Attitude, Culture, and Teaching Mathematics

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Abstract

International assessments have revealed discrepancies between the mathematics achievement scores of various countries and also between the scores of male and female students around the world. Although math education may look similar on the surface in different countries, there are subtle differences in the methods and the attitudes that teachers and students adopt from their cultures. These differences may be explained in part by the theory of mindsets, specifically the contrast between fixed and growth mindsets. Mindset theory illuminates the impact of beliefs and values on academic achievement, particularly in the area of math. These same principles also seem to apply to the gender gap in mathematics that exists in varying degrees throughout the world.

Attitude, Culture, and Teaching Mathematics

Introduction

Despite the common misconception that math is strictly factual and disconnected from emotion and creativity, attitude is deeply intertwined in how educators and students approach mathematics. Defined as a set of attitudes shared by a group, it follows that culture has an immense impact on the realm of mathematics education. This influence extends to views on the nature of mathematics as well as beliefs about math ability. The mindsets that permeate each unique culture can shape the outcomes of math achievement, though their effects may be difficult to directly measure. Teachers and students of different nations bring different approaches and attitudes to the classroom which become evident in observing their daily routines. Even the different subcultures of males and females influence the way math education is perceived and how students of different genders respond to various facets of their academic or professional environments. Many of these differences, when viewed through the lens of mindset theory, can bring helpful insights into the world of mathematics education.

Comparison of Instruction and Curriculum in a Sampling of Countries

Researchers have conducted many studies in an attempt to record key details about mathematics lessons taught in different countries, searching for the strategies and methods that work best or yield the most satisfactory results on international achievement exams, such as the scores recorded by Trends in International Mathematics and Science Study (TIMSS). Many individuals and corporations have analyzed these studies in an attempt to adopt and incorporate better teaching strategies into America’s education system, whether to reform the entire educational process or simply to better U.S. scores (Holliday & Holliday, 2003). However, it is important to note the significance of culture and overarching attitudes which determine the effectiveness of various strategies used in any given classroom. Each country and culture has its own unique perspectives and attitudes which distinguish it from others, and these distinctions have a profound impact on the way that educational changes are received in each unique culture. In other words, the nuances of culture and its influence on education ensure that there is no universal solution to bettering the field of education.

Culture is more deeply engrained than simple teaching strategies – even changing the methods and strategies used in the classroom may not have the same effect in one country as another due to cultural differences (Andrews, Ryve, Hemmi, & Sayers, 2014). Teachers who have been raised and taught in a particular culture will likely teach using the methods they observed as students (Stigler & Hiebert, 1999), perpetuating that culture's unique teaching methods and attitudes. Nevertheless, teachers in any culture can benefit from reflection and learning why the strategies of other cultures are effective or ineffective. Despite the challenges presented in adapting cultural methods, there remains much to be learned about how teachers of various cultures approach the task of teaching mathematics.

Similarities

Overall, various countries' approaches to mathematics have more similarities than differences (Hiebert et al., 2003). The results of the TIMSS 1995 and 1999 studies of seven different countries – Australia, the Czech Republic, Hong Kong, Japan, the Netherlands, Switzerland, and the United States – showed that a typical math lesson in each of these countries shared a pattern or structure that was very similar. In every country studied, math lessons generally included the following components: the review of prior knowledge or skills, the introduction of new concepts and material, the use of a textbook or some extraneous material, some mix of whole-class instruction and individual or small-group work, and the majority of class time spent solving problems (Hiebert et al., 2003). The same study found that, no matter the country, teachers in the math classroom spent more time speaking than students did, by a proportion of at least eight to one (Hiebert et al., 2003). Overall, it appears that teaching mathematics looks remarkably similar across vastly different cultures. However, international studies have shown consistent differences in the mathematics scores of several countries, implying that these similarities are not the determining factor in mathematics achievement.

Differences

The results of international assessments reveal that subtle differences in teaching can have a significant impact. Though all cultures tend to teach mathematics via solving problems, the types of problems each culture presents to its students are of greater consequence. This is not the same when one compares a nation such as Japan with the U.S. Another key factor to consider is the manner in which the teacher guides students in approaching a problem, whether directly or conceptually, and how much flexibility is allowed for questions and critical thinking (Sawada, 1999). Although classes in all nations review old information and introduce new concepts, they may differ in how much time is spent on each (Ginsburg, Leinwand, Anstrom, & Pollock, 2005). When these similarities are analyzed through a narrower lens, the cultural differences become much more apparent.

Language. A notable difference between various nations' classrooms is language. While it may seem that concepts may be translated from one language to another with little difference in the content or connotation, this may not be the case. Even basic procedures such as counting can reflect the influence of a learner's native language. Sousa (2015) noted the distinctions between English and Chinese words for numbers and the effects of such seemingly trivial differences on the way children learn to count and associate numbers. Western studies have found that seven appears to be the "magic number" of digits that the average English-speaker can remember in sequence at a time (Cowan, 2010). However, people who speak Cantonese appear to have a much higher average recall of about ten digits in the same span of time (Sousa, 2015). It has been hypothesized that this is mainly due to the length of time it takes to speak each number aloud in any given language, as the working memory only holds a span of about two seconds of speech at a time (Cowan, 2010). The average English number takes roughly one-third of a second to pronounce as opposed to one-fourth of a second for the Cantonese equivalent (Sousa, 2015). This simple difference seems to make a significant impact on the efficiency of the working memory in remembering digits.

Sousa (2015) also suggested that children in China get more practice with numerical concepts from an early age because of China's numerically-derived names of weekdays and months of the year, as opposed to English-speaking children who learn these same concepts with names derived from Roman mythology. Even the way that numbers are written in English as opposed to other languages can add up to major differences in children's counting abilities in the early years of their development (Sousa, 2015). Uy (2003) noted that the structure of numerical vocabulary in the Chinese and Japanese languages is more conducive to understanding the concept of place-value in the base ten number system, whereas English requires more extensive and less intuitive vocabulary. For example, the suffix *–teen* is used in place of the word *ten*, and the outliers eleven and twelve fit no apparent logical pattern. English is not the only culprit in adopting counterintuitive numbering rules, as both French and German also have notable idiosyncrasies in their linguistic forms of numbers (Houdement & Tempier, 2018). Beyond simply learning numbers and counting, language affects how students learn to think mathematically (Sun & Zhang, 2001), as well as students' interpretation of word problems (Moseley & Okamoto, 2008) and of place values, which can impact understanding of many other concepts (Hiebert & Wearne, 1992). Right from the start, language has a significant impact on the way that students of different cultures learn the very building blocks of mathematics. Unfortunately, it also seems that English-speaking students are at a disadvantage, at least compared to Chinese and Japanese students.

Japan and the United States: A Comparison. Japan is one of the most consistently high-scoring competitors in international mathematics assessments. Stigler and Hiebert (1999) described the Japanese classroom as a relationship between students and mathematics, with teachers stepping in as mediators. The problems presented to students are typically more challenging and are dealt with more conceptually than in other nations (Starr, 1998). Teachers in Japan tend to give the class fewer problems to work on during class time (sometimes even just one key problem), and often let the students come up with their own strategies for solving the problems (Stigler & Hiebert, 1999; Sawada, 1999). This follows a method known as *productive failure*, in which students attempt to use their prior knowledge solve a new problem before receiving direct instruction on the topic (Kapur, 2014). Kapur (2014) showed that this strategy increased students' cognitive load and led to greater conceptual understanding than when students were given direct instruction and then asked to solve problems. This type of instruction forces students to think conceptually about problems and their connections to prior knowledge, as opposed to merely recognizing a certain type of problem and following a predetermined procedure.

The underlying beliefs that teachers in Japan tend to hold about the nature of both mathematics and how learning should occur also shapes some major differences in the ways that they implement classroom strategies and procedures. In Japan, mathematics is not seen as a set of rules or skills, but rather as “relationships between concepts, facts, and procedures” (Stigler & Hiebert, 1999, p. 89). While many teachers in the U.S. tend to shrink away from problems that involve frustration and struggle, Japanese teachers embrace these as natural and necessary elements of the learning process (Spiegel, 2012). Making mistakes is seen not as a barrier, but rather as a stepping-stone to increased learning (Stigler & Hiebert, 1999). Teachers in Japan also treat mathematics as if students are inherently interested in the subject, rather than resorting to non-mathematical applications and entertaining add-ons to pique students’ interest (Sawada, 1999). Surprisingly, rather than focusing on individual differentiation as is the trend in the U.S., Japanese teachers focus on commonalities among students and make problems both challenging and accessible to a variety of learners (Corey, Peterson, Lewis, & Bukarau, 2010). This seems to be a major motivator behind the belief commonly held by Japanese teachers that students should not be tracked or separated by academic level, since “all students should have the opportunity to learn the same material” (Stigler & Hiebert, 1999, p. 94).

Japanese teachers also make use of an integrated curriculum, meaning that in each grade connections are made between many different areas of mathematics, as opposed to separating content each year into classes such as Algebra I, Geometry, Algebra II, and so on (Brahier, 2016). Japan’s national curriculum is known as the Course of Study (COS), which includes various standards and objectives for teachers to follow. The COS for mathematics indicates that teachers can make connections between content covered from year to year, and the objectives for each grade specified in the COS fall under the same broad categories: numbers and mathematical expressions, geometric figures, functions, and data handling (Takahashi, Watanabe, & Yoshida, 2008). Compared to U.S. teachers, Japanese teachers in the TIMSS Video study made more connections between mathematical concepts (Starr, 1998). Instead of learning Algebra I, Geometry, and Algebra II as distinct and separate subjects as is currently the standard practice in the U.S., Japanese students learn some algebra and geometry in each grade level. By the end of secondary education, Japanese students will cover material comparable to the Common Core State Standards in Mathematics (Achieve, 2010).

The way that the typical Japanese math classroom works on a daily basis also differs vastly from classrooms in the U.S. Rather than focusing on step-by-step procedures and practicing the solutions to many similar problems, Japanese teachers encourage their students to see the big picture of the concepts involved in the lesson and use these to figure out different methods for solving problems (Corey et al., 2010). This is why, rather than giving exact solutions for problems, Japanese math teachers use less direct intervention and guide their students in thinking about and representing the problem to find their own solutions (Moseley & Okamoto, 2008). Japanese teachers often set distinct goals for their students to both understand concepts being taught in conjunction with their connections and to be able to demonstrate skill in solving problems (Corey et al., 2010). In the classroom itself, Japanese math teachers tend to prefer the use of a simple blackboard over newer alternatives like an overhead projector, often because they want students to be able to see the cumulative record of ideas given in class, enabling them to refer back to previous concepts in order to figure out new methods for finding solutions (Stigler & Hiebert, 1999). Additionally, interruptions appear to be far less common in the typical Japanese school during math class than in other countries (Starr, 1998).

In contrast, schools in the United States generally give the mathematics classroom a much different treatment and hold it in a different view. American classrooms seem to foster a dynamic that is focused more on the students and the teacher, but not inherently on the subject of mathematics itself (Stigler & Hiebert, 1999). The materials covered in American classrooms are often less advanced than those covered in other nations, and the process is mainly focused on straightforward skills, definitions, and procedures (Cox, 2015). Rather than connecting lessons and topics conceptually, learning is broken down into distinct modules, and many practice problems are given to the students, often very similar to problems that the teacher has already given a full solution to (Sawada, 1999). Teachers typically try to alleviate frustration and stress, often giving solutions to students when they cannot work out the problem the first time around.

Out of the three countries focused on by Stigler and Hiebert (1999), the United States tended to give the greatest percentage of classroom time to reviewing materials that students had already covered. The curriculum of the United States has tended to favor breadth over depth, as U.S. math textbooks cover significantly more topics than do comparable textbooks used in other nations (National Research Council [NRC], 2001). As a result, the American math classroom typically covers more topics for a shorter period of time, and then reintroduces these same topics over the course of several years (Ginsburg et al., 2005). This may explain why U.S. math teachers were shown to cover an average of two topics per class period, as opposed to only one topic for Japanese teachers (NRC, 1997).

Singapore. Singapore's education system should certainly not be ignored, as students from Singapore scored the highest in both math and science TIMSS and PISA (Programme for International Student Assessment) assessments in 2015 (Gurney-Read, 2016). According to Vasagar (2016), this may be a result of the fact that Singapore's education has been more recently developed than that of many other nations, with a special emphasis on understanding and the practical use of math and science. Singapore's schools favor math and science over humanities since these subjects are more relevant to job acquisition, thus aiding the nation's economy (Vasagar, 2016). In Singapore, education is held in high regard as a pathway to higher social standing (Menon, 2000), while U.S. students, especially those from low-income families or lower-performing schools, value finding a job over academic achievement (Usiskin, 2012). Unlike the U.S., curriculum in Singapore is narrowed to fewer subjects, which are taught in greater depth (Ginsburg et al., 2005). According to Vasagar (2016), mathematical thinking and diligence were attributed as major components of the success of Singaporean math education, although Usiskin (2012) asserted that Singapore's achievement may be due more to achievement tracking, its citizens' high socioeconomic status, and intense after-school tutoring programs.

Learning to think mathematically rather than gaining simple knowledge of facts and procedures is the goal of math education in Singapore, and diligence is held in higher regard than is raw talent (Vasagar, 2016). The Singaporean mindset towards learning mathematics is that all children are capable of learning the required materials, though some may learn at different paces and with different levels of support (Ginsburg et al., 2005). In contrast, U.S. teachers are more prone to watering down mathematics curriculum for students who are slower learners (Ginsburg et al., 2005). Visual aids and representations are abundant, as Singapore's math teachers strive to help students picture and model mathematical concepts (Ginsburg et al., 2005). Much like Japan, the motivation for learning math and the process by which it is taught are vastly different from the United States.

China. China is another top-scoring nation for mathematics according to the TIMSS assessment results (Gurney-Read, 2016). In China, teachers approach education and the classroom differently and are viewed in a different cultural light than in many Western cultures. Stemming from Confucian teachings, Chinese culture holds a deep respect for teachers and education as a whole to a higher degree than many other nations (NRC, 2010). Math classes in China usually have a greater number of students than most math classes in the U.S. (Organization for Economic Co-operation and Development [OECD], 2012), and emphasis is placed on whole-class learning much more than small group and individualized teaching (NRC, 2010). Teachers also seem to have more opportunities to interact and collaborate, as teacher workspaces are set up in Chinese schools specifically for the purpose of working and collaborating among teachers of similar subjects, a model which stands in stark contrast to the traditional U.S. concept of a teacher breakroom (Yang & Ricks, 2012). Math teachers in China are highly encouraged to hold discussions and reflect on their teaching experiences (NRC, 2010). In fact, it is a common practice in China for teachers to present their lessons in front of fellow teachers in order to improve their skills and strategies (Huang, Fang, & Chen, 2017). This practice originated and is still common in Japan (Brahier, 2016). Chinese teachers tend to teach fewer classes per day than U.S. teachers (Yang & Ricks, 2012), giving them more time to focus on correcting homework, planning lessons with other teachers, and helping students outside of class (NRC, 2010).

Testing and Assessments

Aside from everyday classroom procedures, another central aspect of any nation's education system is the way students' learning is assessed. Most nations have their own individual examinations for all students, especially in relation to assessing college readiness. In addition to the differences in style and format for standardized assessments in each nation, the attitudes of students and parents and the overall importance placed upon each assessment can be vastly different from one nation to another. To be explored are two nations whose treatment of standardized testing are nearly polar opposites: Finland and China.

Finland

Finland is a particularly interesting nation to investigate since international studies have shown Finnish students to rank exceptionally highly compared to students from other nations (Dickinson, 2019). Finland ranked first in mathematical literacy according to the PISA assessment results from 2003 (OECD, 2004). Finland's stance on education may be a major factor in its success, as the nation espouses a unique view of education that sets itself apart from that of most other countries in the world. Whereas many nations tend to require several standardized tests and place high stakes on assessments for each individual student, Finland's system of education does the opposite (Hendrickson, 2012). The emphasis in Finland is on learning and growing as a community, and less weight is placed upon the individual student to compete for success (Kasanen, Räty, & Snellman, 2003). Additionally, in stark contrast to most other high-achieving nations where students spend hours outside of regular school time cramming for exams, Finnish students actually spend less time in the classroom, with an average of only 5500 hours in school as opposed to 7500 hours in the U.S. over a span of eight years

(Seaberg, 2015). Further, Finland's education involves no tracking, meaning that students are not separated into high- and low-achieving groups throughout their high school career, giving equal opportunities to succeed to all students (Ruzzi, 2005; Hendrickson, 2012).

As for assessments, Finland is also unique in that it administers only one high-stakes standardized test, called the national Matriculation Examination, or ME (Paasonen, 2004). This assessment is taken by all 12^{th} grade students in the upper-secondary schools in Finland, as opposed to the technical schools which students can choose as an alternative for grades 10-12, and is required for admission into education at the university level (Seaberg, 2015; Hendrickson, 2012). The exam itself also differs greatly from standardized tests typically given in the U.S., as the ME is not a multiple-choice test, but rather contains open-ended and essay-based questions to probe students' higher-order thinking (Seaberg, 2015). Students are allowed to refer to a booklet of formulas and tables to which they have referred throughout high school, reflecting Finland's emphasis on problem-solving and higher-order thinking skills rather than rote memorization (Seaberg, 2015). This allows educators in Finland to de-emphasize lower-levels of thinking such as memorization and recall in class and on the ME, and Finnish math textbooks coincide by providing answers to practice problems so that students can check their work (Seaberg, 2015). Finland's math curriculum in general tends to focus on "real-world" problems, rather than contrived math problems which focus attention back on formulas and memorized algorithms (Loveless, 2013). In addition to the ME, teachers use portfolios of student work to assess student learning at the end of compulsory education (Hendrickson, 2012). Most other forms of assessment are not graded on a numeric scale and are rather viewed as feedback for students to assess their own learning, fostering self-efficacy in students (Hendrickson, 2012).

Finland's ME is also distinguished from other standardized tests in that it requires students to think critically about relevant issues such as ethics, religion, culture, and real-life issues that exams like the California High School Exit Examination purposely avoid (Strauss, 2014). Questions on the exam do not skirt sensitive topics, but prompt students to reflect and give their views on potentially controversial subjects (Strauss, 2014). Finnish schools also give their own year-end examinations; however, these assessments, unlike most end-of-year tests in the U.S., are not meant to determine the comprehensive achievement levels of individual students for the year (Dickinson, 2019; Hendrickson, 2012). Rather, these assessments use samples of content provide overall feedback to schools for improvement of curriculum and classroom strategies (Dickinson, 2019).

Nevertheless, comparing Finland's education and assessments with those of countries like the U.S. may not be as straightforward as one might hope. Aside from some major differences in approach, Finland's total population is less than that of New York City alone (Dickinson, 2019), meaning that what works for Finland may not be directly comparable to what works for larger countries. This also means that Finland's leaders in education have greater ability to make smaller class sizes a reality and to implement higher standards for training teachers, as Finnish high school teachers are required to have at least a master's degree, as opposed to only a bachelor's degree for U.S. teachers (Seaberg, 2015). In addition, Finland's apparent success in the PISA studies may not carry over to others, such as TIMSS (Loveless, 2013) and the 2013 International Mathematics Olympiad (Seaberg, 2015), as Finland did not score nearly as well on these, leading to some ambiguity in its true international mathematical standing.

China

China's norms for assessment are on the opposite side of the spectrum, stressing standardized testing as highly important. Among China's assessments, the most influential is the *gaokao,* a nickname for China's National Higher Education Entrance Exam meaning *high test* (Pinghui, 2017). This assessment is arguably the most important factor in a student's academic career, as each individual's score determines eligibility for entering different levels of higher education, which can greatly impact a student's opportunities for attaining a successful career in the working world (Lu, Shi, & Zhong, 2018). The assessment takes two to three days, and covers content from Chinese, mathematics, and English, as well as one or more elective subjects (Gu & Magaziner, 2016). The assessment is administered every June to millions of students, mostly those who have just completed high school (Pinghui, 2017). Questions on the assessment include multiple choice, short answer, and essay items, and the content of each year's *gaokao* exam is notoriously difficult (Ma, 2018).

For students from a low socioeconomic background, this exam can be a major chance to open doors to a better education and a better life, placing immense academic and social pressure on students taking the *gaokao* (Pinghui, 2017). An outstanding score on the test can even offer opportunities abroad (Min, 2018). Since the assessment can carry so much weight for a student's future, immense precautions are taken against cheating, with police in the vicinity of each testing center to ensure that students arrive on time and the use fingerprint scanners to confirm the identity of each test-taker (Gu & Magaziner, 2016). Cities where the test is held may even ban noisy construction in the vicinity of the testing centers (Ruiqing, 2013).

Though in theory the *gaokao* is similar to Finland's ME in that it assesses graduating high school students' readiness for university, the attitudes and cultural norms surrounding China's biggest standardized exam are a stark contrast to Finland's laid-back stance. Finland and China, both considered to be high achievers in education, take distinctly different approaches to testing students. While Finland stresses the importance of real-world problem-solving skills, many of China's exams and assessments have historically relied upon rote learning and students' ability to memorize large quantities of material, leading many educators in China, especially in lower-income areas, to teach to the test rather than teaching applicable skills and critical thinking (Ruiqing, 2013). Rather than backing away from standardized testing, China's education specialists have embraced these assessments as a central facet in the endeavor to propel low-income students out of poverty (Rotberg, 2006). Recent reforms in Chinese education have attempted to move towards assessing higher-order thinking and integrating multiple areas of thought in the standardized exam questions, with a surprising level of success in a relatively short time (Rotberg, 2006).

International Assessments

Though individual nations differ in their treatment of mathematics assessments, leading to different ways of interpreting scores and viewing achievements, recently global interest has increased in comparing and contrasting the academic progress of students around the world. The resulting studies have sparked debate about everything from classroom teaching methods to the politics of each nation's educational authority. Two such studies which are commonly referenced in this ongoing international conversation are PISA and TIMSS. Both of these studies administer assessments, which may be devoted to mathematics alone or in addition to other subject areas, to samples of students from various nations and then analyze the results to determine a global objective standard for effectiveness in education.

Although TIMSS and PISA seem similar on the surface, and scores from both these studies tend to be correlated in each country where they are administered, they still have some notable distinctions (Else-Quest, Hyde, & Linn, 2010). While most countries' students score similarly on the two assessments, there are a few outliers for which TIMSS and PISA scores differ significantly (Loveless, 2013). For example, Finland (which has an educational emphasis on problem solving) was the top scorer in the 2003 PISA assessment, but lagged behind significantly in the comparable TIMSS assessment, ranking about the same as the U.S. in this more curriculum-focused assessment (Andrews et al., 2014). Loveless (2013) argued that the two assessments actually examine different kinds of learning, despite their apparent correlation.

TIMSS tends to focus on curriculum-based learning, placing emphasis on schools and the content covered in each nation, while PISA assesses students' individual abilities to understand and apply math strategies to solve real-world problems (Else-Quest et al., 2010). While TIMSS measures the effectiveness of each country's curriculum and how content is taught, PISA assesses students' broader mathematical abilities in the areas of space and shape, change and relationships, quantity, and uncertainty, presenting these via questions involving plausible scenarios relating to education, occupations, community issues, or scientific situations (OECD, 2004). Loveless (2013) speculated that this difference may explain Finland's score gap between the two international assessments. Additional discrepancies include the fact that PISA assesses students at age fifteen regardless of grade, while TIMSS administers its assessments to students in grades 4, 8, and 12 (Loveless, 2013). Even among these international assessments, cultural and national differences have a major influence on the results of each study.

Culture and Mindsets

As seen from several different countries' approaches to teaching math, culture and a nation's perspective on the nature of mathematics and learning can have a huge impact on the way that students learn and what they achieve. According to the National Research Council (1989), "the social context of education has a greater influence on student performance than does actual classroom practice" (p. 90). The dynamics of student-teacher interactions in the classroom and the role of the teacher seem to be some of the key cultural factors in distinguishing between the perspectives of Japan and China as opposed to those of the United States. Generally, it seems that higher-achieving countries have a greater focus on conceptual understanding in the classroom rather than simply practicing and repeating steps in a predetermined process. In addition, teachers in many high-achieving countries appear to hold the common belief that a student's success in mathematics is dependent on effort rather than fixed natural ability. These beliefs, reflected in the teaching methods and everyday responses of teachers, may be a significant factor in the success of students on assessments as well as students' overall relationship with mathematics in compulsory education and beyond.

The Myth of the *Math Person*

Perhaps nowhere more than in the United States, the attitude of attributing both poor and excellent mathematical achievement to fixed, innate ability is pervasive (NRC, 1989). Psychological researcher Catherine Good both summed up and refuted this popular misconception, as cited in Campbell (2011), stating, "People assume that math is somehow linked to genes: either you are a math person or not. The reality is that math is an ability and a skill set that can be nurtured and developed over time" (p. 27). This math person belief may be held in varying degrees throughout other nations and cultures as well, but what difference can it make in students' achievement in mathematics? The effects of a mindset based on fixed math ability are subtle, but can be widespread and influence many different factors that contribute to math achievement. The National Research Council (1989) suggested that this type of belief can influence the attitude of parents, students' self-concept and self-efficacy, and even how students are tracked within an education system.

Growth and Fixed Mindsets: Dweck's Mindset Theory

This belief in math people has deeper roots in theories of overall intelligence. Psychologist Carol S. Dweck (2006) defined these underlying beliefs about intelligence as mindsets and distinguished two main types: *fixed mindsets* and *growth mindsets*. According to Dweck's (2006) framework, those holding a fixed view believe (whether consciously or unconsciously) that a person's intelligence is determined by factors beyond that person's control. Also known as an *entity* theory, the fixed mindset is a view in which individuals are born with a certain measure of intelligence that remains constant throughout their lives, and practice or training can do very little to expand or improve this innate level of intelligence (Dweck, 2012). When faced with challenges or failures, those holding a fixed mindset tend to attribute these setbacks to inherent inadequacy and are less likely to make additional attempts (Dweck, 2006). Conversely, the growth or *incremental* mindset encompasses the view that intelligence is malleable, and that it can be increased and refined through effort and hard work (Dweck, 2006). Those who have a growth mindset more often view challenges and even failures as opportunities to learn and grow, and thus are more resilient in the face of adversity (Dweck, 2010). These two opposing mindsets are especially impactful in the setting of mathematics education, deeply affecting not only students' learning behaviors, but also the ways that teachers shape classroom culture and students' experiences with math (Boaler, 2016).

Boaler's applications of mindset to mathematics. Jo Boaler has become a prominent expert in this field, emphasizing the significance of Dweck's mindset theories specifically within the realm of mathematics. She has found that negative attitudes about math are so pervasive that even individuals who hold a growth mindset about other areas can still be curtailed by fixed views about math intelligence in specific (Boaler, 2016). Boaler (2016) identified some harmful effects of the fixed theory of intelligence concerning math in particular, including: the belief that math is devoid of the creativity and flexibility inherent in other subjects, the belief that math ability is an indicator of a person's general intelligence, and the belief that math is not relevant or necessary for socially adept people. Boaler (2016) prescribed incorporating specific math mindset strategies into the classroom, extending her ideas to the structure of math curriculum and its presentation and even the ways that parents and teachers praise students.

According to Boaler's (2016) mathematical mindset stance, students, teachers, and parents must work to discard a fixed mindset, as these beliefs about math ability are not only detrimental to those who believe they are bad at math, but also to those who believe themselves to be gifted at math. Teachers should be careful to examine their own mindsets and ensure that they do not hold to negative beliefs about students or the nature of math and how it is learned, as even unconsciously held beliefs can influence teachers' instructional practice (Thompson, 1984). Such beliefs held by teachers can even impact the mindset and performance of students, particularly girls in elementary grades (Beilock, Gunderson, Ramirez, & Levine, 2009). In addition, teachers or parents who praise students for personal qualities rather than effort or actions can also unintentionally promote a fixed mindset (Mueller & Dweck, 1998; Boaler, 2016). Rather than complimenting intelligence, educators can promote a growth mindset by complimenting hard work or specifying what the student did in order to achieve success.

Neurological findings. Boaler (2016) used several neurological studies as the basis for her various workshops and development sessions for teachers and students alike, including findings on the extensive development of the hippocampus in London's Black Cab drivers as a result of their intensive training, the case of Cameron Mott's nearly complete recovery from the removal of half of her brain, and a study in which structural growth and changes were recorded in the brains of subjects who practiced certain mental exercises for only ten minutes a day over the course of three weeks. These examples illustrate Boaler's point that the human brain can and does grow and develop in response to mental training and correct educational techniques. Boaler, Dieckmann, Pérez-Núñez, Sun, and Williams, (2018) and Dweck (2006) have demonstrated that becoming aware of the fact that learning can positively affect neural pathways helps students develop a growth mindset, which can raise the performance of students from any background. Boaler (2016) also cited studies which found that making mistakes caused increased electrical activity in the brain which led to the formation of new synapses (Moser, Schroder, Heeter, Moran, & Lee, 2011). Further, while this initial brain *spark* occurred when any individual made a mistake, activity involving conscious response was observed more often and at higher levels in those who held a growth mindset as opposed to those with a fixed mindset (Boaler, 2016; Moser et al., 2011).

Impact. As a result, Boaler has published a multitude of works for students and educators alike, including a free online resource called Youcubed, in which she teaches strategies for promoting and developing a growth mindset and provides math interventions that help learners to visualize and conceptualize mathematics rather than relying on rote memorization and isolated procedures (Boaler et al., 2018). In her writings, Boaler (2016) emphasized the need for change in American and British mathematics classrooms, stressing the importance of teaching math as an inherently interesting and deeply connected subject involving beauty and creativity, fostering healthy collaboration and discussion among students, and promoting good critical and mathematical thinking while deemphasizing calculational speed as an indicator of success. She also advocated for tasks that are not mere repetition of applying an established formula, but that are creative, able to be pictured and visualized, and *low floor, high ceiling,* meaning that any student can grasp the basics and engage with the problem or project, but that is still challenging and initiates high levels of critical thinking (Boaler, 2016). These problems are often open-ended and usually do not have only one routine solution, but are meant to get students to think about concepts and allow students to discover different aspects of a mathematical relationship for themselves, fostering rigorous mathematical thinking and strategies (Boaler, 2016). On the whole, Boaler's theories and suggested practices strike a considerable difference when compared to the traditional classroom procedures in most U.S. schools. Her ideas are similar to those implemented by math teachers in the highest-achieving nations in the world (for example, China and Japan), indicating that the mindset which teachers hold (and which they foster in their students) may be a determining factor in the mathematical success of students around the world (Boaler, 2014; Stigler & Hiebert, 1999).

Mindsets of Teachers

Although it is clear from Boaler and Dweck's research that one's personal beliefs and mindset have an impact on learning and achieving in mathematics, another important factor in students' lifelong mathematical journeys is the mindset their teachers hold. Francome (2016) highlighted the importance of teachers' mindsets and expectations, declaring, "Teachers' beliefs are powerful and perhaps the most important aspect of teaching mathematics is for teachers to believe that any pupil can improve their mathematics with hard work, effort and good teaching and achieve at the highest levels" (p. 17). Rattan, Good, and Dweck (2012) found that the mindset which university professors adopted had a heavy influence on both their own instructional strategies and the motivation of their students to continue in post-compulsory mathematics courses. Those who held an entity (fixed) theory of intelligence were shown to be more likely to attribute even one instance of poor performance to an inherent lack of ability and to have lower expectations for students who performed poorly (Rattan et al., 2012). Further, university instructors with an entity view were more likely to advise students who performed poorly in introductory mathematics courses to choose a course of study which required fewer courses in mathematics or to focus on other strengths, causing students to perceive less investment from the instructor and decreasing their motivation to work hard in math classes (Rattan et al., 2012). Students who dropped out of these introductory math courses were forced to exit the sequence of mathematics required for most careers in science, technology, engineering, and mathematics (STEM) and were consequently unable to pursue higher education in math-oriented fields (Rattan et al., 2012). Even though many entity-view instructors intended to comfort their students by suggesting other fields of study, these strategies often resulted in decreased student motivation and lower self-expectations for students (Rattan et al., 2012; Mueller & Dweck, 1998). This is significant considering that Boaler (2016) declared math to be the field in which instructors "held the most fixed ideas about who could learn" (p. 5).

The reverse was true for instructors who held an incremental (growth) mindset, as they tended to attribute poor performance to variables such as lack of effort instead of lack of ability, maintained high expectations for students' future performance in class, and were more likely to encourage students to persevere in math classes by focusing their efforts on practicing and improving weak areas, reinforcing behavior attributed to a growth mindset in students (Rattan et al., 2012). This resulted in students' perception of greater investment from their instructors and higher expectations for their own achievement in the class (Rattan et al., 2012). In a related vein, these results may also reflect a phenomenon known as the Pygmalion effect, which posits that teachers' expectations of students become a self-fulfilling prophecy of student performance (Ellison, 2015). Both Dweck's intelligence theory and the Pygmalion effect demonstrate how vital teachers' beliefs and attitudes are to their students' success.

Tying Together Mindsets and Culture

Dweck and Boaler's insights into mindsets illuminate some common threads from the classroom. Finnish teachers took a stance similar to Boaler's in emphasizing problem-solving and conceptual understanding over memorization of formulas and procedures (Dickinson, 2019). Top-scoring nations such as Japan and Singapore also appeared to exhibit a growth mindset, treating all students as capable of learning math and giving students challenging problems that foster conceptual understanding and collaboration (Boaler, 2013). Boaler (2016) noted that student achievement on recent PISA assessments was highest for those holding a growth mindset, and that teachers in high achieving countries such as China treated mistakes as valuable learning experiences, rather than penalizing mistakes as is typical in America. Additional cultural differences in mindset include the attitude of students towards school and homework, as Chinese and Japanese students in Stevenson, Lee, and Stigler's (1986) study recognized the high value placed on education in their cultures. These same students responded more positively to homework than did their peers in the U.S. (Stevenson et al., 1986). Parents from Chinese and Japanese cultures tended to value effort and expect more from their children and schools, as opposed to American parents who valued natural ability and were more satisfied with the performance of their children and schools (Stevenson et al., 1986). The growth and fixed mindsets revealed by these findings may help to explain American students' lower achievement.

Even the effects of math anxiety, an increasingly important aspect of attitudes and mindsets toward math, can affect students differently across cultures. Cognitive math anxiety affected the performance of Taiwanese students to a lesser degree than that of American and Chinese students, possibly since Taiwanese students tended to direct this anxiety into a desire to perform well, while other students tended to let this anxiety manifest in self-detrimental attitudes (Ho et al., 2000). While American and Taiwanese students demonstrated a significant gender gap in affective math anxiety, Chinese students did not, possibly due to cultural ramifications of China's policy of one child per family, leading parents to have high expectations for their children regardless of gender (Ho et al., 2000). In general, students from Asian cultures tended to have higher levels of math anxiety and lower self-efficacy in math despite higher academic achievement, whereas students from Finland and other similar cultures maintained high achievement with much lower levels of math anxiety (Lee, 2009). These are only a few examples of ways that mindset theory may interact with cultural differences in math education, and the findings on math anxiety also highlight another dimension of culture outside of national boundaries: the culture of females in math-related fields.

The Culture of Females in STEM

In addition to cultural differences between citizens of different nations, the culture of females differs in subtle ways from that of males, specifically in the areas of math and its related subjects, commonly referred to as STEM. Both in the U.S. (Noonan, 2017) and abroad (OECD, 2015), the gender gap in STEM fields remains significant, impacting female representation in math-oriented fields at the university level and beyond. Interestingly, in many mathematically high-achieving countries, gender gaps in student scores are significantly smaller than in other nations, implying that differences in culture and overarching school systems may play a role in the matter (OECD, 2015). While there are many theories as to why women are not equally represented in STEM fields, some of the reasons may be related to mindsets, as teachers of various nations may adopt and foster certain mindsets stemming from their cultural treatment of education. Just as Dweck's growth and fixed mindsets were shown to impact student achievement in mathematics, these same principles may have a different effect on women than on men (Dweck, 2014). Good, Rattan, and Dweck (2012) showed that among female university students, the more women perceived a fixed mindset in their academic environment the less they felt they belonged. This sense of belonging, particularly the type that is based in effort rather than achievement or social factors (Campbell, 2011), was shown to be a determining factor in students' intent to continue in math courses and in how they perceived their own abilities in math (Good et al., 2012). Conversely, the more women perceived a growth mindset in their academic environment, the more they felt like they belonged. As a result, these women were more likely to continue in math and were more resilient even in the presence of harmful stereotypes (Good et al., 2012). At the faculty level, Leslie, Cimpian, Meyer, and Freeland (2015) observed that in fields where the belief that innate talent was required, females were less represented in academic departments. The same mindsets that are embedded in various cultures, whether national or academic, have a powerful influence on women and their representation in the STEM field.

Along a similar vein, stigma and stereotypes may also be partially responsible for gender gaps in education and STEM. Though as early as 1978 it has been shown that there is no significant difference in math ability between males and females (Fennema & Sherman, 1978), the stereotype remains that males are inherently better at math than females. Although gaps in mathematics achievement around the globe remain, these may be attributed not to differences in ability, but to differences in mindset and self-efficacy (Else-Quest et al., 2010). Results of a recent PISA assessment have shown that low self-efficacy in math is strongly correlated with lower achievement (OECD, 2015). In all but a handful of countries, girls reported higher levels of math anxiety, which caused a difference in scores equivalent to almost one year of schooling (OECD, 2015; Else-Quest et al., 2010). In addition, the average girl in China scored higher than the average boy in any other nation, and China, Finland, and Singapore were among the nations where no math gender gap was observed (OECD, 2015).

Clearly, females are capable of performing just as well as males, but outside factors (many attributable to mindset and culture) can still cause underachievement. Fennema and Sherman (1978) showed that when harmful stereotypes were present, girls' confidence tended to be lower, as did their math achievement. Ellis, Fosdick, and Rasmussen (2016) showed in a university calculus course that when students' confidence dropped, females (whose confidence levels were lower from the start) were more likely to drop out of the course (and therefore the STEM pipeline). Beilock et al. (2010) also demonstrated that girls in elementary school who believed harmful stereotypes about women and math performed worse than boys in general and girls who did not believe the same stereotypes. The same study also correlated female elementary teachers' math anxiety with increased endorsement of harmful stereotypes, resulting in higher levels of math anxiety and lower math scores for their female students, but only females who accepted gender stereotypes (Beilock et al., 2010). The female culture is worth noting and studying in the STEM field, as understanding why the gender gap exists can help educators and world leaders to close it.

Conclusion

In exploring the classroom procedures and assessment norms of a few key countries, a pattern emerges which points to mindset as an important factor in how teachers and students around the world approach mathematics. Countries such as Japan, China, Singapore, and Finland have outpaced the U.S. in international assessments, and the methods which teachers from these nations use as well as the overarching attitudes prevalent in these countries may be partially responsible for this. Dweck's mindset theory and Boaler's application of it to the field of math education reveal the impact of mindsets and cultural attitudes on students' academic performance. Notably, in several countries where citizens generally valued math more and where educational strategies resembled those described by Boaler, math achievement was higher than in America. These same principles may also be underlying causes in the gender gap between males and females in math and other STEM areas. Though the learning process is too multifaceted to be explained by any one variable, it may still benefit parents, students, and educators alike to study factors which are correlated with greater math achievement and confidence in order to give every student the opportunity to enjoy and succeed in math, regardless of nationality, race, or gender.

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