Mathematics for Visually Impaired Students: Increasing Accessibility of Mathematics Resources with LaTeX and Nemeth MathSpeak

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Abstract

The ability to accurately read and subsequently comprehend a mathematical expression is vital to the understanding of mathematics. The Braille illiteracy rate has steadily risen among individuals with visual impairments, increasing the need for effective resources in mathematics education for such individuals. A literature review was conducted, compiling information from various resources which demonstrate the gaps in this area. Audio System for Technical Readings (AsTeR) is an existing software which can read aloud technical documents created in LaTeX, the most used typesetting application for the creation of mathematic documents. A collaboration between Abraham Nemeth's MathSpeak guidelines for vocalizing mathematical text and AsTeR software could increase accessibility in math documents. This accessibility would create resources for students with varying visual disabilities.

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Introduction

Mathematical thinking and reasoning skills are valuable tools which are crucial to any well-rounded education. Mathematics helps students develop critical thinking skills, attention to detail, and essential knowledge which will be of benefit when striving to become a productive member of society. Mathematic concepts are used by every person daily when filing taxes, creating budgets, or calculating tips. Furthermore, the problem solving and critical thinking skills learned in a math class are resources which translate into broader areas of life when it comes to decision making and more. The ability to accurately read and subsequently comprehend a mathematical expression is vital to the understanding of mathematics and mathematical concepts. While most mathematics students have a plethora of accessible resources that can be beneficial in understanding mathematical concepts, this is not the case for students with visual impairments.

As of 2017, more than 60,000 blind students were in kindergarten through high school in the United States ("Statistics About Children," n. d.). These students are lacking in assistive technologies in mathematics education. One of the greatest deficits in math education for blind students is in the area of accessible textbooks. The current process for creating a textbook which is compatible with screen-reader technology is inefficient and limits the ability for publishers to create accessible tools. Visually impaired students experience many gaps in accessibility, but the accessible translation of textbooks provides the greatest opportunity for improvement. Therefore, a study will be conducted examining the methods which could improve the accessibility of mathematics resources within this area.

Review of Literature

The following literature review will provide a summary on the existing literature concerning tools and programs available to assist in the teaching of mathematics to students with visual impairments. The literature on both established techniques and newly proposed methods will be discussed, along with an analysis of the strengths and weaknesses of each program. Specifically, research on Braille in education, implemented methods of instruction, non-technological strategies for instruction, and other applications available will be discussed. It is important to understand established practices for teaching math to students with visual impairments; a thorough understanding of the available resources must precede any appropriate intervention. While this review is not exhaustive, it will present some crucial gaps in the literature and will highlight areas for future research.

Critical Analysis of Literature

The use of braille in education. Unified English Braille and its subgroups are the original resources which allowed those with visual impairments or low vision to read material. However, it is questionable if that is the best solution to educating such students, especially in mathematics (Jernigan Institute, 2009). Unified English Braille was developed by the International Council on English Braille; this system sought to unify the variants of Braille codes among each English-speaking country in the subjects of literature, mathematics, and computing (International Council on English Braille, 2017). There has since been developed Nemeth Braille Code for Mathematics and Science Notation, also referred to as Nemeth Code (Kendrick, 2013).

Published in the *Journal of Visual Impairment & Blindness*, a study in late 2017 investigated the implementation of Braille by teachers of students with visual impairments in the United States (Hong, Rosenblum, & Frank, 2017). This study analyzed 141 survey responses from teachers of students with low vision. Authors Hong, Rosenblum, and Frank (2017) noted that while many educators attended workshops discussing word-based Braille, far fewer attended workshops on math-based Braille. This implied a lack of confidence of teachers in the possibility of using Braille as a strategy in mathematics education. Not only did the teachers exhibit doubt as to their ability to implement effective instruction of mathematics using Braille or Nemeth Code, in this survey they also expressed concern for the students' ability to comprehend instruction using Braille resources (Hong et al., 2017). The implications of this study are twofold: (1) there is a lack of confidence in educators concerning their ability to teach mathematics to students with visual impairments and (2) the use of Braille or Nemeth Code in education of mathematics is unlikely to be successful, in part due to the implied lack of confidence in educators concerning this subject, but also due to high rates of Braille illiteracy.

Further developments in technology have created a Braille illiteracy which has rendered these educational strategies nearly useless. It was estimated in 2009 that fewer than ten percent of those who are legally blind in the United States were Braille readers (Jernigan Institute, 2009). This is in part due to technological developments giving text-to-speech conversion or advanced communication abilities to those with visual impairments. For this reason alone, the previous literature has interesting implications about teacher preparedness for mathematics education; Unified English Braille and Nemeth Code are not sufficient strategies for mathematics education for students with visual impairments going forward.

The use of technology-based strategies in education. For students with visual impairments, the most used primary reading media include print, large print, or Braille. However, newer technologies are being developed and implemented which allow for text-to-speech or text-to-diagram conversions. These auditory or tactile diagram methods could be useful in implementing effective instruction in the mathematics subject. Many individuals with visual impairments are already familiar with iPhone or tablet technology which works in a similar fashion. Thus, the implementation of such technologies would be a relatively smooth transition for the students.

Studies have also been conducted which evaluate the effectiveness of instruction using text-to-speech or text-to-diagram conversions. A recent study proposed a low-cost teaching aid for visually impaired students (Mukherjee, Garain, & Biswas, 2014). The text-to-diagram conversion allows students to input a geometry word problem into a system attached to a traditional Braille printer and then receive the underlying diagram on a Braille printout (Mukherjee et al., 2014). This proposed solution to costly, ineffective technologies was evaluated through a seven-month study at a blind boys' school in India (Mukherjee et al., 2014). While the results of this study showed a general comprehensiveness of the geometric shapes by the students, the results were ultimately inconclusive as to the effectiveness of such technology (Mukherjee et al., 2014). The authors suggest there would have to be further research conducted based in the United States to determine the validity of the technology used in this study (Mukherjee et al., 2014). Although Unified English Braille is not a sufficient tool for teaching mathematics, a simple adaptation could be useful in understanding printed graphs, shapes, or figures through this study's proposed method.

Other computer programs have undergone testing which may provide partial solutions to the lack of tools and programs available for mathematics education of visually impaired students. The conclusions based on these studies suggest that audio features in general are more effective than Braille text, although some studies noted in feedback from participants that providing Braille text in addition to auditory speech could be useful (Beal, Rosenblum & Smith, 2011). In addition, if hints are available throughout a problem, student involvement and motivation are more likely to be maintained (Beal & Rosenblum, 2018). However, hints being provided is not the most notable feature of tested technology, as these programs were primarily tested on lower-level mathematic subjects where teachers were available in residential classrooms to assist students when necessary.

Many researchers noted the desire of students to have the program interface be “more game-like” (Beal et al., 2011, p. 166). This was expressed by study participants ranging from fifth grade to high school. It was inconclusive whether a more game-like interface would promote content retention, as this was simply the most common feedback from tested technology, but it would likely increase student participation and enthusiasm for completing material through the applications.

Overall, the literature suggests a note-worthy potential for tablet and technology-based tools as students are shown to be more encouraged and focused when using such programs compared to solely Braille-based methods of instruction (Beal & Rosenblum, 2018). This technology also assists in visualization of graphs, figures, and other highly visual components of mathematical instruction.

The use of non-technological strategies in education. Although this review is primarily focused on technological tools available for teaching mathematics to those with visual impairments, it is important to make note of non-technological strategies for teaching mathematics to students with visual impairments. Even technological tools used in most classrooms will have some degree of instructor involvement, and the presented strategies could be beneficial when paired with technologically-based programs. By evaluating various learning theories regarding the teaching of secondary mathematics and the available assistive technologies for teaching math to students with visual impairments, educators can demonstrate more effective teaching practices within this context.

The most prominent learning theories include behaviorism, cognitivism, constructivism, connectivism, and humanism (“The Five Educational Learning Theories,” 2016). Simply put, behaviorism is the implementation of the phrase practice makes perfect. Within the learning theory of behaviorism, there is no focus placed on the mental activity or social aspect of learning (Clark, 2018). Many see this as a fundamental drawback of solely practicing behaviorism in the classroom. Social interaction and development is critical for students and is a part of their education, consciously or not. Social belongness is often an issue for students with disabilities, especially those with blindness or low vision. Various studies have reported that students with visual disabilities feel increased levels of loneliness and feel less accepted at school (Jessup, Bundy, Broom, & Hancock, 2018; Coduti, Herbert, Chiu, & Doke, 2017). For this reason, instructors who rely solely upon instruction based on the behaviorist learning theory could be doing harm to students with disabilities. Behaviorism’s lack of emphasis on the importance of social learning adds to the negative social impact of staff and faculty doing nothing to assist in the socialization of students in a math classroom (Jessup et. al, 2018).

Instead, constructivism and connectivism are learning theories which focus on the social side of learning, a strategy which could be of great benefit to students with visual impairments. Constructivism is the “study of a learner’s own construction of knowledge” (Clark, 2018, p. 180). This approach suggests that students are responsible in creating their own understanding and using that knowledge to acquire understanding of more information by linking prior and new concepts (“The Five Educational Learning Theories,” 2016). Essentially, students are constructing their own meaning with guidance from the teacher. Connectivism suggests that people process information by forming connections (“The Five Educational Learning Theories,” 2016). Furthermore, people do not stop learning after formal education; rather, they continue to acquire knowledge through jobs, experience, and more (“The Five Educational Learning Theories,” 2016). Connectivism is relatively new compared to other learning theories, as it focuses on how learning is shaped by technology and the digital age (Goldie, 2016). Teaching with these two learning theories in mind would allow such students to have more specialized instruction. Students experiencing blindness or low vision face “considerable challenges”, particularly when it comes to the more visual aspects of math (Rosenblum, Cheng, & Beal, 2018, p. 475). Thus, students with visual impairments would benefit from individualized guidance involving manipulatives and systematic instruction (Rosenblum et al., 2018). Lessons based on the cognitivist approach especially could allow for students with little to no vision to receive the level of specialized instruction necessary to be successful in mathematics.

Along with learning theories, a handful of broader nontechnological techniques can be helpful in instructing blind students. The presentation of a multi-step mathematics problem to a student with a visual impairment has been identified by researchers as needing improvement (Gulley, Smith, Price, Prickett & Ragland, 2017; Maćkowski, Brzoza, Zabka & Spinczyk 2018). Proposed solutions include a process-driven math technique and a method based on the decomposition of a mathematical problem into a series of simple sub-exercises (Gulley et al., 2017; Maćkowski et al., 2018). These two methods, if proven effective, could be used by a teacher in the classroom or by developers of software that could incorporate these methods into the tools and programs already available.

The process-driven math technique was developed for a single college student unable to utilize Braille or Nemeth Code (Gulley et al., 2017). This method essentially begins with describing a complex algebraic expression in general and vague terms, such as “rational divided by rational” (Gulley, et. al, 2017, p. 466). From there, the student is able to describe the steps necessary to solve the problem in general terms without being overloaded with information at the start of the problem (Gulley et al., 2017). This method was proven successful for the two students it was used on, but, as it exists at the moment, would be difficult to scale to a full classroom or education system (Gulley et al., 2017). A trained reader-scribe is someone who has been taught the proper sequence of translating a regular math problem to a process-driven problem and is necessary for proper implementation of this method (Gulley et al., 2017). This method has not been used on a whole classroom setting; even with a class size of ten students, this method would be difficult to implement, given the level of required individualized instruction and student response. Therefore, it is unclear whether or not it would be effective or even possible to use in such a fashion. Further research could be conducted using this method in small group or classroom settings, as well as in different subjects, as the method itself has potential but is perhaps incomplete.

Another alternative proposal to presenting math problems involves breaking down a mathematical exercise into several sub-exercises (Maćkowski et al., 2018). This method describes another way to present the problem both auditorily and using Braille to a student through a system called Lambda software (Maćkowski et al., 2018). The literature on this method found a good understanding of mathematics concepts by students using this approach and found it to be useable both in a class setting and through an e-learning platform for distance education (Maćkowski et al., 2018). Further investigation into this software and possible applications is recommended, as the study supports potential use in the field of teaching mathematics to those with visual impairments.

Other applications and future directions. Several additional software systems are mentioned in current research, but the most widely known tool is Assessment and Learning in Knowledge Spaces (ALEKS) (Course Products, n. d). ALEKS is a web-based assessment and learning system which continually assesses student understanding and delivers instruction based on that assessment (Course Products, n. d.). While ALEKS is not specifically targeted towards students with visual impairments, it has potential to be adapted through partnerships with online mathematics education software meant for those with low vision, such as MiniMatecaVox or MathPlayer (Course Products, n. d).

Current literature suggests that there may not be a single solution to the issue of teaching mathematics to students with visual impairments. While many have tried to propose holistic solutions, none have yet been successful and usable in both an online and residential learning environment (Course Products, n. d.; Maćkowski et al., 2018). In the reviewed literature, there has also been a heavy focus on the K-12 mathematics courses, rather than collegiate courses. This not surprising since the educational foundation has to be built before a student can reach the collegiate level mathematics courses. Further research into the Lambda software used in the 2018 research study could provide information into how mathematics needs to be broken down for effective understanding by students (Maćkowski et al., 2018). This method of decomposition could then be used in assistive technologies, both in residential and online settings.

The process by which technology that holistically serves the mathematics education of students with visual impairments is created has many steps. It begins with understanding how those students can visualize formulas, charts, graphs, etc. and how instructors can most clearly communicate those concepts and equations to the student. While authors Maćkowski et al. (2018) could be on the right track concerning the decomposition of mathematics problems, the overall method needs to be scalable for any grade level and any setting. Once the communication aspect is resolved, programs and educational courses could be designed, similar to Lambda software that could be implemented in distance learning or classroom settings. It is currently unclear what this technology would look like in a classroom setting, but some online learning systems have already been developed. Additionally, existing typesetting software not specifically directed towards students with visual impairments provides an interesting potential to be used in collaboration with guidelines for verbalizing mathematical information in a way which would be beneficial to blind students and teachers of blind students (“Accessibility of Information”, n. d.).

The Use of LaTeX and MathSpeak in Education

Background of LaTeX and MathSpeak

Although not heavily mentioned in the research, LaTeX and Nemeth MathSpeak are integral for the creation of accessible resources for visually impaired students when it comes to technical subjects such as mathematics and science. LaTeX (pronounced “Lah-tech” or “Lay-tech”) is actually an extension of TeX, a typesetting system (“A document preparation system”, n. d.). Typesetting is a part of typography which involves preparing text for reading (Brown et al., 2018). TeX has the ability to typeset and format technical documents by typing standard text; this program can be used to create complicated or simple mathematical symbols, equations, and diagrams (Maneki, n. d.). Since LaTeX is meant for typesetting rather than word processing, the author of the text does not have to deal as much with the appearance of the document; rather, he or she is able to concentrate on the content and LaTeX will, for the most part, design the document in a logical fashion (“A document preparation system”, n. d.). LaTeX is also text-based and not visual in nature (Maneki, n. d.). As of today, LaTeX is the primary programming language used in publishing technical textbooks for math and science education (Maneki, n. d.).

Along with the author not having to fret over the layout of the document in LaTeX compared to in a word processor, LaTeX has many features that demonstrate why it is the most used programming language in mathematical textbook publishing. Some of these features include typesetting journal articles, technical reports, books, and slide presentations, advanced typesetting of mathematics with the add on AMS-LaTeX, multi-lingual typesetting, and automatic generation of bibliographies and indexes (“A document preparation system”, n. d.).

When LaTeX was first introduced, it was beneficial to visually impaired students, in part due to its non-graphical nature (Maneki, n. d.). As well, LaTeX is compatible with Nemeth Braille Code for Mathematics and Science Notation (Maneki, n. d.). Existing LaTeX-Nemeth Braille translation software allows for the production of a text in Braille from the LaTeX document (Maneki, n. d.). When the Braille illiteracy rate was lower, this was a beneficial development for visually impaired students. However, with the rise in Braille illiteracy, the availability of Braille textbooks based on LaTeX typesetting has been rendered ineffective and not as useful.

While LaTeX is useful in the creation of mathematics textbooks, the resultant PDF files are lacking in their accessibility to visually impaired students. Today students often rely on text-to-speech tools in order to read a technical textbook. Many word processing software, such as Microsoft Word, have both the ability to type mathematical equations and some type of text-to-speech converter. However, there is a lot of room for improvement with these devices and the process of verbalizing mathematics (Nemeth, n. d. a.). Across the board, Microsoft and other companies offering text-to-speech conversion differ greatly in the standards used to translate complicated mathematical equations and the translations are still ambiguous. In response to this inconsistency, Abraham Nemeth developed an unambiguous format for translating mathematical equations called “MathSpeak”.

Nemeth himself was a blind college mathematics professor for over forty years (Kendrick, 2013). Growing up in Manhattan, and later in Brooklyn, Nemeth struggled in his early years with arithmetic (Kendrick, 2013). This struggle is not uncommon for visually impaired individuals; as previously mentioned, the Braille illiteracy rate and lack of accessible tools for teaching mathematics to students with visual impairments only increases as the level of difficulty of a math class goes up. In addition, the social expectations of the abilities of a blind person were more limited at that time than in the modern day (Kendrick, 2013). Rather than being encouraged to excel in school, children with visual impairments were taught more menial tasks, such as basket weaving and chair caning (Kendrick, 2013). The idea behind this mentality, according to Kendrick (2013), was that “if a blind man were to have any gainful activity beyond begging, it would be in menial tasks and manual labor” (para. 3). However, with the help of Evanderchild’s High School in the Bronx, Nemeth was able to overcome his math difficulties and receive top scores in a first-year algebra course (Kendrick, 2013). This mastering of mathematics inspired him to want to teach math to others.

When discussing this desire to teach math with his university counselors, Nemeth (n. d. b.) received a plethora of reasons why this would simply not be possible. . Among those reasons were “mathematics [is] too technical a subject . . . notation [is] specialized, . . . no material [is] available in Braille, . . . readers would be difficult to recruit, and . . . no employer would be likely to consider a blind person for a position related to mathematics” (para. 10). In response, Nemeth decided to major in psychology while taking as many mathematics electives as he had room for (Navy, 1991). In these electives he began to improvise symbols in Braille to use as mathematical notation (Kendrick, 2013). This was the beginning of Nemeth Code. In addition to developing Nemeth Code during his undergraduate education, Nemeth (n. d. a.) also relied on volunteer or paid sighted readers for accessing text materials. It was during this time Nemeth (n. d. a.) realized there was no “standard protocol . . . for articulating mathematical expressions as [there is] . . . for articulating the words of an English sentence” (para. 2). Hence, he decided to make one, referred to as “MathSpeak,” which he claimed he could teach to anyone in about fifteen minutes (Nemeth, n. d. a.).

By keeping these symbols consistent from course to course, along with help from readers, Nemeth (n. d. b.) completed courses in analytic geometry, differential and integral calculus, modern geometry and statistics at the college level. With a degree in psychology Nemeth went on to work sporadically as a musician and for the American Foundation for the Blind doing menial labor (Kendrick, 2013). It wasn’t until he volunteered to tutor World War II veterans in calculus that Nemeth was able to show mastery of mathematics to society which soon led to his big break (Kendrick, 2013). Nemeth was unaware that one night another professor was observing him tutor calculus; impressed by what he saw, the professor quickly asked Nemeth to fill in for a teacher who had fallen ill at the mathematics department (Kendrick, 2013).

At some point Nemeth (n. d. b.) became acquainted with another blind mathematician in need of a table of integrals. In response, Nemeth (n. d. b.) quickly taught him his own code. Unknown to Nemeth, this other mathematician, Clifford Witcher, served on the joint Uniform Braille Committee at the time and asked Nemeth to prepare a guide explaining his code to be presented to the committee (Kendrick, 2013). Following the presentation of Nemeth’s code, Nemeth Code was officially adopted in 1951 (Navy, 1991). For the first time there was a uniform Braille system by which students with visual impairments could read and comprehend mathematical equations and the like. While this was an enormous hurdle that Nemeth overcame for the blind community, there was another large one to come: the Braille illiteracy rate. Today very few blind individuals can adequately read and understand simple Braille, let alone specialized Braille code directed towards science and mathematics (Jernigan Institute, 2009).

Thus, Nemeth created MathSpeak as a set of guidelines to allow students, teachers, and live readers to verbally articulate a mathematical expression with little to no ambiguity ("What is MathSpeak?", 2019). Originally, Nemeth (n. d. a.) developed these protocols during his undergraduate degree as a way for his live readers to consistently translate mathematical content in a way he could understand. Since his undergraduate education, Nemeth (n. d. a.) has spoken on these ground rules so that other blind students and readers may benefit from them as well.

Explanation of MathSpeak Guidelines

Below is a brief summary of exactly how MathSpeak articulates different types of text, taken from "A Talk on Verbalizing Math" by Abraham Nemeth (Nemeth, n. d. a.).

Letters. When read aloud by MathSpeak lowercase letters are read as the letters are routinely pronounced. Uppercase letters, such as "X", are read as "Upper X". A word in all caps is read as "upword" and then the word. When it comes to trig functions such as sine and cosine, each individual letter is read aloud. For instance, sin is read as "s i n". The same is for cosine, tangent, or log. Greek letters are read as "Greek" followed by the English name of the letter.

Digits and punctuation. Digits in MathSpeak are pronounced by each individual digit. For instance, instead of pronouncing 15 as "fifteen", in MathSpeak this would be pronounced "one five". Just as translating numbers is fairly self-explanatory, so is most punctuation. Punctuations such as period and comma are pronounced "period" and "comma". The two most common punctuations that are not taken at face value are semicolons and exclamation points, pronounced "semi" and "shriek" respectively.

When it comes to brackets, parenthesis, or other grouping symbols, MathSpeak is intentional about announcing when the grouping begins and ends rather than one or the other.

For instance, “L-pare” and “R-pare” describe left and right parenthesis. Similarly, left and right brackets are identified as “L-brack” and “R-brack.”

Operators and other math symbols. Many math symbols could be addressed. However, only the more commons ones will be highlighted at this time. Addition and subtraction are denoted as “plus” and “minus” while multiplication is either described as “dot” for dot multiplication or “cross” for cross multiplication. As well, the equal sign is pronounced as “equals”. So far these are fairly self-explanatory and intuitive. When it comes to inequalities the less-than sign (or “right-opening wedge”, as Nemeth refers to it) is said as simply “less” and its counterpart, the left-opening wedge is “greater”. Nemeth also specifies “less-equal” and “not-less” when the less-than sign has been modified to have such a meaning. The slightly less intuitive translations for symbols include “crosshatch” for the pound/number sign symbol and “joint” and “meet” for the up-opening wedge and down-opening wedge, respectively.

Fractions and radicals. Just as with grouping symbols, MathSpeak takes care to denote when a fraction or radical is beginning as well as when it ends. The beginning of a fraction is specified as “B-frac”, short for “begin-fraction”. Similarly, the end of a fraction is denoted by “E-frac”. According to Nemeth (n. d. a., p. 3), “Even the simplest fractions require “B-frac” and “E-frac.”” Additionally, “over” describes the fraction line. In the same manner, “B-rad” and “E-rad” signify the beginning and end of a radical. For nested fractions or radicals Nemeth uses “B-B-frac”, “E-E-frac”, “B-B-rad”, and “E-E-rad”.

For example, “B-B-rad a plus B-rad a plus b E-rad plus b E-E-rad” is the translation:

$$\sqrt{a+\sqrt{a+b}+b} \tag{1}$$

Another example is of the nested fraction

$$\frac{1+\frac{a}{b}}{1+\frac{1}{1+\frac{1}{a}}} \tag{2}$$

which, by Nemeth’s guidelines, is translated as “B-B-B-frac one plus B-frac a over b E-frac over one plus B-B-frac one over one plus B-frac one over a E-frac E-E-frac E-E-E-frac.” From (2), it is evident that MathSpeak has the ability to concisely and accurately verbalize even complicated nested fractions.

Subscripts and superscripts. To introduce the beginning of a superscript say “sup”; to introduce the beginning of a subscript say “sub”. Thus, instead of saying “x squared”, in MathSpeak, this would be “x sup 2”. Whether using a superscript or subscript, say “base” to return to the base level and signify the end of the superscript or subscript.

Below is an example of an expression with multiple superscripts:

$$e^{x^i} \tag{3}$$

This would be translated as “e sup x sup-sup i”. This could be used for any number of superscripts by adding a “sup” for each one. Superscripts are also used when dealing with radicals that are not a square root. An example of this is the cubed root of x which would be read as “B-rad sup 3 base x E-rad”.

Mapping LaTeX to MathSpeak

The semantics of MathSpeak are fairly intuitive but could still take a few examples to get used to. As well, Nemeth has provided through MathSpeak a widely comprehensive and definitively unambiguous system for translating written mathematics verbally. MathSpeak is currently used to adapt traditional mathematics textbooks into tools able to be read by students with visual impairments. This has widely increased the accessibility of mathematics instruction to such students. Unfortunately, this process is currently highly inefficient. Although many textbooks are available in electronic form, the audio copy of textbooks that would assist a visually impaired student is not easily accessible (“Accessibility of information”, n. d.). As well, the form an electronic textbook can take varies greatly, making it more difficult to ensure that any given textbook is either able to be read aloud itself through the use of the publisher’s resources or able to be read aloud with an outside resource (“Accessibility of information”, n. d.). The current steps to making a mathematics textbook compatible with MathSpeak can be summarized as follows: (1) publishers write a textbook in full using software such as LaTeX, (2) the textbook is published in traditional print form, (3) the newly published textbook has to be translated into a format compatible with MathSpeak (Allan, n. d.).

While this may not jump out as an inefficient method to many, it’s important to note that if a direct mapping from LaTeX to MathSpeak were to exist, the process of publishing a textbook able to be used by students with low vision could be made much more efficient, opening up the doors for students to access even better instruction and tools. If a mapping between the LaTeX and MathSpeak were to exist, the time in which it takes to create an accessible math textbook for students with visual impairments would decrease significantly. Instead of writing the textbook in LaTeX, publishing it, then trying to convert the published textbook into a form compatible with MathSpeak, publishers would be able to create an accessible file or document as soon as the textbook is finished in LaTeX. Specifically, if a verbalization of the technical document being produced were able to be embedded along with the file's metadata, the audio file would not need to be added separately from the file. Ideally, each document created in LaTeX would incorporate an audio component, allowing the document to be accessible to visually impaired students immediately. The development of an add-on or discovery of direct mapping between LaTeX and MathSpeak would greatly benefit the blind community.

Not only would a direct mapping cut down on the time it takes to make a mathematics textbook accessible to students with visual impairments, it would also significantly cut down on resources used in the conversion process. This would cut out the middle-man and allow publishers the ability to go straight from the coded textbook to an audio file which can be made available for students and professors. Having more textbooks accessible for students and teachers opens the door for better math education, something that every student needs in order to improve problem solving and critical thinking skills. This same potential software or add-on to LaTeX has broader implications as well for any textbooks in a technical field. This gives students more accessible resources for science, technology, engineering, or math (STEM) subjects. Not only would they be able to access more textbooks, but also worksheets, tests, quizzes, etc. would be more readily available in a format which is more efficient than the standard practices.

Programs which offer a read-aloud feature of mathematical content often do not follow MathSpeak guidelines and are ambiguous and leave the reader confused. To demonstrate this, consider the quadratic formula. The quadratic formula is a formula involving square roots and fractions, used to determine the roots of a quadratic equation. It is brought up frequently in math starting in algebra, a course nearly every student is required to pass in order to graduate from high school in the United States. The LaTeX code for this equation:

```
$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
```

.

Figure 1. A snippet of the LaTeX code which typesets as the quadratic formula.

The code in Figure 1 would typeset as (4):

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{4}$$

Given the prevalence with which many students and teachers use Microsoft Word for creating documents, the default for a read aloud program would be to use the Microsoft Word Read Aloud feature listed under the Review tab. This feature gives the user the option to speed up or slow down the reading, as well as the ability to choose from four different voices, some male and some female. Microsoft Read Aloud verbalizes the quadratic equation as “x equals numerator minus b plus or minus square root of b-squared minus four a c end square root end numerator over two a”. For this example, Microsoft provides a fairly unambiguous translation of the given equation. Considering that Microsoft provides the quadratic equation as one of the commonly used equations, it makes sense that the Read Aloud would provide an accurate translation. Now consider the following LaTeX code shown in Figure 2:

```
\[x^3 + 6x^2 + x = 30\]
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Figure 2. The LaTeX code which typesets as shown in Equation 2.

The code displayed in Figure 2 typesets as the equation shown in (5):

$$x^3 + 6x^2 + x = 30 \tag{5}$$

Microsoft Word reads this equation aloud as “x-cubed plus six x-squared plus x equals thirty”, much like a sighted individual would read the equation aloud if asked. However, this translation does not specify if the equation is meant to be interpreted as: $6x^2$ or $(6x)^2$. In fact, if the creator of (5) wanted to change the equation from $6x^2$ to $(6x)^2$, Read Aloud would then read that term as “open-parenth six superscript base x close-parenth end base squared.” This translation may be technically correct, but is overall insufficient in consistency and clarity. While

Microsoft Read Aloud can occasionally provide adequate translations of mathematical equations, it is not sufficient in itself as a way for a visually impaired students to consistently have technical content verbalized.

In contrast, MathSpeak is able to translate both equations with complete clarity through its detailed guidelines. Following Nemeth MathSpeak rules, (4) is translated as “x equals B-Frac minus b plus or minus B-Rad b Sup two Baseline minus four a c E-Rad over two a E-Frac.” This is a far more detailed translation, demonstrating that MathSpeak is able to be used for complex equations. Along with the most complicated equations in mathematics, MathSpeak is also successful in clearly translating simpler equations. (5) is translated in MathSpeak as “x Sup three Base plus six x Sup 2 Base plus x equals three-zero.” Adding “Baseline” or “Base” after ending the superscript or subscript signals to the reader that they are returning to the baseline, giving a clear understanding of when the superscript or subscript begins and ends. These kinds of consistent and unambiguous translations are necessary for teaching mathematics to students with visual impairments.

Therefore, if a mapping between LaTeX and MathSpeak were to exist, the code in Figure 1 would map to the translation on (4) and the code in Figure 2 would map to the above translation of (5). A complete mapping between LaTeX and MathSpeak would take some time to compile, but is possible to do. The more difficult part would be embedding such a mapping into LaTeX so that the translation is able to, for all intents and purposes, “ignore” the code including the “\” and words such as “frac”.

Audio System for Technical Readings (AsTeR)

The existing technology which is closest to being able to verbalize LaTeX coding is Audio System for Technical Readings (AsTeR) (Raman, n. d.). AsTeR is a computing system which can read aloud technical documents created in LaTeX (Raman, n. d.). AsTeR was developed by T. V. Raman, a blind mathematician who currently works for Google, for his Ph.D. at Cornell University (Raman, n. d.). Included on Raman’s website are sample readings of eighteen different sections of mathematics: simple fractions and expressions, superscripts and subscripts, Knuth’s examples of fractions and exponents, a continued fraction, simple school algebra, square roots, trigonometric identities, logarithms, series, integrals, summations, limits, cross referenced equations, the distance formula, quantified expression, exponentiation, a generic matrix, and Faa di Bruno’s formula (Raman, n. d.). It is evident that Raman has covered a wide variety of mathematics categories, ranging from simple algebraic expressions to lesser-known formulas.

In the process of verbalizing complex technical documents from LaTeX, AsTeR relies on inflection and tone to display information to the user (Raman, n. d.). For example, when reading aloud a complex fraction, AsTeR will start out at a normal volume and lower its volume for every new fraction within a fraction (Raman, n. d.). This signals to the user that the fraction is continuing without the user getting confused about where the numerators or denominators begin and end. This can be a useful technique in some cases. However, it is a more effective practice to keep the same tone throughout a reading for the purposes of mapping from LaTeX to MathSpeak, as MathSpeak does not need any changing inflection to be interpreted correctly.

There are also two more prominent issues with this software: First, it is not easily accessible to content creators. AsTeR is not available for download or use anywhere online, while, in comparison, LaTeX software can be downloaded for free from the Internet. Second, AsTeR does not translate LaTeX using MathSpeak guidelines. These two issues combine to greatly limit the ability of every blind student being able to use this software for his or her education. Also, it actually appears that AsTeR is not completely finished software, but instead is still in the beginning stages of development. While AsTeR covers multiple bases and some common examples ranging from simple to complex, it does not adequately translate all mathematical equations.

AsTeR does not abide by Nemeth MathSpeak's guidelines for interpretation of mathematical text. For instance, AsTeR is able to translate the following LaTeX code:

```
\[
a + \frac{b}{c} + d
\]
```

Figure 3. LaTeX code

as "a plus fraction b over c plus d". This translation could be interpreted in at least two different ways, as shown in (6) and (7), with (6) being the proper interpretation of the LaTeX code:

$$a + \frac{b}{c} + d \tag{6}$$

$$a + \frac{b}{c + d} \tag{7}$$

Comparatively, MathSpeak would translate the equation as “a plus B-Frac b over c E-Frac plus d”, a correct and far less ambiguous translation which leaves no room for misinterpretation. For this reason, AsTeR is not currently a sufficient tool for going straight from LaTeX to MathSpeak. However, there could be room for adjustments which would allow this to become the key to creating accessible technical education resources for students with visual impairments.

A Collaboration using LaTeX, MathSpeak, and AsTeR

The overarching purpose of creating a mapping from LaTeX to MathSpeak is to increase the accessibility of technical documents for visually impaired students. AsTeR is currently the closest technology to achieving this goal, but evidently still needs adjustments. AsTeR takes a LaTeX document and renders an audio recording from that; a more efficient process would involve implementing software straight into LaTeX that automatically converts a LaTeX document into MathSpeak as it is being created, rather than creating a separate audio file after the fact. Thus, it is proposed that further research be conducted into the development of a type of collaboration among LaTeX, AsTeR, and MathSpeak. AsTeR may work as a bridge between LaTeX and MathSpeak through its own adaptation of the MathSpeak guidelines and adjustments to make AsTeR an add-on of LaTeX. More information should be gathered on the mechanics of AsTeR to determine its ability to be implemented directly into LaTeX.

Conclusion

The ideas presented in this thesis express the profound benefits of having more accessible materials for students with visual impairments. While past societies have deemed blind people as only fit for unspecialized menial tasks, the perceptions of the abilities of blind people have shifted in recent decades. Now many understand that with the right tools and education blind people can be teachers, scholars, mathematicians, and more.

The information given in this paper could provide assistance for those interested in learning how to make technical subject material more accessible to students with visual impairments. While accessibility has radically improved over the past thirty years, there is still more that could be done. Given the importance of a foundational education, especially in mathematics, the work to create resources available to every student with varying disabilities should continue to grow.

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