TRADITIONAL VS. PROJECT-BASED LEARNING:

THE EFFECTS ON STUDENT PERFORMANCE AND MOTIVATION

IN HONORS LEVEL MATHEMATICS COURSES

by

Sunletha Carter

Liberty University

A Dissertation Presented in Partial Fulfillment

Of the Requirement for the Degree

Doctor of Education

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ABSTRACT

Since the charge by the National Council for Teachers of Mathematics (NCTM) to move away from the traditional learning (TL) method of instruction to more learner-controlled techniques, project-based learning (PBL) has been on the rise. This quasi-experimental nonequivalent control group design study compared the academic achievement of 122 eleventh and twelfth grade students who were instructed using PBL techniques, with a control group of their counterparts who were instructed using TL techniques, in honors level mathematics courses. Pretest and Posttest data collected from both groups of students were statistically analyzed using independent t-tests, and Analysis of Covariance (ANCOVA), respectively, since the independent t-tests on the pretest data yielded significant differences between the two groups. The ANCOVA results showed a statistically significant difference between the mean performance of the PBL group and the mean performance of the TL group. Similar results were obtained when the mean performance of male students only, as well as the performance of female students only, were compared between the two groups, thereby revealing that PBL instructional method helped to improve student achievement in honors level mathematics courses. In addition, students' motivation data obtained from the Instructional Materials Motivation Survey (IMMS) were analyzed using Multivariate Analysis of Variance (MANOVA) and Analysis of Variance (ANOVA). The results showed that there was no statistically significant difference in the motivation of students instructed using PBL method and students instructed using TL method. The study further discussed implications for teaching and learning, as well as made recommendations for future research.

Keywords: Constructivists, traditional learning, project-based learning (PBL), ARCS model (attention, relevance, confidence, and satisfaction), motivation, motivational theories.

Dedication

I dedicate this study to my precious grandsons, Jamison, Hudson, Maison, and Carson. You may be too young, at this time, to understand the concept of hard work and perseverance, but soon you too will come to realize that with God all things are possible. Always Be Confident and reach for the stars.

I dedicate this study, also, to the parents of the *sons*, my children Rondell, Candace, and their spouses, for their love, respect, encouragement, and unwavering support. Indeed, “children are a gift from God” (Psalm 127:3, NLT).

Acknowledgements

I have always believed that “I can do all things through Christ who strengthens me” (Philippians 4:13, NAS); and so I thank God for blessing me with his love and grace, and for giving me the strength to complete this monumental task.

I would also like to express thanks to my team of committee members, Dr. Jerry Westfall (chair), Dr. Steve McDonald, and Dr. Fred Decovsky. I am truly grateful for your patience, constructive criticism, support, and encouragement.

In addition, to my professors from whose expertise I benefitted immensely, I wish to express special thanks to my professional colleagues for their invaluable contribution by way of data collection and the advice they offered to me throughout my studies.

Finally, I wish to express thanks to my family and friends, for their unconditional love, unwavering support, and continual encouragement. God bless you all.

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Analysis of Covariance (ANCOVA)

Analysis of Variance (ANOVA)

Attention Deficit/Hyperactivity Disorder (ADHD)

Attention, Relevance, Confidence, and Satisfaction (ARCS)

Central Limit Theorem (CLT)

Common Core State Standards Initiative (CCSSI)

Every Student Success Act (ESSA)

Instructional Materials Motivation Survey (IMMS)

Mean (M)

Multiple Intelligence (MI)

Multivariate Analysis of Variance (MANOVA)

National Council of Teachers of Mathematics (NCTM)

No Child Left Behind (NCLB)

Partnership for Assessment of Readiness for College and Career (PARCC)

Project-based Learning (PBL)

Quartile 1, Quartile 3 (Q_1, Q_3)

Race to the Top (RTT)

Science, Technology, Engineering, and Mathematics (STEM)

Specific, Measurement, Attainable, Relevant, and Time-bound (SMART)

Standard Deviation (SD)

Traditional Learning (TL)

Variance Inflation Factor (VIF)

Zone of Proximal Development (ZPD)

CHAPTER ONE: INTRODUCTION

Education in America continues to experience a myriad of reforms with renewed emphasis on mathematics. These reforms took center stage after the government's proclamation of an educational crisis when Russia launched *Sputnik*, the first space satellite, in 1957. However, it was the 1983 publishing of *The Nation at Risk* by the National Commission on Excellence in Education, however, that catapulted America's reform. The *Nation at Risk* report contained alarming statistics regarding the low academic performance of American students, relative to their international counterparts (U.S. Department of Education, 1983). Twenty-five years later, *A Nation Accountable* reported that America was at an even greater risk due to rising demands in the global economy, and in order to keep pace with these demands, American students needed to be educated at higher levels. (U.S. Department of Education, 2008). Petric (2011) cited Schmidt, Houang & Wolfe (1999) who warned that "the American students lack not only logical and analytical skills needed to figure out complex mathematical problems but also the knowledge of the fundamental theoretical concepts and how to discover those concepts through investigations" (p. 2).

The 'New Math' period (1950s to 1970s) which heralded in the introduction of calculus courses at high school levels, was conceived by many to be suitable only for college capable students (Ellis & Berry, 2005). In the 1990s, mathematics education policies in American public schools experienced much criticism due to the introduction of new books, diminished content, and a dearth of basic skills (Klein, 2003), but it was the National Council of Teachers of Mathematics (NCTM) report entitled "An Agenda for Action" (NCTM, 1980) that challenged mathematics educators to re-evaluate the significance of calculus in the mathematics program. The recommendation of this action report included, but was not limited to, more emphasis on problem solving, and more options in mathematics so as to accommodate the diverse needs of the students. Algebra, and Precalculus, at the average and accelerated levels not only provide the foundation for the higher level calculus courses but also provide students with the opportunity to engage in real-life application and integration problems.

This quantitative study compares instructional strategies for their effectiveness in promoting students' understanding and motivation in foundational honors level mathematics classes. This is in keeping with the NCTM goal about understanding the value of mathematics through exposure to a variety of learning experiences. All such experiences should be representative of the cultural, historical, and scientific evolution of mathematics (NCTM, 1989).

In this research presentation, the background information on educational reforms is followed by the problem and purpose statements, and a brief statement that described the significance of the study. The research questions, hypotheses, description of the variables, assumptions, results and limitations of the study are also defined. A summary of the research is included.

Background

Mathematics education in America's public schools has been heavily influenced by both the traditionalist and constructivists theories. It is important to understand, though, that whatever the viewpoint, the aim has always been the same; to enhance students achievement in mathematics. Views differ, however, on how to achieve this aim. Traditionalists contend that mathematics is best learned in a drill and practice manner with memorization and recall of mathematical facts (Ellis & Berry, 2005; Davison & Mitchell, 2008), whereas, the constructivist's viewpoint is one in which students are actively involved in the learning process. Ellis & Berry (2005), upon examining the research on the past century mathematics reform movements, discovered that past practices still led “inherently to inequitable practices and outcomes” (p. 14).

The 1965 Elementary and Secondary Education Act (ESEA), which was enacted by then President Johnson was a reform measure to address the inequitable practices and thus, fight the ‘war on poverty’ (The Social Welfare History Project, 2014). In 2002, the No Child Left Behind Act (NCLB), a reauthorization of the ESEA, was signed into law by then president Bush (Burke, 2012). The NCLB Act is a federal law that was enacted to provide additional financial assistance for the education of poor children. The standards and accountability measures of NCLB allowed for the accumulation and analysis of data regarding students, teachers and general school performance, particularly in the secondary schools. Ten years after this mandate, there was evidence that NCLB was not functioning as expected. Teachers and other educators claimed that “NCLB has set impossible high standards and has narrowed curricula, forcing teachers to teach to the test” (Webley, 2012). Still concerned about the students’ performance, especially in the core subject areas of mathematics and reading, current president Obama’s most significant educational initiative in 2009, Race to the Top (RTT), was an attempt to kick start yet another education reform to improve teaching and learning in school. According to President Obama, “America will not succeed in the 21st century unless we do a far better job of educating our sons and daughters” (Boser, 2012).

The grants obtained by states, which are voluntary participants of the RTT initiative, focus on reforms in four main areas that would enable students to be successful and competitive in the global economy. These include (a) designing and implementing standards and assessments for college and success, (b) increasing efficiency in the acquisition of data so as to provide adequate information for improving instruction, (c) recruiting, evaluating, and retaining highly effective teachers in an effort to help close the achievement gap and get more students into college and a career, and (d) improving performance in the lowest achieving schools (The White House, 2009). In 2015, President Obama reauthorized the 50 year old ESEA as the Every Student Success Act which will become fully operational in 2017-2018 academic year. This bipartisan bill will 'fix' the NCLB while addressing the current initiatives of the RTT.

The newly adopted common core state standards in mathematics is a response to this call as it aims to make the mathematics curriculum "more focused and coherent in order to improve mathematics achievement in this country" (Common Core State Standards Initiative (CCSSI), 2011, p. 3). The standards for practice require that mathematically proficient students "make sense of problems and persevere in solving them" (CCSSI, 2011, p.6). The instructional method that will be researched and analyzed in this study, allows for the demonstration of this principle.

The most recent initiative targets assessment and is known as the Partnership for Assessment of Readiness for College and Career (PARCC). This is a partnership of local educators and states to develop assessment that would serve as tools for enhancing teaching and learning (PARCC, 2015). The intent of this partnership is for teachers and educators to customize their instructions in mathematics and English language arts literacy, in order to meet students' needs as they relate to the rigorous Common Core State Standards introduced in 2010 and 2011. PARCC assessments are the resulting high quality assessments which are aligned to these standards.

Problem Statement

"Mathematics classes such as pre-calculus serve as 'gatekeepers' to educational and vocational opportunities in STEM disciplines" (Olson, Cooper, & Lougheed, 2011, p. 747). These mathematics classes lay the foundation for careers in the field of science, technology, engineering, and mathematics (STEM). The problem is that many students, who enroll in the honors level courses with the intention of going on to the higher level calculus classes, fail to do so. Among the reasons cited for this change is the decline in mathematics achievement of many of these students who must now challenge themselves even more in the accelerated classes. Marcoulides, Gottfried, Gottfried, & Oliver (2008), who analyzed achievement and motivation in at-risk, intermediate, and gifted students, theorized that "declining mathematics achievement was associated with a decline in intrinsic mathematics motivation. Other researchers such as Olson et al., 2011), who conducted a longitudinal study to investigate teaching approaches to mathematics, hypothesized that the problem was more one of the type of instructional method used. The active learning which is inherent in student-centered learning methods such as PBL, is antithetical to the traditional teacher-centered learning method of instruction. Zydney, Diehl, Grincewicz. Jones, & Hasselbring (2010) cited other research which indicated that increasing learner control was better suited for students with high prior knowledge, such as those enrolled in advanced placement and honors level mathematics courses.

In addition to motivation, there are conflicting reports as to whether or not differential cognitive ability exists between male and female students. For example, Ding, Song, and Richardson (2006) found that, while both male and female students demonstrated the same growth trend in mathematics performance over time, females reported significantly higher grade point averages than males. They also found that, as the content became increasingly more difficult and challenging, the gender difference favored females, in terms of grade. This is contradictory to other studies including those conducted by Kyriakides & Antoniou (2009) and by Liu & Wilson (2009). In both of these studies it was found that boys outperformed girls on the most difficult test items. This study seeks to test the afore-mentioned claims and provide answers to the problem stated.

Purpose Statement

The purpose of this proposed quantitative study was to test the theory of learning and motivation that relates instructional methods to students' achievement and motivation, controlling for difference in academic ability among students enrolled in the honors level mathematics courses at a New Jersey suburban high school. A non-equivalent control-group design was employed to investigate this relationship. The independent variable, instructional method, was defined as *traditional learning*, a teacher-centered technique in which mastery of knowledge through drill and practice is emphasized, and *project-based learning*, which is a student-centered technique requiring students to complete tasks based on challenging questions and problems that engage their interests. The dependent variables were student achievement, and student motivation.

Significance of the Study

The literature base on the effectiveness of the student-centered project-based learning (PBL), and instructional method is limited to the elementary levels and other subject areas, particularly, the sciences and web-based courses. Research which compares PBL instructional method with Traditional Learning (TL) at the high school level, particularly at the honors or advanced level, is non-existent. Consequently, this study will contribute to the body of knowledge for high school level and beyond.

The researcher, who is also a mathematics teacher, has encountered many instances in which students have expressed the need for more engaging activities instead of the overused traditional direct methods of instruction. This is especially critical for schools that operate an extended time schedule such as an eighty to ninety minutes block. This study will further provide teachers and other educators with empirical evidence to support modifications in the quality, type and duration of mathematics instruction that high school students find motivating and engaging.

Research Questions and Hypotheses

The following research questions and accompanying hypotheses will guide the researcher throughout this project.

RQ1: Is there a difference in the mean performance scores of students instructed using project-based learning techniques and students instructed using traditional learning techniques, in an honors level mathematics class?

$H_0$1a: There will be no statistically significant difference in the mean performance score of students in an honors level mathematics class instructed using project-based learning techniques and students instructed using traditional learning techniques, as shown by the expert-validated end-of-unit posttest.

$H_0$1b:There will be no statistically significant difference between the mean performance of male students in an honors level mathematics class instructed using project-based learning techniques and male students instructed using traditional learning techniques, as shown by the expert-validated end-of-unit posttest.

$H_0$1c: There will be no statistically significant difference between the mean performance of female students in an honors level mathematics class instructed using project-based learning techniques and female students instructed using traditional learning techniques, as shown by the expert-validated end-of-unit posttest.

$H_0$1d: There will be no statistically significant difference between the mean performance of male students instructed using project-based learning techniques and female students instructed using project-based learning techniques in an honors level mathematics class, as shown by the expert-validated end-of-unit posttest.

RQ 2: Is there a difference in the motivation of students instructed using project-based learning techniques and the motivation of students instructed using traditional learning techniques, in an honors level mathematics class?

$H_0$2a: There will be no statistically significant difference in the motivation scores between students instructed using project-based learning techniques and the motivation scores of students instructed using traditional learning techniques in an honors level mathematics class, as indicated by the Instructional Materials Motivation Survey (IMMS).

$H_0$2b: There will be no statistically significant difference in the mean attention subscale scores between students in an honors level mathematics class instructed using project-based learning techniques and the mean attention scores of students instructed using traditional learning techniques, as indicated by the IMMS.

$H_0$2c: There will be no statistically significant difference in the mean relevance subscale scores between students in an honors level mathematics class instructed using project-based learning techniques and the mean relevance scores of students instructed using traditional learning techniques, as indicated by the IMMS.

$H_0$2d: There will be no statistically significant difference in the mean confidence subscale scores between students in an honors level mathematics class instructed using project-based learning techniques and the mean confidence scores of students instructed using traditional learning techniques, as indicated by the IMMS.

$H_0$2e: There will be no statistically significant difference in the mean satisfaction subscale scores between students in an honors level mathematics class instructed using project-based learning techniques and the mean satisfaction scores of students instructed using traditional learning techniques, as indicated by the IMMS.

Identification of Variables

For this study, the independent variable is identified as the instructional method. The two factors of this variable are project-based, and traditional. Project-based learning, also called project-based pedagogy, refers to instruction that emphasizes students' selection and investigation of authentic or real-world problems (Colley, 2008). Traditional method, on the other hand, consists of mostly note-taking, drills and memorization. Students are expected to be cognitively active, while being physically inactive, except when taking notes (Hagnighi, Vakil, & Wetiba, 2006). There are two factors of the dependent variables, namely, student's mathematics achievement, and motivation. Student mathematics achievement measure will be obtained through comparison between the mean posttest scores. Motivation will be measured using the IMMS, a 36-item Likert scale instrument based on Keller's Attention, Relevance, Confidence, Satisfaction (ARCS) model of motivational design.

Definitions of Key Terms

Following are key term that is included in this research study.

At-Risk Students: Students who are 'at-risk' of failing for one or more reasons, such as, socio-economic status, learning disability, negative peer pressure.

Behavioral Learning Theory: A theory that focuses on change of behavior based on whether or not the outcomes and consequences are pleasant or unpleasant (Slavin, 2003).

Common Core State Standards: Standards for mathematics (and English language arts) that stress conceptual understanding of key ideas, sequentially. These standards replace individual states standards. (CCSSI, 2011).

Constructivist Theory: "The idea that learners must individually discover and transform complex information if they are to make it their own" (Slavin, 2003, p.257).

External Locus of Control: External factors such as luck, task difficulty, and other people's action being offered as the cause of success or failure (Slavin, 2003).

Extrinsic Motivation: Use of praise, rewards and incentives to encourage students to perform at their best (Slavin, 2003).

Intrinsic Motivation: Motivation that comes from within an individual and driven by his or her own interest, as well as the enjoyment derived from learning or completing the task.

Project-based learning: Learning that takes place when students select and work cooperatively to investigate authentic or real world problems as projects, under the supervision of the teacher (Bell, 2010; Colley, 2008).

Race to the Top: An education initiative which requires states to compete for financial grants in order that they use such grants for greater educational innovations (Boser, 2012).

Scaffolding: The process of providing complete support at the early stages of learning then diminishing the support to allow the learner to take on increasing responsibility for his or her learning (Slavin, 2003).

Student Success Act: A bill introduced by Representative John Kline (R-MN), to rewrite Title I of NCLB. (Burke, 2012).

Traditional Instruction: Instruction that is teacher-centered and involves passive involvement of students in whatever is said by the teacher or is printed in a textbook. It involves whole-group learning and students remain physically inactive, except for note taking. (Ornstein & Hunkins, 2004).

CHAPTER TWO: REVIEW OF THE LITERATURE

Introduction

There are many non-traditional instructional methods that are available to teachers of mathematics yet the predictable traditional method of memorization and regurgitating of facts prevail in American schools (Nesmith, 2008). Many teachers, who are experts in their disciplines at the secondary level, continue to provide instructions through examples, demonstrations and guided practice, with the aid of the textbooks and/or workbooks, to the class as a whole. Although revisions are constantly being made to the mathematics curriculum so as to meet the scientific and technical challenges, teaching and learning by the traditional lecture-based method persists.

The contemporary approach to teaching mathematics involves a more progressive and constructivist focus whereby students are actively involved in constructing their own understanding based on their prior knowledge and their experiences (Nesmith, 2008; Ornstein & Hunkins, 2004; Schifter & Fosnot, 1993) affirmed that "once one accepts that the learner must herself [sic] actively explore mathematical concepts in order to build the necessary structures of understanding, it follows that teaching mathematics must be reconceived as the provision of meaningful problems designed to encourage and facilitate the constructive process". (Nesmith, 2008, p. 9).

The National Council of Teachers of Mathematics (NCTM), like many other organizations concerned about pedagogy and mathematics reform, implemented standards based on constructivist principles. These standard-based principles include specific requirements for teachers to provide a variety of learning experiences that represent the cultural, historical, and scientific evolution of mathematics, while simultaneously ensuring academic rigor and accountability (NCTM, 1989; Newell & Van Ryzin, 2007). These principles are identified and outlined by McKinney & Frazier (2008), as follows.

NCTM (2000) vision of the *equity* principle is based on the premise that, despite the learning differences among students, and differences in their socio-economic background, age or gender, all students are capable of learning mathematics. By holding students to high expectation and giving them the same high level of support, they are able to improve their mathematics competencies.

The *curriculum* principle places emphasis on the development of a well-designed curriculum across all levels of instruction. This means that the curriculum should be "coherent and integrate and link mathematical ideas" (McKinney & Frazier, 2008, p. 205).

The *teaching* principle is based on the understanding on how students learn, what they already know and need to learn. This is crucial for teacher in order for them to provide the necessary support to the learners. Teachers must acknowledge and prepare their lessons to appeal to the different learning styles of students.

The *learning* principle suggests that students' understanding is critical to their learning as is "actively building new knowledge from experience and prior knowledge" (McKinney & Frazier, 2008, p. 202).

The *assessment* principle is based on the preparation of formative and summative assessments which support the mathematical content studied. "Authentic assessment is best achieved through teaching: interactions between both teacher and student, and student and student; and observing students in meaningful tasks" (Ishii, 2004, p.3). The feedback from multiple assessment practices must inform both teacher and student.

The *technology* principle is a critical component in mathematics education as it serves to enhance students' content learning and critical thinking skills, by making it possible for students to produce, collect, and analyze data they need for problem-solving.

According to Brooks and Brooks (1999), the typical constructivist classroom is one in which teachers pose problems of emerging relevance to students. This is a guiding principle of constructivist pedagogy. Often when students enter a classroom they do so with varying interests and they are not all interested in the list of academic goals already set out in the curriculum as necessary and relevant for their academic development. It is therefore the responsibility of the teacher to invite relevance through mediation. Mediation takes the form of posing questions that pique the interest of the learners, and allow them to appeal to their own thoughts and conceptions, as a point of departure from which to engage their interest and be self-motivated.

Since students are more engaged when problems and ideas are posed "holistically rather than in separate, isolated parts" (Brooks & Brooks, 1999, p. 46), the curriculum for the constructivist classrooms must be structured around main ideas and primary concepts. This is unlike the curriculum of the traditional classroom in which more emphasis is placed on acquiring basis skills in parts, focusing on each part separately. Many students encounter problems as they try to put the parts back together, in order to understand the big concepts. If however, students are presented with the concepts as a whole, they can find their own meanings as they look at the parts within the whole. They do this by comparing, contrasting, analyzing, and synthesizing.

Of much importance in the constructivist classroom also, is the importance of teachers valuing the points of view of students whom they continually challenge to make learning meaningful. "Students' points of view are windows into their reasoning" (Brooks & Brooks, 1999, p. 60). Hence, teachers must demonstrate their willingness to listen to students by providing them adequate opportunities to do so. Also critical in the constructivist classroom, and based on the seminal works of Jean Piaget, is the teachers' adaptation of the curriculum to address students' suppositions. This refers to the matching of the cognitive demands of the curriculum with the cognitive abilities of the students.

In this chapter, the theories of some prominent constructivist, as well as motivation theories, are examined for their relevance to the learner centered instructional technique of project-based learning that is emerging in the mathematics classrooms. A review of the literature pertaining to the traditional lecture-based method and project-based learning is followed by a discussion of motivational techniques and the applicability of Keller's ARCS (attention, relevance, confidence, and satisfaction) model of motivation to students' achievement in mathematics.

Theoretical Framework

The process of project-based learning has its roots in constructivist theory, which posits learners takes an active part in generating meaning and constructing their own understanding (Garmston & Wellman, 1994; Ishii, 2003; Ornstein & Hunkins, 2004). This is antithetical to traditional lecture-based learning in which the learner adopts the role of passive observer, absorbing mounds of information from an external source, the teacher. According to this theory, learners are in a continual process of checking for new information by using old rules which they revise as they (rules) become obsolete (Slavin, 2003). There are several pedagogical implications of the constructivist theory, as discussed by Ernest (1996). These include

- The teachers' sensitivity to the learners' previous conceptions, previous knowledge, and their informal knowledge, a critical step in the construction of new knowledge.

- Students use of cognitive conflict techniques, which afford them the practice of thinking through their problems, and developing their own meanings as they work through rectifying and finding solutions.
- Students assume responsibilities for their learning through self-regulation.
- Students make use of multiple representations to facilitate the connection between previous learning and new constructs.
- Students develop more awareness of the importance and value of the goals that they are working to achieve. This may differ from the teachers' goal of acquiring knowledge and skills.
- Students develop understanding and appreciation of the importance of the social context of learning. While some learning may occur in the formal setting of the classroom, some learning may occur in a more social setting, which may be outside of the classroom.

To fully appreciate the relevance of constructivism in the teaching and learning of mathematics, it is necessary to explore the cognitive theories of the earliest and most notable constructivists.

Constructivist Theorists

Jean Piaget (1896-1980). He was considered to be the original constructivist, and contended that the cognitive development of children from birth to adolescence occur in four stages each marked by "the emergence of new intellectual abilities" (Slavin, 2003, p.32). He (Slavin) listed these stages as (a) sensorimotor (birth to 2 years), (b) preoperational (2 to 7 years), (c) concrete operational (7 to 11 years), and (d) formal operational (11 years to adulthood). He (Piaget) also believed that children are constantly constructing new knowledge of the world around them, and as such, they must be encouraged to learn through self discovery (Edwards, Hopgood, Rosenberg, & Rush, 2000; Siegler & Ellis, 1996). However, his principle that development precedes learning has been largely criticized, not only his research method, but also for his belief that the developmental stages he posited were fixed. Researchers disputed this latter tenet, suggesting instead that children can be taught many Piagetian tasks at earlier developmental stages, and citing environmental factors, including teaching methods, to be the influential factors (Cherry, 2012; Slavin, 2003).

Bhattacharya & Han (2001) identified *adaptation* and *organization* as two of Piaget's major principles which guide intellectual growth and biological development, facilitating the self discovery process. With regards to the adaptation principle, they explained that, the learner must assimilate information into an existing cognitive structure or schema which he/she modifies to accommodate new experiences. As the learner makes the necessary adjustment of his/her mental structures and accommodations, the process of organization follows as they attempt to make sense of new information

The educational implications of Piaget's theory to student's mathematics classes center on providing them with an environment and resources that are developmentally appropriate (Berk 2001, Slavin, 2003). A summary of the main teaching implication for teaching include the following.

1. Understanding the thought process in the mathematical computation instead of focusing only on correct answers. This technique involves planning to include the kinds of problems and projects that will foster critical thinking and "build the child's level of cognitive functioning" (Edwards et al., 2000).

2. Encouraging students to be active participants in their own learning by engaging them in discovery type activities such as is the case with project-based learning.

3. Being mindful of students' cognitive abilities and refraining from incurring practices aimed at speeding up their development.

4. Ensuring that class activities appeal to the individual differences of students. Small group activities for groups, formed by ability instead of age, tend to address these differences appropriately.

Lev Vygotsky (1896-1934). There is a great deal in common between Piaget's cognitive constructivism and the social constructivist theory of Lev Vygotsky. Chen (n.d.), in her overview of social constructivism, discussed the emphasis that Vygotsky placed on the importance of culture and the social context for cognitive development. She noted that unlike Piaget, Vygotsky placed more emphasis on the role of the teacher and other adults. For Vygotsky, the most important role of the teacher in mediating the learning process is that of creating a social environment (i.e., classroom culture) through the use of appropriate material tools, symbolic systems, and other human beings, while functioning as a participant-observer and assisting students in the construction of new knowledge (Eun, 2010). This begins with the careful diagnostic assessment of the learner's category system in order to facilitate the development of appropriate learning experiences in the proper sequence, in the curriculum (Marsigit, 2009).

According to Slavin (2003), Vygotsky also believed that learning could only take place when children were working on tasks within their *Zone of Proximal Development* (ZPD). The ZPD is the gap between what the child is able to do independently and that which he/she can do with the guidance and assistance of a competent adult or peer(s). Hence, according to this view "effective instruction neither exceeds the learner's current level of understanding nor underestimates the learner's ability to learn independent of the teacher" (Parkay, Hess, & Anctil, 2010, p. 193). This is a concept widely embraced by many teachers and researchers. Hence, the tasks are the ones that require input from the teacher in order for students to attain full mastery. The teacher is able to provide assistance in the ZPD by being sensitive to the needs of the learners and by being willing to assist them in creating the opportunity to do so. Critical also in Vygotsky's ZPD is the role of assessment in the process of social interaction.

According to Newman et al. (1989, in Marsigit, 2009, the ZPD provides an alternative to traditional standardized test. Whereas traditional tests focus on rightness and wrongness, on how well students do, and how poorly they fail, for Vygotsky it is more about giving the students tasks, observing them as they perform the tasks, and giving them guidance and help as needed for successful completion. In the latter case, the student is not assessed alone but instead, in a social context with the teacher and their peers thereby measuring the degree of progress. This type of performance assessment further provides students with the opportunity for developing their ability to assess their own progress. This is a zone, as described by Vygotsky because as learners become more and more involved in self assessment they will be able to take over the task from their teachers, complementing the other skills that have acquired.

In planning a developmentally appropriate mathematics curriculum, it is incumbent on teachers to recognize and plan activities that would appeal not only to those who are capable of working on their own, but also to those who learn best with help from other competent peers and adults. The cooperative learning arrangements that are part of project-based learning is an effective way to meet the learning needs of the diverse group of students commonly found in most classrooms. Vygotsky insisted that cooperation and collaborative instructional practices are paramount to the development of higher psychological functioning in students (Eun, 2010). Byrnes (2001) also suggested that students who worked together are usually more engaged in their learning. They learn how to analyze and solve problems, thereby acquiring a deeper understanding of the problem situation and the overall concept. This is achieved with the guidance, encouragement and advice of the teacher, who diminishes support so that the student can take on more of the responsibility for his/her own learning. This *scaffolding*, as it is termed by Vygotsky, is a principle that supports the use of projects and other real-life authentic tasks. The teacher is more than an instructor in this case. Rather, he or she works along with the learners, helping in the process of constructing meaning. At this time, too, much emphasis is placed on communication.

The constructivist approach to teaching mathematics places emphasis on presenting students with real-life problem situations for them to formulate their own inquiry questions, explore multiple interpretation and multiple intelligences through collaboration. According to Ernest (1996), the essentials of teaching for this purpose have several components, which include discussion between and among students, between students and their teachers, projects and investigative type problems for critical skills development, problem solving, and creativity. Ernest (1996) further suggested that because of the varied, active, socially engaged and self-regulatory nature of learning, the resources of teaching should (a) be of a wide variety to facilitate many different approaches to teaching, (b) include authentic materials such as newspaper, official statistics, and so on, so as to make learning more socially relevant, and (c) be relevant and easily accessible.

There is a vast array of technological resources now available to facilitate students' collaborative and research efforts, and giving them some responsibility for their own learning. No longer is the teacher the only person with the keys to learning. Computers, word processors, multimedia programs, and compact discs databases are some of the tools of technology that students can use to demonstrate their learning.

Howard Gardner (1943-). For teachers to provide meaningful learning activities and problem situations in an environment that is conducive to students' learning needs, they must take students' interests into account. Howard Gardner proposed eight multiple intelligences (MI) inherent in each student, although some may be more dominant than others. With collaborative learning being part of the project-based learning environment, so does the opportunity for interdisciplinary learning (Lamie, 2000). It is also an opportunity for students to showcase their stronger intelligence(s). Lamie (2000) further suggested that stronger intelligences can be used to nurture weaker ones. Although Gardner's MI theory was developed from brain research its principles are constructivist. It posits that the teacher must not only create a suitable working environment, but must also develop the kind of purposeful problem-related projects and other activities that engage students higher order thinking skills of analysis, synthesis, and application. According to Gardner (2000), "all human possess at least eight forms of intelligences which call linguistic, logical-mathematical (the two favored in school), musical, spatial, bodily-kinesthetic, naturalist, interpersonal, and intrapersonal" (Gardner, 2000, p.32). A brief explanation of the intelligences, as discussed by Armstrong (2009), Heiss (2002), and Smith (2002, 2008) follows.

Linguistic Intelligence (word smart). Being able to learn and master the use of language to express oneself. Students with this type of intelligence find pleasure in activities such as poetry, reading and storytelling. They are also likely to excel in subject areas that require reading and research. In the collaborative group setting of the mathematics class, these are the students who would most likely opt to take on the written report of performance tasks.

Logical-Mathematical Intelligence (number smart). Also called “scientific thinking”, is described by Gardner as the ability to engage in inductive and deductive reasoning, and to express thoughts in logical and sequential steps. Students who possess this type of ‘number smarts’ will derive much satisfaction from logic problems and brainteasers and more likely to lead an organized life (Heiss, 2002). They are usually the ones who excel in the traditional mathematics classroom because “the kind of processes used in the service of logical-mathematical intelligence, include categorization, classification, inference, generalization, calculations, and hypothesis testing” (Armstrong, 2009, p.6).

Musical Intelligence (music smart). This intelligence concerns appreciation and involvement in all kinds of music. For learners manifesting this kind of intelligence, it is usually a way of self-expression. Students with this dominant intelligence think in notes, beats, rhythms, and would likely tap to beats or prefer the sound of music as they engage in learning. They would necessarily create their own music if there is none in the background. They are the ones who most likely incorporate appropriate and meaningful sounds into their projects (Lamie, 2000). Gardner felt that this MI was parallel to the linguistic or verbal intelligence since many students are able to create impressive pieces of writing and convert them to music.

Bodily-Kinesthetic Intelligence (body smart). This intelligence involves the use of the entire body in the learning process. Learners of this intelligence type may do great in physical education but have difficulty sitting still for a forty minute class. They need to move around, and for this reason they are often mislabeled as ‘hyperactive’. Varying the learning activities to include movement, appeal to these learners. For example, having students measure the dimension of a room in a geometry lesson on calculating surface area is better than supplying them with formulas and expecting them to quietly contemplate computations.

Spatial Intelligence (art smart). Involves the ability of the learner to visualize patterns and use the mental images they form to find solutions to abstract problems. According to Gardner, these learners will benefit from visual aids such as puzzles, pictures, manipulables, and art media. Computer-Assisted Design (CAD) programs and video programs will also appeal to learners with this dominant intelligence.

Naturalist Intelligence (nature smart). This intelligence is manifested through the ability to categorize plants, animals, and inanimate object such as cultural artifacts. Nature smart learners will invariably demonstrate sensitivity to the environment, and as such, would function best in outdoor learning situations such as nature walk, field trips, and video recording of natural events.

Interpersonal Intelligence (people smart). Described by Armstrong (2009) as the ability to discern feelings in others and make distinctions in their moods, intentions, and motivation. Students with this type of intelligence are able to collaborate with others in any learning situation. Not only do they enjoy working with others but they are able to increase their own learning by doing so. They are very influential and will function very effectively as project leaders because of their confidence and influential status.

Intrapersonal Intelligence (self smart). A learner with this dominant intelligence has a heightened sense of self. That is, understanding personal strengths, weaknesses, and limitations. Activities such as the writing of journals, autobiographies, and designing projects, provide ample opportunities for students' reflections, allowing them to develop the capacity to regulate their lives.

In the mathematics classroom, all of these multiple intelligences are usually present, and as such, activities that encourage students to use these intelligences should be included on a regular basis. It is not necessary to employ all intelligences simultaneously but problems and projects may vary over time so that all students are engaged meaningfully by way of their particular multiple intelligence(s).

Jerome Bruner (1915-2016). Bruner was considered to be one of the founding fathers of constructivism. Like Vygotsky, he posited that learning is an active social process during which students construct new ideas based on previous and current knowledge and experiences. This process involves selection and transformation of the information selected. The learner must formulate hypotheses, discover meanings, and make decisions from the information they amass, a process that is facilitated by their own experiences. At this time too, it is incumbent on the teacher to encourage students in discovery type and in Socratic learning.

Bruner's 'enactive' and 'iconic' stages of development can be compared to Piaget's sensorimotor and pre-operational stages, respectively. By the time the learner reaches adolescence and is enrolled in secondary or high school, he/she has attained Bruner's symbolic stage and is therefore ready for interpreting and representing abstract ideas, as well as extrapolating to create new ideas. In his landmark book entitled *The Process of Education* (1960), the following themes emerged (Smith, 2002).

- Teaching and learning structures should focus on more than just the mastery of facts and techniques. More emphasis should be placed on activities that give the learner opportunity to categorize. In his theory, Bruner stressed the importance of categorization in perception, conceptualization, forming new knowledge, and in decision making.
- A child's readiness to learn should be taken into account when planning the curriculum. Bruner believed that by using a spiral curriculum it is possible to present any subject to children, at any stage of development. Such a curriculum allows concepts to be presented

in a way that builds on students' previous knowledge, in a manner commensurate with their cognitive abilities.

- Not enough emphasis is placed on the development of students intuitive and analytical skills. As such, teachers should make every effort to incorporate techniques that would provide students with ample opportunities to develop these underdeveloped skills.
- Students' interest in the material to be learned is a better motive for learning, and should be encouraged over competition for grades.

Bruner later published *The Culture of Education* (1996) in which he made some revision of his thinking. In this publication he expressed greater appreciation for the role of culture in development, stating that "culture shapes the mind . . . it provides us with the toolkit by which we construct not only our words but our very conception of ourselves and our powers" (Bruner, 1996, x). In accordance with this changed viewpoint, he then listed four features of effective instruction, as follows:

- Predisposition to learn: This is influenced by motivational, cultural, and personal experiences. Teachers and parents can be instrumental in this process by engaging children in exploratory type activities, as these would increase learning and general problem solving skills.
- Structure of knowledge: The content should be presented in an ordered and structured so that all learners have a fair chance of understanding. Categorization is important in this process.
- Modes of representation: These should vary to include visual, words, and symbols.
- Effective sequencing: The varying needs the learners can be met by using an instructional technique of moving through contents from easy to difficult.

To ensure learning, Bruner supported the idea of appropriately paced rewards and punishment, for concept reinforcement.

Motivational Theories

Several theories of motivation are applicable and pertinent to understanding the mathematics achievement of students involved in project-based learning. These include, but are not limited to, behavioral theories of motivation, attribution theory, expectancy theory, and goal theory.

Behavioral theories of motivation. Slavin (2003), describes the concept of motivation as being closely linked to the principle of behaviors being more likely to be repeated if they are reinforced, than those that have not been reinforced. This is a simple behavioral learning principle for classroom practice. Traditional instructional method is based on this premise by using rote and repetition. Reinforcement can be positive or negative. Examples of positive reinforcement include rewards, praises, and grades are commonly given to students in schools. A negative reinforcement is any action that releases or excuses someone from an unpleasant or undesirable task or situation. Both of these types of reinforcements can strengthen students' learning behavior. This is essentially the position of behavioral learning theorists such as Albert Bandura (1986) and B.F.Skinner (1953) as cited in Slavin (2003).

According to Bandura, people learn by observing others such as role models. The type of behavior involved in this modeling process are: Paying attention to learn something new; retaining what was acquired while paying attention, with the aid of imagery and language; and being able to reproduce or translate the images in desired behavior (Careersnz, 2012).

Slavin (2003) further identified the following practices for teachers to consider when using reinforcement to elicit appropriate learning behaviors.

1. Make a decision on the type of behaviors for which reinforcement will be needed before they actually occur. This would prevent rewards and praises be given for work that is below students' capabilities.

2. Inform students about what is considered acceptable forms of behavior. When reinforcement is used they should be notified of the reason(s) so that they can use the feedback to evaluate their strengths and weaknesses.

3. Reinforcement should take place as soon as possible after the need arises, for maximum benefit.

Other behavioral scientists, for example Maslow (1954, as cited in Slavin, 2003), focus on motivation as the drive which involves satisfying a hierarchy of basic individual needs, desires for self-actualization, and the perceived necessity for achieving success but avoiding failures (Middleton & Spanias, 1999; Rabideau, 2005; Slavin, 2003). Thus, an individual learner is motivated to satisfy the basic physiological, safety, belonging, love, and self-esteem needs in order to attend to growth needs such as wanting to obtain knowledge, and the need for creativity and self-actualization. In the classroom, the teacher can help to create an environment and a feeling of acceptance, trust, fairness among students, as being a precursor to self-directed learning.

Behavioral theory aptly provides a framework for students' achievement in the study of mathematics at an advanced level. For example, students opting to study a mathematics course at the honors and advanced placement levels will normally do so with the expectation that they will be successful. This perception in itself is motivating but it must be coupled with equally engaging and meaningful tasks in order to sustain the motivational effort and realize the long term benefits. To this end, however, behavioral researchers caution about the overuse of extrinsic incentives to individuals, and recommend instead, group rewards for individual learning. Motivating students in a group allow them to help each other while they seek to challenge themselves individually. While Middleton and Spanias (1999) confirmed that the "expectation of tangible task-contingent rewards tend to weaken the intrinsic desire to learn"(p. 69) it is important to understand that individuals who are intrinsically motivated may seek reward incentives but not as a condition for evoking acceptable study habits and behaviors.

Attribution theory. This theory, which has implications for academic motivation, concerns how individual learners interpret and explain their successes and failures at tasks they undertake. According to Bernard Weiner's (1935-) model of attribution, learners are affected by environmental factors such as their homes and schools, as well as by personal factors such as their prior knowledge and experiences (Anderman & Anderman, 2012). Attribution theory assumes that people will interpret their environment in a way to maintain a positive self-image (Slavin, 2003; Vockell, n.d.). Thus, if learner experiences an unexpected achievement-related outcome then he/she will undertake an attributional search to determine the cause, since their perception of the cause will affect motivation for engaging in similar tasks in the future. The level of the learner's motivation will depend on whether the cause was external such as environmental factors or internal - originating from within. Task difficulty, other people's actions, and luck are identified as external factors whereas one's own ability and effort are the internal factors which are related to attribution theory. This personality trait is referred to as the *Locus of Control*, a concept that was originally developed by social learning theorist Julian Potter (1916 - 1985) in 1954 (Neill, 2006) and is one of three sets of characteristics used for analyzing the explanations for a learners success or failure. The other characteristics refer to the *stability* - whether the cause(s) is the same over time and situation, and the controllability - the degree to which the learner can alter the cause of such successes and failures.

Students tend to be successful if they attribute their successes to ability, and fail if they attribute their failures to lack of ability (Middleton & Spanias, 1999). Consequently, it is incumbent on teachers to establish confidence within their students throughout each step of their project tasks and other learning activities. It is not uncommon to find some teachers who unwittingly convey to their students that they are not capable of realizing academic excellence. This happens, for example, when (teachers) place more emphasis on competitive grading. Emphasizing effort on the cause of success and failure, and rewarding students' efforts rather than their ability is more likely to provide greater motivation (Slavin, 2003). It is also necessary for teachers to impress upon students that other factors such as the difficulty of the task, as well as inadequate effort, are credible reasons for failure. Understanding that occasional failure is acceptable in learning is necessary for building students' self confidence and competence, especially in the mathematics classroom. Thus, when students encounter problems in their self-directed learning environment they can be confident to recognize and accept these problems as phases in the learning process rather than as failures.

Another factor that has yielded mixed results in explaining the causes of students' successes and failures is that of gender. In a study conducted by Meyer and Fennema (1998), they found significant gender differences in students' perception of the reasons for success and failure (Middleton & Spanias, 1999) . That is "girls tend *not* to attribute their successes to ability but *do* attribute their failures to lack of ability (p. 70). Their findings for boys were not so pronounced. It is not uncommon to find teachers who attribute successes and failures in their mathematics classrooms in such a manner that they perpetuate these differences in motivation between the genders.

Expectancy theory. Unlike other theories of motivation (Maslow, Herzberg), expectancy theory as posited by Vroom (1974), makes reference to the conscious behavior made by individuals, based on factors that would yield maximum benefit (Your Coach BVBA, 2016). Hence, the effort a student puts into his work is linked to his or her motivation. The student would make greater effort based on his/her expectation of a greater reward. In other words, "children's achievement performance, persistence, and choice of achievement tasks are most directly predicted by their expectancies for success on those tasks and the subjective value they attach to success on those tasks (Widfield, 1994, p. 50). Thus, if a student believes that with more effort he/she can be placed on the high honor roll, then it is likely that he/she will work hard towards that end. The perceived probability of success is what motivates the student. If on the other hand the chances of being placed on the honor roll is perceived as an unlikely task to accomplish then the motivation to make the effort in non-existent (Slavin, 2003; Widfield & Eccles, 2000).

It is not surprising then that students with high motivation tend to be successful in almost all tasks. The question is whether high achievement motivation leads to success in school, or whether success leads to high achievement. In studies conducted by Widfield, Eccles, & Rodrigues (1998), it was concluded that each contributed to the other.

To discuss the influence of motivation on students' performance, Oliver (1995), summarized the requirements Vroom's expectancy theory as nine Cs, which are as follows.

Challenge – questions the amount of effort the student needs to make to succeed.

Criteria – refers to whether or not the student is able to differentiate between good and bad performance.

Compensation – students will always want to know if they are going to be rewarded for their good work.

Capability – refers to how well the student can perform the task.

Confidence – does the student believe that he/she can perform the task, adequately?

Credibility – the student must believe that the teacher is capable of presenting the information which they must use in order to construct their knowledge.

Consistency – is about the students' assurance that the reward for their performance, good or poor, is consistent over person and time.

Cost – refers to knowing how much effort needs to be put into a task in order to get the desired outcome of performing well.

Communication – this concerns the teacher's ability to communicate to ensure the other eight Cs.

Teachers should have an understanding of this theory in order that they can create the learning environment in which students are actively engaged in their learning, and in which positive social interaction are encouraged so as to enhance motivation, and subsequently, achieve academic success.

Goal theory. This theory makes a distinction between learning (or mastery) goal, and performance goal. While learning goals are set by individuals to improve their knowledge and competency, performance goals are set to gain favorable judgments and prove one's competency. (Ferlazzo, 2010; Vockell, n.d.). Dr. Edwin Locke, who carried out a study on goal setting and motivation in the 1960s, concluded that setting a goal and working towards it is the motivation that is needed to achieve this goal and improve one's performance (Mindtools, 2012). In his published works three decades later, he identified five principles for goal setting, which are summarized as follows:

Clarity - a goal should be clear, specific, measurable, and have a definite time set for completion. Hence, before students become engaged in their self-directed projects and other cooperative learning activities they should be provided with clear instructions on what is expected and how it will be measured.

Challenge - the level of the challenge of a goal is of utmost importance. If the task is too easy it will be viewed as unimportant. At the same time, goals must be realistic. There should be a balance between a challenging and a realistic goal. Rewarding students for their efforts on more difficult tasks will boost their enthusiasm and motivate them to do their best.

Commitment - understanding and agreement of set goals leads to a commitment for learners to achieve same. Locke (Mindtools, 2012) felt that there is a positive association between difficulty and goal commitment. That is, the harder the goal, the more commitment is required. Involving students in goal setting for their learning tasks not only lend credibility to the goals but ensures more commitment from them.

Feedback - providing consistent feedback is just as important as selecting the right goals. It provides the learner with the opportunity to seek clarifications, make adjustments and engage in self-assessments. The teacher should ensure that time is built in for formal and informal feedback to each student or team of students involve in a performance task. This feedback must include a discussion on goal performance.

Task Complexity - In an effort to set goals that are challenging care must be taken to ensure that the tasks assigned are not overwhelming. Locke (Mindtools, 2012) again suggested that enough time be given to meet and improve performance, as well as for individuals to practice and learn what is expected and required for success. Including students in the setting of their goals also provide them with the opportunity to agree on a suitable time frame for meeting the required expectations.

Each of the five aforementioned principles of Locke's goal setting theory incorporates at least one aspect of SMART (Specific, Measurable, Attainable, Relevant, and Timebound) goals characteristics, an acronym that is familiar in human resource management. These characteristics are just as valid and useful for motivating students to increase their academic achievement.

Students enrolled in an honors level mathematics course usually possess superior learning goals as they are willing to work hard, take risks, if necessary, to achieve mastery of the tasks to which they are assigned. Because these students are motivated by their own desire to learn and improve, they are more likely to view mistakes as just another learning opportunity. According to the expectancy value theory, another mastery or goal theory, past experiences of success, complexity of the task, encouragement of others, feedback, and the skill of the individual student are some of the factors that will affect the students' expectations of success (Slavin, 2003). This is not so for the performance oriented students who play it safe by sticking with only what is familiar to them and take precautions to avoid mistakes (Svinicki, 2012). There is a third orientation referred to as 'work avoidance', in which the individual thrive on exhilarating 'nice' behavior at the expense of academic work. Middleton and Spanias, (1999) stated that "work avoidance is often developed as a coping method for preserving feelings of adequacy by eliminating any threatening or difficult activities so that a legitimate negative evaluation of one's ability cannot be made by others (p. 73). In the mathematics classroom one or the other of these orientations may exist and structuring the classroom to incorporate project-based learning in a cooperative environment, and setting SMART goals may discourage work avoidance just as it would discourage ego goals among students.

Traditional Based Instruction

Traditional based instruction has its foundation in behaviorist theory which "emphasizes conditioning behavior and altering of the environment to elicit selected responses from the learner" (Ornstein & Hunkins, 2004, p.101). This method of instruction is characterized as a system that encourages students to "bank" knowledge from teachers who are authorities on their fields, and who control the learning environment. Brazilian educator Paulo Freire (1921-1997), who was an advocate for liberation pedagogy, denounced this method in which he called the teacher the 'talking head' supplying every piece of information, and giving little or no opportunity for students to develop their critical thinking skills (Gutek, 2005). The curriculum is organized with the aim of mastery of the content. Hence, the emphasis is on knowledge, information, uniformity of classroom experiences, instructional situations, and to a lesser degree, on conceptual understanding. Behaviorist Edward Thorndike (1873-1949) and his colleagues contended that mathematics is best learned in a drill and practice manner, and viewed mathematics as a 'hierarchy of mental habits or connections' (Thorndike, 1923, p.52, as cited in Ellis & Berry, 2005). This was also the view of Dewey (1859-1952) who contended that:

> Since the subject matter as well as standards of proper conduct are handed down from the past, the attitude of pupils must, upon the whole, be one of docility, receptivity, and obedience. Books, especially textbooks, are the chief representatives of the lore and wisdom of the past, while teachers are the organs through which pupils are brought into

> effective connection with the material. Teachers are the agents through which knowledge and skills are communicated and rules of conduct enforced. (Parkay et al., 2010, p. 40)

In discussing the behaviorist approach to teaching and learning, Ornstein and Hunkins (2004) listed four traditional instructional models, among them the *Good and Brophy model*, a systematic approach that was originally developed for mathematics instruction, and which is still prevalent in many mathematics classrooms today. The steps involved in this models are (a) *review* of the skills and concepts from previous work, (b) *development* during which the teacher presents examples, provides explanations and demonstrations, (c) *assessment of comprehension* through questioning and controlled or guided practice, (d) *seatwork*, also referred to as independent practice for the reinforcement of the concepts taught, (e) *accountability* by the students as the teacher methodically check their work, (f) *homework* for additional practice, and (vii) *special review* or assessment. Reteaching is necessary if the desired level of mastery is not attained.

The afore-mentioned mathematical model describes what Ellis & Berry (2005) termed "thinking within the traditional paradigm" (p.11), a practice with which many teachers of mathematics can identify. Enquiring on the pedagogical and instructional practice of mathematics teachers, McKinney & Frazier (2008) confirmed that approximately 78% of middle school principals reported that mathematics was taught in their school with mostly traditional methods. This method relies on intellectual learning and excludes experiential learning. For students who do not possess a logical-mathematical intelligence or learning style, this method can make learning difficult. According to the Association of Supervision and Curriculum Development, "the process of human learning consists of (a) confronting new information or experiences, and (b) personally discovering the meaning of the experiences (Bondelli, 2012).

However, the traditional system does not allow for personal discovery of meaning. Instead, more and more emphasis is placed on the acquisition of information for mastery, and more recently, mastery on standardized tests.

Project-Based Learning

This technique is one of many alternative methods for students to learn mathematics, as a result of reforms in education, particularly the shift to the standards-based principles of the NCTM. The idea of project-based learning (PBL) is to use real world problems to capture students' interests, as they acquire and apply their new knowledge in problem-solving contexts (David, 2008). According to Harada, Kirio, and Yamamoto (2008), "Project-based learning frames an approach to learning that actively engages students in deeper levels of comprehension and interpretation about what and how they study" (p. 14). This type of learning puts students at the center of the learning process and help them in developing the thinking and collaboration skills they will need to be successful in school and when they enter the workplace. They will be involved from the initial development of their research question to working collaboratively, under the supervision of the teacher, to completing their project. Students may even be allowed to formulate their own groups. This learning technique, which is quite flexible, provides for adequate interaction and cooperation among the students, between the students and the teacher, and between the students and specialists or resource personnel (Bell, 2010). Bell stressed further that teachers must not view this technique as a supplementary activity but rather as the basis of the curriculum development. In the EdVision model of secondary schools, Newell and Van Ryzin (2007) discussed a school culture in which a PBL program is embraced for the opportunity it provides " for students to develop autonomy, belongingness, and a learning goal orientation" (p. 4). In these secondary schools, students' choice and self sufficiency are reinforced while academic rigor and accountability are ensured (Newell & Van Ryzin, 2007). In one Iowa elementary school and one Boston middle school that implemented PBL, the results were similar, with the latter also realizing significant increase in reading achievement.

In a more recent study conducted by Ada and Kurtulus (2012) in Turkish primary schools PBL was used to get students to use non-Euclidean geometry to better understand Euclidean geometry. At the end of the project students commented that they only found the project enjoyable but it attracted their attention and strengthen their interpersonal relationships and improved their academic outcomes. Their also recommended that PBL be included mathematics teacher education programs so as to prepare new teachers for incorporating these techniques at all levels of mathematics classes

Using PBL is not without its challenges. One such challenge is that teachers experience enormous pressure to cover curriculum content (David, 2008, Marx, Blumenfeld, Krajcik, & Soloway, 1999). They are sometimes inadequate length of class periods giving rise to the issue of breadth versus depth of study. Working with technology, a growing emphasis in education, also presents another challenge. Sometimes it is necessary to collaborate with others in remote areas and this makes additional demands on teacher. Hence, teachers involved in PBL must fully understand the concepts and procedures involved so that they can serve as effective problem-solving role-models for their students

Concerned that many teachers were engaging students in 'meaning-lite' assignments they billed as projects, Larmer and Mergendoller (2010) stressed that well-designed and well-implemented projects had to be perceived as personally meaningful to students, as well as fulfill an educational purpose, for them to serve their intended purpose. They further listed the following seven essentials for meaningful PBL to take place.

1. A need to know - activated by introducing the project with an event or activity which engages students' interest and initiates a question or discussion.

2. A driving question - for capturing the essence of the project while lending a sense of purpose and some challenge to students. "A project without a driving question is like an essay without a thesis" (Larmer & Mergendoller, 2010, p. 36).

3. Student voice and choice - giving students autonomy makes the projects more meaningful to them. They can select topics within a general driving question and chose their own methods for developing and presenting their projects.

4. Twenty-first century skills - collaborating, role-playing, team-building, critical thinking, self-assessing, and using technology are all skills are developed in PBL and which will prove useful in the workplace and in life.

5. Inquiry and innovation - conducting real inquiry using books, websites and other resources allow students to view project work as more meaningful. "With real inquiry comes innovation - a new answer to a driving question, a new product or an individually generated solution to a problem" (Larmer & Mergendoller, 2010, p. 36-37).

6. Feedback and revision - providing regular feedback emphasizes that high quality performance and end product are requirements of meaningful learning.

7. A publicly presented product - presenting the final product to an audience other than the teacher make students more caring about the quality of their production. They may even choose to emulate professionals in the field of their study.

Motivation in Mathematics

Based on research done by Middleton and Spanias (1999), the following findings, are evident, or developing regarding students' motivation in the area of mathematics.

(1) Students' motivation is heavily influenced by their perception of success, which they formulate as they evaluate the demands of the tasks at hand. Consequently, teachers should assign mathematics problems and projects that are challenging enough to encourage interest and involvement but not too difficult that students become bored and exacerbated by feelings of helplessness and failure.

(2) Students tend to be motivated to do mathematics in their early years and are heavily influenced by teachers' actions and attitudes. In his study Reeve (2009) found that many teachers adopted a controlling motivating style towards students. This could be attributed to factors such as having dual responsibilities and accountability roles, social and cultural values and expectations, reacting to students' passivity, or they may be holding on to traditional educational philosophies which cause them to equate control with structure in the classroom.

According to Flink, Boggiano, & Barrett (1990) this belief is erroneous as research showed that students who were subjected to controlling strategies performed significantly worse than those who were allowed the autonomy and support that define learner-centered techniques such as project-based learning (Reeve, 2009).

(3) "Providing opportunities for students to develop intrinsic motivation in mathematics is generally superior to providing extrinsic incentives for achievement" (Middleton & Spanias,1999, p. 81). Although extrinsic incentives may not necessarily undermine intrinsic motivation it is still necessary for teachers to provide activities that stimulate students' interest so as to increase their intrinsic motivation.

(4) There still exist inequities in how students have been taught to view mathematics. In their research, Asante, Al-Mahrooqi, and Abrar-ul-Hassan (2012) made reference to other studies in which gender differences in student-teacher interaction adversely affected girls' interests in

mathematics. Compounding this situation is the stereotypical behavior of teachers who sometimes unwittingly reinforce helplessness and low confidence in female students (Asante et. al., 2012; Middleton & Spanias, 1999). To alleviate this problem, teachers should encourage positive relationship with students so that they all feel confident in participating in all learning activities.

In their study on extracting factors for student motivation in studying mathematics, Teoh, Koo, & Singh (2010) were able to identify seven factors but it was (a) significance of learning, (b) confidence, (c) interest, and (d) effort that were found to be the most statistically significant. This was in accordance with the reviews of Middleton and Spanias (1999). Other minor factors included family support, independence or self-directed, and teacher attention. Teoh et al. recommended that all these factors be given appropriate consideration when planning instruction for mathematics.

Keller's ARCS Model of Motivation

The well-known Keller's ARCS model of motivational design is applicable for motivating students to success in mathematics. Originating in 1979 by John Keller, this method for improving motivational appeal of instructional materials is grounded in expectancy value theory (Shellnut, 1996). According to this theory people's motivation to complete a task is dependent upon their positive expectation to be successful, and upon the extent to which their needs are satisfied. Shellnut further identified three distinct features of the ARCS model. First, it has a connection with motivational theory, using four essential strategy components discussed below. Second, a complete set of strategies are included to enhance the appeal of the instruction, and third, it utilizes a systematic design. Keller (1987a) emphasizes that "ARCS is a problem-solving empirical approach to applying motivation to instructional design" (Shellnut, 1998, p.5).

Small (1997) is one of many researchers who have discussed the four essential strategy components of this model. These are as follows:

Attention strategy. The instructional material, in this case the projects, must attract or hold the students' attention. Posing a problem about a topic of students' interest is one way to gain such attention, and interest can be maintained if the elements of instructions are varied (Keller, 1999). Other methods that may be used to grab the learner's attention include, visual stimuli, a story, or actively involving students. Making statements which are contrary to student's previous knowledge, statements of inquiry, and interjecting some mild humor are all ways to get students' attention.

Relevance strategy. By taking into account students interests, motives, and needs the answer to why should I care about this content is revealed to students. "Relevance results from connecting the instruction to important goals of the learners, their past interests, and their learning styles" (Keller, 1999, p.14). Hence, a good strategy is to involve students in by getting them to provide examples from their own life experiences, that are relevant to their project. By examining the present worth and future usefulness of a project, students will be more readily accepting of the task or be more motivated to learn (Clark, 2004).

Confidence strategy. Students should be provided with a map that guides them successfully towards successful completion of the project. Keller (1999) suggests that this is achieved by helping them (students) to establish a positive expectancy of success. Because many students tend to have low self-confidence about their own ability, clear objectives, expectations, and examples are needed in order to help build confidence. Along the learning journey they would understand the process to be one that builds upon itself, realizing small number of successes along the way. As success gets more challenging they will begin to understand that there is a positive correlation between the energy they expend and the learning gain from the experience. They will also need regular feedback from the teacher who serves as the facilitator.

Satisfaction strategy. To sustain students' motivation, satisfaction is required. This refers to "a positive feeling about one's accomplishments and learning experiences" (Keller, 1999, p.14). Teachers encourage this feeling by giving students recognition for their efforts, and by providing appropriate extrinsic rewards such as grades, certificates, privileges, and other tokens to celebrate their achievements upon completion and mastery of the skill set. Keller (1999) cautions teachers about over rewarding simple behaviors and using too many extrinsic rewards. He advises that satisfaction is closely related to confidence since students who have built up their confidence are more likely to experience satisfaction as they complete challenging tasks. By allowing students to overtly reflect on their experiences while completing the task, and providing opportunities for them to apply what they have learned, also provide students with a sense of satisfaction. Keller again cautions teachers about the importance of manifesting a sense of equity and fairness in achieving student satisfaction.

In a non-equivalent control group quasi-experiment with junior high school, Calahan and Smith (1990) conducted a study in psychology using traditional lecture format and instruction based on Keller's ARCS model. At the end of the unit, posttest results revealed significantly higher scores for the group that was instructed with the ARCS model. This result in not unlike those of similar studies done with elementary and middle school students. as well as with college students involved in web-based courses. Several other claims have also been made in support of this model for enhancing existing instructional materials and for developing new materials (Shellnut, 1998). It has also been used successfully across systems and across cultures.

Summary

Research on the type of instruction that is best suited for teaching mathematics subjects continue to permeate the literature base. Both traditional and the contemporary constructivist methods have their merits. While modern science, technology, engineering, and mathematics demand some sequencing and structure, as posited by the traditionalist camp, they also require student to be able to think outside of the box, constructing their own knowledge so as to adequately meet these demands.

Education reform and the shift to standard-based principles in the mathematics subjects have their foundations in constructivism, which gave rise to a school culture of autonomy, flexibility, and two-way interaction between students and teachers, without compromising accountability and the academic rigor of the course (Bell, 2010; NCTM, 1989; Newell & Van Ryzin, 2007). The success of these reform efforts is contingent upon the teacher’s ability to motivate students. Factors which were found to affect students’ motivation included students’ perception about the relevance of the subject matter, teachers’ actions and attitudes, which included stereotypical behavior towards the genders (Middleton & Spanias, 1999). These factors will be addressed through the use of Keller’s ARCS model of motivational design.

While there is limited research on the use of project-based learning technique being used in the advanced and accelerated mathematics classes, there are even fewer on the use of ARCS model at these levels. In addition, no studies have been found in which these techniques and models have been studied simultaneously. This existing gap in the literature will be addressed through the use of a non-equivalent control group quasi-experimental research, which is described in the following methodology chapter.

CHAPTER THREE: METHODOLOGY

Overview

The approach to teaching a high school mathematics course at the honors level is one of the challenges faced by teachers who must prepare students for the challenges of becoming the scientists, technicians, engineers, and mathematicians of the future. To do this effectively they are charged by the National Council of Teachers of Mathematics (NCTM) to move away from the traditional instructional method of memorization and the prevalence of worksheets to involving students in investigations of "authentic problems" (Blumfeld, Soloway, Marx, Krajcik, Guzdial & Palincsar, 1991).

This chapter describes the design and methodology that were used to test the hypothesis that students instructed using project-based learning (PBL) method in an honors level mathematics course perform better and are more motivated than those instructed using traditional learning (TL) methods. It also contains a description of the participants and the setting for the study. These are followed by an explanation of the data gathering method and the instrumentation employed. The chapter culminates with a description of the data analysis procedure. Following the design of are the research questions and the accompanying null hypotheses, which guide the research.

Design of the Study

For this study, a nonequivalent control group design was preferred over the nonequivalent group posttest only. This widely used quasi-experimental design is similar to a true pretest-posttest experimental design but without the critical feature of random allocation of subjects to the treatment and control groups. The researcher was not able to randomly assign each student to either the treatment or control group, as they were already assigned to their classes by the school personnel. However, random assignment was used to determine which class will be instructed using PBL techniques, and which class will be instructed using TL technique.

Without random allocation of subjects, it is difficult to determine whether the groups were equivalent before the study begins. Ary, Jacobs, & Sorensen (2010) points to the existence of an inherent initial selection bias that can cause serious threat to the internal validity of this design. Gall, Gall & Borg (2007) described the internal validity of an experiment as “the extent to which extraneous variables have been controlled by the research, so that any observed effect can be attributed solely to the treatment variable” (p.383). They further suggested that, if properly carried out, this quasi-experimental design can effectively control for eight threats to internal validity, with some sacrifice to the external validity or generalizability of the findings. Much later, Cook & Campbell (1979) expanded the list to include four more extraneous variables (Gall et al., 2007).

In addition to administering a pretest for checking on the equivalence of the groups before the experiment begins, and for eliminating initial differences among subjects, the researcher chose to include only groups of students who were similar in age and ability. This was done so as to reduce the effects of the aforementioned extraneous variables on the results of the experiment.

Research Questions and Hypotheses

RQ1: Is there a difference in the mean performance scores of students instructed using project-based learning techniques and students instructed using traditional learning techniques, in an honors level mathematics class?

$H_0$1a: There will be no statistically significant difference in the mean performance score of students in an honors level mathematics class instructed using project-based learning techniques and students instructed using traditional learning techniques, as shown by the expert-validated end-of-unit posttest.

$H_0$1b: There will be no statistically significant difference between the mean performance of male students in an honors level mathematics class instructed using project-based learning techniques and male students instructed using traditional learning techniques, as shown by the expert-validated end-of-unit posttest.

$H_0$1c: There will be no statistically significant difference between the mean performance of female students in an honors level mathematics class instructed using project-based learning techniques and female students instructed using traditional learning techniques, as shown by the expert-validated end-of-unit posttest.

$H_0$1d: There will be no statistically significant difference between the mean performance of male students instructed using project-based learning techniques and female students instructed using project-based learning techniques in an honors level mathematics class, as shown by the expert-validated end-of-unit posttest.

RQ2: Is there a difference in the motivation of students instructed using project-based learning techniques and the motivation of students instructed using traditional learning techniques, in an honors level mathematics class?

$H_0$2a: There will be no statistically significant difference in the motivation scores between students instructed using project-based learning techniques and the motivation scores of students instructed using traditional learning techniques in an honors level mathematics class, as indicated by the Instructional Materials Motivational Survey (IMMS).

H_02b: There will be no statistically significant difference in the mean attention subscale scores between students in an honors level mathematics class instructed using project-based learning techniques and the mean attention scores of students instructed using traditional learning techniques, as indicated by the IMMS.

H_02c: There will be no statistically significant difference in the mean relevance subscale scores between students in an honors level mathematics class instructed using project-based learning techniques and the mean relevance scores of students instructed using traditional learning techniques, as indicated by the IMMS.

H_02d: There will be no statistically significant difference in the mean confidence subscale scores between students in an honors level mathematics class instructed using project-based learning techniques and the mean confidence scores of students instructed using traditional learning techniques, as indicated by the IMMS.

H_02e: There will be no statistically significant difference in the mean satisfaction subscale scores between students in an honors level mathematics class instructed using project-based learning techniques and the mean satisfaction scores of students instructed using traditional learning techniques, as indicated by the IMMS.

Participants

The participants in this study consisted of 11th and 12th grade students ranging in ages from 15 to 17 years, who are enrolled in honors level Algebra II/Trigonometry, and Precalculus courses, in a large New Jersey suburban high school. The courses were taught by a total of six teachers who used one of two teaching methods. Students who were selected to be in the experimental treatment group were instructed by teachers using project-based lesson plans, while students in the control group were instructed by the traditional teacher-centered method. All participating teachers are certified to teach mathematics at junior and senior high schools in the state of New Jersey, and have a minimum of five years of experience teaching mathematics in high schools. They all provided instruction in the same unit of study on polynomial functions, during the first semester of the 2014-2015 academic year, as described in the New Jersey Core Curriculum Content Standards.

Project-based lesson plan. The project-based algebra II lesson plan was entitled "this pool is too cool", and it was used with permission from the contributing teacher on the West Virginia Department of education *Teach 21* project-based lesson path (West Virginia Department of Education, 2009). The lesson plan contained a suggested timeline for completing each section of the unit. In addition, it contained a detailed list of the prerequisite knowledge and skills that students should possess, as well as the knowledge and skills they will acquired through their self-discovery. Teaching strategies and evidence of success in achieving the identified learning targets, were also included with each plan. The researcher provided the participating teachers with the lesson plans and other supporting materials, which included the pretest, posttest, and the IMMS, which the participants completed at the culmination of the unit of study.

Project-based classes. In the project-based classes, teachers allowed students to form their own groups of 3 or 4 students, with roles defined as stated in the West Virginia Department of education *Teach 21* project-based lesson plan (2009). These roles were as follows.

- The *project manager* who was responsible for keeping all records of the team progress, and for ensuring that each team member was fully utilized. He/she also assisted the design engineer and the research engineer in completing other tasks relating to the project.

- The *design engineer* conceptualized and examined ideas that were related to the project, as well as, conducted test and experiments to decide on the most efficient problem solving strategy. He/she provided assistance to the project manager with scheduling, organization, and providing quality reports from the preliminary tests and experiments.
- The *research engineer* assisted the project manager with the research phase of the project. He/she used mathematical concepts to help the design engineer in developing applications that were easy to test.

The project-based cooperative learning sessions allowed students to engage in investigative type learning, which required them to use higher-order critical thinking skills (David, 2008). They were also able to apply the 21st century information, communication, personal, and workplace skills, as described in the lesson plans. During each PBL work session, students engaged in discussions that were guided by a checklist of requirements for answering the driving question. At this time, the teachers monitored the discussions within each group, listening and facilitating the discussions by asking questions that encouraged critical thinking. In the final 10 minutes of each session, the project manager gave a synopsis of the group accomplishment, as well as, that which needed to be completed.

Setting

The setting for this study was a northern suburban New Jersey senior high school. This is a four-year comprehensive public high school in a school district that with a total of seven schools for students from kindergarten to the 12th grade. The high school was fully accredited since 1935, by the Middle States Association of Colleges and School Commission on Secondary Schools.

According to the 2012-2013 public school report data (National Center for Education Statistics, 2014), the school population comprised of approximately 1279 students, 114.40 total teachers (Full Time Equivalent) with a faculty mobility rate of 2.1, a student/faculty ratio of 9.6, and an average class size of 20.5. The race/ethnicity composition is shown in Table 3.1.

Table 3.1

Race/Ethnicity Composition

Race/Ethnicity	%
Black, non-Hispanic	46.3
Hispanic	28.1
White, non-Hispanic	12.7
Asian/Pacific Islander*	11.0
American Indian/Alaska Native	0.3
Two or more Races	1.6

* Combined Asian and Native Hawaiian/Pacific Islander categories

The 2009 - 2010 report card (State of New Jersey Department of Education, 2010) school demographic and performance indicators are shown in table 3.2.

Table 3.2.

Demographic and Performance Indicators.

Indicator	%
Attendance	92.0
Dropout	0.9
Suspension	10.0
Graduation	97.0
Faculty attendance	95.6
HSPA Math (Language Arts)	24.8 (9.4)[1]

[1]Students classified as partially proficient with a score of 200 or below in the content area of the High School Proficiency Assessment (HSPA), a New Jersey state mandatory standardized test.

In addition to the regular class schedule that takes place in a school day lasting 6 hrs. 40min., students at this school have access to a number of before and /or after school programs, which include peer tutoring. Many of the peer tutors are students who have successfully completed the Pre-calculus honors course. Other special program options include, but are not limited to, peer leadership, peer mediation, and job placement.

Data Gathering Procedure

Upon receiving permission from the University Internal Review Board (IRB, see Appendix A), the signed agreement of the Application for the Use of Human Research Participants (Appendix B), the school's principal, and the district superintendent approval (Appendix C), and permission to use Keller's IMMS instrument (Appendix D), the researcher distributed Parent and Student Information Letters and Consent forms (Appendix E).

Subsequently, the researcher began data collection using the expert validated pretest, which the teachers, who were involved in the research, administered to the experimental treatment and control groups. This pretest was a version of the end-of-unit test, designed to determine whether there was any initial difference in the performance of students instructed using PBL techniques and the performance of students instructed using TL techniques. Another version of this expert-validated end-of-unit test was administered as a posttest (Appendix F) so that the researcher could determine whether there had been any significant difference in the group's performance due to instruction using the project based technique.

At the end of the unit study the researcher distributed Keller's IMMS (Loorbach, N., Peters, O., Karreman, J., & Steehouder, M., 2014) to all teachers involved in the study, for them to administer to their classes. The data obtained from the survey was used to determine whether there was a difference in the motivation of students instructed using PBL techniques and the motivation of students instructed using TL techniques.

To preserve the integrity of this study, care was taken to maintain confidentiality throughout the study, and thereafter. This included the assigning of arbitrary numbers to student participants, so as order to protect their identities. All collected data was secured in locked filing cabinets in school, and at the home of the researcher.

Instrumentation

To measure student performance end-of-unit tests were compiled from the publisher created test bank, which accompany the primary Common Core State Standard aligned textbooks used in the Algebra II/Trigonometry, and Pre-calculus classes. One test was used as a pretest, while another version, similar in design and level of difficulty was administered as a posttest. Since both classes covered the same area of study, they completed the same pretest, and posttest at the end of the study. The test bank items have been in use since the adoption of the texts, all of which have been in use for at least five years prior to this study. Both versions of the test had undergone face and content validity by expert teachers with five to 25 years of experience in the teaching of mathematics. These teachers also assisted in the administration of the pretest and posttest, from which reliability estimates based on the Cronbach's alpha, were calculated at 0.85 and 0.86, respectively.

To measure student motivation, the IMMS was employed. This 36-Likert scale instrument is based on the Keller ARCS model. Small (1997) cited Keller (1983) in describing the four essential strategy components for motivating instructions as follows:

> [A]ttention strategies for arousing and sustaining curiosity and interest;
>
> [R]elevance strategies that link to learners' needs, interests, and motives;
>
> [C]onfidence strategies that help students develop a positive expectation for successful achievement; and
>
> [S]atisfaction strategies that provide extrinsic and intrinsic reinforcement for effort.

According to Keller (2010), the IMMS has internal consistency (reliability) estimates, based on the Cronbach's alpha, is shown in Table 3.3.

Table 3.3

IMMS Reliability Estimates

Scale	Reliability Estimate (Cronbach α)
Attention	0.89
Relevance	0.81
Confidence	0.90
Satisfaction	0.92
Total Scale	0.96

Keller (2010) reported that the validity of the IMMS instrument was established through a study with undergraduate pre-service teachers. In addition, the wide use of this instrument in many studies, and its translation into several other languages are cited as confirmation of its utility and validity (Keller, 2010).

Sampling Procedure

Since the participating students were already assigned to their different classes, some by ability, as is the case with students in honors and other advanced level courses, it was not possible to employ random allocation of students to the two experimental groups. Ary et al. (2010) suggested that "in a typical school situation, schedules cannot be disrupted to accommodate a research study" (p. 316). Hence, the method of sampling could best be described as convenience sampling, a non-probability sampling technique, which has an inherent selection bias that poses a threat to the internal validity of this design (Gall et al., 2007). By employing the pretest, however, the researcher was able to "check on the equivalence of the groups on the dependent variable, before the experiment began" (Ary et al., 2010, p. 317). Although it was not possible to use complete random allocation to groups, the researcher was able to randomly determine which class will be given the treatment of project-based instruction. The group that was instructed using the traditional lecture method served as the control group. The sampling frame was the list of students enrolled at the target New Jersey high school.

Data Analysis Procedure

For this quasi-experimental research design, the presumed cause or independent variables are the instructional methods - traditional and project-based. The presumed effect or dependent variables are (a) the difference in students' achievement, as determined by expert validated end-of-unit tests, and (b) students motivation, as measured by the IMMS instrument.

The first step in the analysis was the computation of the preliminary descriptive statistics of the pretest data from both groups. The achievement measures consisted of the pretest means and standard deviations, for both the treatment and control groups. At this time too, the researcher examined the overall pattern of the data, including the test for outliers, gaps, and deviation from the overall pattern.

The next step in the analysis was to "test the statistical significance of the observed differences in the mean scores of the treatment and control group" (Gall et al., 2007, p. 408). As such, the researcher conducted independent samples t-test, using the 0.05 level of significance, on the pretest data, to determine whether the mean pretest scores differ between the two groups. The mean scores for the pretest represented the covariate used to control for students' previous achievement in mathematics. The t-test was appropriate since empirical evidence confirms its robustness (Weaver, 2004). The result showed that there was significant difference between the treatment group mean and the control group mean. Consequently, a one-way ANCOVA was applied to answer research question one. This statistical technique is used "to control for initial differences between groups before a comparison of the within-group variance and between-group variance is made" (Gall et al., 2007, p. 320). Trochim (2006) demonstrated, through the use of hypothetical examples, that this method of analysis is biased due to "the attenuation of the slope that results from pretest measurement error coupled with the initial nonequivalence between the groups". However, Gall et al. (2007) suggested that ANCOVA can reduce the effects of initial group differences which pose a threat to the internal validity of a non-equivalent control-group design, and as such can be used if the assumptions are not violated. If one or more of the conditions for using ANCOVA are not met, then the researcher can employ the alternative procedure of transforming the data values for the explanatory and response variables to provide a better analysis. By transforming the explanatory variable data any non-linearity in the data will be corrected. Similarly, by transforming the response variable data, the presence of heteroscedasticity as well as non-normality of the error terms will be corrected. In addition, the researcher conducted t-tests to determine whether there were differences in the performances of female and male students between groups, and within the experimental PBL group.

To determine if there is significant difference in the motivation of students in the two groups, the researcher conducted a MANOVA, as well as ANOVA, using the means of the data collected from the IMMS. MANOVA, which is a multivariate extension of ANOVA, is a powerful statistical technique for determining group differences on more than one dependent variable (Gall et al., 2007; Montgomery, 2009). ANOVA was used to determine difference between the dependent motivation subscales for each group of students.

Finally, to get a better understanding of how large a mean is or how different the means of the groups are, a measure of effect size using Cohen's d statistics, was computed.

CHAPTER FOUR: RESULTS

Introduction

This chapter contains the results of the statistical analysis of the data collected to test the research on Traditional vs. Project-based learning: The effects on student performance and motivation, in honors level mathematics courses. The analysis was performed using the IBM®®SPSS Version 22 software, and the data included the pretest and posttest scores from female and male participants, in the control (traditional learning [TL]) and treatment (project-based learning [PBL]) groups. The two instructional methods (TL and PBL) are the factors of the independent variable, whereas student mathematics achievement and motivation are the factors of the dependent variable.

The information presented in this chapter is as follows: (a) demographic information for the participants, (b) statistical analysis (t-test and ANCOVA) to answer research question 1, and (c) statistical analysis (MANOVA and ANOVA) to answer research question 2.

Demographic

The 122 participants in this study consisted of 11th and 12th grade students enrolled in honors level mathematics courses in a comprehensive high school in northern New Jersey. Of this total, 58 (47.5%) were in the control group, and 64 (52.5%) were in the treatment group, for both the pretest and posttest. The 68 female students represented 55.7% of the participants while the 54 male students represented 44.3%. The control group had a mean pretest score of 47.8448 (SD = 16.2522) and a posttest score of 56.9828 (SD = 16.7787), which represented a 19% increase in the number of questions answered correctly. The experimental or treatment group had a mean pretest score of 53.2813 (SD = 16.861), and a posttest score of 71.17 (SD = 12.304), which represented a 33.6 % increase in the number of questions answered correctly. The preliminary descriptive data is presented in Table 4.1.

Table 4.1

Descriptive Statistics for the Control and Experimental Groups

	Pretest			Posttest		
Variable	N	M	SD	N	M	SD
Group						
Control	58	47.8448	16.2522	58	56.9828	16.7787
Experimental	64	53.2813	16.861	64	71.17	12.3040
Total	122	50.6967	16.7297	122	64.4262	16.1884
Gender						
Female	68	50.5147	16.3255	68	65.6618	14.5044
Male	54	50.9259	17.3769	54	62.8704	18.1087

Note. N = number of participants, M = mean, SD = standard deviation

A comparison of the pretest and posttest scores for both groups is represented in the bar graph (Fig. 4.1). The control group which was taught using the TL method, showed an increase of 9.138% whereas the treatment group, which was taught using PBL methods, showed an increase of 17.889%. Table 4.2 shows the original means and variability, after adjusting for the pre-scores between the groups.

Table 4.2

Post Scores Unadjusted and Adjusted Group Means and Variability

	Unadjusted			Adjusted	
Variable	N	M	SD	M	SE
PBL	64	71.1719	12.3038	71.060	1.842
TL	58	56.9828	16.7787	57.106	1.937

Note. N = number of participants, M = mean, SD =standard deviation
SE = standard error

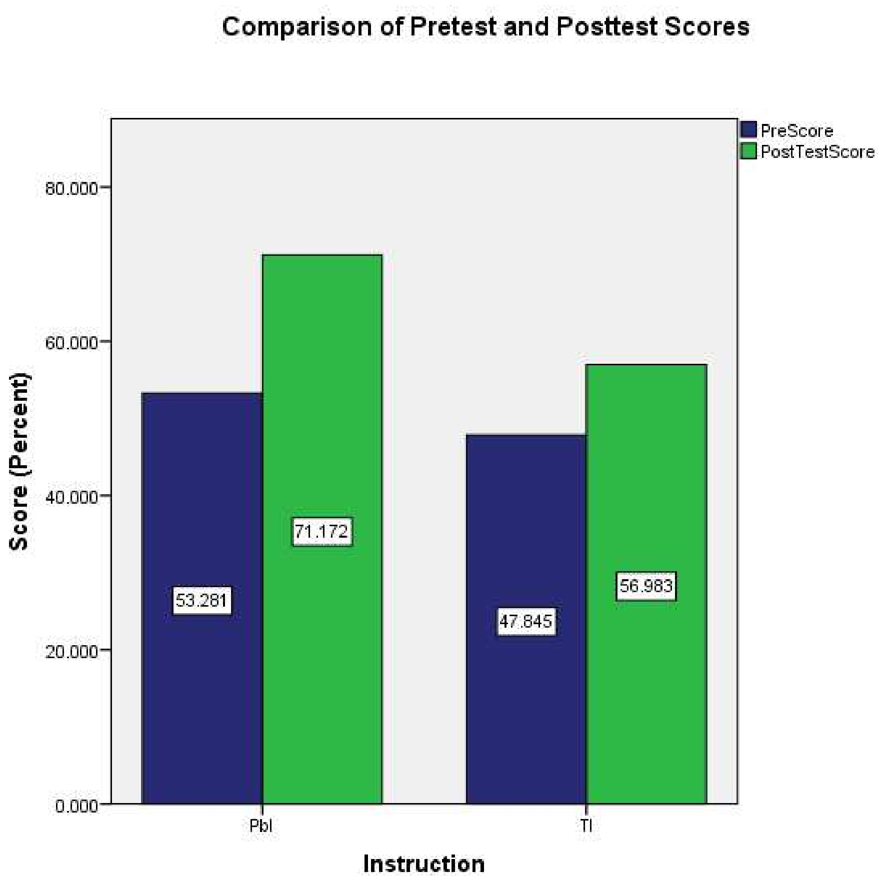

Figure 4.1. Bar Graph of Means of Pretest and Posttest scores for Control and Experimental Groups

In addition, 119 students completed the IMMS at the end of the research. The response rate for this survey was 64/64 (100%) for the treatment group, and 55/58 (94.8%) for the control group. Table 4.3 represents the descriptive statistics for the overall data, as well as for each of the strategy components, which were obtained from the 5-point Likert scale IMMS survey.

Table 4.3

Descriptive Statistics for Motivation

	Control (TL) (n = 55)		Treatment (PBL) (n = 64)	
IMMS Subscale	M	SD	M	SD
Attention	2.9061	0.6168	2.7643	0.7058
Relevance	2.8949	0.7347	2.8212	0.6721
Confidence	3.1939	0.6139	2.8420	0.6911
Satisfaction	2.6879	0.9639	2.6771	0.8067

Research Question One and Null Hypotheses

In this study, research question one asked if there was a difference in the mean performance scores of students instructed using project-based learning methods and the performance scores of students instruct by traditional learning methods. The null hypotheses are: (1a) There will be no statistical significant difference in the mean performance score of students instructed using project-based learning methods and the performance scores of students instruct by traditional learning methods, enrolled in honors level mathematics courses. (1b) There will be no statistical significant difference in the mean performance score of male students instructed using project-based learning methods and the performance scores of male students instructed by traditional learning methods, enrolled in honors level mathematics courses. (1c) There will be no statistical significant difference in the mean performance score of female students instructed using project-based learning methods and the performance scores of female students instruct by traditional learning methods, enrolled in honors level mathematics courses. (1d) There will be no statistical significant difference in the mean performance score of female students instructed using project-based learning methods and the performance scores of male students instruct using project-based methods, enrolled in honors level mathematics courses.

To test the statistical significance of the observed differences in the mean scores of the groups, as outlined in null hypothesis (1a), an examination of the histograms of the pretest data was first conducted (Figure 4.2). It shows that, despite two obvious peaks, the assumption of normality is not violated. This is supported by the result of the Kolmogorov-Smirnov test which yielded a significance value of p = .180 for the control group and p = .052 for the experimental group.

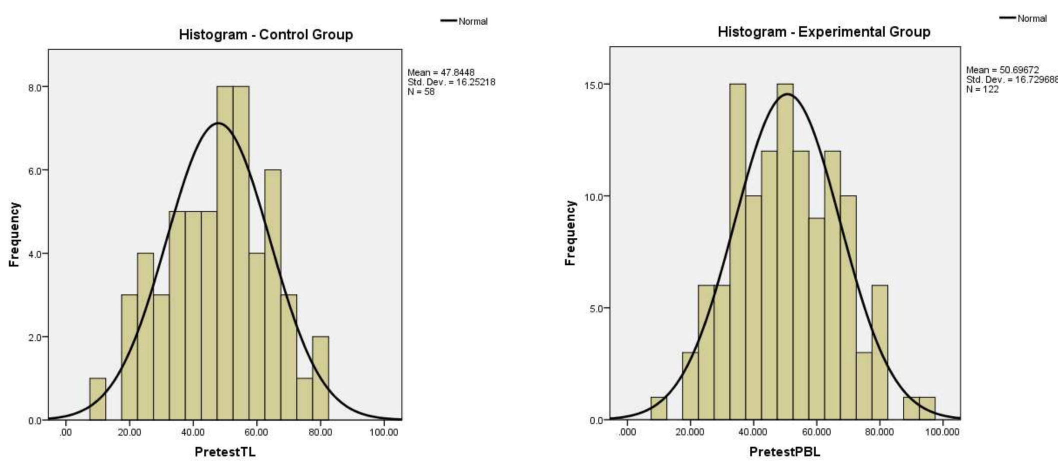

Figure 4.2. Histogram of Pretest

No outliers were found in the two experimental groups after being analyzed by the criterion [Q_1 - 1.5(Q_3 - Q_1), Q_3 + 1.5(Q_3 - Q_1)] (Yates, Moore, & Starnes, 2008). The researcher conducted an independent samples t-test, using the conventional level of significance ($\alpha = 0.05$) on the pretest data. The result was found to be highly significant ($t = 14.1349$, $p < .001$). Significant difference in the pretest scores justified the use of ANCOVA, since it allows for the difference caused by the treatment group to be differentiated more clearly. ANCOVA removes the portion of each subject score that is common to his/her pretest score (Ary et al., 2010). The researcher then conducted the ANCOVA on the posttest data, using the pretest as a covariate, after the following assumptions were testing.

Assumption Testing

Before conducting the ANCOVA procedure on the posttest data, the assumptions were tested to verify the appropriateness of the procedure for the analysis. The assumptions tested were (a) The existence of a linear relationship between the covariate (pretest), and posttest for each type of instruction, (b) homogeneity of the regression slopes, (c) normality, (d) homoscedasticity, and (e) homogeneity of variance.

(1) The linearity between the covariate (pretest) and the response variable (posttest) was confirmed by a visual inspection of a scatterplot (Fig. 4.3). It showed a somewhat weak linear relationship (Pearson coefficients, $r_{pbl} = .045$, $r_{tl} = .055$, $r_{overall} = 0.114$).

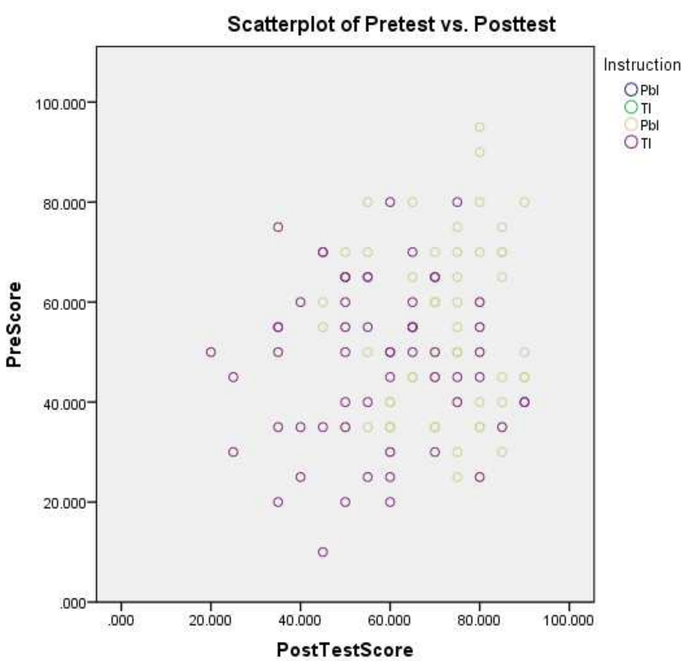

Figure 4.3. Scatterplot of Pretest vs. Posttest for Control and Experimental Groups

(2) ANCOVA requires homogeneity of the regression slope (Elashoof, 1969). The researcher confirmed that this assumption was met, by including and assessing the interaction term in the general linear model of SPSS. The interaction term was found not to be statistically significant ($F_{1, 119}$) = 0.034, p = .853). In addition, the estimated effect size was extremely small (η^2 = .000014), indicating little or no effect of the interaction variable on the response variable. Ary et al. (2010) describes effect size as "a useful measure of the strength or magnitude of a relationship between experimental and control groups". Hence, a small effect (0 - 0.1) indicates a weak or trivial relationship.

(3) The assumption of normality was checked by a visual inspection of the histograms of the posttest scores for all participants, as well as for male and for female participants. (See Figures 4.4, 4.5, and 4.6).

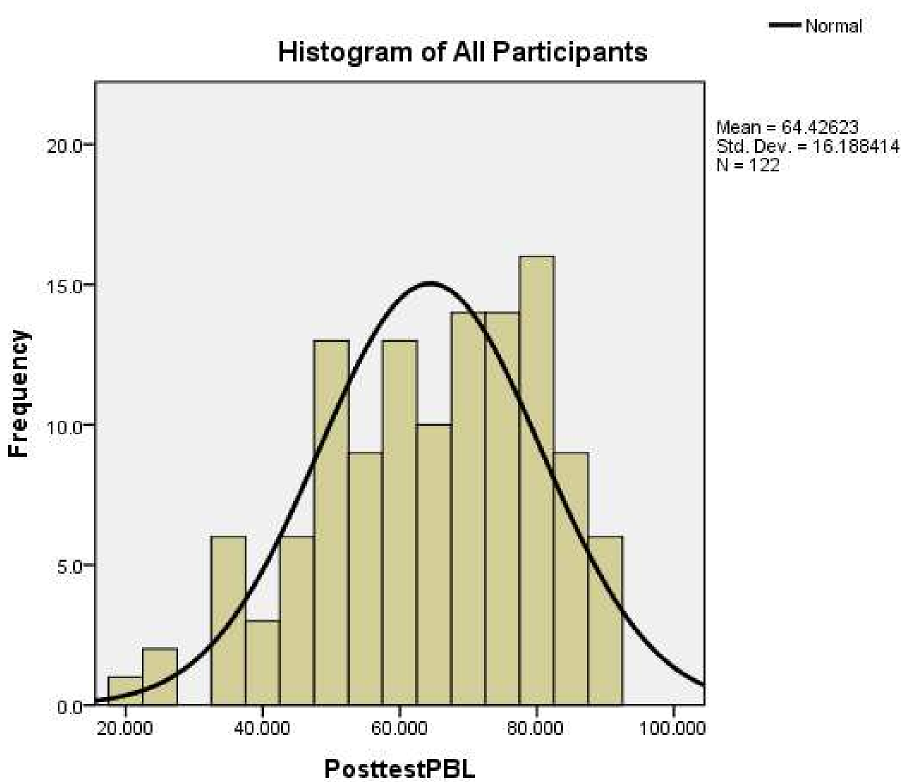

Figure 4.4. Histogram of posttest of all participants

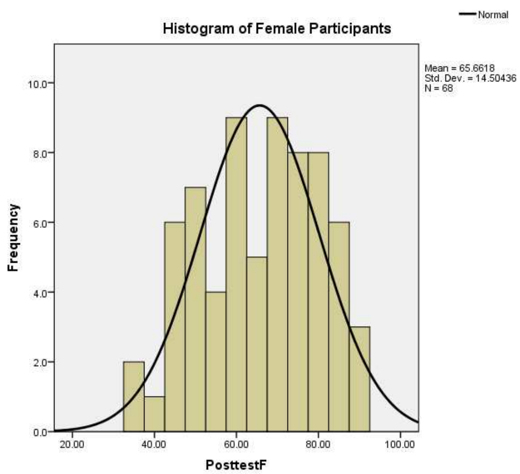

Figure 4.5. Histogram of posttest of female participants

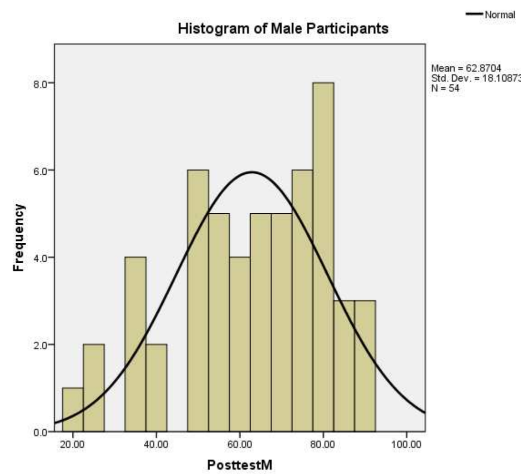

Figure 4.6. Histogram of posttest of male participants

Despite small gaps, the histograms show approximate normal distributions, which are supported by the skewness and kurtosis z-scores, all of which fall within the conservative ± 2.58 boundary. However, the test for normality, as determined by the Shapiro-Wilk test (Laerd Statistics, 2013) showed p = .024 for females, and p =.127 for males (Table 4.4, 4.5).

Table 4.4

Tests of Normality[a]

	Kolmogorov-Smirnov[b]			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
PosttestPBL	.169	34	.015	.926	34	.024

a. Gender = female, Group = Pbl
b. Lilliefors Significance Correction

Table 4.5

Tests of Normality[a]

	Kolmogorov-Smirnov[b]			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
PosttestPBL	.135	30	.168	.945	30	.127

a. Gender = male, Group = Pbl
b. Lilliefors Significance Correction

It should be noted here that the ANCOVA procedure can still be conducted if $p < .05$ since, like the independent t-test, it is fairly robust to deviations from normality (Montgomery, 2009).

(4) A plot of the standardized residuals versus the predicted values of the posttest data (Figure 4.7) reveals a random distribution, thereby indicating approximately the same variance for all values of the predicted scores. Hence, the assumption of homoscedasticity is not violated.

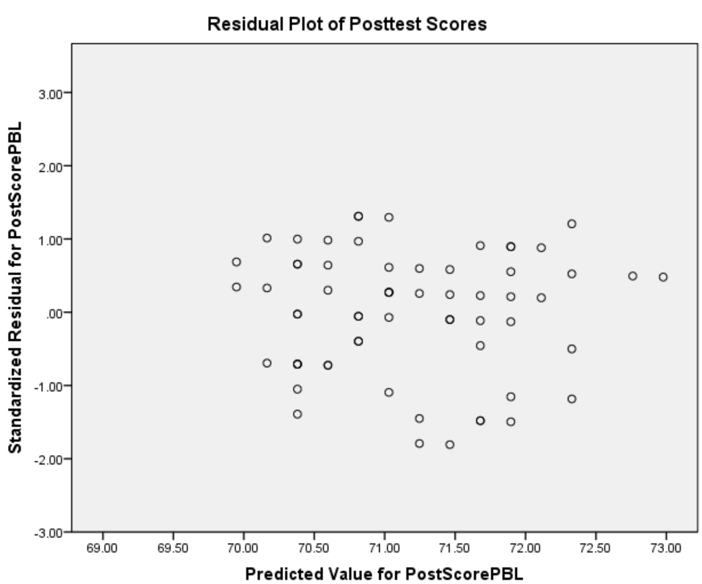

Figure 4.7. Plot of standardized residuals vs. predicted values of posttest scores

(5) It is assumed that the variance of the residuals is equal for different groups of the independent variable. To test this assumption the researcher examined the results from the

Levene's Test of Equality of Error variance at $\alpha = .05$ level of significance. Table 4.6 shows a p-value of 0.029, which suggests that this assumption was violated. This violation persisted despite a variance-stabilizing transformation, which the researcher conducted on the posttest.

Table 4.6

Levene's Test of Equality of Error Variance

F	df 1	df 2	Sig.
3.100	3	118	.029

Dependent Variable: Posttest PBL

After adjusting for the pretest scores between the groups, the researcher conducted the ANCOVA to test null hypothesis 1(a), which stated that there would be no statistical difference between the mean performance scores of students in the control group and experimental group. The researcher rejected this hypothesis, concluding that there was a statistically significant difference between the mean posttest scores, and that the PBL group had a higher mean score than the TL instructional group. ($F_{1, 121}$) = 29.187, $p < .001$, partial $\eta^2 = .20$ (See Table 4.7).

The ANCOVA examined the following effects: (a) group (control vs. treatment, (b) gender (female vs. male), and (c) interaction of groups and gender. The pretest scores were entered as a covariate to correct for any differences in the control and experimental treatment group, while gender was entered as a fixed factor.

Table 4.7

Test of Between-Subjects Effects with Dependent Variable: Posttest

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial η^2	Observed Power[b]
Corrected Model	6951.210[a]	4	1737.803	8.212	.000	.219	.998
Intercept	43637.525	1	43637.525	206.251	.000	.638	1.000
Pretest Score	75.774	1	75.774	.358	.551	.003	.091
Group	6176.423	1	6176.423	29.187	.000	.200	1.000
Gender	435.443	1	435.443	2.058	.154	.017	.296
Group*Gender	375.828	1	375.828	1.776	.185	.015	.262
Error	24758.626	117	211.612				
Total	538100.000	122					
Corrected Total	31709.836	121					

Note. a. R Squared = .219 (Adjusted R Squared = .193); b. Computed using alpha = .05

Hypothesis 1(b) stated that there would be no statistical difference between the mean mathematics performance scores of male students in the control and male students in the experimental groups. The researcher rejected this hypothesis, concluding that there was a statistically significant difference between the mean posttest scores, and that the PBL group had a higher mean score than the TL group. ($t_{52} = 4.0067$, $p < .001$).

Hypothesis 1(c) stated that there would be no statistical difference between the mean mathematics performance scores of female students in the control and female students in the experimental groups. The researcher rejected this hypothesis, concluding that there was a statistically significant difference between the mean posttest scores, and that the PBL group had a higher mean score than the TL group. ($t_{52} = 3.4736$, $p = .001$).

Hypothesis 1(d) stated that there would be no statistical difference between the mean mathematics performance scores of female students and male students, both in the experimental groups. The researcher failed to reject this hypothesis concluding that there was no difference between the mean posttest scores of male and female students in the experimental group ($|t_{52}|$ = .1038, p = .918).

Research Question Two and Null Hypotheses

Research question two asked if there was a difference in the motivation of students instructed using project-based learning and the motivation of students who were instructed using traditional learning methods. This question provided the null hypothesis 2(a) that there will be no statistically significant difference in the motivation scores between students instructed using project-based learning techniques and the motivation scores of students instructed using traditional learning techniques in an honors level mathematics class, as indicated by the IMMS. In addition, there were four strategy components or dependent variable for motivation, each providing a null hypothesis and tested for statistical significance between the control and experimental groups. The null hypotheses for the four components of motivation are as follows.

Hypothesis 2(b) stated that there will be no statistically significant difference in the mean *attention* subscale scores between students instructed using project-based learning and students using traditional learning techniques, in honors level mathematics courses.

Hypothesis 2(c) stated that there will be no statistically significant difference in the mean *relevance* subscale scores between students instructed using project-based learning and students using traditional learning techniques, in honors level mathematics courses.

Hypothesis 2(d) stated that there will be no statistically significant difference in the mean *confidence* subscale scores between students instructed using project-based learning and students using traditional learning techniques, in honors level mathematics courses.

Hypothesis 2(e) stated that there will be no statistically significant difference in the mean *satisfaction* subscale scores between students instructed using project-based learning and students using traditional learning techniques, in honors level mathematics courses.

To test null hypothesis 2(a), the researcher conducted a one-way MANOVA, which simultaneously assessed the linear combination of the individual dependent subscale variables. Prior to conducting this analysis, however, it was necessary to authenticate its appropriateness by verifying several assumptions.

Assumption Testing

The assumptions for the MANOVA include (a) testing for univariate outliers, (b) normality, (c) multicollinearity, (d) linearity, (e) multivariate outliers, (f) homogeneity of variance.

(1) There were no univariate outliers in the data, as assessed by inspection of a boxplot.

(2) Preliminary assumption check of normality using the Shapiro-Wilk's test showed that the data was normally distributed for the *attention*, *confidence*, and *satisfaction* subscales of Motivation ($p > .05$) (see Table. 4.8). For *relevance,* however, there was significance ($p < .05$), indicating nonnormality for the Treatment group. Since, $n > 30$, this violation is not critical as the Central Limit Theorem (CLT) applies, thereby allowing the researcher to assume approximate normality of the data for this group. According to Yates, Moore, and Starnes (2008), a large sample size from any population with a finite standard deviation, has a sample mean that is approximately normal.

Table 4.8

Tests of Normality: ARCS

	Instruction	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
		Statistic	df	Sig.	Statistic	df	Sig.
Attention	PBL	.087	64	.200*	.984	64	.591
	TL	.069	55	.200*	.979	55	.447
Relevance	PBL	.145	64	.002	.958	64	.028
	TL	.109	55	.155	.970	55	.192
Confidence	PBL	.104	64	.082	.972	64	.157
	TL	.088	55	.200*	.972	55	.232
Satisfaction	PBL	.082	64	.200*	.981	64	.427
	TL	.123	55	.036	.969	55	.174

*. This is a lower bound of the true significance.
a. Lilliefors Significance Correction

(3) To test for multicollinearity among independent variables, the researcher examined the Variance Inflation Factor (VIF). This statistic provides a quantifier of variance inflation that is attributed to a specific independent variable. A rule of thumb is that a VIF of 5 or more indicates that a high multicollinearity exist between independent variables. Tables 4.9, 4.10, 4.11, and 4.12 provide the VIF values for the respective ARCS dependent variables. This shows all VIF values to be less than 5 and greater than 1, which indicates moderate multicollinearity between variables, a condition that is not critical to the test.

Table 4.9

Variance Inflation Factor[a]

	Collinearity Statistics	
Model	Tolerance	VIF
Relevance	.514	1.945
Confidence	.842	1.188
Satisfaction	.488	2.048

[a]. Dependent Variable: Attention

Table 4.10

Variance Inflation Factor[a]

	Collinearity Statistics	
Model	Tolerance	VIF
Confidence	.820	1.220
Satisfaction	.508	1.969
Attention	.508	1.969

[a]. Dependent Variable: Relevance

Table 4.11

Variance Inflation Factor[a]

	Collinearity Statistics	
Model	Tolerance	VIF
Satisfaction	.410	2.438
Attention	.485	2.061
Relevance	.478	2.091

[a]. Dependent Variable: Confidence

Table 4.12

Variance Inflation Factor[a]

	Collinearity Statistics	
Model	Tolerance	VIF
Attention	.574	1.743
Relevance	.604	1.656
Confidence	.836	1.196

[a]. Dependent Variable: Satisfaction

(4) MANOVA requires that a linear relationship exist between each pair of dependent variable for each group. The scatterplot matrices in Figures 4.8 and 4.9 show the linear pattern, with a much stronger linear pattern for the treatment group than the control group.

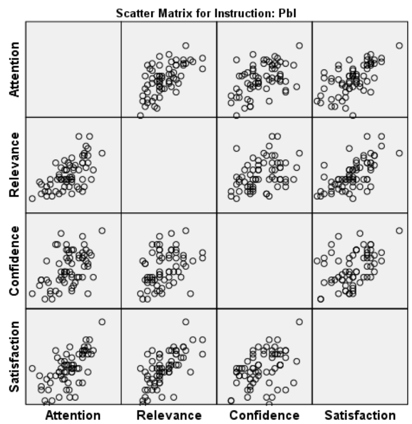

Figure 4.8. Scatter Matrix for Instruction: PBL

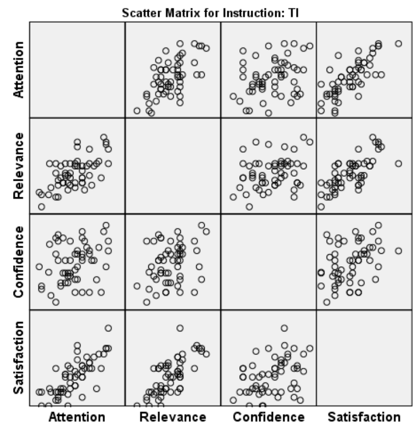

Figure 4.9. Scatter Matrix for Instruction: TL

(5) There were no multivariate outliers in the data, as assessed by the *Mahanlanobis* distance ($p > .001$). This distance is calculated and compared to a Chi-squared statistic. For the four dependent variables, the Chi-square critical value is 18.47 ($\alpha = .001$). However, the computed test statistic value for the independent subscale data was only12.36226, thereby indicating that there were no multivariate outliers in the data.

(6) The final assumption test is the test of homogeneity of variance-covariances. There was homogeneity of variance-covariance matrices as assessed by Box's M test of equality of covariance matrices. The result of the Box's test is $M = 12.689$, $p = .271$, which was not significant at $\alpha = .001$. The researcher therefore, failed to reject the null hypothesis that the observed variance-covariance matrices of the dependent variables are equal across groups.

Upon verifying the assumptions, the MANOVA was conducted and yielded results which were not statistically significant based on the Wilks' Lambda ($\Lambda = .984$, $F_{(4,114)} = .458$, $P = .766$, partial $\eta^2 = .016$), at the $\alpha = .05$ significance level. The researcher failed to reject the null hypothesis, thereby concluding that there was no statistical significant difference in the linear combination of motivation scores between students who were instructed using project-based learning methods and those who were instructed using traditional learning methods, at the honors level.

To determine whether any particular dependent subscales of motivation were statistically significant, a one-way ANOVA was conducted for each dependent variable, the results of which were contained in the 'test of between-subjects effects' results. The researcher applied the Bonferroni correction procedure. This multiple comparison procedure in which the conventional error allowance, $\alpha = .05$, is divided by the number of comparisons (4, in this case), is used to reduce the likelihood of a type I error (Howell, 2008). Hence, $\alpha = .01$ was used for this test. Since $p > .01$ (See table 4.13) for each of the dependent subscale, the researcher failed to reject the null hypothesis, concluding that there was no significant difference in the attention, relevance, confidence, and satisfaction between the students in the control group and the students in the experimental group. The results are presented in Table 4.13.

Table 4.13

Tests of Between-Subjects Effects-Motivation Subscale

Dependent Variable	F	Sig.	Partial Eta Squared
Attention	1.024	.314	.009
Relevance	.735	.393	.006
Confidence	.037	.848	.000
Satisfaction	.148	.701	.001

Finally, the researcher computed effect size measures for both the control and treatment group. The statistic used was the Cohen's *d*, which, according to Howell (2008), give "a meaningful indication of how large a mean was, or how different two means were" (p. 295). The effect size is calculated, as follow

$$d = \frac{\overline{X}_{posttest} - \overline{X}_{pretest}}{S_{\overline{X}\ pretest}}$$

and Cohen's rule of thumb for effect size is *small* (d = .20), *medium* (d = .50), and *large* (d = .80) (Howell, p. 358). Accordingly, the effect size measures calculated for the control group was of medium effect (d = 0.56), which meant that the average gain in the posttest scores for the control group was approximately one half times the standard deviation of that group's pretest score, over the course of the study. For the treatment group, the effect was large, and almost doubled that of the control group (d = 1.06). That is, the average gain in the posttest scores was more than one standard deviation of the pretest score, over the course of this study.

CHAPTER FIVE: DISCUSSION

Introduction

The education curriculum continues to undergo reforms aimed at increasing student learning in an actively engaging manner, as they are prepared for productive and effective roles in today's global economy (Berends, Boersma, & Wegeman, 2003; Center for Mental Health in Schools, 2008; Mergendoller, Markham, Ravitz &Lerner, 2006; Rogers, 1983). Consequently, more and more classrooms are student-oriented, in which learning is constructed, active, reflective, collaborative, and inquiry based. Project-based learning is one type of student-oriented learning technique that is credited with providing students with a deeper understanding and increased achievement because of its emphasis on student autonomy and collaborative learning (English & Kitsantas, 2013; Thomas, 2000). Hence, the purpose of this nonequivalent control study was to investigate the difference in the achievement, and motivation, of students who were enrolled in honors level mathematics courses, and instructed either by project-based method or by traditional learning method, in a suburban New Jersey high school. This chapter consists of the findings of the study, a discussion of the result, limitations of the study, recommendation for future research, and concluding remarks.

Restatement of the Research Questions

RQ1: Is there a difference in the mean performance scores of students instructed using project-based learning techniques and students instructed using traditional learning techniques, in an honors level mathematics class?

RQ2: Is there a difference in the motivation of students instructed using project-based learning techniques and the motivation of students instructed using traditional learning techniques, in an honors level mathematics class?

Summary of Findings for Research Question One

Research question one focused on comparing the mathematics achievement of eleventh and twelfth grade students enrolled in honors level mathematics, as measured by expert-validated end-of-unit tests. The treatment group was instructed using project-based method while the control group was instructed using traditional learning method. To obtain baseline measurements for the analysis, preliminary descriptive statistics were computed for both experimental groups, after which, formal statistical analysis by way of a one-way ANCOVA, was computed. The ANCOVA revealed a highly significant difference between the mathematical performance between the two groups, as indicated by a p-value less than .0005. This difference is also evident in the summary statistic which showed a greater increase in the mean mathematics post score for the treatment group. That is, a 17.889% increase over the pretest score, whereas for the control group there was only a 9.138% increase over the pretest score for that group. This statistic is displayed in Figure 4.2. Independent t-test showed that there was also a difference between the performance of the female students in the treatment group and the female students in the control group. This was the same result for the female in the two experimental groups. However, when comparing the results of female students with male students, both in the experimental group, no significant difference was found between them

Summary of Findings for Research Question Two

The focus of research question two was to compare the impact of project-based learning, and traditional learning method, on the motivation of students who were enrolled in honors level mathematics courses, as measured by the IMMS, a valid self-reported measure. The response rates were 94.8% and 100%, for the control and treatment groups, respectively. The data was analyzed using MANOVA, as well as ANOVA, using Bonferroni procedure.

Discussion of the Results

Student Achievement

The findings of this study for research question one support other research studies which have credited project-based learning methods with improving students' academic achievement (Boaler 2002 in David, 2008; English & Kitsantas, 2013; Thomas, 2003; Vega, 2012; Verma, Dickerson & McKinney, 2011). The study showed that the students who were instructed using project-based learning method outperformed their traditional learning counterparts in mathematical skills as well as in conceptual knowledge. The PBL group had a more impressive improvement which was represented by approximately 34% in the mean posttest scores in comparison to the TL group which realized an approximate 19% increase in their posttest scores. This significant difference in the overall mean posttest scores between the two groups may be attributed to the fact that the PBL group was engaged in more research type activities consistent with constructivist principles. Unlike the TL group which was involved in the mastery of the content through worksheet drills and skills practice, students in the PBL group frequently collaborated in formulating hypotheses, discovering meaning, and making decisions in answer to a driving question. The results indicated that student learning is enhanced when "they are taught to focus their attention on the processes and strategies that help them acquire knowledge and skills" (Camahalan, 2006, p.200). The students were able to use elements of their past experiences, as well as technology to aid in their categorization and conceptualization, as needed. Piaget describes and compares these students to a scientist who discovers and learns through activities which require investigation rather than memorization of facts, while Vygotsky suggests that it is this kind of cooperative and collaborative practices that support higher psychological functioning (Eun, 2010). This is critical since students have to be able to go beyond basic knowledge and comprehension to the use of abstraction, and other higher order thinking skills of analysis, synthesis, and evaluation in order to compete with their counterparts in the global economy. As the students in the PBL discuss and negotiate the requirements of their project they were able to engage other 21st century skills which included, but not limited to, communication, creativity, critical thinking, cross-culture understanding, problem-solving, reasoning, time management, and reasoning. They were also able to devise an organized approach to the project idea, to self assign tasks, and to solve the problem in a logical and methodical manner. Learning mathematics require this kind of thinking about abstract concepts. The schema shown in Fig. 4.10 (Educational Technology and Mobile Learning, 2013) shows the different features of the 21st century pedagogy which contribute to students' higher psychological and cognitive functioning in the PBL classroom. Many of the skills described in this schema (figure 4.10) are not allowed to develop in the traditional teacher-centered pedagogy which offers little or no autonomy to the students.

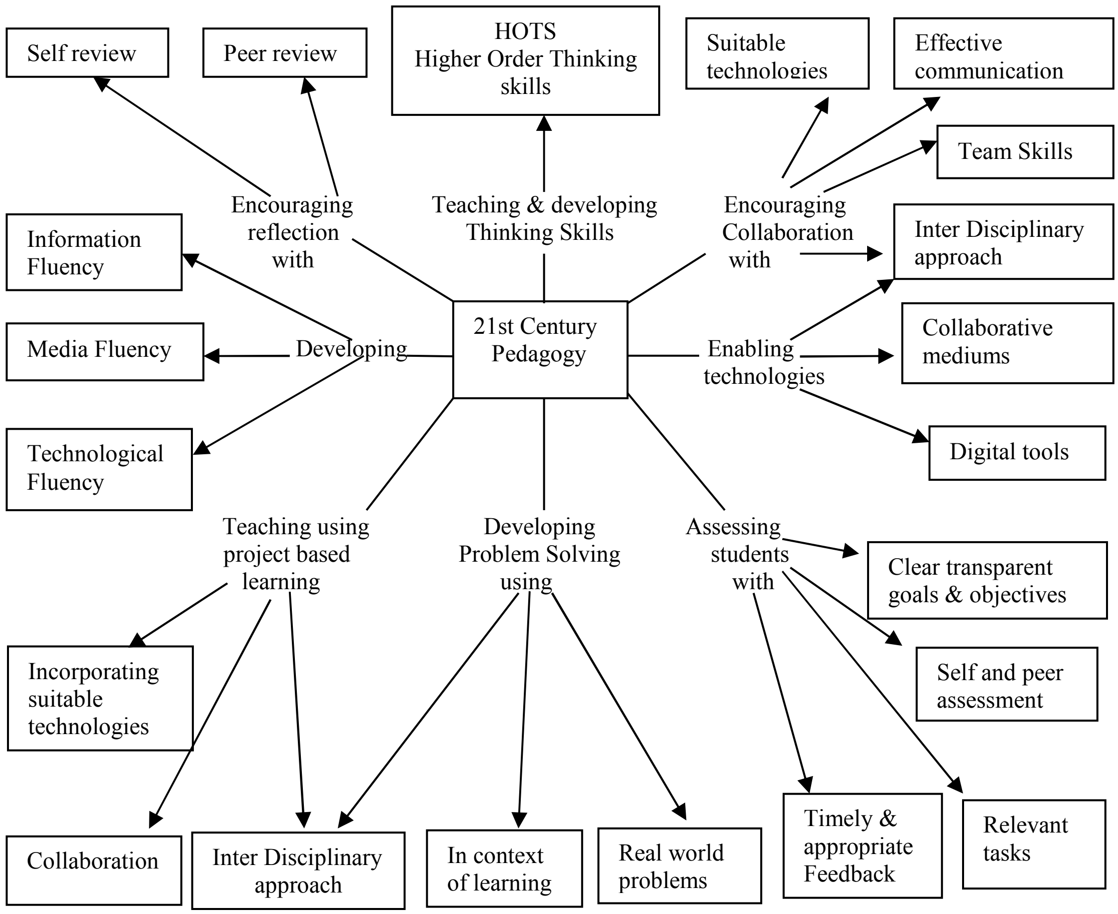

Figure 4.10. A Diagram of 21st Century Pedagogy (http://www.educatorstechnology.com/2013/03/awesome-graphic-on-21st-century-pedagogy)" by A. Churches (http://www.educatorstechnology.com/2013/03/awesome-graphic-on-21st-century-pedagogy/) is licensed under CC BY-SA 2.5 (http://creativecommons.org/licensed/by-sa/2.5/). Reprinted with permission (see Appendix G).

The importance of collaboration and teamwork was evident as indicated by the improved mathematical achievement of PBL students in this study. While students worked individually on their self-assigned tasks, they often worked collectively to synthesize and optimize their learning. Throughout the project lesson, students were provided with valuable feedback from their peers and from their teachers, and this enable them to make revisions as needed, as well as to enhance their confidence.

A notable feature of the PBL classroom is the ability of students to learn from each other, to be accountable to each member of the group as they work towards a common goal, and to engage their multiple intelligences, as posited by Howard Gardner (Helle, Tynjala, Olkinuora, & Lonka, 2007; Lamie, 2000). This study was consistent with that of Bas and Beyham (2010), who stated that “students who are educated by multiple intelligences, supported by project-based learning methods, are more successful and have a higher motivation level than students who are educated by the traditional instructional method. “Socialization and collaboration allowed students the opportunity to engage their particular multiple intelligences as well as nurture the weak ones, and help in the development of intellectual, emotional, moral, research, and social skills (Bilgin & Karakuyu, 2015; Lamie, 2000).

In answering research question one also, it was discovered that the difference in the mathematics achievement between the PBL and TL groups, was also significant, when gender was considered. That is, female students in the PBL group score significantly higher than females in the TL group. Similar results were obtained for the male students, when the two groups were compared. However, when gender difference was considered only among students in the experimental PBL group, there was no significant difference between the mean mathematical performance of female and male students. This was in direct contradiction to the widely held hypothesis that male students outperformed female students in mathematics (Leahey & Guo, 2001; Preckel, Goetz, Pekrun, & Kleine, 2008). However, this study was consistent with that of Benbow & Stanley (1980) whose study with a large national sample of high school students found that the difference in academic performance between the genders was only slight, if at all. Consistency was also determined with the study by Ding et al. (2006), whose study concluded that there was no significant difference in the academic achievement of female and male students, even more so for the high performing and gifted students. In comparing PBL and TL groups, it is likely that the gender difference performance observed was situated dependent, as surmised by Ding et al. (2006). That is, the socialization perspective provided an explanation for the difference in mathematics achievement between male students in the PBL group and male students in the TL group, as well as for the mathematics achievement of female students in the PBL group and female students in the TL group. Students, both male and female in the PBL group, were engaged in the collaboration and socialization process which are critical to learning. The lack of difference between male students' performance and female students' performance in the PBL group supports this socialization perspective.

Student Motivation

The findings of this study yielded results which were inconsistent with studies done by other researchers. For example, Bas and Beyham (2010), and Pederson, 2003) concluded that students who were taught via PBL method had a high motivation level than those who did not, because of the autonomy they had over their class activities. In this study, however, there was no significant difference in the overall motivation between the treatment group of students who were instructed by PBL method, and the control group of students instructed using TL method. While this result may be different from expectation, based on previous studies, it has been an understanding that students who enroll in advanced mathematics classes are usually persistent and possess a desire to succeed. Consistent with the expectancy value theory these students are known to show a positive correlation between their educational experiences and how they perceive their successes (Roval, Ponton, Wighting & Baker (2007).

Since all students in this study were enrolled in the honors level mathematics course, they were already exposed to a more intensive and challenging academic experience than their non-honors counterpart. As such, they possessed a greater desire to learn and succeed. The results of this study support the statement that "as long as the level of the challenge is at, or just above, the ability level, learning and intrinsic motivation go together" (Scager, Akkerman, Pilot & Wubbels, 2014, p. 661). This result is not surprising as it is a fact that many students who enter honors classes have not experienced much failure in mathematics before doing so. Often it is found that they have the preconceived notion that they are more academically gifted, and as such, they make a conscientious effort to live up to this external expectation (Schreiber, 2002; Seifert, Pascarella, Colangelo, & Assouline, 2007). Both groups of students exhibit motivation knowing that their efforts were going to be rewarded with a grade, not necessarily below their expectation. However, the results from this study supports the findings of the study conducted by Pedersen and Williams (2004), who concluded that although grading is used extensively to enhance student motivation, it was of lesser importance in student-centered learning where students exert more ownership of their work. Other studies also concluded that intrinsic motivation of students lead to achieving better results in learning mathematics than extrinsic motivational factors such as grading (Middleton & Spanias, 1999; Teoh, Koo, & Singh 2010).

Though the self-reported IMMS report showed no difference in the motivation of students between the two groups, it was evident that the PBL method provided students with the opportunity to perform at a higher functional level than the TL group, as evident from the mean performance scores. By their own effort, through collaboration, and the guidance of the teacher, who acted as facilitators, students in the PBL group planned and executed their research. With the aid of the media and technology they used the 21st century skill shown in Figure 4.10 to aid in their discovery and in achieving the objective of the lesson. In addition, students in this group expressed their satisfaction at successfully completing what some termed 'a complex project'.

The feeling of ownership of their learning, coupled with teamwork, was instrumental in enhancing their sense of efficacy. While students in the TL group did show some improvement theirs was not as rich an experience as the PBL group.

Conclusion

The myriad of reforms that have been introduced in American education system over the past 50 years have included the ESEA, NCLB, RTT, ESSA, CCSS, and most recently, the PARCC, have all focused on improving students achievement. This is particularly the case in mathematics with not much emphasis placed on those students enrolled in honors level mathematics courses. Too often this group of high achievers is overlooked because the consensus is that they are already motivated to excellence. This study confirmed that the instructional method used is significant in affecting student achievement. The study showed that students enrolled in honors level mathematics courses who were instructed using PBL methods performed significantly better than those who were not instructed using PBL method. The results showed that the experimental PBL group had a mean overall post score that was doubled that of the control TL group. This significant difference was also observed between the groups when gender was considered, although gender comparison showed no significant difference when this comparison was made within the PBL group. This study confirms the superiority of the PBL method as it relates to student achievement.

There was no difference in the motivation of students between the groups, which was unlike results of previous PBL studies with elementary students and web based course. This was not surprising since students who enrolled in honors classes do so with the expectation to succeed, and as such, possess motivation to do so. Engaging in PBL, however, enhanced the motivation of students, thereby allowing them to realize greater increase in their mean mathematics post score.

The findings of this study, along with its implications for teaching, justify the implementation of PBL pedagogy, which incorporates all of the 21st century skills students need to become lifelong learners, and to be able to function efficiently and effectively in the global economy.

Implications

Based on the findings of this study, PBL is shown to be effective in enhancing the achievement of students enrolled in the honors level mathematics classes. It is not uncommon to find that teachers have unusually high expectations for honors students, and as such expect them to excel with little effort. But while it was shown that students in honors classes do come to those classes with more intrinsic motivation than their non-honors counterpart, this study highlights the need for more cooperative learning, as was used with the PBL group, which showed significantly higher results over the TL group. Feedback is an integral part of learning and is the basis of cooperative learning. The PBL environment challenges students to constantly conduct peer-peer discussion, as well as teacher-peers discussions, which leads to ongoing student and teacher evaluation, and which is not based simply on a standardized test grade.

The use of PBL in the classroom requires not only conceptual knowledge of the teacher but also confidence in understanding and using the technology that this instructional technique often requires. Teacher who are ill-equipped in this respect are usually reluctant to facilitate this kind of student-led inquiry. Consequently, school administrators have the responsibility to provide the necessary professional development during which deficiencies in the use of technology can be addressed. In other words, school administrators should be willing to embrace this 21st century pedagogy. This study should be instrumental in encouraging teachers to be more receptive to the change from their traditional perceptive of learning to more progressive methods such as PBL and other self-regulated methods, for the greater and sustained achievement by all students.

Although this study focused on PBL in honors level mathematics classes, the findings could be used to encourage its use in non-honors mathematics classes. Keller's (1984) ARCS model of motivation, which the researcher used to test the difference in motivation between the PBL and TL groups, is a widely used model (Calahan & Smith, 1990; Cook, Beckman, Thomas, & Thompson, 2009; Huett, Moller, Bray, Young, & Huett, 2006; Shellnut, 1998; Small, 1997; Visser & Keller, 1990) that teachers can incorporate into their lesson planning, as an aid for stimulating and sustaining students' motivation and achievement.

Limitations

Although this study provides some insight into the differences between students with and without the student-centered PBL method, there are several limitations that should be mentioned. Because students were already assigned to the honors classes, it was not possible to use a completely randomized design. Instead, a nonrandomized pretest-posttest design was used, and as such generalization to other or similar settings should be considered with caution.

Another limitation concerns the data on student motivation which was obtained by a self-reported survey, and which may have caused some threat to the internal validity of the study. Although the students responded to the survey anonymously, it is not uncommon to find students responding in a manner which they deem consistent with their particular group. This behavior was explained in Vroom's expectancy theory of motivation, which discusses how an individual's selected behavior is based on expected outcomes. In addition, some items on the survey were left unanswered by some students, particularly in the TL group. This non-response by may have given rise to results that are biased.

A further limitation of this study pertained to the participants involved, the scope and duration of the study. The participants were all enrolled in their third or final year in a suburban school high school, and as such, caution should be exercised when generalizing to the school population. Also, only one project unit was completed in approximately six weeks.

A final limitation consisted of the quality of the teachers involved in the study. Since their knowledge, experience, and educational philosophy differ, it is possible that their influence on the students' effort, motivation, and achievement varied between the groups. The researcher was cognizant of the subjective nature of this aspect of the study, and took precautions, as much as was possible, to minimize this teacher quality effect, by providing them with lesson plans, supporting materials, and comprehensive guidelines for overseeing the project.

Recommendation for Future Research

In this study a group of students was instructed using the student-led PBL method. Analysis of the data obtained from this study indicated that there is need for a more comprehensive study, in order to improve its generalizability. While the study did reveal improved mathematical competency among the students in this treatment group, future research should seek to involve a larger sample group of participants, in areas of more cultural diversity and socio-economic challenges.

In most cases when a researcher attempts to conduct a study involving students in a school after the school session for the year has started, students are already assigned to classes and the researcher is not able to conduct a truly randomized experiment. The next best option is a quasi-experiment which has several threats to the internal validity of the research data, despite the effort of the researcher to control for these threats. Future researchers should consider this challenge and seek to work with education officials ahead of their class assignments to conduct the randomization process for the classes that would be involved in the study, and hence improve the generalizability of the results obtained.

Due to the aforementioned limitations of this study, which pertains to the design method, future studies should be replicated to further improve generalization. It is possible that replication over time, and with students in other levels of mathematics classes, may produce results that conflict with previous research or challenge a general accepted theory, giving rise to new areas of investigation (Gall et al., 2007). On the other hand, it is possible that through replication, “the development of a formal theory of academic monitoring” (Gall et al., 2007) is obtained.

A further recommendation which the researcher considered pertains to students who are classified as having attention deficit/hyperactivity disorder (ADHD), and who often have poor grades, particularly in mathematical standardized test scores (Loe & Feldman, 2007). Future research on the use of PBL techniques with ADHD students may provide the answer to the question often asked about improving their academic ability. The researcher recognized that while instructional change in the classroom is necessary, it will be unlikely to affect any positive change in student achievement without the support and collaboration of educational administrators, who are responsible for putting an efficient program in place. The support and collaboration of family members and health care professionals who provide the medical care for ADHD students will be critical and should be carefully considered in future research which seeks to employ this recommendation. PBL may be the answer to engaging students with ADHD while increasing their achievement in mathematics. Future results would provide the empirical evidence to support or refute this claim.

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APPENDICES

Appendix A: Institutional Review Board Approval

The Graduate School
(434) 592-5530
Liberty University | Training Champions for Christ since 1971

From: IRB, IRB <IRB@liberty.edu>
Sent: Friday, October 17, 2014 1:29 PM
To: Carter, Sunletha
Cc: Westfall, Jerry (School of Engineering and Computational Sciences); Garzon, Fernando (Center for Counseling and Family Studies); IRB, IRB
Subject: IRB Approval 1983.101714: Traditional Versus Project-Based Learning: The Effect on Student Performance and Motivation in Honors-Level Mathematics Classes

Dear Sunletha,

We are pleased to inform you that your above study has been approved by the Liberty IRB. This approval is extended to you for one year from the date provided above with your protocol number. If data collection proceeds past one year, or if you make changes in the methodology as it pertains to human subjects, you must submit an appropriate update form to the IRB. The forms for these cases are attached to your approval email.

Your IRB-approved, stamped consent form is also attached. This form should be copied and used to gain the consent of your research participants. If you plan to provide your consent information electronically, the contents of the attached consent document should be made available without alteration.

Please retain this letter for your records. Also, if you are conducting research as part of the requirements for a master's thesis or doctoral dissertation, this approval letter should be included as an appendix to your completed thesis or dissertation.

Thank you for your cooperation with the IRB, and we wish you well with your research project.

Sincerely,

Fernando Garzon, Psy.D.
Professor, IRB Chair
Counseling

(434) 592-4054
Liberty University | Training Champions for Christ since 1971

Appendix B: Investigator Agreement and Signature Page

[redacted]

[redacted]

IV. INVESTIGATOR AGREEMENT & SIGNATURE PAGE

BY SIGNING THIS DOCUMENT, THE INVESTIGATOR AGREES:

1. That no participants will be recruited or entered under the protocol until the investigator has received the final approval or exemption email from the Chair of the Institutional Review Board.
2. That no participants will be recruited or entered under the protocol until all key personnel for the project have been properly educated on the protocol for the study.
3. That any modifications of the protocol or consent form will not be initiated without prior written approval, by email, from the IRB and the faculty advisor, except when necessary to eliminate immediate hazards to the participants.
4. The PI agrees to carry out the protocol as stated in the approved application: all participants will be recruited and consented as stated in the protocol approved or exempted by the IRB. If written consent is required, all participants will be consented by signing a copy of the approved consent form.
5. That any unanticipated problems involving risks to participants or others participating in the approved protocol, which must be in accordance with the Liberty Way (and/or the Honor Code) and the Confidentiality Statement, will be promptly reported in writing to the IRB.
6. That the IRB office will be notified within 30 days of a change in the PI for the study.
7. That the IRB office will be notified within 30 days of the completion of this study.
8. That the PI will inform the IRB and complete all necessary reports should he/she terminate University association.
9. To maintain records and keep informed consent documents for three years after completion of the project, even if the PI terminates association with the University.
10. That he/she has access to copies of 45 CFR 46 and the Belmont Report.

Principal Investigator (Printed) Principal Investigator (Signature) Date

FOR STUDENT PROPOSALS ONLY

BY SIGNING THIS DOCUMENT, THE FACULTY ADVISOR AGREES:

1. To assume responsibility for the oversight of the student's current investigation, as outlined in the approved IRB application.
2. To work with the investigator, and the Institutional Review Board, as needed, in maintaining compliance with this agreement.
3. That the Principal Investigator is qualified to perform this study.
4. **That by signing this document you verify you have carefully read this application and approve of the procedures described herein, and also verify that the application complies with all instructions listed above. If you have any questions, please contact our office (irb@liberty.edu).**

[redacted] [redacted] [redacted]

Faculty Advisor (Printed) Faculty Advisor (Original Signature) Date

*The Institutional Review Board reserves the right to terminate this study at any time if, in its opinion, (1) the risks of further experimentation are prohibitive, or (2) the above agreement is breached.

Appendix C: Approval by School Superintendent to Conduct Research

September 28, 2012

Ms. Sunletha Carter
25 Sherman Avenue
East Orange, NJ 07017

Dear Ms. Carter:

After reading your dissertation proposal and discussing it with you, I give you approval to use Teaneck High School for your research, with the limitations that you specified in the proposal.

Best Regards.

[redacted]

Superintendent of Schools

Appendix D: Permission to Use Keller's IMMS Instrument

Monday, September 10, 2012 [redacted]

To:

M

Carter, Sunletha

Attachments:

(4)Download all attachments

Keller 1999 Motivation in ~1.pdf (463 KB)[Open in Browser]; Keller 2010 ARCS Measureme~1.pdf (4 MB)[Open in Browser]; Kim & Keller 2010 Motivati~1.pdf (948 KB)[Open in Browser]; Kim & Keller 20071224 bjet~1.pdf (213 KB)[Open in Browser]

Dear Sunletha,
Thank you for your courteous letter. You are most welcome to use the IMMS or CIS (see attached document) according to which is most relevant to your study. I have attached an excerpt from my book. It contains two instruments together with scoring and psychometric information. It also explains how you can revise the referent in the items to make them situation specific.
I have also attached three articles, and please let me know if there are others you would like to have.
Best wishes,

[redacted]

Educational Psychology and Learning Systems
Florida State University
9705 Waters Meet Drive
Tallahassee, FL 32312-3746
Phone: [redacted]
Official ARCS Model Website: http://arcsmodel.com
Professional Website: http://mailer.fsu.edu/~jkeller/JohnsHome/
Keller, J.M. (2010), ***Motivational Design for Learning and Performance: The ARCS Model Approach***. New York: Springer. Now available in English and Japanese. Will soon be available in Korean.

**"Do not seek to follow in the footsteps
of the men of old. Seek what they sought."**
Bashō (1644 – 1694)

Appendix E: Parent and Student Information Letter

The Liberty University Institutional Review Board has approved this document for use from 10/17/14 to 10/16/15
Protocol # 1983.101714

STUDENT PARTICIPANT CONSENT FORM

Traditional vs. Project-based Learning:

The Effects on Student Performance and Motivation

in Honors-Level Mathematics Courses

Sunletha Carter

Liberty University

Department of Education

Introduction: Your child/you (if you are 18 years old or older) is/are being asked to participate in a research study, which is a doctoral class assignment, under the direction of Dr. Jerry Westfall and conducted through Liberty University. In accordance with the policy regarding the protection of human research subjects, the University asks that you give your signed consent for your child to participate in this project. The researcher intends to compare project-based learning techniques with the traditional lecture-based technique in honors level mathematics courses in order to assess the impact on students' achievement and motivation. Your child was selected as a possible participant because he/she may fit the criteria for this study. That is, he/she is currently enrolled in one or the other of the honors level algebra II/trigonometry or pre-calculus courses. This informed consent provides an outline of the facts, implications, and consequences of the research study. I ask that you read this form and ask any questions you may have before agreeing to let your child participate.

Procedure: Your child's teacher will administer a 20-questions pre-and post end-of-unit assessment over the core curriculum standards, taught in one unit of mathematics using project-based or traditional lecture method. If your child is in the Project-Based Learning Group, then he/she will be introduced to a unit of study using a project idea. He/she will then work with 2 or 3 others in a small group to provide the answer to a driving research question, which pertains to the project. On completion of the project, your child will have covered each major concept in the unit. Those who are in the traditional group will also be taught the same unit of study but without a project. They will also be given the same assessment. At the end of the unit, your child will be asked to complete a 36-question instructional material motivation survey. The length of time to complete this survey is estimated to be approximately 20 minutes of class time.

Risks/Benefits: The risks in this survey are no more than the participants will encounter in everyday life. The potential publication of the findings from this study may be beneficial to teachers, administrators, and other educators seeking ways to encourage more students to enroll in honors and advanced placement mathematics courses, as well as improve the achievement and motivation of those who are already enrolled in these courses.

The Liberty University Institutional Review Board has approved this document for use from 10/17/14 to 10/16/15 Protocol # 1983.101714

Compensation: Participants will not receive any compensation for their participation.

Confidentiality: The records of this study will be kept private. In any sort of report I may publish, I will not include any information that will make it possible to identify a subject. Research records will be stored securely and only the researchers will have access to the records. Records will be kept for a mandatory period of three years after which all information collected will be shredded.

Voluntary Nature of the Study: Participation in this study is voluntary. Your decision whether or not to have your child participate will not affect his/her relationship with Liberty University, grade, or future services to which he/she is entitled from the school system. If you decide to allow your child to participate/participate, your child/you are free to not answer any question or withdraw at any time without affecting those relationships. If you agree for your child to participate in this study and decide later that you wish to withdraw, you will be free to do so without any penalty to your child.

Contact and Questions: The researcher conducting this study is Sunletha Carter. You may ask any question you have now. If you have questions later, you are encouraged to contact me directly at (201)306-5803 or through my email, scarter8@liberty.edu. You can also contact my advisor, Dr. Jerry Westfall at (434) 592 - 4681or through his email, jwestfall @liberty.edu. If you have any questions or concerns regarding this study and would like to talk to someone other than the researcher, you are encouraged to contact the Institutional Review Board, 1971 University Blvd., Suite 1837, Lynchburg, VA 24515 or email at irb@liberty.edu

Statement of Consent: I have read and understood the above information. I have asked questions and have received answers. I consent to have my child participate in the study.

Signature of Student:_______________________________ Date:_____________

Signature of Parent or Guardian:________________________ Date:_____________

Signature of Investigator:____________________________ Date:_____________

(NOTE: DO NOT AGREE TO PARTICIPATE UNLESS IRB APPROVAL INFORMATION WITH CURRENT DATES HAS BEEN ADDED TO THIS DOCUMENT.)

Appendix F: Posttest

Name: ______________ Class: ______________ Date: ________ ID: B

Test

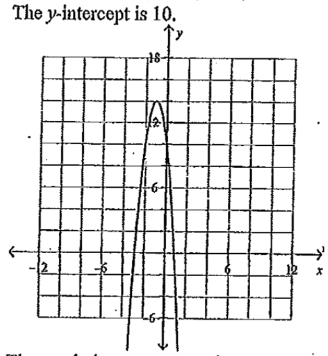
Multiple Choice
Identify the choice that best completes the statement or answers the question.

____ 1. Identify the axis of symmetry for the graph of $f(x) = x^2 + 2x - 8$

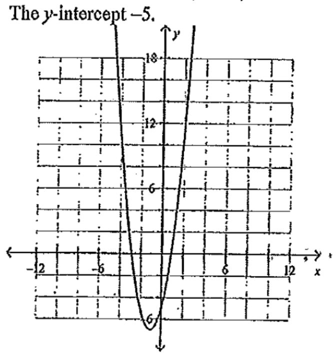
a. $x = -1$
b. $y = -4$
c. $y = -1$
d. $x = -4$

____ 2. Consider the function $f(x) = -2x^2 - 4x + 5$. Determine whether the graph opens upward or downward. Find the axis of symmetry, the vertex and the y-intercept. Graph the function.

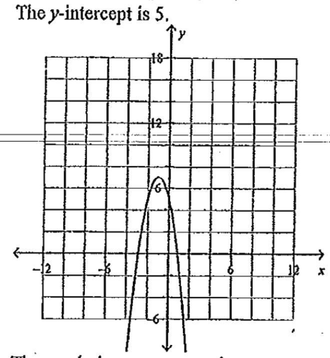
a. The parabola opens downward.
The axis of symmetry is the line $x = -1$.
The vertex is the point $(-1, 14)$.
The y-intercept is 10.



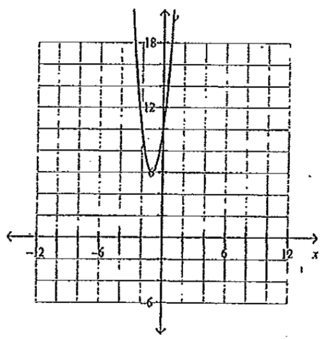
b. The parabola opens upward.
The axis of symmetry is the line $x = -1$.
The vertex is the point $(-1, -6)$.
The y-intercept -5.



c. The parabola opens downward.
The axis of symmetry is the line $x = -1$.
The vertex is the point $(-1, 7)$.
The y-intercept is 5.



d. The parabola opens upward.
The axis of symmetry is the line $x = -1$.
The vertex is the point $(-1, 14)$.
The y-intercept 10.



____ 3. Find the minimum or maximum value of $f(x) = -x^2 - 2x - 5$. Then state the domain and range of the function.

a. The maximum value is -4. D: {all real numbers}; R: $\{y \mid y \le -4\}$
b. The minimum value is -7. D: $\{x \ge -7\}$; R: {all real numbers}
c. The maximum value is 4. D: $\{x \ge -4\}$; R: {all real numbers}
d. The minimum value is -7. D: {all real numbers}; R: $\{y \mid y \le -7\}$

____ 4. The distance d meters traveled by a skateboard on a ramp is related to the time traveled t in seconds. This is modeled by the function: $d(t) = 5.3t^2 - 2.2t + 3$. What is the maximum distance the skateboard can travel, and at what time would it achieve this distance? Round your answer to the nearest hundredth.

a. 0.21 meters in 2.77 seconds
b. 0.23 meters in 4.73 seconds
c. 4.73 meters in 0.23 seconds
d. 5.00 meters in 0.47 seconds

____ 5. Find the zeros of $f(x) = x^2 - 4x - 5$ by using a graph and table.

a. 5 and –1
b. –5
c. –2 and –9
d. –5 and 1

____ 6. Find the zeros of the function $h(x) = x^2 - 23x + 60$ by factoring.

a. $x = 4$ or $x = 15$
b. $x = -20$ or $x = -3$
c. $x = -4$ or $x = -15$
d. $x = 20$ or $x = 3$

____ 7. Find the roots of the equation $-36x - 162 = 2x^2$ by factoring.

a. $x = 9$
b. $x = -9$
c. $x = 3$
d. $x = -3$

____ 8. Solve the equation $x^2 - 10x + 25 = 54$.

a. $x = 5 \pm 3\sqrt{6}$
b. $x = 5 + 3\sqrt{6}$
c. $x = 5 - 3\sqrt{6}$
d. $x = 5 \pm 6\sqrt{3}$

____ 9. Complete the square for the expression $x^2 - 32x +$ ____. Write the resulting expression as a binomial squared.

a. $(x - 8)^2$
b. $(x + 8)^2$
c. $(x + 16)^2$
d. $(x - 16)^2$

____ 10. Solve the equation $x^2 = 3 - 2x$ by completing the square.

a. $x = 2$ or $x = -2$
b. $x = 1$ or $x = -3$
c. $x = -1$ or $x = 3$
d. $x = 2$ or $x = -6$

____ 11. Write the function $f(x) = -5x^2 - 120x - 901$ in vertex form, and identify its vertex.

a. $f(x) = (x + 12)^2 - 181$; vertex: $(-12, -181)$
b. $f(x) = -5(x + 6)^2 - 1$; vertex: $(-6, -1)$
c. $f(x) = (x + 6)^2 - 1$; vertex: $(-6, -1)$
d. $f(x) = -5(x + 12)^2 - 181$; vertex: $(-12, -181)$

____ 12. During the eruption of Mount St. Helens in 1980, debris was ejected at a speed of over 440 feet per second (300 miles per hour). The height in feet of a rock ejected at angle of 75° is given by the equation $y(t) = -16t^2 + 425t + 8200$, where t is the time in seconds after the eruption. The rock's horizontal distance in feet from the point of ejection is given by $x(t) = 115t$. Assuming the elevation of the surrounding countryside is 0 feet, what is the horizontal distance from the point of ejection to the where the rock would have landed? Round your answer to the nearest foot.

a. 1,117 ft
b. 4,546 ft
c. 1,380 ft
d. 8,933 ft

____ 13. For $h(x) = 2x^2 + 6x - 9$ and $k(x) = 3x^2 - 8x + 8$, find $2h(x) - k(x)$

a. $x^2 + 4x - 26$
b. $-4x^2 + 22x - 25$
c. $-4x^2 + 14x - 17$
d. $x^2 + 20x - 26$

____ 14. Find the product $(5x+3)(x^3 + 5x - 2)$.

a. $5x^4 + 3x^3 + 25x^2 + 5x - 6$
b. $5x^4 - 3x^3 - 25x^2 + 25x - 6$
c. $5x^4 - 3x^3 - 25x^2 + 25x - 6$
d. $5x^3 + 22x^2 - 5x - 6$

____ 15. A florist delivers flowers to anywhere in town. d is the distance from the delivery address to the florist shop in miles. The cost to deliver flowers, based on the distance d, is given by $C(d) = 0.04d^3 - 0.65d^2 + 3.5d + 9$. Evaluate $C(d)$ for $d = 6$ and $d = 11$, and describe what the values of the function represent.

a. $C(6) = 15.24$; $C(11) = 22.09$.
$C(6)$ represents the cost, \$15.24, of delivering flowers to a destination that is 6 miles from the shop.
$C(11)$ represents the cost, \$22.09, of delivering flowers to a destination that is 11 miles from the shop.

b. $C(6) = 62.04$; $C(11) = 179.39$.
$C(6)$ represents the cost, \$62.04, of delivering flowers to a destination that is 6 miles from the shop.
$C(11)$ represents the cost, \$179.39, of delivering flowers to a destination that is 11 miles from the shop.

c. $C(6) = 23.43$; $C(11) = 49.62$.
$C(6)$ represents the cost, \$23.43, of delivering flowers to a destination that is 6 miles from the shop.
$C(11)$ represents the cost, \$49.62, of delivering flowers to a destination that is 11 miles from the shop.

d. $C(6) = 22.09$; $C(11) = 15.24$.
$C(6)$ represents the cost, \$22.09, of delivering flowers to a destination that is 6 miles from the shop.
$C(11)$ represents the cost, \$15.24, of delivering flowers to a destination that is 11 miles from the shop.

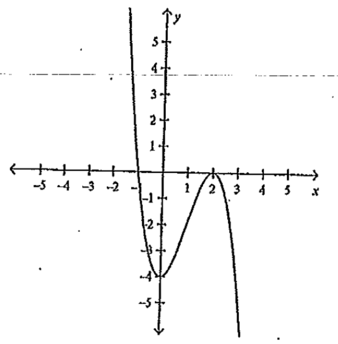
____ 16. Ms. Ponce owns a company that makes specialized race car engines. From 1985 through 2005, the number of engines produced can be modeled by $N(x) = 4x^2 - 2x + 100$ where x is number of years since 1985. The average revenue per engine (in dollars) can be modeled by $R(x) = 8x^2 + 50x + 2{,}000$. Write a polynomial $T(x)$ that can be used to model Ms. Ponce's total revenue.

a. $12x^4 - 100x^2 + 2{,}100$
b. $180x^4 + 300x^3 + 14{,}720x^2 + 17{,}100x + 300{,}000$
c. $32x^4 + 39x^3 - 580x^2 + 9{,}700x + 200{,}000$
d. $32x^4 + 184x^3 - 8{,}700x^2 + 1{,}000x + 200{,}000$

____ 17. Solve the polynomial equation $5x^5 - 10x^4 - 120x^3 = 0$ by factoring.

a. The roots are 0, -6, and 4.
b. The roots are -18 and 12.
c. The roots are 0, 6, and -4.
d. The roots are -6 and 4.

____ 18. Computer graphics programs often employ a method called *cubic splines regression* to smooth hand-drawn curves. This method involves splitting a hand-drawn curve into regions that can be modeled by cubic polynomials. A region of a hand-drawn curve is modeled by the function $f(x) = -x^3 + 3x^2 - 4$. Use the graph of $f(x) = -x^3 + 3x^2 - 4$ to identify the values of x for which $f(x) = 0$ and to factor $f(x)$.



a. $x = -1; x = 2; f(x) = (x+1)(x-2)^2$
b. $x = 1; x = -2; f(x) = -(x-1)^2(x+2)$
c. $x = -1; x = 2; f(x) = -(x+1)(x-2)^2$
d. $x = -1; x = 2; f(x) = -(x+1)^2(x-2)$

____ 19. Factor $(2x-1)^3 - 27$ as the difference of two cubes. Then, simplify each factor.

a. $(2x-4)(2x+2)^2$
b. $(2x-4)(4x^2 - 10x + 13)$
c. $(2x-4)(4x^2 + 4x + 13)$
d. $(2x-4)(4x^2 + 2x + 7)$

____ 20. A jewelry box has a length that is 3 inches longer than the width and a height that is 2 inches smaller than the width. The volume of the box is 216 cubic inches. What is the width of the jewelry box?

a. 2 in.
b. 4 in.
c. 6 in.
d. 5 in.

Appendix G: Permission to Use Diagram in Figure 4.10

Sunletha Carter <srcarter221@yahoo.com>
To achurches@kristin.school.nz May 31 at 9:07 PM

Mr. Churches,
Good evening. My name is Sunletha Carter, and I am writing you at this time about the above mentioned diagram, which I originally located on Google images. I am currently writing my dissertation manuscript and would love to include this diagram which has all the features of 21st century pedagogy, including Project-Based Learning, the focus of my study. I must have written permission to use this diagram but I have not been able to locate its creator. If you are the original designer, do I have your permission to reproduce it in my manuscript? If it is not yours, I would appreciate if you would direct me to its creator, if you are able to do so. I appreciate your assistance, immensely.
Thank you.
Sunletha Carter

Reply, Reply All or Forward | More

Andrew Churches <achurches@kristin.school.nz>
To Sunletha Carter May 31 at 9:49 PM

Hi Sunletha
the documents were created under the creative common share and share-alike license. As such please feel free to make use of this. Thank you for asking

A

Andrew Churches
achurches@kristin.school.nz
Head of faculty Technology and Design,
Kristin School
Albany, Auckland

[redacted]

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