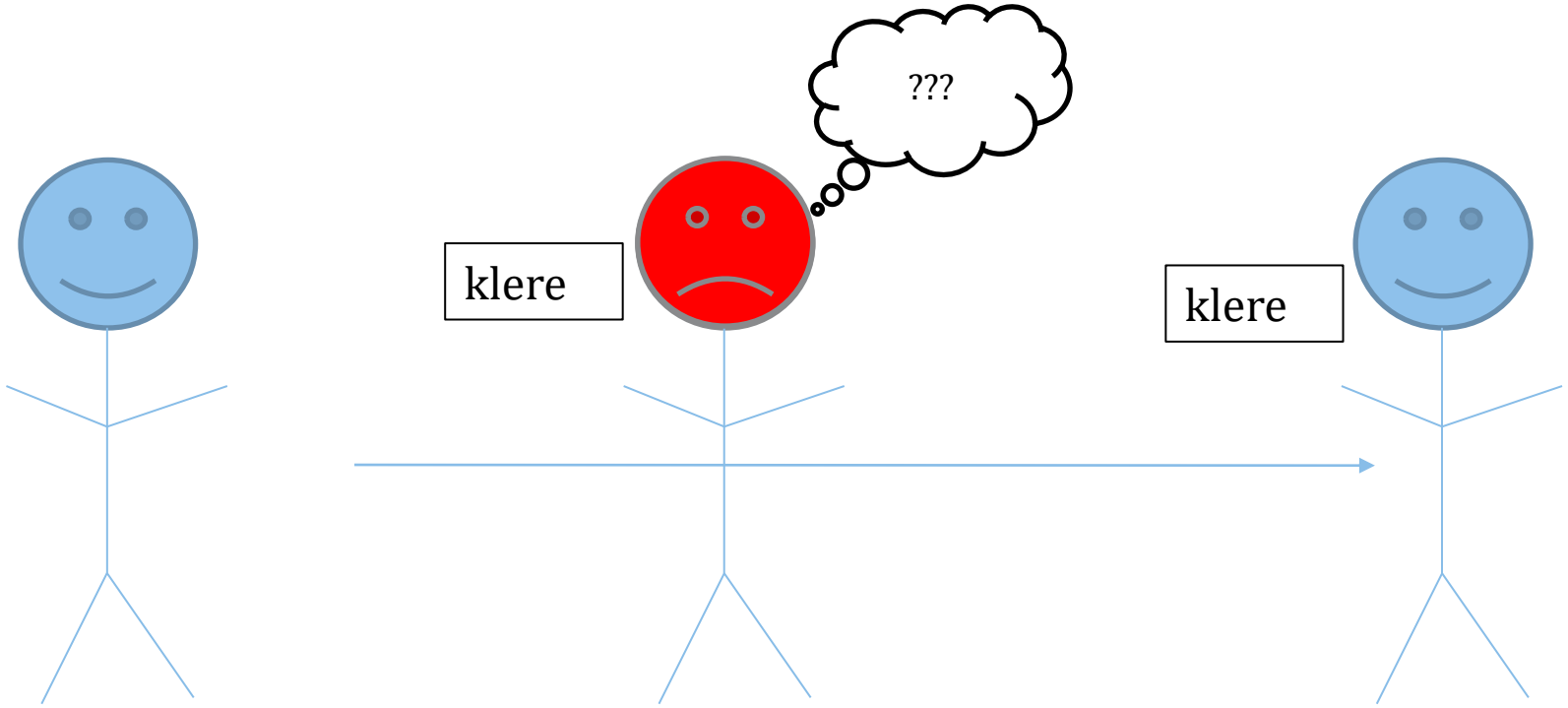

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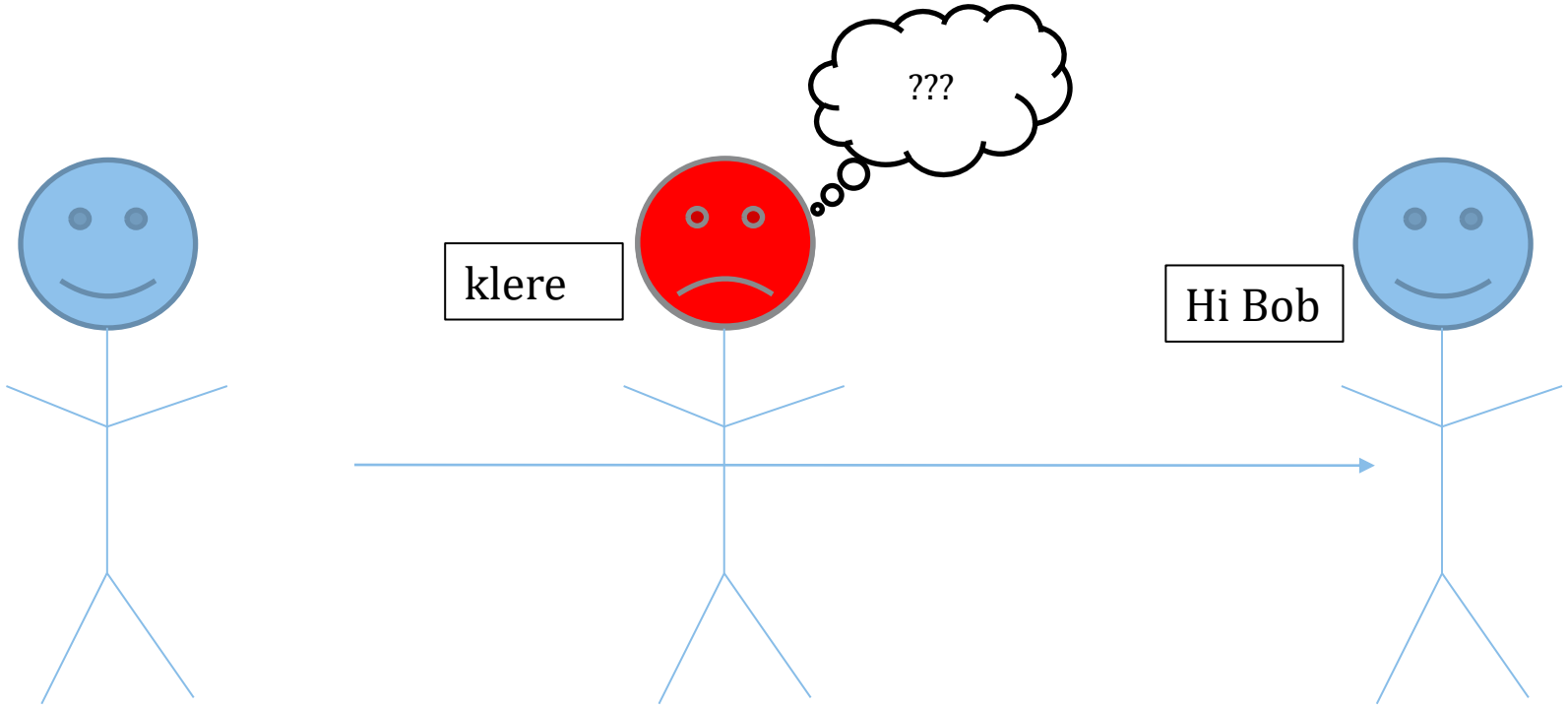

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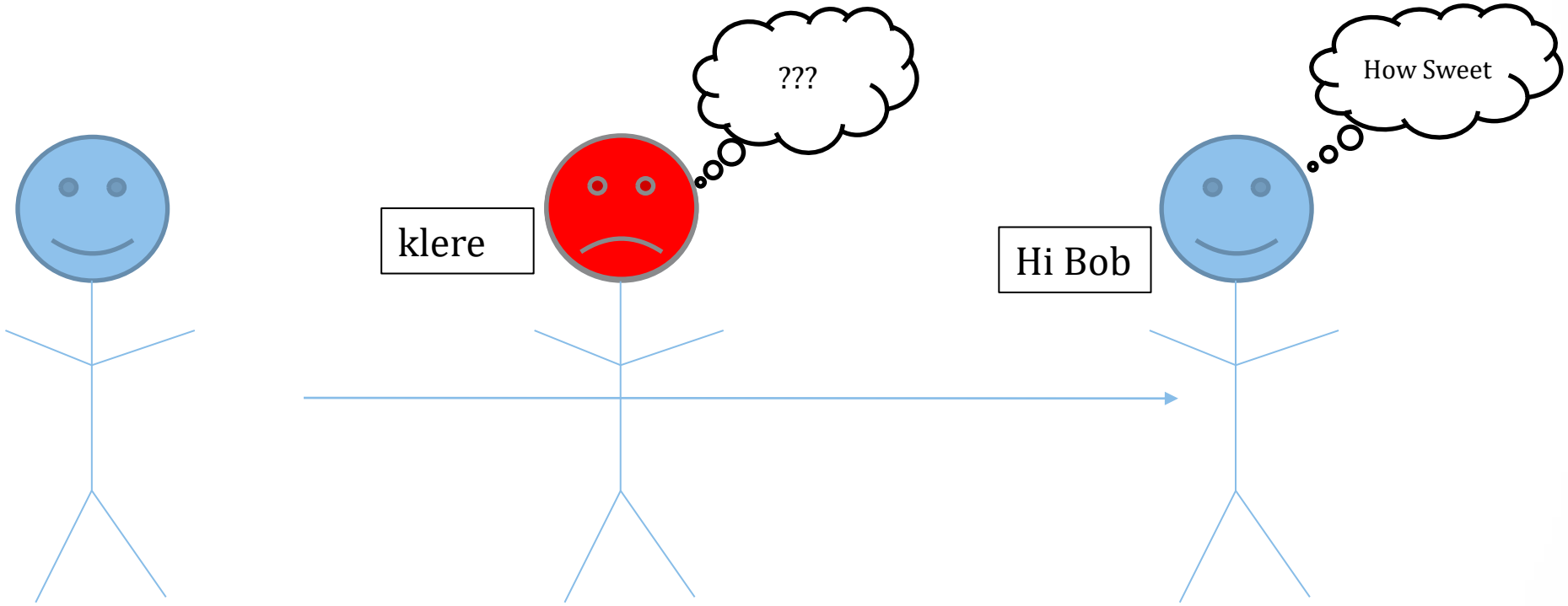

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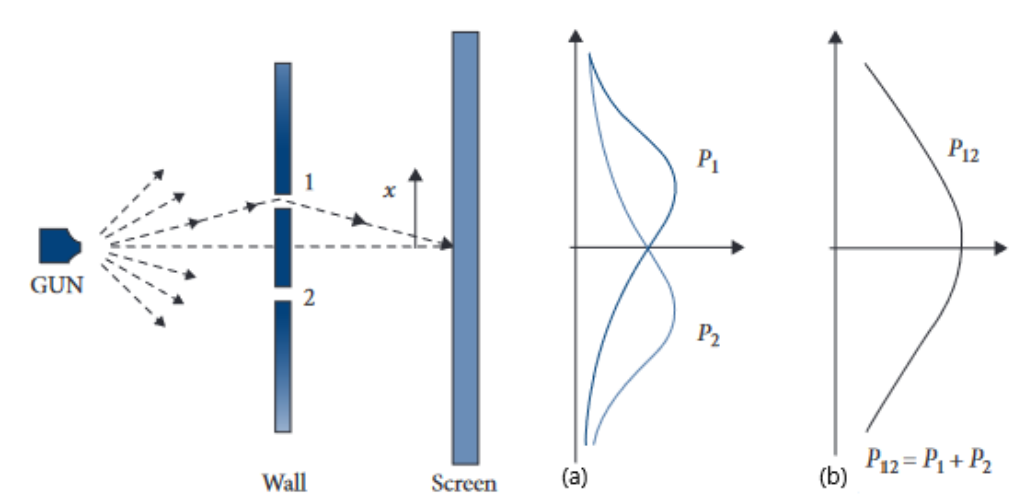
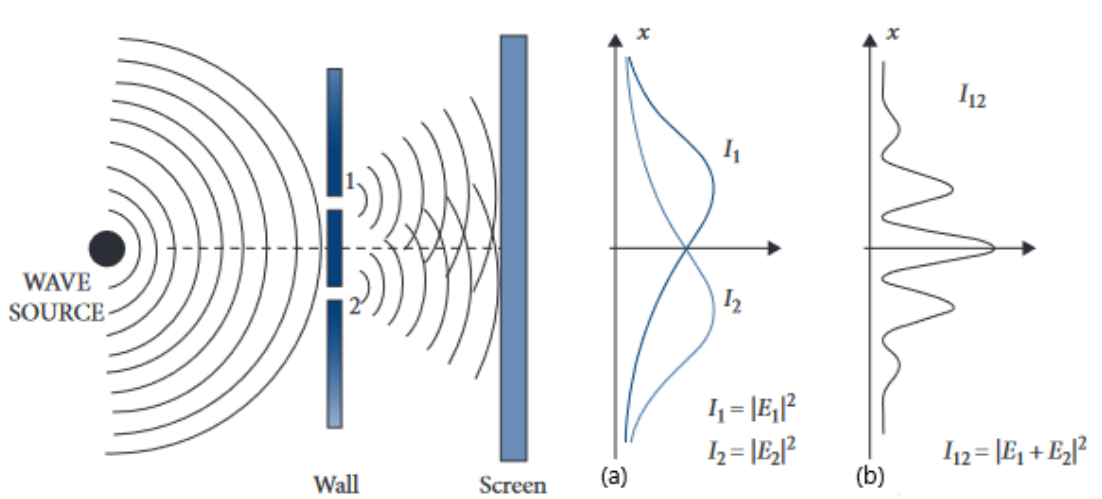
- Shor's Algorithm provides an exponential speedup

Double Slit Experiment





Quantum States and Superposition

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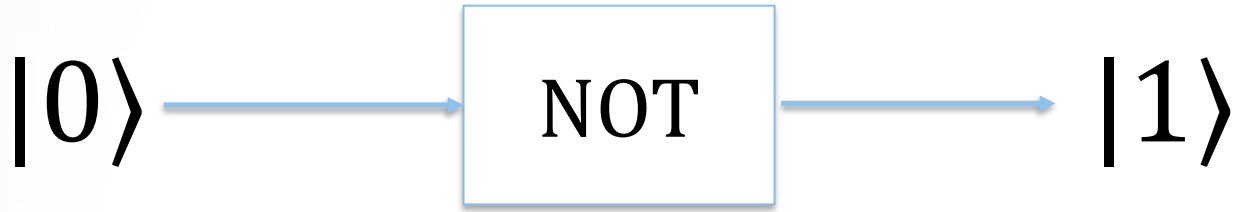
- $|ab\rangle = c_{00}|00\rangle + c_{01}|01\rangle + c_{10}|10\rangle + c_{11}|11\rangle$

Quantum Gates

Quantum Gates



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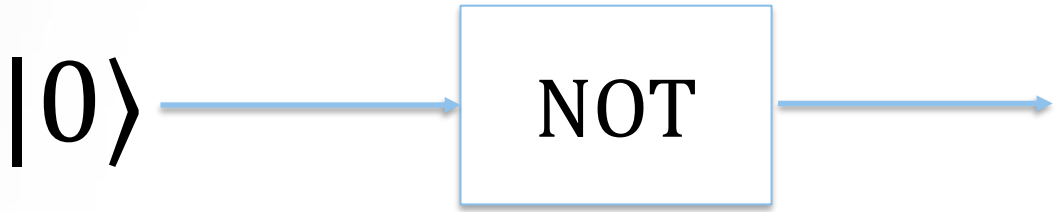

Multi-Qubit Gates

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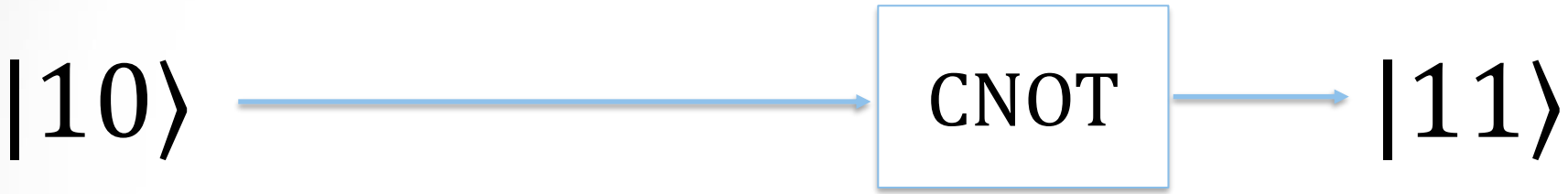

Multi-Qubit Gates

$|10\rangle$ → CNOT → $|11\rangle$

$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|10\rangle$ → CNOT → ???

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- Finding information about one gives us information about the other

Phase Encoding

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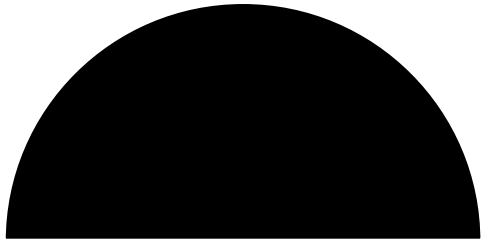

Phase Encoding

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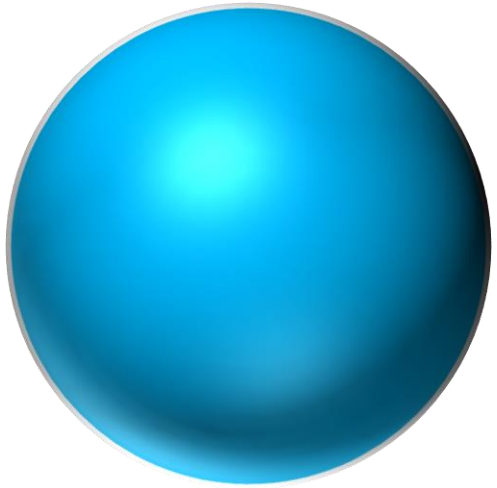

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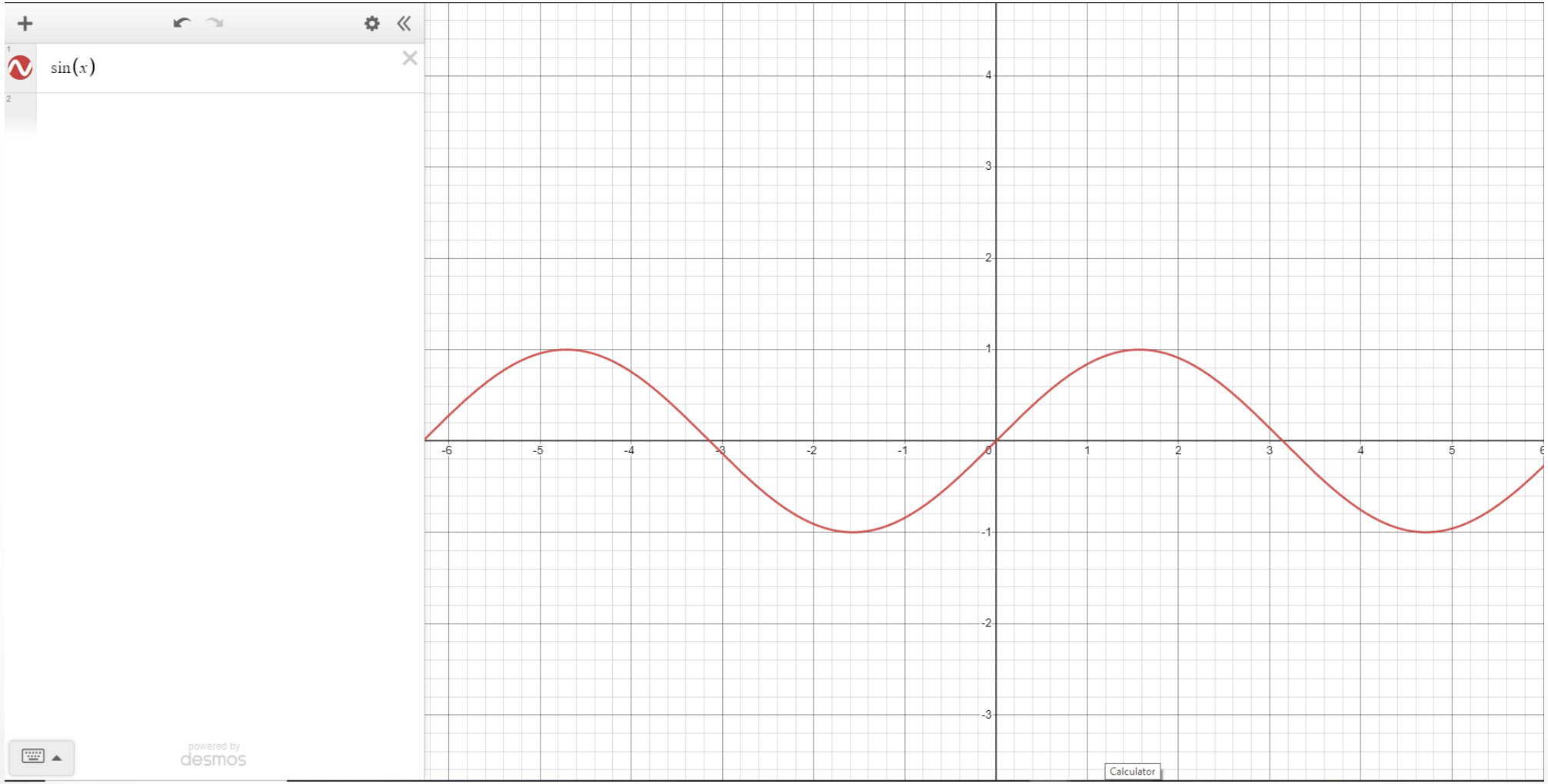

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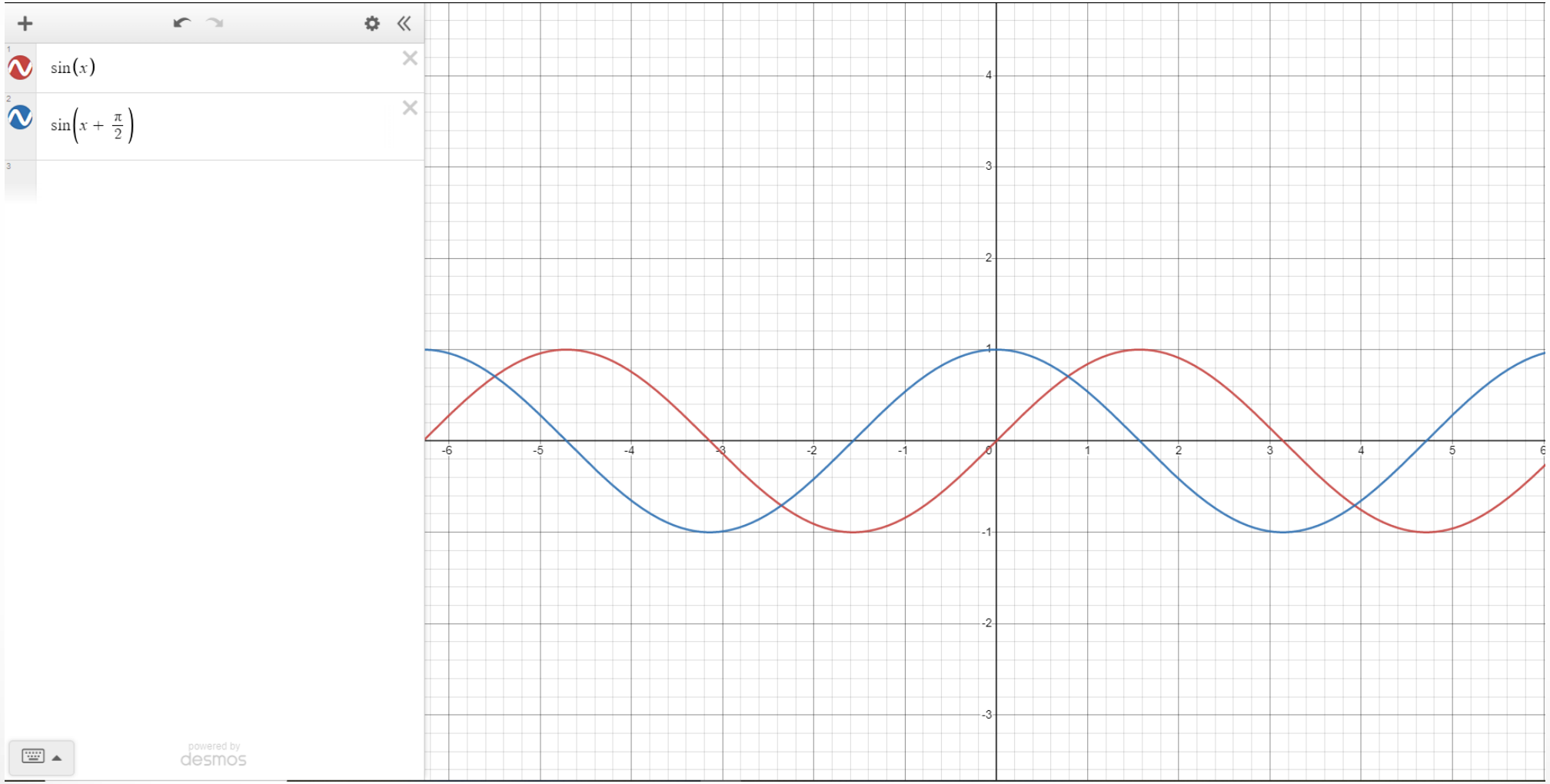

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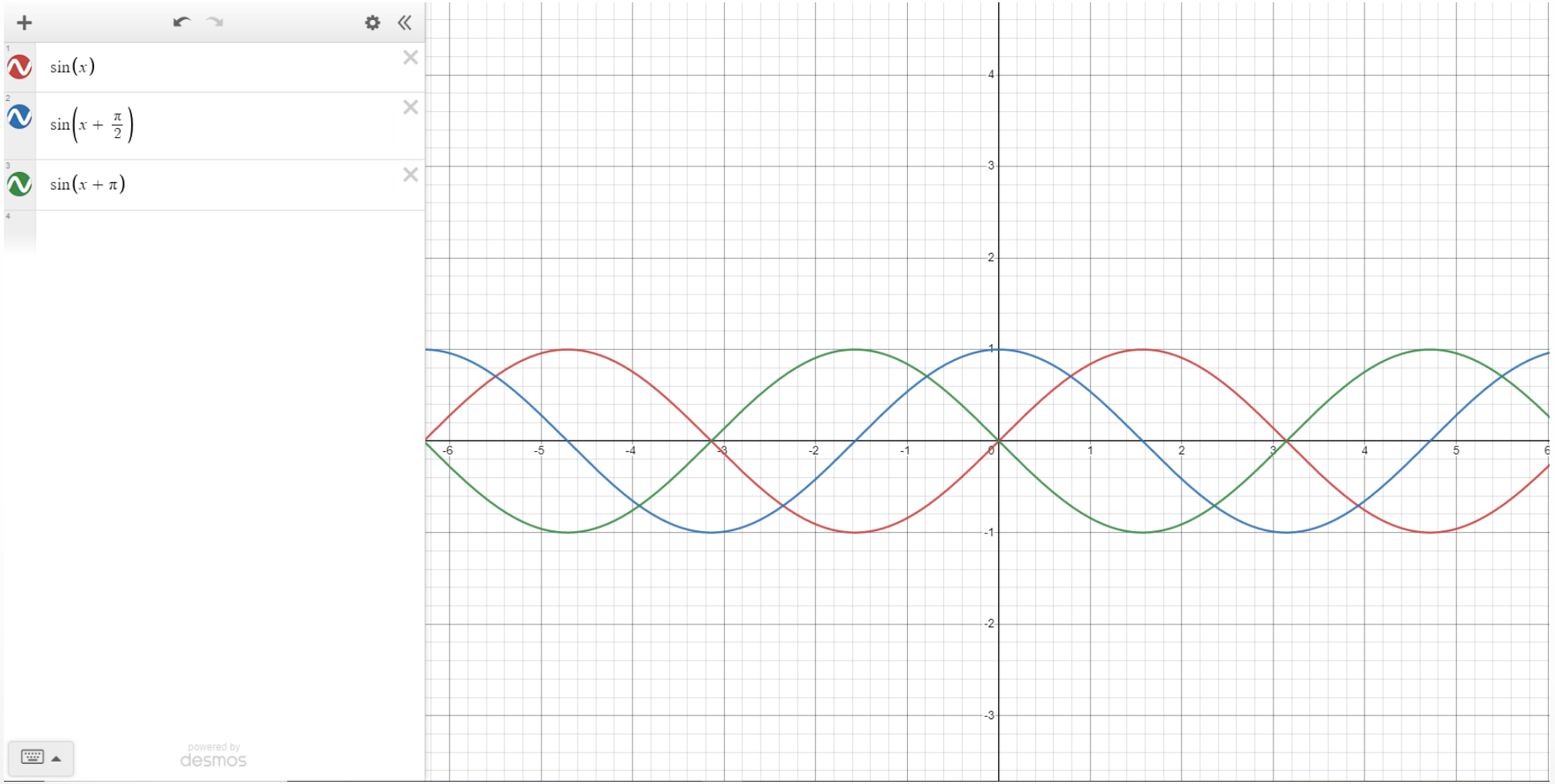

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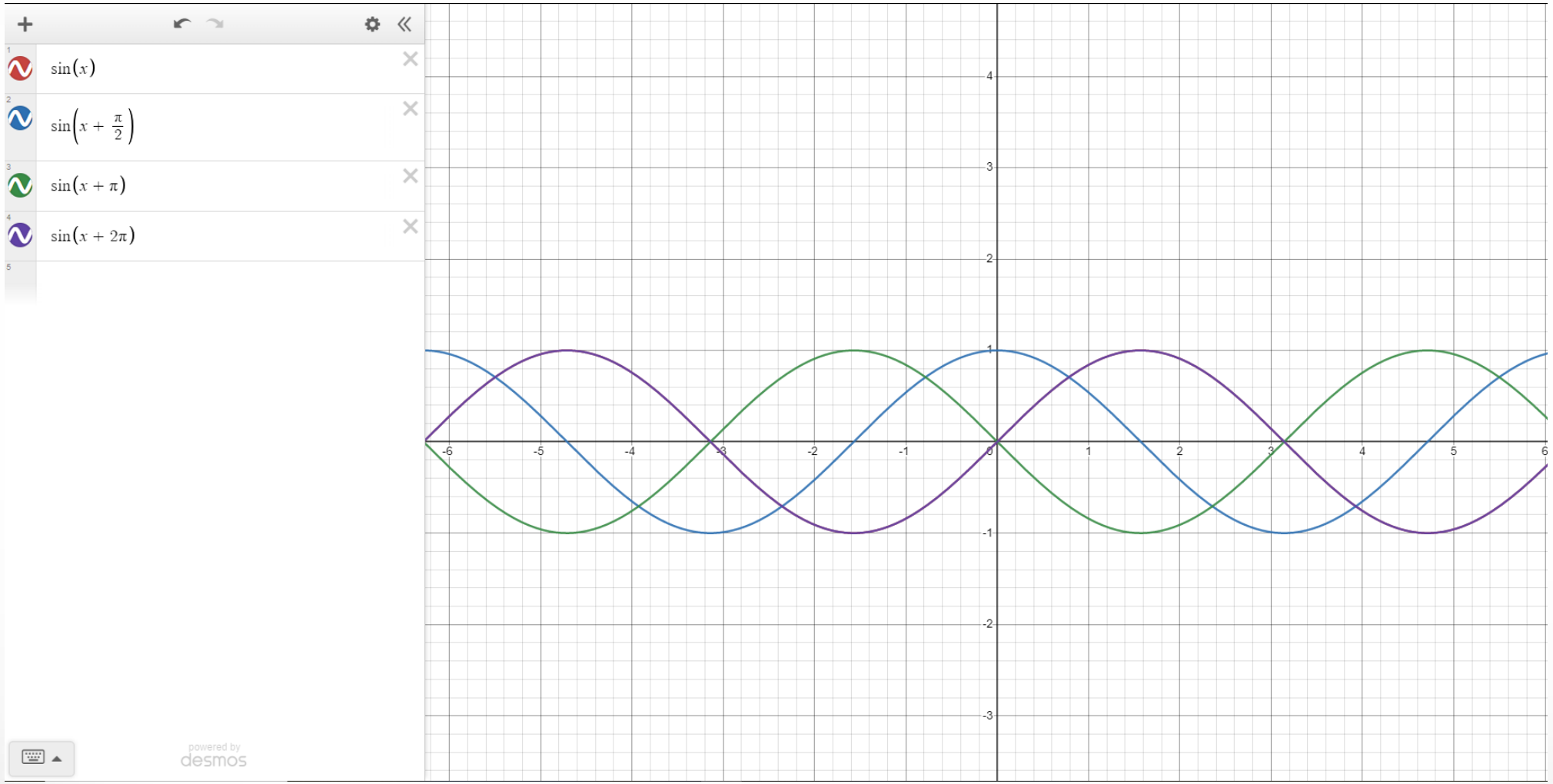

Shor’s Algorithm

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- Create an Entanglement with a second register by mapping a to $x^a \ (mod\ n)$
- Encode the values in the phase of the first register
- Observe the value of the second register

McEliece

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- Polynomial of degree s allows correcting up to s errors

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- Permute codewords using permutation matrix P
- Add t errors where t is the degree of the generator polynomial

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Security and Cost

- Security rests upon multiple difficult problems
  - Crack a linear code with errors
  - Or Decode a matrix into three matrices
- Cannot be reduced to the Hidden Subgroup Problem
- The large keys make it much more expensive than current schemes

Conclusion

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- Current encryption is based upon integer factorization
- Shor's Algorithm takes advantage of superposition, entanglement, and phase encoding to provide an exponential speedup
- The McEliece Cryptosystem is not currently susceptible to quantum attack

References

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