On the submultiplicative constant of an algebra

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Abstract

If \( A \) is a finite dimensional commutative associative real algebra and \( \| \cdot \| \) denotes a given norm on \( A \) then it was shown in [1] that there exists a constant \( m \) such that \( \| xy \| \leq m \| x \| \| y \| \). However, the \( m \) given in [1] is not sharp. We prove there exists an optimal smallest choice for \( m \) which we denote as \( m_A \). Furthermore, we prove if \( A \) is the real group algebra of the cyclic group of order \( n \) given the usual Euclidean norm then \( m_A = \sqrt{n} \). We also find the submultiplicative constant for the complicated numbers \( A = \{ x_0 + \cdots + x_{n-1}i^{n-1} \mid x_i \in \mathbb{R}, i^n = -1 \} \) with the Euclidean 2-norm. Additional results concerning numbers generated from nilpotent elements are also discussed. Applications of our theorem to the study of power series in \( A \)-variables are briefly discussed.

References


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