Cost Functions of Crabs: Applications of Hermit Crab Shell Exchange Behavior to Vacancy Chain Modelling

Hannah Miele

A Senior Thesis submitted in partial fulfillment
of the requirements for graduation
in the Honors Program
Liberty University
Spring 2021

Acceptance of Senior Honors Thesis

This Senior Honors Thesis is accepted in partial
fulfillment of the requirements for graduation from the
Honors Program of Liberty University.

____
Robert Rich, M.S.
Thesis Chair

____
Hector Medina, Ph.D.
Committee Member

____
David Schweitzer, Ph.D.
Assistant Honors Director

____
Date

Abstract

Vacancy chain systems function as a method of resource distribution in domains such as housing and labor markets. Hermit crabs also employ vacancy chains as a method of shell exchange. Application of vacancy chain modelling in engineering has been attempted, but numerous flaws exist in the developed vacancy chain scheduling algorithm. This work addresses the lack of an appropriate vacancy chain cost function by developing a generalizable cost function based on hermit crab shell exchange behavior. The cost function's purpose is enabling development of realistic engineering experiments and models based on real-world vacancy chain systems.

Cost Functions of Crabs: Applications of Hermit Crab Shell Exchange Behavior to Vacancy Chain Modelling

Introduction

Vacancy chain systems redistribute resources effectively in a variety of domains. The housing and labor markets exhibit vacancy chains, and hermit crabs actively construct vacancy chains as a method of shell exchange. Attempts have been made to apply vacancy chain modelling to engineering domains. Specifically, Dahl et al. (2009) propose a vacancy chain scheduling algorithm for application to multi-robot task allocation problems. Although their work proposes to solve relevant real-world problems, the experimentation methods used by Dahl et al. (2009) fail to consistently apply vacancy chain modelling to their problem domains. The circularity in their cost function and definition of optimality renders their work incomplete; this work attempts to address that gap by examining how hermit crab shell exchange behavior may inform the development of an appropriate cost function for vacancy chain modelling.

In this work, an overview of vacancy chain systems in human domains and their specific exhibition in hermit crab shell exchange behavior are provided. A detailed examination and critique of the work of Dahl et al. (2009) follows. Research questions regarding the development of a cost function and analogizing hermit crab behavior are proposed. An overview of this author’s research methodology, including databases used, search terms employed, and constraints applied, are also provided. The development of a hermit crab vacancy chain cost function is discussed, and this work concludes with recommendations for potential application of the hermit crab vacancy chain cost function.

Vacancy Chains

General Theory

Vacancy chains are a method of resource distribution in which a resource that becomes available triggers a further exchange of resources (Fioretti, 2009). Chase et al. (1988) denote the criteria for identifying vacancy chains:

> First, the resource must be reusable, discrete, and used by only one individual (or social or ecological unit) at a time. Second, a vacancy is required before an individual takes a new resource unit, and individuals must need or desire new units periodically. Third, vacant resource units must be scarce, and many individuals must occupy sub-optimal ones. (p. 1265-1266)

Chase (as cited in Fioretti, 2009) also noted that vacancy chains require most units in the queue to already possess a resource in order to leave an available resource behind.

Models involving vacancy chains have primarily been applied in the realm of economics. Specifically, vacancy chain models have been developed for applications in the housing market (Nordvik, 2004). Ferrari (2011) as well as Ben-shahar and Sulganik (2011) analyze mobility in the housing market using vacancy chain modelling. In addition to their applications in the housing market, vacancy chain models have also been constructed for labor markets. Fioretti (2009) explores the relationship between vacancy chains and competition in the context of internal labor markets. Interestingly, vacancy chains have also been observed as a method of resource allocation among criminal organizations. Friman (2004) remarks that “vacancy chain arguments suggest that short of significantly altering demand patterns in drug markets, the current emphasis on disrupting large-scale drug trafficking organizations and domestic distribution networks will lead to unintended mobility in the criminal economy" (p. 71). In other words, even criminal economies demonstrate the effectiveness of vacancy chains as a method of resource allocation.

While vacancy chain modelling has some applications in economic theory, there appear to be fewer applications of vacancy chain modelling in engineering. Dahl et al. (2009) develop a vacancy chain model for robot task allocation. A detailed examination and critique of their work is presented in a separate section of this work; a significant gap in their application of vacancy chain modelling involves the lack of an appropriate cost function to determine optimality.

Surprisingly, vacancy chain modelling overall has received sparse attention in academia. Pârvulescu (2020) explains, "after a strong showing in elite sociology journals from the late-1980s to the mid-1990s, vacancy chain analysis has all but died" (p. 2). Nordvik (2004) also mentions that even with its uses in economics, "vacancy chain models are utilized to a quite low degree in the analysis of local housing markets" (p. 155). Pârvulescu (2020) also argues that vacancy chains, unlike other models, enable the analysis of absences in resource distribution. Though the reasons for the infrequency of vacancy chain models in literature are not evident, the merits of vacancy chain models invite further exploration.

Hermit Crab Behavior

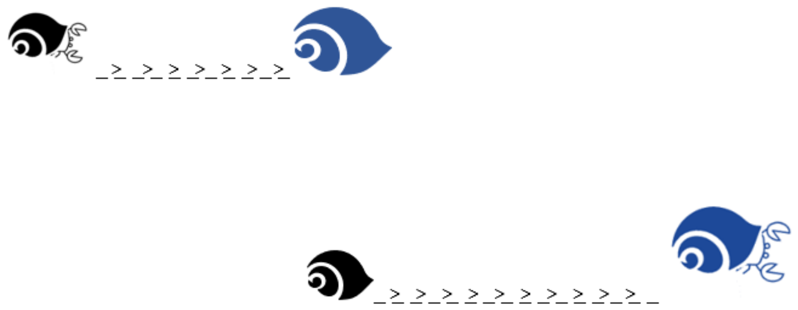
The literature presents few applications of vacancy chain models in human fields. Hermit crabs, however, exhibit vacancy chains naturally as a method of shell exchange. Lewis and Rotjan (2009) explain that hermit crabs depend on shells for protection but must obtain shells created by other creatures. As hermit crabs grow, they must seek new shells in which to live (Lewis & Rotjan, 2009). Hermit crabs employ vacancy chains as a method of shell exchange in order to gain newer and better shells. Rotjan et al. (2010) provide an overview of the two types of vacancy chains found among hermit crabs, the synchronous and the asynchronous.

Synchronous vacancy chains, according to Rotjan et al. (2010), involve exchanges that happen after crabs have queued near an available shell in descending order. Once the largest crab occupies the vacant shell and leaves its own unoccupied, the smaller crabs in the queue rapidly switch into the appropriately sized shells in the queue (Rotjan et al., 2010). Asynchronous vacancy chains, however, involve crabs encountering vacant shells individually and swapping without an immediate social context (Rotjan et al., 2010). Rotjan et al. (2010) note that for both synchronous and asynchronous vacancy chains, “vacancy chains are terminated when the last shell discarded is of such low quality (too small or damaged) that all crabs reject it” (p. 639).

It should be noted that synchronous vacancy chains involve a deliberate queueing behavior, whereas asynchronous vacancy chains operate passively. Figure 1 illustrates a simple asynchronous vacancy chain in which a crab finds and obtains a single shell, leaving its previous shell behind.

Figure 1

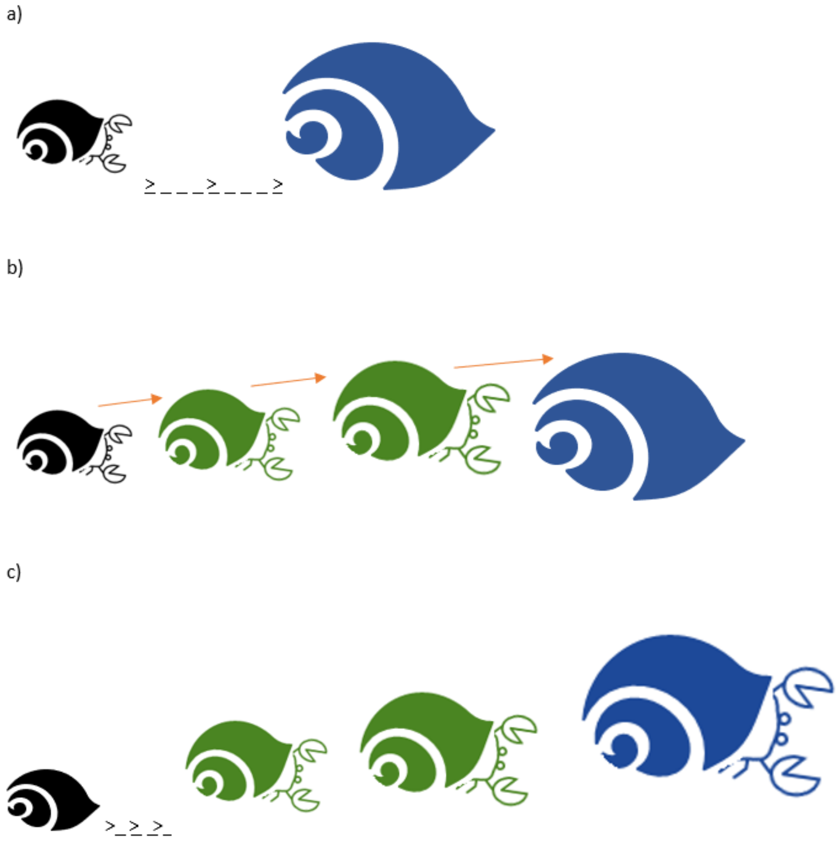
Asynchronous Hermit Crab Vacancy Chain



The exhibition and nature of synchronous vacancy chain behavior depend on a variety of factors. Rotjan et al. (2010) found that synchronous vacancy chains were stimulated by crabs choosing to wait near an unoccupied shell. Figure 2 shows the simplified stages of a synchronous vacancy chain.

Figure 2

Synchronous Hermit Crab Vacancy Chain



Note. This figure demonstrates three stages: a) a crab finds a shell too large for itself and chooses to wait for b) the arrival and queueing of other larger crabs until c) the crabs rapidly switch shells, leaving the smallest shell behind.

Briffa and Austin (2009) found that the presence of a predator cue reduced the number of skips and backward shell switches in hermit crab vacancy chains. To determine the influence of habitat on shell exchanges, Edquist and Rotjan (2012) observed hermit crab populations from two locations, a rocky beach and a mudflat, in Massachusetts. Although the vacancy chains did not involve the ideal queuing behavior, and the shell quality differed between the two populations, it was found that the average length of the chains was approximately the same for both populations (Edquist & Rotjan, 2012). These results are especially interesting when it is considered that the average of those two chain lengths is approximately 3.8, which correlates with observations of vacancy chain lengths in human systems (Edquist & Rotjan, 2012).

Through experimental study in the laboratory and the field, Lewis and Rotjan (2009) found that vacancy chains provide aggregate benefits to the hermit crabs involved in the shell exchanges. The results of experiments by Briffa and Austin (2009) also support the fact that vacancy chains provide aggregate benefits. By studying the personalities of hermit crabs, Briffa (2013) discovered that the multiplier effect increased gradually for shy hermit crabs, but during the same time period, the multiplier effect increased and then levelled for bold hermit crabs. “Thus, in bold groups, the multiplier effect accrues relatively quickly but then does not change. In shy groups, the multiplier effect accumulates more slowly but increases to a greater extent than in bold groups” (Briffa, 2013, p. 1020).

In sum, hermit crab shell exchange behavior demonstrates that vacancy chains provide an effective method of resource distribution. The complex factors involved in the formation, characteristics, and results of these vacancy chains illustrate that the systems remain effective in a variety of situations. Systems of hermit crab shell exchange may also be studied more readily and easily than human systems, enabling the development of detailed, practical models. An example of such detail involves the criteria for competition over a resource. Lewis and Rotjan (2009) found that when two similarly sized hermit crabs compete for a single vacant shell, crabs in damaged shells are likely to win against crabs with ill-fitting shells. Future vacancy chain models may need criteria for allocating similar resources, and hermit crab behavior provides one potential solution. Thus, deriving models from hermit crab behavior and finding new applications of vacancy chain theory warrant further investigation.

Literature Critique

Overview of Literature

In an article entitled “Multi-Robot Task Allocation Through Vacancy Chain Scheduling,” Dahl et al. (2009) attempt to demonstrate the applicability of vacancy chain modelling to the domain of robotic scheduling problems. They note that previously developed algorithms do not adequately address the factors of group dynamics and robotic communication (Dahl et al., 2009). Group dynamics refers to the synergistic results of interaction between robots (Dahl et al., 2009). Dahl et al. (2009) provide further clarification on the effects of group dynamics and describe how both congestion and cooperation exemplify group dynamics. In other words, a model must account not only for the actions of an individual robot but also for the results of those actions on the group as well as the results of the group’s actions on the individual robot.

Dahl et al. (2009) specifically define group dynamics by making the task processing time a function of a task’s allocation:

> We suggest that the effects of group dynamics on job processing times can be included in the formal scheduling framework by making the job processing time, p_i, from the time

> the job is started, t_s, to the time the job is finished, t_f, a function of the allocations during this time, $A_{ts,tf}$ . . . . (Dahl et al., 2009, p. 678)

It is important to note that Dahl et al. (2009) define allocation as the assignment of a task to a robot at a given point in time. Dahl et al. (2009) argue further that they seek to address spatial problems and indicate that a robot's position also constitutes a portion of the allocation definition. Thus, according to the model established by Dahl et al. (2009), group dynamics refers to the fact that the processing time of a task is dependent on the robot receiving that task, the point in time at which the robot receives the task, and the robot's location in a defined space.

In addition to the complexities of group dynamics, the need for communication between robots poses a problem of efficiency as well as effectiveness in various domains. Dahl et al. (2009) mention that in certain operations, such as reconnaissance and underwater missions, communication among robots or between robots and control systems may not be practical. Thus, developing an algorithm that effectively allows robots to self-organize without the need for a centralized control system would provide significant practical benefit. Dahl et al. (2009) attempt to prove that vacancy chain modelling enables such decentralized organization.

Inconsistencies Between Problem Parameters and Experimentation

In their attempts to apply vacancy chain modelling to the problem of multi-robot task allocation, Dahl et al. (2009) present several inconsistencies in their work. Dahl et al. (2009) argue that most algorithms and linear models for robot task allocation do not address the significant challenges of group dynamics and situations where robots cannot communicate. However, their experimentation does not incorporate the essential parameters of either of those problems.

As previously discussed, Dahl et al. (2009) define processing time in group dynamics as a function of task allocation. They state for their experimentation, though, that they “empirically estimated a range of relevant average traversal times” (Dahl et al., 2009, p. 681). In other words, rather than developing a function that would derive task processing times from task allocation dynamically, task processing times were predefined. Furthermore, the methods of empirical estimation are not discussed. Thus, the inputs for the experimental systems are unknown and cannot be repeated based on the information provided by Dahl et al. (2009).

The group dynamics portion of the experimentation also intentionally avoids the very complications of group dynamics previously mentioned by Dahl et al. (2009). They note that the problem addressed by their experimentation, “has a very restricted interaction function which reduces the scheduling complexity” (Dahl et al., 2009, p. 686). Dahl et al. (2009) also acknowledge that an algorithm constructed only on the basis of vacancy chain systems without incorporation of other redistribution methods fails to reflect real resource distribution. They even directly note that hermit crabs exhibit some competition in shell exchange (Dahl et al., 2009). Fioretti (2009) comments directly on the weakness of the algorithm produced by Dahl et al. (2009), stating that, “interestingly, in order for vacancy chains to work it was necessary to add a constraint that impaired too many robots from servicing the same route at a time” (p. 55). The importance of this constraint is that it effectively eliminates the possibility of competition, which is an element that may be found in both human and hermit crab vacancy chains (Fioretti, 2009). Fioretti (2009) remarks, “In the case of the above robots, impairing competition was simpler than endowing them with a criterium to establish a winner” (p. 56).

Thus, Dahl et al. (2009) forced their algorithm to reflect a perfect vacancy chain that does not reflect problems occurring in natural vacancy chains. Rather than improve an initial algorithm to account for congestion and competition, Dahl et al. (2009) intentionally ignore the group dynamics that they intended to address. The constraint impairing competition renders the algorithm unrealistic and inapplicable.

Additional Problematic Elements

Dahl et al. (2009) argue that vacancy chain scheduling can provide an effective method for allocating tasks to robots. However, they fail to provide a comparison of their model's performance to similar multi-robot task allocation algorithm performances. Although Dahl et al. (2009) discuss a variety of multi-robot task allocation algorithms, their experimentation only compares their own algorithm to randomness and hand-coded solutions. It is important to mention that the term “hand-coded” is not defined by Dahl et al. (2009), and it is unclear whether the hand-coded algorithms refer to hard-coded values or basic linear models. Significantly, in all except one experiment, the hand-coded system consistently outperformed the vacancy chain scheduling algorithm (Dahl et al., 2009).

Furthermore, Dahl et al. (2009) compare performances among their chosen experimental algorithms by examining two metrics: the rate of task completion and the amount of time spent in a particular system state. Regarding the state space, Dahl et al. (2009) argue that their results demonstrate the superiority of the vacancy chain scheduling algorithm because “the group's set of Q-tables have converged to promote the state defined as optimal according to the VCS [Vacancy Chain Scheduling] model” (p. 685). In other words, because the state space gave the appearance of a vacancy chain scheduling system, it is deemed optimal. This presents an unfortunate circularity which weakens the argument for the vacancy chain scheduling algorithm’s effectiveness.

A final problematic element is the oversimplification of vacancy chain distribution presented by Dahl et al. (2009). Dahl et al. (2009) state that vacancy chains do not “take into account the quality of the consumer” (p. 678), and thus, “the vacancy chain distribution process cannot exploit the possible advantages of distributing particular resources to particular consumers” (p. 678-679). However, this notion of complete impartiality does not comport with actual distribution logic of vacancy chains that exist in human or hermit crab systems. In the labor market, a job vacancy can only be filled by a candidate with proper qualifications; similarly, houses can only be purchased by consumers with an appropriate budget. Hermit crabs likewise cannot move into new shells that are too large for them to carry. Dahl et al. (2009) do not account for the fact that consumers must be appropriately fit to the vacancies they intend to fill. The concept of fitness is critical for vacancy chain systems and must be considered in the development of vacancy chain model cost functions.

Summary of Problems

While Dahl et al. (2009) recognize the uniqueness and usefulness of vacancy chains as a method of resource distribution, they do not adequately address the problems they intend to solve with their experimentation. The problem parameters of group dynamics are intentionally avoided in their experimentation, and they do not provide a proper comparison of their model to existing task allocation models. Dahl et al. (2009) do acknowledge that their model is oversimplified and could be improved. In particular, a straightforward cost function based on the complexity of empirical vacancy chain systems should be developed.

Research Questions and Methodology

Research Questions

The following research questions are proposed for study:

- Can hermit crab shell exchange behavior provide insight to cost function development for vacancy chain modelling?
- Which parameters of hermit crab shell exchange may be analogized to fit known problems?

Hermit Crab Cost Function

The development of a hermit crab cost function, which will be denoted as F_{HC}, requires consideration of the costs involved in the hermit crab's decision to initiate or join a synchronous vacancy chain. Since asynchronous vacancy chains are merely a simple shell exchange, only synchronous vacancy chain patterns will be considered for cost function development.

Another consideration is whether a cost function based on hermit crab behavior should be additive or multiplicative. For the sake of simplicity, this work uses an additive model. Weights may be applied to the different cost factors to emphasize their importance.

A final assumption in the development of the cost function is that shell switches are instantaneous. This assumption is based on the empirical observations of Rotjan et al. (2010) who note that "immediately after the largest crab had switched into the vacant shell, a rapid cascade of sequential shell switches by each queued crab followed until the smallest crab in the queue had discarded its shell" (p. 644). Thus, the processing time of a synchronous vacancy chain shell exchange is considered an insignificant cost.

Overview of Costs

Various factors influence the development of synchronous vacancy chains. Hermit crabs determine individually that participation in a vacancy chain increases the likelihood of obtaining a higher quality shell than could be found individually. Costs that may be considered in the formation of individual utilities include the length of time required for a vacancy chain to form, time required for shell investigation, the quality of a crab’s shell fit, presence of a predator cue, and whether the crab possesses a damaged shell.

Waiting Behavior

One factor, as noted by Rotjan et al. (2010), is waiting behavior, which is the choice of a hermit crab to remain near a vacant shell too large for itself. According to Rotjan et al. (2010), observations of hermit crabs in the field showed the typical wait times for hermit crabs prior to the instigation of synchronous vacancy chains:

> After investigation of a vacant shell that was too large, hermit crabs would remain near (within 50 cm) the shell rather than moving away immediately: crab waiting times ranged from several minutes to >1 h, and up to 20 waiters at a time were present near the empty shell. Crabs exhibited waiting behavior at 55% (6 of 11) of stations with large vacant shells and at 100% (9 of 9) of stations with medium vacant shells. Waiters were observed at all stations where synchronous vacancy chains eventually occurred. (p. 643)

The observations of wait times indicate that the hermit crab considers the cost of waiting at the larger shell to be lower than the cost of further exploration up to a given wait time. Using the empirical data from Rotjan et al. (2010), wait time cost may be estimated with a negative exponential variable. Given a waiting period, the cost of waiting for a queue to develop is as shown in Equation (1).

$$c_{\text{wait}} = p_{\text{wait}} = \lambda e^{-\lambda m} \tag{1}$$

where m is the number of minutes spent waiting for the queue to form, and p_{wait} is the probability that a crab will continue to wait, and c_{wait} is the waiting cost. The coefficient λ is simply a constant that may be adjusted based on the needs of the model. The reciprocal of p_{wait} may be multiplied by m to give the expected cost of waiting, c_{wait}, in time units. However, to make the overall cost function generalizable, it is helpful to simply equate c_{wait} to the probability p_{wait} to keep the cost dimensionless; the number of minutes only serves as a multiplier for the cost and is likely not necessary.

Inspection Time

Like waiting time, inspection time reflects a cost of participation in a vacancy chain. Hermit crabs will inspect a shell regardless of whether the vacancy chain is synchronous or asynchronous, and Rotjan et al. (2010) found that the duration of inspections does not differ significantly between the two types of vacancy chains. According to their data, hermit crab shell inspections lasted approximately between 60 and 100 seconds (Rotjan et al., 2010). Thus, inspection time may be represented as a fixed cost, $c_{\text{inspection}}$, in the cost function. This cost may be given time units if necessary, but a dimensionless inspection cost is likely more useful.

Quality of Fitness

According to Lewis and Rotjan (2009), synchronous vacancy chains benefit hermit crabs through a reduction in shell crowding. Through observations they found that “hermit crabs that participated in experimental vacancy chains benefited by significantly reducing their shell crowding (an average of approximately five fewer appendage segments exposed). In hermit crabs, reduced shell crowding is likely to translate directly into fitness benefits” (Lewis & Rotjan, 2009, p. 362).

As hermit crabs continuously grow, the cost of exposed appendages is likely to increase exponentially over time. Thus, the cost of an ill-fitting shell may be determined through a correlation with time and the rate of growth of the hermit crab. Alternatively, a simple ratio of new shell fitness to current shell fitness may be used to estimate the cost of poor-quality shell fit, c_{fit}, as shown in Equation (2).

$$c_{\text{fit}} = \frac{a_n w_n}{a_c w_c} \tag{2}$$

where a_n is the number of appendages exposed in the new shell, a_c is the number of appendages exposed in the current shell, w_n is the weight of the new shell, w_c is the weight of the current shell, and c_{fit} is the cost of poor-quality fit.

Importantly, the quality of shell fit cost, c_{fit}, is a dimensionless number. It should be noted that this cost ratio also contains constraints on the size of the new shell. Because the weight of the new shell is in the numerator, excessive additional weight will have a multiplicative effect on the cost of poor fit. Thus, the cost of fit includes not only improved spaciousness in a new shell but also the minimization of unnecessary weight. The weight elements allow the cost of fit to function better than a simple greedy algorithm, which accepts maximum profit or minimum cost. In the case of a shell, covering the fewest number of appendages may be possible with several shells of different weights; the inclusion of weight as a factor of cost enables a more nuanced and realistic approach to resource allocation.

Predation

Another influence on vacancy chain formation is the presence or absence of a predator cue. Briffa and Austin (2009) describe ideal synchronous vacancy chains as those where crabs switch directly to the next-largest shell, and that new shell is of higher quality than the previously occupied shell. When a chemical predator cue was introduced to the experimental environment, Briffa and Austin (2009) found that the predator cue significantly influenced shell exchange behavior:

> The presence of a predator cue also had an effect on the chain structure. It is interesting that while the pattern of vacancy moves was not random in either treatment, ideal vacancy chains occurred in the presence but not the absence of the predator cue. It therefore appears that significant levels of skipping and backwards moves are features of hermit crab vacancy chains but that these features are reduced in the presence of predator risk. (p. 1034)

The reduction of a synchronous vacancy chain to the ideal form is explained as a method of avoiding the notice of predators (Briffa & Austin, 2009). Interestingly, Briffa and Austin (2009) conclude that higher risk levels drive vacancy chains to operate ideally. “The current data suggest that although vacancy chains can supply benefits to multiple individuals, it is only under risky conditions that movements in discrete reusable resource units occur through ideal vacancy chains” (Briffa & Austin, 2009, p. 1034).

Although the overall structure of a synchronous vacancy chain converges to an ideal scenario in the presence of a predator cue, Briffa and Austin (2009) found that the level of individual benefits varied depending on whether the predator cue existed. All hermit crabs ultimately gained shells of higher quality, but smaller crabs in the queue benefitted from the increased caution levels – that is, reluctance to switch shells – of larger crabs. Thus, when considering how to model the predator cue effects, a crab’s queue position ought to be considered.

Given the complex influence of predation on synchronous vacancy chain shell exchanges, incorporating the presence of a predator cue into the hermit crab cost function presents a challenge. One method of representing the cost of a predator cue is to use a binary variable which indicates the presence or absence of a predator cue. Another potential method is to establish a risk probability, $c_{\text{predation}}$, which indicates the probability of suffering from predation. The risk probability would be higher in the presence of a predator cue and lower in the absence of a predator cue, as shown in Equation (3).

$$c_{\text{predation}} = \begin{cases} p_s, & \text{if predator cue} \\ 1 - p_s, & \text{if no predator cue} \end{cases} \tag{3}$$

where p_s is the probability of suffering, and $c_{\text{predation}}$ is the cost of a predator’s presence. Importantly, the risk cost is dimensionless to maintain generalizability.

Shell Damage

Damaged shells increase a crab's vulnerability and present a significant cost for hermit crabs (Lewis & Rotjan, 2009). In fact, damaged shells may incur the highest cost since Lewis and Rotjan (2009) found through experimentation that damaged shells ended vacancy chains. In addition, crabs in damaged shells won competitions for vacant, high-quality shells more frequently than crabs in poorly fitted shells (Lewis & Rotjan, 2009). Thus, it may be argued that shell damage presents the highest cost for the hermit crab.

The cost of shell damage may be represented mathematically through a weighted sum, as shown in Equation (4). As with other cost function elements, the weighted sum is dimensionless; this is due to the fact that the weights represent severity of damage and not a direct count of damaged points on the shell. For *n* damage points, the cost of shell damage, c_{damage}, is

$$c_{\text{damage}} = \sum_{i=1}^{n} w_i \tag{4}$$

where w_i is the weight corresponding to the damage severity at point *i* on the shell.

Final Cost Function

The final hermit crab vacancy chain cost function is presented in Equation (5). As previously discussed, a key assumption of this hermit crab vacancy chain cost function is an additive relationship among the elements. This assumption may not be valid; a more accurate model may be non-linear.

$$F_{HC} = c_{\text{wait}} + c_{\text{inspection}} + c_{\text{fit}} + c_{\text{predation}} + c_{\text{damage}} \tag{5}$$

where F_{HC} represents the total cost function, c_{wait} is the cost of queue development, $c_{\text{inspection}}$ is the cost of resource inspection, c_{fit} is the cost of resource fit, $c_{\text{predation}}$ is the cost of a threat's presence, and c_{damage} is the cost of resource damage.

Cost Function Analogizing

In attempting to use the hermit crab vacancy chain cost function, it is important to consider how elements may be analogized to real problems. Avoiding illogical connections is helpful for preventing the development of algorithms which are inapplicable or inaccurate.

Although all illogical connections between hermit crab behavior and real-world problems cannot be addressed in advance, this issue should be addressed as a general principle. For example, processing time cannot be analogous to hermit crab size – that is, shell fitness – because multiple time variables represent the corresponding costs of time in the hermit crab cost function. Even if the variables are dimensionless for generalizability, it is nonetheless important to remember the elements they represent and avoid what may be called overlapping analogization. The consequences of overlapping analogization may include circularity or redundancy in the objective function as well as inaccuracies in results or inapplicability of an algorithm.

Comparison of the Cost Function to Previous Literature

The development of the hermit crab cost function was inspired by the inadequacies of the work of Dahl et al. (2009). While their experimentation contains many flaws, the key flaw may be summarized as a disconnect between their proposed real-world problem and their proposed algorithmic solution. Because the inputs of the experiment, namely, the traversal times, are predefined, the allocation of robots to tasks appears to operate according to a simple greedy algorithm. Furthermore, the constraints of the experiment force the allocations of the robots to appear as a vacancy chain rather than allow a vacancy chain to emerge freely as a result of group dynamics. The lack of potential competition among robots also reduces the applicability of the model in a real-world group setting where robots may interfere with each other and cause congestion.

Although not all of these problems may be remedied by the hermit crab cost function, the intention of the hermit crab cost function is to prevent simple flaws in experimental designs in future work. Dahl et al. (2009) equate resources – that is, shells – to processing times. As the hermit crab cost function clarifies, resources should not be considered as a time cost but as fit and damage costs. In addition, Dahl et al. (2009) severely restricted their problem scenario in experimentation to the point that the scenario no longer reflected the problem they wished to address. By developing a generalizable cost function that reflects real-world scenarios prior to experimentation, it is possible to establish the applicability of the model in advance of obtaining results.

Finally, the hermit crab cost function attempts to reflect the nuances of vacancy chains and avoid the oversimplification of vacancy chains as greedy algorithms. In particular, the concept of fit considers both the quality of the resource and the capacity of the user simultaneously. A resource of acceptable size may be available, but it may not ultimately be optimal to allocate that resource to the currently available customer or recipient. Thus, the hermit crab cost function provides a foundation for future vacancy chain modelling that avoids the oversimplistic modelling of Dahl et al. (2009).

Recommended Applications of Vacancy Chain Modelling

Humanitarian Supply Chains

While many algorithms and heuristics have been developed for commercial supply chains, humanitarian supply chains pose unique problems. The hermit crab cost function may be beneficial for the development of algorithms that address humanitarian supply chains specifically. The importance and characteristics of humanitarian supply chains must be examined before they are considered for application of the hermit crab cost function. A comparison of the situations faced by humanitarian supply chains and hermit crabs will also be provided.

Defining Characteristics of Humanitarian Supply Chains

Before discussing the key differences between the strategies of humanitarian supply chains and those of commercial supply chains, the practical significance of humanitarian supply chains should be emphasized. Behl and Dutta (2019) note that crisis situations "whether man-made or natural, are increasing year by year throughout the world" (p. 1002). In addition, Behl and Dutta (2019) comment that crises arise from various causes and that "the rate of growth of natural disasters (droughts, hurricanes, floods, famines, earthquakes, etc.) and manmade disasters (conflicts among and within nations, refugee crises, wars, etc.) has been impacting the social existence of mankind" (p. 1002). Unfortunately, as noted by Oloruntoba and Gray (2006), humanitarian supply chains often suffer from a lack of planning. Hirschinger et al. (2015) emphasize that logistics is the second-largest cost for humanitarian organizations, with only personnel costs exceeding those of logistics. Developing models that account for the unique values and characteristics of humanitarian supply chains could help mitigate planning problems, reduce costs, and ultimately alleviate suffering in crisis situations.

Although both humanitarian supply chains and commercial supply chains face some similar challenges, the goals of humanitarian supply chains involve additional values. Equity, or fairness, is a primary value for humanitarian supply chains in addition to efficiency and effectiveness. As Anaya-Arenas et al. (2018) explain, “fairness is important, if not the most important, principle common to any humanitarian intervention seeking impartial access to assistance” (p. 1145). The importance of equity in humanitarian supply chains is also discussed by Lien et al. (2014) who note that with non-profit organizations, “objectives are often more difficult to quantify since issues such as equity and effective use of donations must be considered, yet efficient operations are still crucial” (p. 301). Whereas cost-effectiveness or timeliness often serve as the highest value in commercial supply chains, humanitarian supply chains require more complex considerations. Thus, when developing models for humanitarian supply chains, it is important to account for the value of equity as well as the factors of cost-effectiveness and efficiency.

Van Wassenhove and Martinez (2010) provide a thorough discussion of the different characteristics of humanitarian supply chains and commercial supply chains. Both humanitarian supply chains and commercial supply chains face uncertainties of demand, but HSCs also experience significant supply risks due to their reliance on donations and volunteers. Unlike military operations, humanitarian supply chains also lack a clear control structure. Humanitarian supply chains must also respect the needs of local cultures and economies (Van Wassenhove & Martinez, 2010).

It is also important to note that humanitarian supply chains may be characterized as either responses to short-term emergencies or continued support of long-term humanitarian efforts.

Falasca and Zobel (2011) explain the differences between the two types of humanitarian supply chains. According to Falasca and Zobel (2011), humanitarian activities that involve long-term development goals more closely resemble commercial supply chains because they can depend on regularity in supply chain lead times. Short-term emergency relief humanitarian supply chains, however, require an emphasis on speed due to the urgency of saving lives (Falasca & Zobel, 2011).

The degree to which a humanitarian supply chain resembles a commercial supply chain depends on the type of the humanitarian effort. Short-term humanitarian supply chains in particular "are characterized by high levels of uncertainty, risk, and urgency, making them a very different field of application for these principles than that of traditional businesses" (Gatignon et al., 2010, p. 102). Thus, developing cost functions and algorithms for application in short-term humanitarian supply chains involves considering different elements than one would apply in commercial settings.

Application of the Hermit Crab Cost Function

Since humanitarian supply chains often operate with significant levels of risk, the predation factor in the hermit crab cost function provides an immediate connection between the two domains. In addition, procurement lead times may be compared to the waiting behavior cost, and transportation time may be compared to the shell inspection time cost of hermit crabs. Matching demand and supply could be analogized to the shell fitness of hermit crabs, and constraints could be applied to prevent overage as would also be required in the case of hermit crabs. Overall, the domain of humanitarian supply chains presents a sufficiently comparable domain to that of hermit crab shell exchange.

Conclusion

Vacancy chain systems effectively redistribute resources in a variety of domains. Although an attempt was made to apply vacancy chain modelling in the realm of engineering, specifically within the domain of multi-robot task allocation, that attempt contains multiple flaws which prevent the algorithm from being practically applicable. To address the flaws in previous engineering experiments, a cost function based on hermit crab shell exchange behavior is developed. The hermit crab cost function enables improved vacancy chain modelling by examining nuanced elements of cost while presenting a generalized model. Application of the cost function to the analogous domain of humanitarian supply chains is also recommended. Further research may include refining the cost function and determining appropriate values for probabilities and constants. In addition, other nuanced factors of hermit crab behavior, such as shyness or boldness levels, may be considered to develop the model further.

References

Anaya-Arenas, A. M., Ruiz, A., & Renaud, J. (2018). Importance of fairness in humanitarian relief distribution. *Production Planning & Control*, *29*(14), 1145-1157. https://doi.org/10.1080/09537287.2018.1542157

Behl, A., & Dutta, P. (2019). Humanitarian supply chain management: a thematic literature review and future directions of research. *Annals of Operations Research, 283*, 1001-1044. https://doi-org.ezproxy.liberty.edu/10.1007/s10479-018-2806-2

Ben-shahar, D., & Sulganik, E. (2011). Vacancy chains and the degree of mobility in the housing market. *The Annals of Regional Science*, *47*(3), 569-583. http://dx.doi.org/10.1007/s00168-010-0395-3

Briffa, M. (2013). The influence of personality on a group-level process: Shy hermit crabs make longer vacancy chains. *Ethology, 119*(11), 1014-1023. https://doi.org/10.1111/eth.12148

Briffa, M., & Austin, M. (2009). Effects of predation threat on the structure and benefits from vacancy chains in the hermit crab *Pagurus bernhardus*. *Ethology*, *115*(11), 1029-1035. https://doi-org.ezproxy.liberty.edu/10.1111/j.1439-0310.2009.01687.x

Chase, I., Weissburg, M., & Dewitt, T. H. (1988). The vacancy chain process: A new mechanism of resource distribution in animals with application to hermit crabs. *Animal Behavior, 36*(5), 1265-1274. https://doi.org/10.1016/S0003-3472(88)80195-7

Dahl, T. S., Mataric, M., & Sukhatme, G. S. (2009). Multi-robot task allocation through vacancy chain scheduling. *Robotics and Autonomous Systems*, *57*(6-7), 674-687. https://doi.org/10.1016/j.robot.2008.12.001

Edquist, S. K., & Rotjan, R. D. (2012). Testing vacancy chain predictions in Pagurus longicarpus hermit crabs: Does ecological gain and behavioral motivation match environmental context? *Journal of Experimental Marine Biology and Ecology*, *430-431*, 78-86. https://doi.org/10.1016/j.jembe.2012.06.018

Falasca, M., & Zobel, C. W. (2011). A two-stage procurement model for humanitarian relief supply chains. *Journal of Humanitarian Logistics and Supply Chain Management, 1*(2), 151-169. https://doi-org.ezproxy.liberty.edu/10.1108/20426741111188329

Ferrari, E. (2011). Conceptualising social housing within the wider housing market: A vacancy chain model. *Housing Studies*, *26*(1), 95-116. https://doi-org.ezproxy.liberty.edu/10.1080/02673037.2010.512786

Fioretti, G. (2009). A model of vacancy chains as a mechanism for resource allocation. *The Journal of Mathematical Sociology*, *34*(1), 52-75. https://doi-org.ezproxy.liberty.edu/10.1080/00222500903355866

Friman, H. R. (2004). Forging the vacancy chain: Law enforcement efforts and mobility in criminal economies. *Crime, Law & Social Change*, *41*, 53-77. https://doi.org/10.1023/B:CRIS.0000015328.82955.f0

Gatignon, A., Van Wassenhove, L. N., & Charles, A. (2010). The Yogyakarta earthquake: Humanitarian relief through IFRC’s decentralized supply chain. *International Journal of Production Economics*, *126*(1), 102-110. https://doi.org/10.1016/j.ijpe.2010.01.003

Hirschinger, M., Moser, R., Schaefers, T., & Hartmann, E. (2015). No vehicle means no aid – A paradigm change for the humanitarian logistics business model. *Thunderbird International Business Review, 58*(5), 373-384. https://doi.org/10.1002/tie.21745

Lewis, S. M., & Rotjan, R. D. (2009). Vacancy chains provide aggregate benefits to Coenobita clypeatus hermit crabs. *Ethology*, *115*(4), 356-365. https://doi-org.ezproxy.liberty.edu/10.1111/j.1439-0310.2009.01626.x

Lien, R. W., Iravani, S. M. R., & Smilowitz, K. R. (2014). Sequential resource allocation for nonprofit operations. *Operations Research, 62*(2), 301-317. http://dx.doi.org.ezproxy.liberty.edu/10.1287/opre.2013.1244

Nordvik, V. (2004). Vacancy chain models: Do they fit into the economist's toolbox? *Housing, Theory and Society, 21*(4), 155-162. https://doi-org.ezproxy.liberty.edu/10.1080/14036090410000499

Oloruntoba, R., & Gray, R. (2006). Humanitarian aid: An agile supply chain? *Supply Chain Management, 11*(2), 115-120. DOI 10.1108/13598540610652492

Pârvulescu, R. A. (2020). Filling the gaps: Vacancy chains and agent-based models. https://doi.org/10.31235/osf.io/nehzj

Rotjan, R. D., Chabot, J. R., & Lewis, S. M. (2010). Social context of shell acquisition in Coenobita clypeatus hermit crabs. *Behavioral Ecology*, *21*(3), 639-646. https://doi-org.ezproxy.liberty.edu/10.1093/beheco/arq027

Van Wassenhove, L. N., & Martinez, A. J. P. (2010). Using OR to adapt supply chain management best practices to humanitarian logistics. *International Transactions in*

Operational Research, 19(1-2), 307-322. https://doi-org.ezproxy.liberty.edu/10.1111/j.1475-3995.2010.00792.x