A COMPARISON OF CHRISTIAN SCHOOL AND PUBLIC SCHOOL GEOMETRY TEACHERS CONCERNING THE BELIEFS AND PRACTICES OF TEACHING PROOFS

by
Benjamin Caleb Lane
Liberty University

A Dissertation Presented in Partial Fulfillment
Of the Requirements for the Degree
Doctor of Education

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ABSTRACT

Theorists contend that mathematics teachers' beliefs influence their practices; consequently, differing Christian and public school philosophies should lead to different practices. However, some researchers have questioned if Christian education is "truly distinct" from public education. Other researchers have noted that this question is still open and that the philosophical differences between Christian and public school teachers might not be translating into differences in practices. A causal-comparative study was conducted between Christian and public school geometry teachers to investigate these differences. This study took place in Florida and Georgia using an instrument designed to measure four different aspects of teaching geometry proofs. An overall difference between the public school teachers ($n_p = 32$) and Christian school teachers ($n_c = 31$) was evaluated by using a multivariate analysis of variance and was found to be statistically insignificant, Wilk's $\Lambda = .926$, $F(4, 57) = 1.137$, $p = .348$.

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Table of Contents

List of Tables and Figures

CHAPTER 1: INTRODUCTION

Background

Hoeksema (1992) has asked if Christian schools provide an education that is "truly distinct" from that offered in the public schools. While there is a clear distinction in religious philosophy, Hoeksema's concern was that this religious philosophy might be the *only* difference between public and Christian education. His own research indicated that this could be the case (Hoeksema, 1991). Hull (2003) revisited this topic and expressed similar uneasiness that "in spite of the things Christian educators know and do, what normally passes for *Christian education* can be more accurately named *Christian educating* . . . [which] stands for a Christianity-enhanced public school brand of education" (p. 204, emphasis in the original).

Thus, the question of "truly distinct" Christian schools is really to ask whether or not they offer an education identical to the public schools merely viewed through a Christian perspective. Boerema (2011) notes several theories that explain why the lack of a distinction is plausible but fails to cite any studies that conclusively establish or eliminate this lack in the first place.

One way to identify a distinction would be to examine a topic that is inclined to be taught similarly in both public and Christian schools. If there is a distinction even in this topic, then that distinction could be extrapolated to subjects with more obvious differences. An appropriate method of identifying a topic that would tend to be taught similarly would be to first examine underlying philosophies.

One inescapable tenet of Christian education is the insistence upon the existence of truth. Christ Himself proclaims in no uncertain terms, "I am . . . *the truth* . . . no man cometh unto the Father, but by me" (John 14:6). Yet modern public education was founded by individuals that denied this notion of truth. Consider John Dewey's (1897/1959) assertion that "education must be conceived as a continuing reconstruction of experience" (p. 27). According to Darling and Nordenbo (2003), "Dewey wanted to replace the prevailing view that knowledge can be uncovered as a definite and permanent truth, with a new understanding: That knowledge is individual, teleological, instrumental, and relative" (p. 293).

Such belief forms a basis for the philosophy of constructivism, yet only the most extreme constructivists (such as von Glasersfeld [1989]) require truth to be relative. More moderate constructivists (such as Piaget [1972]) allow that there may be absolute truths, yet it is impossible to recognize those absolute truths as such.

Common ground between the radical constructivist and Christian philosophies would be that one can construct for himself an understanding of absolute truth. There is certainly a place for the direct transmission of truth. For one, direct instruction is efficient; for another, it may be required. Consider that Christianity is based upon direct revelation and that humanity could never empirically discover certain aspects of biblical truth. On the other hand, truth might be absolute, but human understanding of that truth is limited. For instance, the only way to further an understanding of nature is to build an understanding of new information in the light of existing knowledge. It is at this juncture of traditional and constructivist thought that a comparison of Christian and public school practices can be undertaken. In particular, do Christian and public educators hold convergent or divergent views of the teaching of logical thinking?

"Come now, and let us reason together" (Isaiah 1:18). With this call to the sinner, God is making an appeal to one of the distinguishing characteristics of humanity, the ability to reason. Hannam (2010) notes that Christianity has long had an association with the teaching of logic, extending well back into the Middle Ages. Though much of that medieval education was from a Roman Catholic perspective, the educators recognized that a study of logic is consistent with a study of the Scriptures. For example, even the apostle Paul, "as his manner was, went in unto [the Jews at Thessalonica], and three sabbath days reasoned with them out of the scriptures" (Acts 17:3).

A purely secular approach to learning logic reaches back even farther into history. Since the time of the ancient Greeks, students have studied logic and reasoning by developing an axiomatic system of logic. Typically, this was conducted by proving the theorems of Euclidean geometry (Kline, 1953). Consequently, the study of mathematics in general has grown into the primary educational discipline for the learning of critical thinking skills. For instance, the National Council of Teachers of Mathematics (NCTM) contends that reasoning and proof are still "fundamental aspects of mathematics" (2000, p. 56). Yet it is in the specific discipline of geometry that proof has found a unique home.

However, the actual implementation of the reasoning and proof has been in constant flux. Over a century ago, the familiar two-column form of proof was developed, greatly simplifying the proving process for beginning geometry students (Herbst, 2002). More recently, characterizations of proof have been moving away from Euclid's purely deductive approach to a more inductive approach that relies on experimentation. For instance, the NCTM (2000) often considers demonstrations using concrete examples as a surrogate for formal proof. Knuth (2002) concludes, "The role of proof in school mathematics in the United States has been peripheral at best" (p. 61).

Thus, the perceptions of proof in the geometry class can serve as a tool for gauging perspectives concerning logic and reasoning. If Christian schools are indeed "truly distinct," then Christian education should be exhibiting a strong emphasis on teaching logical thinking. Recalling the historical connection between logical thought and Euclidean geometry, the distinction should translate into Christian education maintaining a strong emphasis on teaching this system of geometry.

Problem Statement

The problem is that there is insufficient information to compare the beliefs and practices of Christian and public school geometry teachers concerning proofs in the geometry class. This lack of information ranges from teachers' beliefs about the purpose of the geometry class to various practices implemented in the actual teaching.

According to Ernest's model (1989), mathematics teachers' beliefs directly influence teacher practices. While much has been researched about the effects of mathematics teachers' beliefs on teaching proofs, much of this research has considered belief to be teachers' opinions concerning students' abilities to learn proofs (Peterson, Fennema, & Carpenter, 1989; Staub & Stern, 2002; Torff, 2005). Thus, the examined beliefs concern cognition, not content. Knuth (2002) further notes that the research on teachers' beliefs that does focus on content typically investigates teachers as mathematicians, not as *teachers* of mathematics. Such research focuses on the role of proof in learning how to create mathematicians instead of how to create broad-minded students who happen to think mathematically.

However, Truelove (2004), investigated teachers' beliefs and practices from a pedagogic perspective, creating a testing instrument for these beliefs and practices in the process (see Appendix A). He identified four characteristics that comprise geometry teachers' beliefs and practices concerning proofs: Concept, approach, usage, and practices. Throughout this study, the term *aspects* will serve as a general reference for these four characteristics.

Purpose Statement

The purpose of this causal comparative study was to test Ernest's (1989) theory of mathematics education that one's beliefs influence one's teaching by comparing public and Christian school geometry teachers on four aspects of teaching geometry proofs. These aspects were defined by the four characteristics of geometry teachers' beliefs and practices given above. The comparison of these aspects was conducted using a causal-comparative research design.

Significance of the Study

The significance of this study stems from Hoeksema's inquiry into whether or not Christian schools offer an education that differs in more than just an injection of Christian views. A superficial survey of Christian education would seem to indicate that a difference does extend into school practices, but more detailed investigations by Christian researchers have failed to conclusively demonstrate this. Boerema (2011) noted that, among the needed areas of research in Christian education, "The research area that was of most interest was that of the gap between Christian school mission and its practice" (p. 44). But even if such a gap exists, would it matter?

The philosophical difference between public and Christian schools is stark. The Christian school is based on and promulgates biblical principles while the public school is compelled to maintain a religiously-neutral atmosphere. But how does this difference play out in actual practice? Are biblical principles independent from other fields of study? Are students taught (perhaps implicitly) to compartmentalize into sacred and secular? Is it possible to effectively learn subject matter regardless of spiritual emphasis? Proverbs 1:7 states, "The fear of the LORD is the beginning of knowledge." Thus, the biblical position is that genuine learning is an outgrowth of biblical truth.

Starting with certain truths and building to others is the domain of axiomatic systems such as geometry. Thus, understanding a teacher's (and, by extension, a school's) perceptions of such systems can elicit underlying conceptions of truth and its application to other disciplines. Knight (2006) remarks, "[A]ssumptions such as the orderliness of the universe and the validity of empirical observation are metaphysical and epistemological presuppositions that undergird science but are rejected by many modern people in both Western and Eastern cultures" (p. 238). Therefore, if a Christian school differs from a public school only in religious instruction, the pedagogic practices could effectively undermine that biblical foundation. A dichotomous view of truth would emerge in which the Bible is taught as true, but all other knowledge is suspect. This view conflicts with a unified biblical perspective in which the Bible is true, and all other truths are then recognized as the products of the same God that authored the Bible.

But offering a distinct education is also important out of a simple pragmatism for the Christian school itself. The religious nature of the Christian school precludes access to public funding due to the separation of church and state. These schools must then seek other funding, usually through tuition. Parents who pay this tuition elect to do so despite the availability of publicly-financed public education. Thus, all privately-financed schools, not just those Christian, must offer a superior education worthy of the additional investment. Consider that one study examined the motives of parents who withdrew their children from public education in favor of private education (Bukhari & Randall, 2009). It is notable that even though this study took place in the highly religious state of Utah (Jones et al, 2011), the most important factor influencing these parents was the quality of the curriculum.

An exhaustive examination of the differences spanning entire curricula in Christian and public environments is too broad for one study. However, Euclidean geometry as practiced throughout the last few centuries is the one subject in which traditional education adopts its most constructivist practices. Thus, if there is evidence of a difference in Christian school and public school aspects of teaching geometry proofs, then there is some evidence that Christian schools are "truly distinctive."

There are indications that a difference *should* exist. For instance, there is a stated difference in goals in certain geometry curricula. Consider the following statement in the *Plane Geometry* textbook from the Christian textbook publisher A Beka Book: "Your success in geometry will be measured directly by the ability and power you develop to logically prove statements yourself. It is the logic that you learn in the process, along with the geometric facts proven, that counts" (McLaughlin, Collins, & Ashworth, 2006, p. *viii*). Conversely, though the NCTM (2000) summarizes the traditional view of geometry as "the place in the school mathematics curriculum where students learn to reason and to see the axiomatic structure of mathematics" (p. 41), proof is just one of the thirteen "expectations" for high school geometry students. González and Herbst (2006) argue that using geometry as a medium for teaching formal reasoning "plays no role in the justification of the study of geometry within the rhetoric of the [NCTM] Standards movement" (p. 24).

The question though, as posed by Hoeksema (1992) and Hull (2003), is if this difference actually *does* exist. Do different curricula translate to different practices? Do teachers ignore differences in curricula and teach a largely homogenous brand of geometry? Is the tuition spent for Christian education producing students who are no different academically from their public school counterparts? Are Christian schools truly distinct? An evaluation of pedagogic beliefs and practices in the geometry classroom should help to inform the answer to this question.

Research Questions

The primary research question that has guided this study is as follows:

RQ1: Is there a difference between Christian and public school geometry teachers on the aspects of teaching geometry proofs in the geometry class?

If a difference were found concerning these aspects, the secondary research question would be evaluated:

RQ2: In which of the four aspects of teaching geometry proofs can a difference between Christian and public school geometry teachers be identified?

Research Hypotheses

The first research question was evaluated by investigating the following research hypothesis:

H_1: As measured by Truelove's (2004) questionnaire, Christian school geometry teachers differ from public school geometry teachers in at least one of the four aspects of teaching geometry proofs.

Because there are four aspects of teaching proof, there were four additional research hypotheses that were used to answer the second research question:

H_2: As measured by Truelove's (2004) questionnaire, Christian school geometry teachers' *concept* of geometry proof is different from that of public school geometry teachers.

H_3: As measured by Truelove's (2004) questionnaire, Christian school geometry teachers' *approach* to geometry proof is different from that of public school geometry teachers.

H_4: As measured by Truelove's (2004) questionnaire, Christian school geometry teachers' *usage* of geometry proof is different from that of public school geometry teachers.

H_5: As measured by Truelove's (2004) questionnaire, Christian school geometry teachers' *practices* involving geometry proof are different from that of public school geometry teachers.

These research hypotheses led to the following null hypotheses:

H_{o1}: As measured by Truelove's (2004) questionnaire, Christian school geometry teachers do not differ from public school geometry teachers in any of the four aspects of teaching geometry proofs.

H_{o2}: As measured by Truelove's (2004) questionnaire, there is no difference between Christian school geometry teachers' *concept* of geometry proof and that of public school geometry teachers.

H_{o3}: As measured by Truelove's (2004) questionnaire, there is no difference between Christian school geometry teachers' *approach* to geometry proof and that of public school geometry teachers.

H_{o4}: As measured by Truelove's (2004) questionnaire, there is no difference between Christian school geometry teachers' *usage* of geometry proof and that of public school geometry teachers.

H_{05}: As measured by Truelove's (2004) questionnaire, there is no difference between Christian school geometry teachers' *practices* involving geometry proof and that of public school geometry teachers.

Identification of Variables

The independent variable in this study was the type of school in which a geometry teacher presents geometry proofs. Generally speaking, there are two types of schools under consideration: Christian and public. However, the immense size of the educational system in the United States (for both public and private schools) made studying the entire nation prohibitive. Thus, this study was restricted to public and Christian schools in Florida and Georgia. As a result, the two types of schools can be more accurately described as follows:

Public school: One of the regular public schools in the states of Florida and Georgia as recognized by the Florida Department of Education (FLDOE, 2011) and Georgia Secretary of State (2011), respectively;

Christian school: Any school in Florida or Georgia that offers a course in geometry and is a member of the Association of Christian Schools International (ACSI), the Florida Association of Christian Colleges and Schools (FACCS), or the Georgia Association of Christian Schools (GACS).

The dependent variables in this study were the four aspects of teaching geometry proofs as given by Truelove (2004). These aspects are as follows:

- Concept: The teacher's belief in proof as foundational to the geometry class or just a topic (among several) within that class;
- Approach: The teacher's preference to emphasize inductive or deductive reasoning;
- Usage: The number of techniques used by the teacher when teaching proof; and
- Practices: The amount of instructional time the teacher devotes to teaching proof.

Measurement of these aspects of teaching proof was conducted using the questionnaire developed by Truelove for his study (2004). This questionnaire is a 32-item instrument which measures the aspects of concept, approach, and usage using a four-point Likert scale. The measurement of practices is conducted using a five-point scale that categorizes the percentage of instructional time devoted to various practices.

Assumptions and Limitations

Assumptions. One key assumption in this study was that each participant would provide an accurate glimpse of that teacher's aspects of teaching proof. Unfortunately, a common issue in experimental designs is compensatory rivalry (Gall, Gall, & Borg, 2007). This situation arises when members of a control group exert additional effort in an experiment because of a perceived competition with the experimental group. Explicit mention of a comparison between Christian and public schools could perhaps induce a similar effect, leading some participants to embellish results. Thus, all contact with participants and their schools minimized reference to the comparison between the types of schools.

Limitations. Because the causal-comparative research design is non-experimental, there is irony in that its main limitation is that it cannot prove causation, only suggest it. Thus, this study was not able to show if where one teaches affects one's beliefs and practices when teaching geometry proofs. It is plausible (if not likely) that one's beliefs actually influence where one teaches.

A key reason that a causal-comparative design is non-experimental is that there is a lack of randomization into the different levels of the independent variable. It would have been impractical and unethical to conduct an experimental study in which teachers are randomly assigned to a teaching environment and then those teachers' aspects of teaching geometry proofs are measured. A more realistic approach was to proceed with a study of existing teachers in the existing environments which those teachers have deliberately chosen. However, this self-selection into groups forfeited the strength of argument that randomization affords.

When conducting the study that created the questionnaire, Truelove (2004) also was confronted with the problem of self-selection as teachers chose whether or not to complete the survey. In a sense, this somewhat confounds the study in that the main goal of comparing Christian and public school geometry teachers became a comparison of Christian and public school geometry teachers *who choose to complete a survey*. However, there are techniques that can be used to mitigate this nonresponse (Gall, Gall, & Borg, 2007).

Another possible source of confounding is that private schools in Florida and Georgia are not subject to state regulation (U.S. Department of Education, 2009). Public school teachers are thus subject to state licensure requirements while Christian school teachers are not. It is possible that some aspect of the licensure process could have influenced teacher beliefs or practices. However, this issue was minimal because the causal-comparative design cannot establish causation. Because the study was not investigating the notion that one's type of school *causes* one's beliefs and practices, an outside causal influence did not appreciably affect the findings.

Research Plan

This study was conducted using a causal-comparative (*ex post facto*) research design. This is a non-experimental design which is useful for determining if groups that differ on some independent variable will also differ on some dependent variable (Gall, Gall, & Borg, 2007). In this study, the type of school (Christian or public) was the independent variable; the aspects of teaching proof (concept, approach, usage, and practices) were the dependent variables.

Statistically significant differences in the aspects between Christian and public school geometry teachers were determined by first conducting a multivariate analysis of variance (MANOVA). This test is useful if comparing multiple groups on multiple dependent variables (Spicer, 2005). If the MANOVA identified a significant difference, *t*-tests for the differences between means were conducted on each of the four aspects to determine in which areas these difference exist (Stevens, 2002).

CHAPTER 2: LITERATURE REVIEW

Introduction

Some Christian educators have wondered if Christian education provides a distinct education from that offered in the public schools (Boerema, 2011; Hoeksema, 1992; Hull, 2003). The purpose of this study was to see if there evidence of a distinction by examining the aspects of teaching geometry proofs in Christian and public school geometry teachers.

This literature review will first examine the philosophical differences between Christian and public education. Assuming such philosophical differences exist, a theoretical framework for geometry teacher beliefs and practices concerning geometry proofs will be identified. The remaining literature will tie into this framework, showing existing research concerning the relationship between teacher beliefs and subsequent pedagogic practices concerning proofs. This will demonstrate that there is a gap in understanding if Christian and public school geometry teachers have different beliefs and practices concerning geometry proofs.

Philosophical Background

Philosophy of education. A comparison between current practices in geometry instruction can be found at the end of a long sequence of related issues. Changes in aspects of teaching geometry proofs result from changes in the geometry class; changes in the geometry class result from changes throughout the entire field of mathematics education; and changes in mathematics education result from the overarching changes in education as a whole.

Changes in education throughout the past do not necessarily require differences in approaches in the present; certain developments could appeal to all educators. Yet it is also possible that some may consider those developments to interfere with educational goals. Certain such developments can be shown to have led to a divergence between Christian and public education.

This split has its roots in the 1800s with the rise of universal public education. Prior to this era, public education had a strong connection with Christian teachings. Consider the inception of public education in 1647 with Massachusetts's Old Deluder Satan Act. This law required towns with suitable populations to construct "public" schools with the stated intention of preventing the "one chief project of that old deluder, Satan, to keep men from the knowledge of the Scriptures." Such a concept is foreign to the modern concept that the public school is to be kept free *from* religious influence. In contrast, Massachusetts sought to create the public school as an instrument *of* religious influence, particularly an instrument with a Protestant character (Cunningham, 1940).

Furthermore, the mode of instruction was of the traditional, teacher-centered format, utilizing the drill of spelling, reading, and basic arithmetic. This traditional pedagogy has enjoyed a historical link with religious education of all types, mainly as the result of the perceived urgency in conveying established, universal truths to the students. Resnick (2008) writes, "The very purpose of traditional—especially religious—education is to induct the young into a unique vision of reality" (p. 107). He continues, "Traditional education . . . has a received vision of the good life to guide its work. Indeed, both parents and educators often elect such education precisely because it offers an ethical anchor in uncertain times" (Resnick, 2006, p. 329).

To the Christian, this "unique vision" is that "all things were created by [the Lord], and for him" (Colossian 1:16). Consequently, the "good life" for the Christian is found in service to Him. Unfortunately, man's "heart is deceitful above all things, and desperately wicked" (Jeremiah 17:9). Thus, it is impossible for man to rationalize his way to God. Instead, God has made Himself known to man by revealing His existence through His creation (Romans 1:20) and His standards for morality through the Bible. Consider Deuteronomy 6:6-7a: "And these words, which I command thee this day, shall be in thine heart: *And thou shalt teach them diligently unto thy children*" (emphasis added). Thus, the direct instruction that typifies traditional education has its roots in the

Judeo-Christian ethic. It is no surprise then that there would be such a strong link between traditional education and religious instruction.

This link is further reinforced by observing that the gradual move from a traditional, teacher-centered education to a more student-centered perspective corresponds with the gradual secularization of education. The first significant push in this direction was to come through the writings and travels of Horace Mann in the mid-1800s. As the first secretary of the newly-created State Board of Education in Massachusetts, Mann was discouraged that the system of education in his state was inferior to that of others—especially considering the leading role that Massachusetts had shown earlier (Cremin, 1957). In his twelve annual reports to the Board, Mann delicately prescribed changes that would change the citizenry's expectations of public education. These changes that would ultimately create a climate that would undermine confidence in traditional education.

One such change that would affect the teacher-student relationship considerably was in reaction to observations from an 1843 tour of European schools. Mann (1957) was struck by the contrast between his perception of the harsh, authoritarian atmosphere in American settings and the "beautiful relation of harmony and affection which subsisted between teachers and pupils" of Prussian schools in particular (p. 55).

A second change was to broaden the scope of education. No longer was education to be the privilege of those fortunate enough to live in a town of sufficient population. Education was now to be the privilege of *all* citizens. Even more so, education was now to be the *right* of all citizens. Mann stated in 1846 in the Tenth Annual Report this conviction:

> I believe in the existence of a great, immutable principle of natural law, or natural ethics,—a principle antecedent to all human institutions and incapable of being abrogated by any ordinances of man,—a principle of divine origin, clearly legible in the ways of Providence as those ways are manifested in the order of nature and

> in the history of the race,—which proves the *absolute right* of every human being that comes into the world to an education; and which, of course, proves the correlative duty of every government to see that the means of that education are provided for all. (1957, p. 63, emphasis in the original)

Such changes in public expectations were peripheral compared to Mann's one true philosophical change to shift the mandate for education from religious to societal purposes. Mann (1957) argued that "our political institutions,—founded, as they are, upon the great idea of the capacity for self-government" can only be preserved by men which have been prepared for self-government by an "apprenticeship [that] must commence in childhood" (p. 58).

These suggestions and resulting changes did not necessarily require abandoning a traditional pedagogy, but several effects of these changes were to sour public sentiment towards traditional education. First, educators no longer saw their mission as a religious commitment with transcendent values, but rather a political duty to prepare individuals for a system of government in which values were subject to the changing impulses of men. Thus, a teacher did not answer to some higher being; instead, through a republican form of government, the teacher indirectly answered to himself. Second, a flood of additional students into the system would require a flood of additional teachers. Combined with the change in mission, the pool of quality traditional-Christian teachers was to be diluted considerably.

Cremin (1964) has chronicled the rise of the progressive movement in the public schools and considers the breaking point of the nation's discontent with traditional education to be 1892's publication of the findings of Joseph Mayer Rice in a series of articles in *The Forum* (Cremin, 1964). Rice traveled the nation, stopping at city after city and observed, "With alarming frequency the story was the same: Political hacks hiring untrained teachers who blindly led their innocent charges in singsong drill, rote repetition, and meaningless verbiage" (Cremin, 1964, p. 5). The irreparable damage to the reputation of traditional education left a vacuum that would be filled by the rise of progressive education. Cremin (1964) describes the transition this way: "In a sense, the revolution Horace Mann had sparked a generation before—the revolution inherent in the idea that everyone ought to be educated—had created both the problem and the opportunity of the Progressives" (p. *ix*).

This "opportunity of the Progressives" was seized by the man generally credited as the "Father of Progressive Education," John Dewey (Graham, 1967). As such, his views have shaped much of the direction of education in America. Even the Christian school movement can trace its history to Dewey's influence in that these schools were formed as a reaction to the progressivism, in particular a purely secular progressivism, that had pervaded the public school environment (Knight, 2006).

The key concept concerning Dewey's philosophy is that "education . . . is a process of living and not a preparation for future living" and the "the school is primarily a social institution" (1897/1959, p. 22). In other words, since working adults do not usually sit passively in a classroom environment, neither should students. Since adults are actively participating in their duties and better understanding ("learning") those duties as a result of the work, so also should students learn by way of physically engaging in activities. Since adults have the freedom to select their own vocations, students should likewise select the activities in the classroom.

Thus, Dewey envisioned the school creating a social environment in which both the teacher and pupil collaborate rather than for the formal passing of information from teacher to pupil (Dewey, 1916). Mann had tried to soften the authoritarian image of the teacher; now Dewey was pulling the student alongside the teacher as an equal. Dewey (1897/1959) stated, "I believe that the discipline of the school should proceed from the life of the school as a whole and not directly from the teacher" (p. 24). Note that this belief does not merely move the student's authority figure from one individual (the teacher) to another (the administration), but it changes the actual *nature* of that authority.

Dewey's vision of the school is a social, democratic institution; thus, the student plays a vital role in the running of school. Ultimately, just as the teacher was to become his own master from the effects of Mann's teachings, the student was likewise to become *his* own master from the effects of Dewey's teachings. One consequence was that the student now was to play a vital role in the determination of the curriculum, choosing only subjects of his personal interest.

Another effect upon the school's curriculum is that the subject matter being presented in the classroom must contain immediate relevance to the students (Dewey, 1916). Recall Dewey's belief that education *is* life, not just preparation for life. Thus, he praises such subjects as literature and fine arts because the master works within these subjects are often readily available for study. While the students may lack the nuance to appreciate the details distinguishing one fine work from another, the works can still be valued on their own merits. However, Dewey (1916) identifies mathematics among the other skill-intensive subjects that do not allow their higher forms to be studied and appreciated so quickly under their traditional development models. Years of practicing the fundamentals of these subjects are usually necessary before the concepts have been developed to the point of useful application. This traditional practice draws the particular ire of Dewey:

> No one can tell in how many schoolrooms children reciting in arithmetic or grammar are compelled to go through, under the alleged sanction of method, certain preordained verbal formulæ . . . Nothing has brought pedagogical theory into greater disrepute than the belief that it is identified with handing out to teacher recipes and models to be followed in teaching . . . Mechanical rigid woodenness is an inevitable corollary of any theory which separates mind from activity motivated by a purpose. (1916, pp. 199-200)

Dewey was to take a particular interest in the field of mathematics, partly because of his association with the philosophy of pragmatism. Charles Peirce was the originator of the pragmatic philosophy and studied mathematics. In fact, his son Benjamin Peirce would later become a famed mathematician and coin what is perhaps the best-known definition of mathematics: "Mathematics is the science which draws necessary conclusions" (Peirce, 1882, p. 1).

According to Campos (2010), Charles Peirce's philosophy holds that "awakening the students' faculties of mathematical reasoning has relative educational priority over teaching definitions, postulates, axioms, and propositions and their demonstrations, without denying the importance of guiding the students to build a body of mathematical knowledge over time" (pp. 434-435). The subtle difference between this attitude and that of traditional education is that traditional educators would consider these basic facts to be part of the body of absolute truths and thus just as worthy of appreciation as the later arrangement of those facts using the principles of logic. Furthermore, the teacher's role in "guiding the students" in the construction of the facts is in striking contrast to the traditional conception in which the teacher directly instructs the students in those facts.

Thus, the pedagogical break with traditional education had been complete. Other changes from the traditional-Christian moorings of education were still to take place throughout the twentieth century (e.g., the removal of prayer from public schools in 1962), but the broad philosophy of education had changed from a Judeo-Christian, teacher-directed model of traditional education to a socially-oriented, student-directed model.

Philosophy of mathematics. Holding competing educational philosophies is not the only difference that one would expect when comparing modern public and Christian schools. One can also expect to find a difference between secular and Christian philosophies concerning mathematics.

Consider first two major works that help to shape the prevailing thought concerning a traditional-Christian philosophy of mathematics. First, Morris Kline, a noted philosopher of mathematics during the twentieth century, addresses the overall question of truth in his *Mathematics: The Loss of Certainty* (1980). Kline has written extensively on a broad range of mathematical topics, including histories of mathematics (1953, 1972) and a treatise on the effects of the abandonment of traditional mathematics education (1973). A Christian philosophy of mathematics can be found in Nickel's *Mathematics: Is God Silent?* (2001). This book was motivated by the constant questioning that Nickel faced as a mathematics instructor concerning the biblical position on mathematics.

In addition to these works, many other writers have addressed the philosophy of mathematics. The early twentieth century mathematician David Hilbert (1983) expressed, "From time immemorial, the infinite has stirred men's *emotions* more than any other question" (p. 185, emphasis in the original). He soon added the remark that "mathematical analysis is a symphony of the infinite" (Hilbert, 1983, p. 187). In other words, the study of mathematics has, at its core, an emotional effect upon finite man comprehending an infinitely complex universe. Thus, there is the tantalizing allure of a certainty in mathematics in a world that many see as containing no absolutes. This perceived paradox has led to many different schools of thought concerning the nature of knowledge in mathematics.

The foremost question concerning knowledge in mathematics is the question of truth in mathematics. For, if there is no truth in mathematics, the question of one's ability to know those truths becomes moot. However, the question runs much deeper than determining if learning mathematics is simply a waste of time. The question runs to the heart of educational philosophy. Consider the comments of Paul Ernest, a leading author on the relationship between mathematical philosophy and mathematics education (White-Fredette, 2010) and the creator of the mathematical philosophy of social constructivism (Ernest, 1991). In his *The Philosophy of Mathematics Education*, Ernest tries to establish a philosophy of mathematics as a prerequisite to a philosophy of mathematics education. He notes that "if mathematics is a body of infallible, objective knowledge, then it can bear no social responsibility" (Ernest, 1991, p. *xii*). Conversely, if "mathematics is a fallible social construct . . . [then] the aims of teaching mathematics need to include the empowerment of learners to create their own mathematical knowledge" (Ernest, 1991, p. *xii*).

Thus, the nature of truth in mathematics has much more than accuracy in application at stake. From an educational standpoint, the nature of truth in mathematics is the very fulcrum upon which the scale between traditional and progressive thought pivots.

The first distinction among the different schools of mathematical thought concerns the nature of mathematical truth as a product of mathematics alone. Ernest (1991) lumps each school into one of two major camps. Absolutism is the view that "mathematical truth is absolutely certain, that mathematics is the one and perhaps the only realm of certain, unquestionable and objective knowledge" (p. 3). Fallibism is the opposite view in which "mathematical truth is corrigible, and can never be regarded as being above revision and correction" (p. 3). Ernest further refines among the various factions within each of these camps, but the distinctions deal more with semantics than the overarching theme of the power of mathematics to determine truth.

Some authors interpret Ernest's dichotomy to hold that absolutism is simply the belief that mathematics contains absolute truth (White-Fredette, 2010). After all, he does make the claim that to reject absolutism is to state that "mathematical knowledge is not absolute truth" (Ernest, 1991, p. 18). He later then the following paradoxical claim: "The rejection of absolutism should not be seen as a banishment of mathematics from the Garden of Eden, the realm of certainty and truth" (1991, p. 20). Ernest's more commonly-used application of the term seems to indicate that there are philosophies of mathematics that permit absolute truth without being under the umbrella of absolutism.

Kline (1980), Nickel (2001), and Ernest (1991) generally agree on the main lines of thought in absolutism: Logicism, formalism, and intuitionism. Ernest (1991) actually labels intuitionism as constructivism, but notes that the leading figures in constructivism were intuitionists. Kline (1980) identifies a fourth branch—the set theorists—that, with minor compromises, could be grouped with the logicists. Regardless of these mild discrepancies, the three writers also agree that absolutism in all of these forms is logically untenable. Any system of logic (even Christianity [Knight, 2006]) requires unprovable postulates to serve as a starting point. In essence, absolutism demands that *all* facts be provable, but there is no way to prove the postulates. The dilemma facing absolutism is that "Deductive logic only transmits truth, *it does not inject it*" (Ernest, 1991, p. 13, emphasis added).

A more formal demonstration of the lack of mathematics to hold truth in and of itself was given by Kurt Gödel in 1931 with his famed paper, "On Formally Undecidable Propositions of *Principia Mathematica* and Related Systems" (Kline, 1980, pp. 260-261). Gödel demonstrated that any axiomatic system is incomplete; there are statements which simply cannot be proven either true or false. Consider Christian Goldbach's famous conjecture that every even integer greater than two can be written as the sum of two prime numbers (i.e., 4 = 2 + 2; 6 = 3 + 3; 8 = 3 + 5). For over 260 years, this concept has defied attempts to be proven either true or false. Nickel (2001) notes that, as a result of Gödel's findings, mathematicians do not know if the conjecture has resisted being proven true because it is actually false or because it is one of the unprovable concepts of arithmetic. Nickel is quick to point out that such conjectures are certainly either true or false—not in some muddied middle—but simply unable to be deduced logically from the given axioms.

However, such observations fall far short of asserting that mathematics is indeed fatally flawed. Mathematics is just fallible in a philosophical vacuum. Even Ernest (1991) admits that external sources could provide certainty within mathematics. He further notes several views within fallibism that allow for such exterior fundamentals in mathematics. The most notable of these beliefs is the Platonic view that "the objects and structures of mathematics have a real existence independent of humanity, and that doing mathematics is the process of discovering their pre-existing relationships" (Ernest, 1991, p. 29).

This view would most closely mirror that of a Christian view of mathematical truth since the broader Christian worldview holds to absolutes. Scripture is full of absolute statements such as "I *am* the LORD, I change not" (Malachi 3:6a) and "No man cometh unto the Father, but by me" (John 14:6b). However, the requirements to hold to an absolutist position in mathematics are simply too strict for the Christian. Using Ernest's dichotomy then, the Christian view of mathematics would fall into the fallibist camp by default.

Nickel (2001) identifies the Platonic basis of Christian mathematics, stating that "the *ultimate* foundation for truth in mathematics must, of necessity, *lie outside the system of mathematics* and *outside the reach of man's mind*" (p. 192, emphasis in the original). The main conflict between absolutism and Christianity would be the source of the absolutes. Absolutists believe that the absolutes of mathematics are derived from mathematics alone while biblical Christians believe that the absolutes of mathematics were established by God ("I am . . . the truth") and can be understood by men having been created in the image of God (Genesis 1:27). Christians thus believe in God as the Source of this existence and man's inquiry as a creation in God's image as this process of discovery.

The establishment of absolute truth is insufficient to construct a comprehensive philosophy of mathematics, for the purpose of mathematics must also be addressed. Biblically, one of the fundamental responsibilities of mankind is to subdue the earth (Genesis 1:28). Therefore, the purpose of mathematics for the Christian is to assist mankind in this process. The famed scientist Johannes Kepler stated, "The chief aim of all investigations of the external world should be to discover the rational order and harmony which has been imposed on it by God and which He revealed to us in the language of mathematics" (as cited in Kline, 1980, p. 31).

Kline (1972) notes further that, not only can the natural world be described mathematically, the natural world serves as the motivation for man's study of mathematics. He remarks, "Nature is the matrix from which ideas are born. The ideas must then be studied for themselves. Then, paradoxically, a new insight into nature, a richer, broader, more powerful understanding, is achieved, which in turn generates deeper mathematical activities" (p. 204). In other words, nature inspires the scientist to discover a mathematical explanation for some phenomenon after which the newly-discovered mathematics serve several roles: Describing the phenomenon; inspiring even deeper mathematics; and ultimately describing yet unforeseen and unanticipated phenomena. Nickel (2001) even proposes that this cycle of connecting nature to mathematics and back serves as the fundamental pedagogical principle of Christian mathematics education.

To summarize, consider Kronecker's famous assertion that "God created the natural numbers; everything else is man's handiwork" (as cited in Gaither & Cavazos-Gaither, 1998, p. 275). The Christian philosophy of mathematics could be described by a twist on Kronecker: God created all of mathematics, and gave man the natural numbers and nature itself as the first clues to its properties. It is no surprise that mathematics can describe natural processes since the same God that created all of nature created mathematics as well.

History of mathematical proof. The history of mathematical proof is virtually indistinguishable from the history of mathematics in general. The earliest historical records of mathematics are the counting techniques developed by the Egyptians and, more notably, the Babylonians who employed a base-60 number system. Kline (1953) remarks that the calculations employed by these ancient civilizations were "of the rule-of-thumb or practical variety" (p. 17). These were empirically derived formulae that served architectural and agricultural purposes rather well. This was especially the case for the Egyptians whose accurate and enduring constructions amaze even modern engineers.

However, these applications lacked a deductive, logical basis to justify their accuracy. It took centuries before the ancient Greeks took the next step and introduced formal logic into their mathematical investigations. The most famous and enduring of the Greek analyses were the geometric principles found in *The Elements*. Though attributed to Euclid, the famed Greek actually served as an editor of the existing research into geometry and collated that material into one coherent volume (Kline, 1953). Yet, his contribution cannot be overstated. Perhaps the most enduring testament to Euclid's ability is that most modern geometry textbooks are merely revisions of his work.

Many of these revisions have been for pedagogic purposes. For one, Euclid's fifth postulate is as follows:

> That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles (Heath, 1956, p. 202).

This description was understandably difficult for students to grasp. However, Playfair's geometry text was to popularize a simpler, but logically-equivalent version of the postulate. He substituted Euclid's postulate as, "Two straight lines which intersect one another, cannot be both parallel to the same straight line" (Playfair, 1819, p. 21).

Another profound addition was the relatively recent development of the famous two-column format for writing proofs. Herbst (2002) has chronicled the rise of this format, starting with the simple numbering of steps by Beman and Smith (1899) and reaching two-column *statement*-and-*reasons* pattern by Schultze and Sevenoak (1913).

But for promoting the study of pure logic, *The Elements* in its original form is in a category all its own. Kline declares, "Western man learned from the Euclidean *Elements* how perfect reasoning should proceed, how to acquire facility in it, and how to distinguish exact reasoning from vague mouthings which carry merely the pretence of proof" (1953, p. 54). The final structure of mathematical proof was thus in place as early as the third century B.C., but this only created a template for formal mathematical inquiry. It has taken the twenty-four centuries since to compile the mathematical knowledge enjoyed today.

There have been serious issues concerning the limits of proof during this time, most notably Godel's famous proof that any consistent axiomatic system must be incomplete. But his conclusion that there are propositions that *cannot* be solved through deductive reasoning has not displaced the role of proof as the singular medium for facilitating skill in logic.

Theoretical Framework

The biblical concept of learning is described in Jeremiah 28:10: "For precept must be upon precept, precept upon precept; line upon line, line upon line; here a little, and there a little." In other words, the individual cannot grasp more involved concepts until the more fundamental concepts have been learned. Scripture also provides the first of these concepts: "The fear of the LORD is the beginning of knowledge" (Proverbs 1:7). It is thus quite evident that the Bible teaches that one's beliefs not only influence the learning process, the proper beliefs are necessary for genuine learning in the first place. This study will be constructed around theories that incorporate the philosophies of belief as foundational and the learning model of "precept upon precept."

The study of Euclidean geometry is well-suited for exploring these theories. As an axiomatic system, the beliefs (the *axioms*) that undergird the system are rather arbitrary. One can more or less pick any starting principles and then explore the logical development of those principles. In the biblical vernacular, one can explore how the founding precepts lead logically to other precepts. Additionally, a study of geometry provides transference of skills into the real world, for the ability to construct mathematical proofs in an axiomatic system is indicative of an ability to construct logical arguments in general.

Ernest (1989) has developed a rather straightforward model of teaching mathematics. He asserts that a teacher's attitudes and knowledge concerning a topic mold that teacher's beliefs concerning instruction on that topic. Those beliefs then work with the underlying attitudes and knowledge to shape the teacher's instructional practices. This entire process operates under the guiding influence of the teacher's background.

On the converse side of teaching philosophy is epistemology, the study of learning processes. The specific epistemological theory for this study is cognitivism, especially the stages of development espoused by Piaget (Piaget & Inhelder, 1969). Though Piaget's description of the progression through the stages underwent modifications throughout his life, the most common presentation of the states is as follows:

1. Sensori-motor (approximately 0-2 years old): The child learns physical movement and develops awareness of sensory signals;
2. Preoperational (approximately 2-7 years old): The child learns to interpret sensory signals and begins to think about relationships between and among objects;
3. Concrete operational (approximately 7-11 years old): The child can solve concrete problems and demonstrates pre-abstract concepts such as categorization; and
4. Formal operational (approximately 11 years old through adulthood): The child can form genuinely abstract thoughts and apply principles of logic.

That these stages would be closely connected to a grasp of logical thought closes a loop in the history of educational psychology. For, just as Peirce exerted a heavy influence upon Dewey's pedagogy, Peirce would also influence Piaget's epistemology.

Piaget acknowledges that his notion of proactive and retroactive implications are borrowed from Peirce's predictive and retrodictive implications (Piaget & Garcia, 1991).

Piaget's stages of development can also be described using mathematical analogs such as the "three worlds of mathematics" described by Tall (2008). Yet, the van Hiele levels present the stages of learning from a purely geometric perspective. The van Hiele levels were developed in the 1950s by the Dutch husband and wife team of Pierre van Hiele and Dina van Hiele-Geldof. Though the Soviet Union expressed an interest in the van Hieles' work, American educators were largely ignorant of the levels until Usiskin (1982) introduced them into the mathematics education literature.

According to Usiskin, the van Hiele levels progress as follows:

1. Recognition: The recognition of shapes, but only if oriented the "correct" way (i.e., a square rotated 45° will not be recognized as a square);
2. Analysis: The recognition of properties of shapes, but not relationships between different shapes;
3. Order: The recognition of abstract properties of shapes and relationships between shapes, but without understanding of the nature of those properties as an outgrowth of more fundamental properties such as formal definition;
4. Deduction: The deductive formulation of new concepts from axioms and definitions, yet an ignorance of the arbitrary nature of those axioms and definitions; and
5. Rigor: The highest level of geometric abstraction in which the nature of axioms is understood; permits an understanding of the consistency of non-Euclidean geometries.

Recall that the concrete demonstrations of geometric concepts espoused by the NCTM are replacing (not merely augmenting) the abstract examples that have served as the basis for geometry study for centuries. According to the van Hiele levels, this practice is stunting the growth into the deduction and consequently the rigor levels of geometric understanding.

In fact, one of the properties of the van Hiele levels is that they operate in *fixed sequence,* meaning that a student *cannot* progress to the next level without having mastered the previous level (Usiskin, 1982). Thus, if training in deductive logic is weakened or even omitted, a student will not be able to grasp the ultimate geometric notion of rigor. Echoing Piaget's levels of learning, this is hindering a student's formal operational development of logic. This problem is magnified by the nearly exclusive hold that geometry possesses on the study of logic in the grade school curriculum.

Note that this cognitive development mirrors the historical discoveries and development of mathematics. The child must first learn what numbers are before learning their basic properties and interactions. The next stage is to learn simple applications that relate those properties to the environment. Finally, the child can be instructed in the abstraction of those applications.

There is empirical support for the van Hiele model. One longitudinal study showed students progressing from perceptual to theoretical thinking (Küchemann & Hoyles, 2006). Furthermore, the reading comprehension of geometry proof (RCGP) model has been shown to be valid (Yang & Lin, 2008). Though the RCGP model evaluated six constructs, the construct of *appreciation* proved too difficult to accurately incorporate into the model (Yang & Lin, 2008). The final RCGP model presented the remaining five constructs as progressing in the following manner:

1. *Basic knowledge*: The transition from a "surface" reading of terms and concepts to "recognizing elements" about those terms and concepts;
2. *Logical Status* and *Summarization*: The recognized elements are chained; and
3. *Generality* and *Application*: The chained elements are "encapsulated" into a final product.

The similarity of the RCGP model to the lower levels of the van Hiele model is obvious. Thus, the empirical verification of this RCGP model provides an indirect verification of the validity of the van Hiele model.

However, while the van Hiele levels present a description of a student learning geometry, this study is concerned with teacher perspectives on the topic. The geometric paradigms described by Houdement and Kuzniak (1999) present a spectrum of views on the role of geometry proofs. The geometric paradigms are as follows:

1. Natural Geometry (Geometry I/Experimental Geometry): Conclusions are connected directly to physical interaction; predictions are based on experiences through the senses; there is no place for proving the obvious
2. Natural Axiomatic Geometry (Geometry II): Truths are deduced logically rather than experimentally; however, the axioms of that system are inferred from physical reality and are crafted to reflect that reality
3. Formal Axiomatic Geometry (Geometry III): Deductive reasoning is the sole method for establishing validity; the axioms of the system are independent of any grounding in physical reality and need only be consistent (thus the establishment of logical validity rather than genuine truth)

A detached view of geometry recognizes that the subject can be legitimately applied in any of these contexts. For one, geometry is used to describe real-world phenomena. In fact, the word *geometry* literally means “earth measure.” Additionally, geometry is used to explain the processes of abstract reasoning—a type of “meta-abstraction.” The question in this study was to determine if public and Christian schools hold different pedagogic perspectives on the place of geometry as a tool for educating students.

To summarize, this study was based upon the biblical principle of “precept upon precept.” Piaget’s levels of cognitive development provided an epistemological framework that mirrors this principle. The van Hiele levels gave a specifically geometric description of student cognitive development, and the geometric paradigms provided a reference for teacher perspectives on the relative value of applications versus rigor.

Review of the Literature

Mathematicians' perspectives of proof. A discussion on the educational role of proof must first begin with a glance at the role of proof as seen by the professional mathematicians that are most familiar and dependent upon proof. Reuben Hersh is a noted philosopher of mathematics, especially on the topic of proof and logic. He notes two different mathematical definitions for proof. His "working" definition of proof is "an argument that convinces qualified judges"; the "logic" definition of proof is "a sequence of transformations of formal sentences, carried out according to the rules of the predicate calculus" (Hersh, 1993, p. 391).

Both definitions are necessary to demonstrate the pure mathematician's dilemma. A sound logical proof is often bogged down in technical minutia while a more informal (but perhaps technically errant) version presents the "spirit" of the argument and illustrates the beauty of the interplay of concepts. Hersh remarks, "Mathematicians prefer a beautiful proof, even if it contains a serious gap, over a dull, boring one" (1993, p. 394).

Thus, there is no one "right" definition of proof, though there are likely many "wrong" ones. The Euclidean geometry studied in high school usually takes the "beautiful but flawed" route. In this approach, precision of terms is often given short shrift and occasional logical shortcuts are taken in order to progress more quickly to the study of the more familiar properties of figures. Beyond a point though, the "gaps" become more infrequent, and technical details are more often required.

Student perspectives of mathematical proof. The key purpose behind the use of proofs (in all branches of mathematics) by the broad mathematical community is the discovery and dissemination of new mathematical discoveries. However, beginning students of mathematics are not going to be generating any of these new discoveries.

And for the most part, the students recognize this. They see two other, main purposes for proofs: The demonstration of one's own understanding and the exchange of ideas (Gfeller, 2010).

Though there will be obvious differences in the depths of thought, students using proofs to exchange ideas is no different than professional mathematicians using proofs to exchange ideas. However, it is expected that those professional mathematicians will have long since demonstrated their understanding of mathematics. In contrast, students are by definition acquiring that understanding and are in the process of convincing both themselves and their instructors of their progress.

This leads to a "didactical contract" in which the teacher and student each have designated roles in the learning of proofs (Herbst & Brach, 2006). This is the simple expectation that teachers are to teach proof methodology and students are to learn how to construct proofs. During this process, there is the tacit understanding that learning to construct proofs is an end in itself and independent from new mathematical inquiry. Within the van Hiele framework, this learning to do proofs is a process whereby students progress through discrete stages of geometric development before finally achieving the ability to think in a genuinely logical manner.

The lowest of the van Hiele levels involve the mastery of geometric shapes and their properties. During these stages, students often can find and learn these properties, but only if they have been shown what they should be looking for (Babai, Zilber, Stavy, & Tirosh, 2010). The methods of showing the students these concepts seem to favor the traditional definition of teaching. Aydin and Ubuz explain that "students should have a rich store of basic facts to adopt adequate procedures to the solution process while they should execute proper algorithms in which they recognize the correct facts" (2010, p. 443).

Regardless of the philosophy of education, however, merely progressing through these lower stages does not mean that students will automatically succeed in the higher levels and ultimately master logical thought. Students must be deliberately taught proof techniques in order to recognize the properties of valid proof (Aydın & Ubuz, 2010). Thus, to hint at the truth of a theorem using concrete examples that draw on students' existing knowledge is insufficient for students to make the cognitive leap to deductive reasoning.

This is not to say that such hints are not useful. Using non-rigorous techniques (such as the use of a protractor) in the midst of a problem can provide clues to the eventual rigorous solution (Bjuland, 2004). Furthermore, employing these techniques gives the students an opportunity to experiment with new concepts. Such experimentation is important, for there is a gradual transition as the teacher's verbal description of that concept is interpreted and understood (Brown & Heywood, 2011). This transition often incorporates incomplete analogues that serve as intermediate stages on the path to full understanding. One example noted students using the four seasons (a discrete description) while learning about planetary motion (a continuous process) (Brown & Heywood, 2011). However, employing a multitude of techniques in the learning of these abstract concepts can be daunting. Students have a tendency to make problems more difficult than necessary by working too fast or being overwhelmed by the number of presented techniques (Bjuland, 2004).

Yet, once the needed basic facts have been established and the higher levels of learning are reached, one technique stands out among others. Students learn best when the teacher questions the students about their existing notions and continually forces the students to re-evaluate those notions (Martin, McCrone, Bower, & Dindyal, 2005). The students' responses and teacher's continued questioning spiral toward the final correct conception of the problem. Such a teaching style enforces the belief that the students' thought processes are the ultimate focus when learning mathematical proof instead of the final proof itself.

Unfortunately, there are a few obstacles to student learning. For one, there is a common complaint among students is that proofs are dull. This is true even among the college students taking upper-level mathematics courses that have shown a proclivity for mathematics (Basturk, 2010). Many students, even prospective student teachers, frankly consider proof to be a "waste of time" for the typical high school student (Varghese, 2009).

One possibility for this attitude is the relative newness of proof to the mathematical curriculum. Prior to the geometry class, classroom mathematics activities typically reduce to solving routing numerical exercises. Students consequently rationalize that calculation-oriented topics are both easier (because of familiarity) and more important (because of years of emphasis) than geometric concepts (Barrantes & Blanco, 2006).

The attitude that proofs are a "waste of time" could also be a result of confusion over the purpose of proof in the first place. Many of these students tend to consider proof a uniquely mathematical exercise (Varghese, 2009). This is especially true in the field of geometry because of its rare use outside of that discipline (Knuth, 2002). However, proof is a tool that spans all fields of mathematics. Even more generally, deductive proofs can exist independently of mathematics altogether. Historically, geometry has been considered the best medium for teaching proof, but proof is not confined to geometry. The ultimate purpose of teaching proof has not been solely to do mathematics, but instead to reason.

Another major difficulty clouding student learning is that the very use of the word *proof* itself can be confusing. Recall that even mathematicians have different interpretations of the term depending on the situation. Students have generally only heard *proof* as synonymous with *argumentation* and thus have a distorted image of genuine logic (Pedemonte, 2007). There is a tendency to equate verification of specific instances with the proof of the general case. This confuses *inductive* thought with *deductive* thought. But even though specific cases may be far from adequate for proving a general case, these specific instances are not irrelevant since students often recognize the general case through the lens of the specific (Pedemonte, 2007).

Yet, this recognition of the general case—and its eventual proof—needs nurturing. The teacher is responsible for guiding the students through this transition to deductive reasoning. This requires using completely different tactics and presentation from even the rough proofs that help validate new concepts in other mathematics classes (Gfeller, 2010). For instance, many algebra teachers present the derivation of the quadratic formula but refrain from discussing this derivation as a genuine proof (Knuth, 2002).

Additionally, much of this transition to formal deductive reasoning takes place in a social setting. Recall that Hersh's working definition of proof involves "convincing qualified judges." There is an element of communication in the process of proving. There is certainly an element of convincing the teacher, but also an element of convincing fellow classmates who are, more or less, training to become "qualified judges." Kuzniak and Rauscher remark, "Geometry, and more generally mathematics, as taught in school, is a human activity that is embedded in a social system and cannot be reduced to abstract signs managed by formal systems" (2011, p. 134). This is not to say that mathematics is *entirely* a social construct (as Ernest would contend). Instead, a successful proof requires successful communication, and successful communication requires an audience with which to communicate.

Such a social setting also allows students to demonstrate the historical social interaction of the mathematical community. In such a setting, students propose and challenge new concepts that remain consistent with existing knowledge. However, Kaisari and Patronis note, "From our point of view, historical situation and conflicts need not be repeated in the classroom but may help in organizing students' epistemological interaction and negotiation of meaning" (2010, p. 267). Thus, geometry as a social activity is more helpful as a tool for gauging (rather than developing) student understanding. Consequently, the employment of social interaction in the geometry classroom does not require traditional educators to surrender the teacher-directed atmosphere. It merely affords the students an opportunity to demonstrate their knowledge in a manner consistent with the totality of the purposes of proof.

Teacher perspectives of mathematical proof. Because the focus of this study is to compare public school and Christian school teachers' aspects of teaching proofs, the current understanding of teacher attitudes towards proof is critical. Fortunately, much of the work in this regard has been accomplished in the previous section, for a considerable portion of teacher attitudes can be explained by the above examination of student attitudes. Despite later exposure to other perspectives and practices, geometry teachers tend to mimic their experiences as students (Barrantes & Blanco, 2006).

However, the role of teaching proof places the teacher in a different position than that of the student. Teachers see proof from the perspective of pedagogy in addition to perspective of students. But these pedagogic perspectives have been in flux for a very long time. Recall the rise of universal public education throughout the late nineteenth and early twentieth centuries chronicled earlier. With Mann's desire to produce an informed citizenry and Dewey's connection with the Peirce's and the Pragmatists, it is no surprise that the role of mathematics was to be discussed intently by educational leaders.

The discussion of the particular role of geometry was to receive special emphasis. Leaders of all types (not just educational) had long believed in the place of geometry for the development of logical acumen. President Lincoln, for instance, attributed his ability to argue entirely to his study of geometry (Robinson, 1918). The earliest formal recommendations concerning the role of geometry in the broad curriculum mimicked these sentiments and sought to incorporate the abstract study of Euclid as a medium for strengthening the mind (González & Herbst, 2006).

However, such a desire was short-lived. The National Committee of Fifteen on Geometry Syllabus (1913) was to conclude that the emphasis on the abstract "has been magnified and extended . . . beyond the interest and appreciation of the average pupil" (p. 52). The subsequent recommendation from the committee was that a "judicious selection of a reasonable number of abstract originals be made in order to leave time for an equally reasonable number of problems . . . stated in concrete setting" (1913, pp. 52-53). Thus was introduced the first formal suggestion that geometry have a problem-based component similar to that of the study of other branches of mathematics. The last hundred years has been the story of constant struggle for the appropriate balance (the "reasonable number") of abstract and concrete problems.

There is a recurring theme in the literature that what a teacher perceives as the purpose of the geometry course directly influences how that course is actually taught. In other words, a teacher's beliefs dictate what a teacher will deem to be this "appropriate balance." Generally, those who lean toward the notion that geometry is a tool for developing logical thinking emphasize the abstract proofs and reasoning. Those who lean toward the belief that the geometry class should teach real-world spatial relationships emphasize calculation-based problems instead.

There have been several efforts to formally categorize these beliefs. For instance, González and Herbst (2006) have studied geometry education throughout the past century and have identified four main themes (*arguments*) that have commonly arisen concerning the purpose of the geometry class:

- Formal Argument: The value of proof lies in the training of deductive reasoning rather than in learning specific mathematical truths;
- Utilitarian Argument: Specific geometric concepts are part of a broad effort to prepare students to enter the workforce;
- Mathematical Argument: Geometry prepares students for the specific working environment of the mathematician; and

- Intuitive Argument: Geometric concepts illustrate the abstract properties of concrete phenomena.

Another categorization of teacher beliefs would be with respect to "authentic mathematics." Though Wiggins (1989) was the first to suggest the notion of authentic learning, Lajoie, Lawless, Lavigne, and Munsie were the first to propose a definition of authentic mathematics. They describe this as "mathematical activities that are meaningful to the learner, represent applications of mathematics to everyday life, and are activities that mathematicians would carry out" (as cited in Graue, 1993, p. 292).

Weiss, Herbst, and Chen (2009) identified four different ways that mathematics educators envision authentic mathematics (abbreviated *AM* in publication):

- AM_W (*w*orld): The use of mathematics in real-world environments;
- AM_D (*d*iscipline): The study of mathematics as a formal discipline; the use of mathematics by professional mathematicians in communication;
- AM_P (*p*ractice): The use of mathematics as practiced by professional mathematicians when discovering new concepts; and
- AM_S (*s*tudent): The treatment of the students as novice mathematicians, subject to the same freedoms that professional mathematicians enjoy.

Weiss, Herbst, and Chen (2009) then studied teacher attitudes toward geometry proofs—in particular, attitudes toward the two-column format—as related to those teachers' view of authentic mathematics. Differences in these views matter since teachers who view a problem from a particular perspective often take issue with student solutions from a different perspective (Kuzniak & Rauscher, 2011). Thus, a student could perhaps solve a problem in a manner that is consistent with that student's perspective, but a teacher with a different perspective might not recognize the validity of that solution.

Many of the differences in teacher attitudes revolved around the amount of liberty to grant the students in making assumptions in the midst of formulating a formal proof.

Those who held to the AM_W perspective varied considerably on this issue. Some remarked that everyday situations involve making assumptions that facilitate interactions; others countered that allowing such assumptions can have serious, negative consequences, especially in these real-world environments that lie outside the relatively safe confines of the classroom.

The teachers who held to the AM_D and AM_P views were more uniform in their attitudes. Those with the AM_D perspective were much more adamant that assumptions have no place within the proving process, contending that formal logic breaks down if each step is not justified before proceeding to the next. In other words, "The *form* of the two-column form is conflated with the *method* of proof" (Weiss, Herbst, &Chen, 2009, p. 284, emphasis in the original). Conversely, the AM_P view corresponded to a more relaxed attitude toward assumptions that allows for the assumption of truth long enough to test if the current line of proof is fruitful in the first place.

It is unsurprising that the teachers with the AM_S perspective did not hold any strong opinions about the place of assumption in the midst of proof. Since the students in this environment are granted a great deal of independence, they may feel their way through their own interpretation of the proving process. The teacher's role is merely to ensure that the students correctly accomplish those tasks which the students have decided are pertinent to their overarching goals.

While Weiss, Herbst, and Chen (2009) were able to determine attitudes toward proof compared with one's perspectives on the ultimate purposes of mathematics, they did not determine how frequently the different perspectives occur in the educational world. This lack of research into the frequency of teacher perspectives is a rather common theme throughout the world of mathematics education.

One notable exception to this issue is the study conducted by Truelove (2004). Truelove identified the four aspects of teaching geometry proofs that serve as the basis for this current study and found the following among public school teachers in the Northwest Arkansas and Southwest Missouri region:

- Concept: There was no preference for proof as foundational or for proof as a topic within the broad geometry course;
- Approach: There was a preference for using the inductive approach as an introduction to deductive proof over the reliance solely on deductive proof;
- Usage: A majority of teachers favor using multiple instructional activities to teach proofs; and
- Practice: A vast majority of teachers spend less than 40% of available time on the teaching of proofs; during the times of teaching, the teachers favored the use of teacher-directed instruction.

This survey of public school geometry teachers presents the very real possibility that there is not a clear-cut difference between Christian and public school approaches to teaching geometry. There appears to be a preference for the teacher-directed instruction that is historically associated with traditional—hence, religious—education. But conversely there is a relatively low amount of class time devoted to proofs as compared with the historical notion of geometry as being primarily the deductive development of an axiomatic system.

Measurement of teacher perspectives. Therefore, it is perhaps most likely that any differences between Christian and public school geometry teachers would be with the aspect of approach though differences in any of the aspects of teaching proof could suggest an overall difference. However, instruments that can gauge these aspects are quite rare.

Certainly there are instruments that measure attitudes about mathematics. For instance, Fennema and Sherman (1976) have developed several scales that measure attitudes toward learning mathematics. From these scales, Utley (2007) has selected the scales most appropriate for specifically gauging attitudes toward learning geometry.

However, such scales focus on attitudes about *learning* mathematics, not *teaching* mathematics.

Truelove (2004) faced the same difficulty when conducting his own research and addressed the issue by incorporating into his study the construction and validation of a new instrument that measures the aspects of teaching proofs (see Appendix A). Of particular interest is the aspect of approach.

While each of the four aspects is measured in Truelove's instrument, there is a striking correlation between the aspect of approach and the geometric paradigms that serve as a framework for this study. The Geometry I perspective describes those favoring an inductive introduction to deductive reasoning that is almost to the exclusion of deduction altogether. Geometry II represents the use of induction to serve as motivation for the axioms that originate the deductive development. Finally, Geometry III describes the purely deductive perspective that could allow for the development of a system from a completely arbitrary beginning. Thus, while it is regrettable that there is no simple instrument that measures teacher beliefs according to the geometric paradigms (A. Kuzniak, personal communication, December 29, 2011), this loss is more than compensated by the breadth of information that can be gathered by Truelove's instrument.

Summary

There are historical evidences that Christian and public schools exhibit a deep philosophical divide. In the field of mathematics, this divide can be expressed as the difference between the acknowledgement and rejection of absolute truths. However, there is no empirical evidence that this has led to real educational differences between the two types of schools. This is compounded by the overall lack of research in Christian education in general.

The many differing perceptions of geometry proofs provide a fertile environment to search for differences between Christian and public school geometry teachers. The questionnaire developed by Truelove serves as a valid instrument for measuring any such differences.

CHAPTER 3: METHODOLOGY

Introduction

The purpose of this study was to explore distinctions between Christian and public education by examining the aspects of teaching geometry proofs in Christian and public school geometry teachers. This chapter will provide the details of the methodology used to guide the causal-comparative research design that was selected to explore this topic.

This chapter will first provide a snapshot of the participants in the study, including the selection of those participants and the setting from which they were drawn. This chapter will then describe the instrument of data collection and the procedures used in its implementation. Next, the appropriateness of the selection of a causal-comparative research design will be discussed. Finally, this chapter will describe the methods of data analysis that permitted the testing of the research hypotheses.

Setting

This study took place by surveying geometry teachers from Florida and Georgia. According to the U.S. Census Bureau (2012a, 2012b), the Black population of these two states is higher than the United States as a whole; otherwise, the combined demographics of Florida and Georgia are comparable to that of the entire United States. Thus, the public school environments in these two states are reasonably comparable to those nationwide.

The Christian school environments in Florida and Georgia are also reasonably comparable to the national Christian school environment, but the term *Christian school* needs clarification. Private schools in the United States are categorized by government agencies as being Catholic, other religious, or nonsectarian (U.S. National Center for Education Statistics, 2011). Schools classified as "other religious" are further subcategorized as conservative Christian, other affiliated, and unaffiliated. This study examined schools that would be considered "conservative Christian."

According to the National Center for Education Statistics (NCES, 2008), the largest non-Catholic religious school association in the United States (in terms of both number of schools and student enrollment) is the Association of Christian Schools International (ACSI). Thus, the "ACSI Core Beliefs" provide a reasonable look at the characteristics of conservative Christian schools. The ACSI Core Beliefs maintain that parents hold the chief responsibility for the education of their children; that academics be held in high regard; and that the Bible is not only permitted, but studied as a core subject (ACSI, 2008). The Florida Association of Christian Colleges and Schools (FACCS) and the Georgia Association of Christian Schools (GACS) are separate associations of Christian schools with standards and beliefs similar to those of ACSI (FACCS, 2008; GACS, n.d.). The schools selected for this study were chosen from the membership directories from these two Christian school associations.

Participants

A multiple analysis of variance (MANOVA) test and the *t*-test for independent means were the two statistical tools used for evaluating the collected data. The *t*-test has the more stringent requirements, setting the minimum number of participants necessary for obtaining statistical significance and observing a desired effect size (Wilson VanVoorhis & Morgan, 2007). The recommendations for this study suggest a minimum of 64 participants, divided evenly between the two groups (Gall, Gall, & Borg, 2007). Thus, this study required at least 32 public school and 32 Christian school teachers.

The Christian school teachers chosen for this study were selected by conducting a random sample of those teachers' schools from the membership directories of ACSI, FACCS, and GACS. Membership in these organizations is voluntary; the associations do not exert any governance over the schools. Instead, these associations typically serve as an educational resource and government liaison. Consequently, these schools are autonomous and make decisions about inclusion in this study on an individual basis. Schools were randomly selected (using *simple* random sampling [Gall, Gall, & Borg, 2007]) from the membership lists of the included associations, and the administrators at those schools were contacted for permission to distribute the survey to their geometry teachers.

In contrast, public school governance is at the district level, generally at the county level. To select public schools for inclusion in the study, schools districts were randomly selected from Florida and Georgia and the superintendents' offices contacted. As a result, permission from the districts filtered down to all of the schools and geometry teachers in that district, and all the geometry teachers in the selected districts were included in the study. This selection process is known as *cluster* random sampling (Gall, Gall, & Borg, 2007).

Truelove (2004), who investigated teachers' beliefs and practices from a pedagogic perspective and creating a testing instrument for these beliefs and practices in the process (see Appendix A),experienced a response rate of approximately 50% which is consistent with typical rates in educational research (Baruch & Holtom, 2008). Thus, the desired minimum of 32 responses each was anticipated by distributing 75 surveys to public school teachers and 75 to Christian school teachers..

The response rate to a survey can be improved through various techniques. One such method is to include a small financial incentive for completing the survey. Studies have shown that a five-dollar incentive is more effective than one- or two-dollar incentives (Doody, et. al., 2003). Furthermore, the effects of an incentive are similar whether the incentive is conditional on the participant's response or if the incentive is unconditional and thus included with the survey (Edwards et al., 2002). However, anonymity would be compromised by the tracking of results required to ensure that only those who complete the survey receive the incentive. This study thus included a five-dollar, unconditional incentive in the form of a Starbucks coffee gift card.

Instrumentation

The data for this study were collected by using the questionnaire (see Appendices A and B) developed by Truelove (2004). In addition to educational background questions, Truelove's instrument contains 32 items that measure the aspects of teaching geometry proofs. However, Truelove expressed dissatisfaction (personal communication) with the performance of two items, believing that they slightly confounded the results for the aspect of approach. He further expressed a desire to see how the instrument would perform with slight revision. Thus, this study used a 30-item revised version of Truelove's survey that omits statements 6 and 14 from the original version.

The four subcategories of the survey correspond to the four aspects of teaching proofs. Each of the 30 questions corresponds to one of these aspects:

- Concept: Questions 1, 7, 10, 13, 14, 19, 21, 22, and 23;
- Approach: Questions 2, 3, 5, 8, 11, 12, 15, and 16;
- Usage: Questions 4, 6, 9, 17, 18, and 20; and
- Practice: Questions 24, 25, 26, 27, 28, 29, and 30.

The responses for the concept, approach, and usage categories range from "[describes me] very well" to "[describes me] not at all." These were given numerical values from 4 (very well) to 1 (not at all) which were averaged into one overall score for each aspect (see Figure 1). Thus, the possibility of scores for *concept* ranged from 1 to 4 with higher values corresponding to the view that proof is foundational to geometry and lower values indicating that proof is viewed as just a topic in the class. For *approach*, scores ranged from 1 to 4 with higher values indicating a preference for an inductive approach and lower values indicating a preference for a deductive approach. *Usage* scores ranged from 1 to 4 with higher values corresponding to the use of a variety of instructional techniques and lower values indicating the opposite. One important

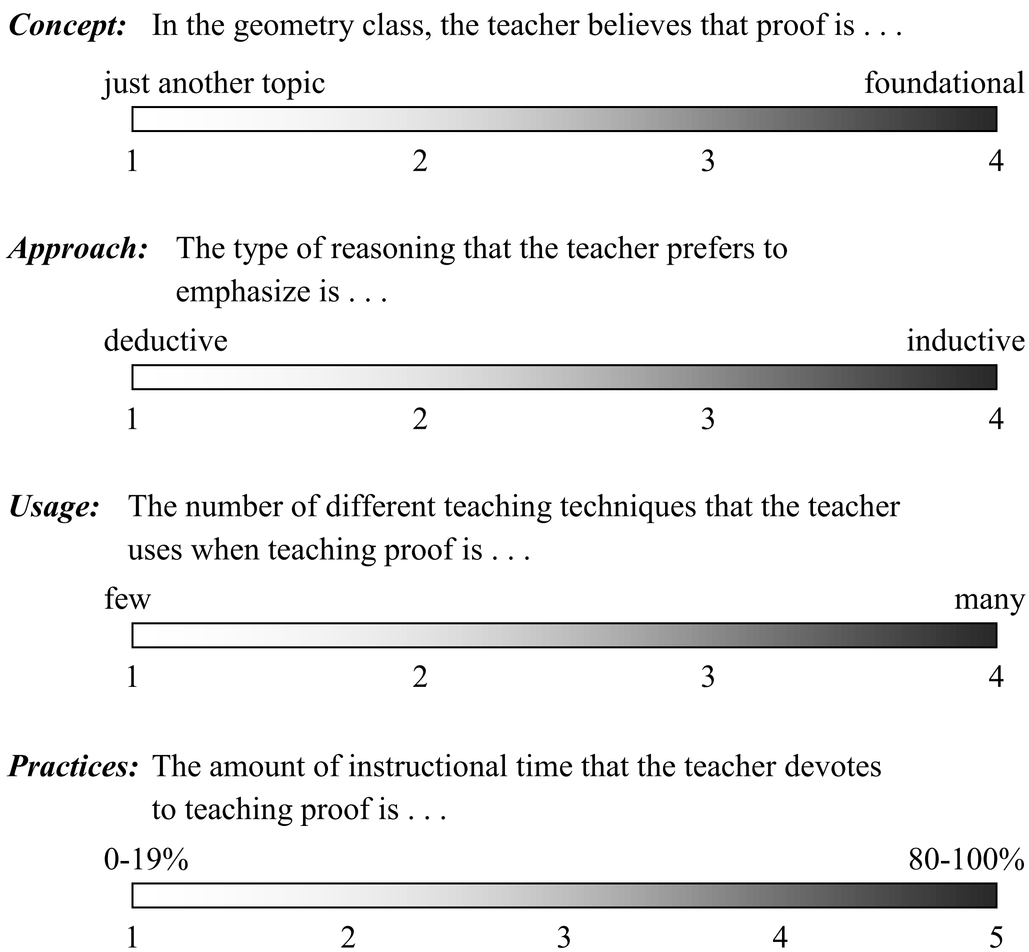

Figure 1. Range of possible scores for each aspect of teaching geometry proofs. Each of these scores is the average of all of the participant's responses for that particular aspect.

consideration is that questions 3, 5, 12, 15, 20, 21, and 22 are reverse-scored, contributing to test validity.

The responses for the *practices* category ranged from “0 to 19” to “80 to 100,” where each value indicated the percentage of instructional time per week devoted to different instructional activities. The numerical equivalents to these values extended from 1 to 5, corresponding to the “0 to 19” and “80 to 100” categories, respectively. Averaging the responses yielded total practice scores ranging from 1 to 5 with higher scores indicating more instructional time devoted to teaching proof and lower scores indicating less time.

To distinguish surveys that were distributed to public schools from those distributed to Christian schools, Question 38 was worded differently. For public schools, the question read as originally written by Truelove, “What was your major in college?” For Christian schools, the question was rewritten, “What was your college major?” Such a change did not affect the content of the question and thus did not affect the teachers’ responses. However, this change distinguished public and Christian responses without compromising the anonymity of the respondents.

Procedures

Before gathering any data, the Institutional Review Board (IRB) was provided the necessary information concerning the protection of participants. After securing IRB approval, the administrative offices of the Christian schools and public school districts selected for participation were contacted by telephone to enlist cooperation in the study and then to determine the number of geometry teachers at each school.

To maintain the anonymity of the participants, the schools were asked to physically deliver the survey to the teachers. Participating schools were then mailed the following: (1) A cover letter with instructions for distributing the survey materials (see Appendix C) and (2) the survey materials pre-packaged into individual packets for each teacher. Each teacher packet included the following: (1) A cover letter introducing the study (see Appendix D), (2) an informed consent document describing the risks and benefits of participation (see Appendix E), (3) the Survey of Geometry Perspectives, (4) a postage-paid return envelope, and (5) the five-dollar unconditional incentive for participation. No signed informed consent form was necessary for participation in this study because it would have compromised the anonymity of the participants (Protection of Human Subjects, 2009). An unsigned informed consent document is thus a preferred alternative when conducting survey research.

The key ethical consideration throughout this study was maintaining the privacy of participants' responses. All questionnaires were identical, except for the rewording of Question 38 to allow distinguishing public school teachers from Christian school teachers. Furthermore, there were no markings on the return envelopes, and the return envelopes were destroyed upon the receipt of completed surveys to remove any information evident from postmarks or return addresses.

After the data collection was completed, the survey results were stored in an encrypted Microsoft Excel file and all of the contact information was destroyed. According to IRB requirements, the survey instruments were securely retained and are to be destroyed after three years.

One of the conditions of the use of Truelove's survey is the forwarding of a copy of the raw data for use in his ongoing research. The original electronic data will be stored in an encrypted format for three years after the defense of this study after which the file will be deleted.

Research Design

The purpose of this study was to explore differences in the aspects of teaching geometry proofs between Christian and public school teachers. This arrangement can be outlined as examining differences in some dependent variable among categorical dependent variables (Christian versus public school geometry teachers). For such a study, Gall, Gall, and Borg (2007) recommend using a causal-comparative research design which they define as follows:

> . . . A type of nonexperimental investigation in which researchers seek to identify cause-and-effect relationships by forming groups of individuals in whom the independent variable is present or absent—or present at several levels—and then determining whether the groups differ on the dependent variable. (p. 306)

An example of a causal-comparative study was conducted by Van de gaer, Pustjens, and Van Damme (2008) exploring the effect of gender roles on mathematical achievement. The two levels of the categorical independent variable (male and female) were examined against two numerical dependent variables (mathematics achievement and mathematics participation). The researchers found that male students are more inclined to participate in high school mathematics, and these students attain higher grades as a result.

The investigation into geometry teachers was similar but instead incorporated four dependent variables (the aspects of teaching geometry proofs). The categorical independent variable (type of geometry teacher) had two levels: Christian school and public school.

After selecting the sample for this study, each participant was given a questionnaire that measures that teacher's aspects of teaching proofs. Once the data were collected, an overall difference in the aspects between the two categories of teachers was examined by conducting a MANOVA. After examining for a statistically significant difference, *t*-tests for independent means were then individually conducted on the dependent variables to identify any specific differences.

Data Analysis

The analysis of a causal-comparative research design begins with examining the descriptive statistics of the data, typically the means and standard deviations of the variables (Gall, Gall, & Borg, 2007). Though these values do not permit any judgments of differences, they provide a snapshot of the data.

A judgment can be made by conducting a test of statistical significance. The MANOVA is useful when comparing two or more independent groups on two or more quantitative dependent variables (Spicer, 2005). There are several statistics that can be used to evaluate statistical significance when conducting a MANOVA, including Wilk's lambda, Pillai's trace, Hotelling's trace, and Roy's largest root (Green & Salkind, 2011). Of these statistics, the most commonly-used statistic in the social sciences is Wilk's lambda (Gall, Gall, & Borg, 2007; Green & Salkind, 2011). This statistic was used to test the following null hypothesis:

H_{o1}: As measured by Truelove's (2004) questionnaire, Christian school geometry teachers do not differ from public school geometry teachers in any of the four aspects of teaching geometry proofs.

There following three assumptions that must be met for the MANOVA to be valid (Green & Salkind, 2011):

- The participants must be randomly selected and respond independently from other participants;
- The population must be multivariately normally distributed on the dependent variable; and
- For each category of the independent variable, the variances of the dependent variables must be equal.

The first two of these assumptions was met by the design of the study. Normality was assumed because the size of the sample was sufficiently large (Green & Salkind, 2011), and randomization was an integral part of the selection process. The final assumption could likewise have been assumed because of the robustness of the MANOVA calculation (Gall, Gall, & Borg, 2007). However, Box's *M* statistic was utilized to ensure the desired equality of variances (Green & Salkind, 2011). The effect size that is associated with Wilk's lambda is the multivariate eta square (η^2) which gives the percentage of variation in the dependent variable that is explained by the association with the independent variable (Green & Salkind, 2011). Small, medium, and large effects are considered to be 1, 6, and 13 percent variation, respectively (Gall, Gall, & Borg, 2011).

If the MANOVA demonstrates a significant difference, analysis of variance (ANOVA) is usually utilized to evaluate which of the dependent variables created the difference (Gall, Gall, & Borg, 2007). However, with only two groups for the independent variable, the *t*-test for the difference between independent groups is equivalent to the ANOVA and was an appropriate *post hoc* technique for this study (Stevens, 2002). The required assumptions for these *t*-tests are similar to those of the MANOVA and were thus considered to have been met. Szapkiw (n.d.) noted that violations of these assumptions are largely insignificant in that the *t*-test is quite robust when the sample size exceeds 30 and the SPSS statistical program can still calculate useful results when equal variances cannot be assumed.

There were four total tests in accordance with the following four null hypotheses:

H_{o2}: As measured by Truelove's (2004) questionnaire, there is no difference between Christian school geometry teachers' *concept* of geometry proof and that of public school geometry teachers.

H_{o3}: As measured by Truelove's (2004) questionnaire, there is no difference between Christian school geometry teachers' *approach* to geometry proof and that of public school geometry teachers.

H_{o4}: As measured by Truelove's (2004) questionnaire, there is no difference between Christian school geometry teachers' *usage* of geometry proof and that of public school geometry teachers.

Because the purpose of the study was to look for differences (either way) between Christian and public school geometry teachers, a two-sided test was used. The effect size of any detected differences was given by using the Cohen's *d* statistic which categorizes effects based upon standard deviations (Szapkiw, n.d.). A medium effect is defined as a difference of at least 0.5 standard deviations (Stevens, 2002). When conducting a *t*-test for independent means, a sample size of 64 participants (evenly distributed between the groups) is recommended to provide a 70% chance of finding a medium effect size (Gall, Gall, & Borg, 2007). This explains the rationale for the minimum sample size of 32 participants from each type of school. The requirements for finding significance using MANOVA are less stringent and did not inflate the sample size further (Wilson VanVoorhis & Morgan, 2007).

One final consideration was that the most commonly-used level of significance is at the .05 level (Gall, Gall, & Borg, 2007; Green & Salkind, 2011). However, conducting four separate tests runs the risk of a Type I error, inadvertently rejecting a true null hypothesis. Consider that with a true null hypothesis, statistical significance is found 5% of the time just by accident. The Bonferroni procedure was used to make a more honest judgment. The simplest application of this technique is to divide the chosen level of significance by the number of tests being conducted (Green & Salkind, 2011; Howell, 2011; Stevens, 2002). This would create a level of significance of .0125 (.0125 = .05 ÷ 4) for each test. A *p*-value that is less than .0125 for one of the aspects of teaching proofs would thus be sufficient to claim that Christian and public school teachers differ in that particular aspect.

However, the Holm's Sequential Bonferroni Method accomplishes the same goal but provides greater statistical power (Green & Salkind, 2011). This Bonferroni technique is a progressive comparison of the *p*-values when dividing by the number of tests. In this study, this method took place in the manner prescribed by Green and Salkind (2011):

1. The overall level of significance (.05) was divided by four (giving .0125). The smallest *p*-value was compared with this level of significance.

2. If significance in the previous comparison is found, the overall level of significance (.05) was divided by three (giving .0167). The next smallest *p*-value was compared with this level of significance.
3. If significance in the previous comparison is found, the overall level of significance (.05) was divided by two (giving .025). The third smallest *p*-value was compared with this level of significance.
4. If significance in the previous comparison is found, the overall level of significance (.05) was maintained. The largest *p*-value was compared with this level of significance.

If, at any point, statistical significance is not found, the process would end and a lack of significance would be considered for that comparison and for the remaining comparisons as well.

Supplemental Data

In addition to the questions that measure the aspects of teaching geometry proofs, the survey instrument includes eight questions (Questions 31 through 38) that provide background information about the teachers. Though these questions were not directly related to the testing of the research hypotheses, they were still valuable for shaping a larger picture of the present state of public and Christian school geometry education.

Question 36 and 37 provide quantitative data on the length of time that the teachers have been involved in teaching geometry. Question 36 asks for the year of the initial license to teach mathematics; Question 37 asks how many years have been spent teaching geometry. For discerning differences in responses of this type, the implied null hypotheses were that there are no differences between public and Christian school geometry teachers. As with the *post hoc* tests mentioned earlier, the hypotheses for Questions 36 and 37 were evaluated by using a *t*-test for independent means (Stevens, 2002). The Bonferroni technique was used to determine statistical significance, albeit dividing $p = .05$ by two this time because of testing only two questions. Note that with only two items being tested, the standard Bonferroni method and Holm's Sequential Bonferroni Method are identical.

Questions 31 through 35 were written to elicit categorical responses. The questions and their associated responses were as follows:

31. Which best describes your pre-service training in geometry?

    Informal (investigation/exploration); Formal (technical/rigorous)

32. Which do you prefer teaching?

    Algebra; Geometry

33. Which best describes how often you teach geometry?

    Frequently; Infrequently

34. Which best describes the approach of the geometry texts you most currently have used?

    Inductive; Deductive; Discovery

35. Which best describes your support of NCTM standards?

    Weak; Moderate; Strong

With categorical responses to a dependent variable, Howell (2011) recommends using the chi-square statistic to determine differences. Each question was tested individually using the null hypothesis that there is no difference the groups comprising the independent variable. Significance was once again determined by using Holm's Sequential Bonferroni Method—this time dividing $p = .05$ by five because of using five questions.

The final question of Truelove's survey (Question 38) asks for the participant's major in college. Though responses to this question are also categorical, the information received from this question was not tested. Quite simply, the open-ended nature of the question permitted responses that were ambiguous. For example, some participants could have responded with "mathematics education" while others could have used a more vague response such as "mathematics." In such instances, it is not possible to ascertain whether or not the teachers completed the same type of program or if the "mathematics" major is a true mathematics degree or an education degree with an emphasis in mathematics. Consequently, these responses were recorded but will not be reported.

In addition to the questions that composed the survey, there was space available for teacher comments. This offered the teachers an opportunity to provide commentary that could be used to frame the teachers' responses in a more open-ended manner.

CHAPTER 4: RESULTS

Introduction

The purpose of this study was to explore distinctions between Christian and public education by examining the aspects of teaching geometry proofs in Christian and public school geometry teachers. This chapter examines the results gathered from the participants' responses.

This chapter will first summarize the data collection then examine the descriptive statistics. Then the main hypothesis will be evaluated, followed by the *post hoc* results.

Data Collection

Prior to collecting data, application was made to the Liberty University IRB for approval of the methodology. Data collection began after receiving IRB approval (see Appendix F).

The district-wide administrative structure of the public schools necessitated using cluster random sampling to select the participating teachers. School districts throughout Florida and Georgia were randomly selected and then contacted for inclusion. If a district agreed to allow participation, all qualified teachers within those districts were sent the survey materials. Sixty-three surveys were distributed to public school geometry teachers and 38 were returned, resulting in a 60% response rate. Of the surveys returned, one had insufficient information; this participant commented on an inadequacy to answer the questions out of a self-perceived inexperience in teaching proof. Five others were rejected due to anomalous responses. Thus, for purposes of calculation, $n_p = 32$.

Conversely, Christian schools are autonomous, and decisions about participation were made on a school-by-school basis. The members of ACSI, FACCS, and GACS were randomly selected and then contacted. This led to 50 surveys being distributed; 31 were returned ($n_c = 31$), giving a response rate of 62%.

Table 1

Length of Career Associated with Teaching Mathematics and Geometry

	Public School			Christian School		
Variable	*n*	*M*	*SD*	*n*	*M*	*SD*
Year of Initial License to Teach Mathematics	31	1998	11.0	27	1999	12.5
Years Teaching Geometry	32	10.0	9.3	31	9.9	8.9

Demonstrating statistical significance among the *post hoc t*-tests required a minimum of 32 responses from each of the public and Christian school teachers (Gall, Gall, & Borg, 2007). Though the total number of returned surveys from the Christian schools just missed this requirement, there were still enough responses received for the MANOVA calculations to be able to evaluate an overall difference (Wilson Van Voorhis & Morgan, 2007).

Background Statistics

The descriptive statistics for this study entail both the aspects of teaching proof and other related information about the participants. Tables 1 and 2 present background information about the teachers. There were no statistically significant differences for when the teachers first received their licenses to teach mathematics ($t\ (56) = 0.525$, $p = .601$) or for the number of years teaching geometry ($t\ (61) = 0.056$, $p = .955$).

Table 2 displays categorical information about other factors that have shaped the teachers' perspectives. When judging whether different groups differ on categorical data, Howell (2011) recommends evaluation by using the chi-square statistic. For example, the survey instrument for this study allows teachers to state their preference for teaching geometry or algebra. The chi-square statistic fails to prove that public and Christian school teachers differ on their teaching preference, $\chi^2(1) = 0.643$, $p = .422$. Table 3 provides the chi-square statistics for all of the categories on Truelove's questionnaire,

Table 2

Additional Characteristics of Geometry Teachers

	Public School		Christian School	
Characteristic	*n*	%	*n*	%
Pre-service Training in Geometry				
Informal (investigation/exploration)	9	28%	9	29%
Formal (technical/rigorous)	21	66%	19	61%
No/Multiple response	2	6%	3	9%
Preference for Teaching				
Algebra	11	34%	17	55%
Geometry	17	53%	11	35%
No/Multiple response	4	13%	3	9%
Frequency of Teaching Geometry				
Frequently	24	75%	30	97%
Infrequently	8	25%	1	3%
No/Multiple response	0	0%	0	0%
Support for NCTM Standards				
Weak	3	9%	2	7%
Moderate	16	50%	18	60%
Strong	11	34%	6	19%
No/Multiple response	2	6%	5	17%
Approach of Geometry Textbook				
Inductive	5	16%	2	6%
Deductive	17	53%	28	90%
Discovery	5	16%	1	3%
No/Multiple response	5	16%	0	0%

Table 3

Testing the Additional Characteristics of Geometry Teachers

Characteristic	χ^2	*d.f.*	*p*
Pre-service Training in Geometry	0.005	1	.944
Preference for Teaching	0.643	1	.423
Frequency of Teaching Geometry	2.654	1	.103
Support for NCTM Standards	1.510	2	.470
Approach of Geometry Textbook	6.396	2	.041

showing that there is insufficient evidence to find any differences among these categorical characteristics. Note that *p* for the Approach of Geometry Textbook characteristic is below .05. However, when conducting multiple tests, a common practice is to use the Bonferroni method (Green & Salkind, 2011; Howell, 2011; Stevens, 2002). In this case with five tests, *p* is divided by five to give .01 as the standard for making a claim. Thus, there is evidence of a difference in the type of geometry textbooks used in public and Christian school geometry classes, yet this evidence is not strong enough to definitively declare that such a difference exists.

Approximately half of the teachers included comments with their survey responses. The qualitative nature of those comments precluded statistical testing. Rather, these comments are better used to help shape the discussion resulting from the findings concerning the two research questions.

Research Question 1

The primary research question that guided this study was the following:

RQ1: Is there a difference between Christian and public school geometry teachers on the aspects of teaching geometry proofs in the geometry class?

This question was examined by searching for differences between Christian and public school geometry teachers on the four aspects of teaching geometry proofs as

Table 4

Descriptive Statistics for the Four Aspects of Teaching Geometry Proofs

	Public School (n = 32)		Christian School (n = 31)	
Aspect	*M*	*SD*	*M*	*SD*
Concept	2.21	0.46	2.49	0.55
Approach	3.01	0.48	3.03	0.65
Usage	3.49	0.44	3.58	0.41
Practices	1.35	0.50	1.56	0.60

measured by Truelove's questionnaire. Table 4 summarizes the data from the teachers' responses to the 30 questions that measure the aspects of a teacher's perspectives on teaching proofs. These aspects are as follows:

- Concept: The teacher's belief in proof as foundational to the geometry class or just a topic (among several) within that class;
- Approach: The teacher's preference to emphasize inductive or deductive reasoning;
- Usage: The number of techniques used by the teacher when teaching proof; and
- Practices: The amount of instructional time the teacher devotes to teaching proof.

Research question **RQ1** was evaluated by testing the following null hypothesis:

H_{o1}: As measured by Truelove's (2004) questionnaire, Christian school geometry teachers do not differ from public school geometry teachers in any of the four aspects of teaching geometry proofs.

This hypothesis was evaluated using MANOVA. This statistical tool required three assumptions to be met. The first two assumptions were independence of participants and normality of the dependent variables; these were met by the conditions of the study. The third assumption was that the variances of the dependent variables were equal, and this was shown to be met by using Box's M statistic, $M = 13.663$, $F(10, 17211) = 1.268$, $p = .242$.

The hypothesis itself was tested using Wilk's lambda and failed to demonstrate a significant difference between the two groups, Wilk's $\Lambda = .926$, $F(4, 57) = 1.137$, $p = .348$. Because of this lack of significance, the calculated effect size (partial $\eta^2 = .074$) is meaningless.

Research Question 2

The second research question guiding this study was the following:

RQ2: In which of the four aspects of teaching geometry proofs can a difference between Christian and public school geometry teachers be identified?

This research question led to the following null hypotheses:

H_{o2}: As measured by Truelove's (2004) questionnaire, there is no difference between Christian school geometry teachers' *concept* of geometry proof and that of public school geometry teachers.

H_{o3}: As measured by Truelove's (2004) questionnaire, there is no difference between Christian school geometry teachers' *approach* to geometry proof and that of public school geometry teachers.

H_{o4}: As measured by Truelove's (2004) questionnaire, there is no difference between Christian school geometry teachers' *usage* of geometry proof and that of public school geometry teachers.

H_{o5}: As measured by Truelove's (2004) questionnaire, there is no difference between Christian school geometry teachers' *practices* involving geometry proof and that of public school geometry teachers.

These four hypotheses were tested by conducting *t*-tests for independent means on each of the four aspects of teaching geometry proofs. However, the main consequence of the lack of significance for **H_{o1}** is that there would have been no significant differences

Table 5

Inferential Statistics for Differences in the Aspects of Teaching Geometry Proofs

Aspect	Levene's Test[1] *F*	Levene's Test[1] *p*	*t*-test for Equality of Means: Mean Difference[2]	*t*	*df*	*p*
Concept	1.505	.225	0.2838	2.221	61	.030
Approach	1.544	.219	0.0202	0.141	61	.889
Usage	0.004	.948	0.0874	0.813	61	.419
Practices	2.844	.097	0.2058	1.459	60	.150

[1]This tests for the equality of variances.

[2]These differences are calculated by $\bar{x}_c - \bar{x}_p$.

among the four aspects anyway. Table 5 gives the statistics for identifying differences between public and Christian school geometry teachers on the four aspects of teaching proofs. Note that the levels of significance for three of these four aspects are above the commonly-accepted standard of $p = .05$ (Green & Salkind, 2011). Though p for the concept aspect was below .05, it was still greater than the .0125 needed when utilizing Holm's Sequential Bonferroni Method. Note further that the *t*-scores were calculated under the assumption that, for each of the four aspects, the variances of public and Christian schools were equal. This assumption is tenable when Levene's test for the equality of variances fails to reject the hypothesis that the variances are equal (Green & Salkind, 2011).

CHAPTER 5: DISCUSSION

Introduction

The purpose of this study was to determine if public school and Christian school geometry teachers had different perspectives on the teaching of proof in the geometry class. This chapter will review the statistics that failed to demonstrate a significant difference between the two types of teachers. There are limitations on the ability to project these results onto all public school and Christian school teachers. These limitations will be examined and suggestions given for improving further research.

The implications of the findings on the foundational theories and classroom practices will then be investigated. This chapter will then conclude by exploring recommendations for future study as a consequence of this study's findings.

Summary of Results

The search for different perspectives on teaching geometry proofs was conducted by investigating differences in at least one of the four aspects of teaching as noted by Truelove (2004): Concept, approach, usage, and practices. There were 32 useable public school responses and 31 Christian school responses. An overall difference was examined by conducting a MANOVA using the type of school as the independent variable and the aspects of teaching proof as the dependent variable. The MANOVA found the difference between the types of teachers to be statistically insignificant, Wilk's $\Lambda = .926$, $F(4, 57) = 1.137$, $p = .348$. Further examination of other characteristics (such as support for NCTM standards) failed to demonstrate statistically significant differences as well.

As expected from this failure to find a statistically significant overall difference, the *t*-tests for independent means on each of the four aspects also failed to identify any statistically significant differences. However, anecdotal observation of several comments from the teachers with long experience in teaching geometry seemed to show these teachers holding a negative opinion of what they perceived as a gradual de-emphasis of proofs in the classroom. The two responding public school teachers with the most years teaching geometry criticized the recent emphasis on testing for this. One of these teachers wrote, “In regular geometry [proofs] are not emphasized and are not tested on the district's subject area exam.” The other experienced teacher commented, “Today, less time is given to formal proofs than before due to increased testing! Not enough class time—however, I believe ‘proof’ (informal and formal) is a very important part of geometry.”

While some teachers with fewer years of experience echoed the sentiments of veteran teachers, they did not appear as uniform in their criticism on the perceived de-emphasis on proof. To see if experience may have influenced the results, the main hypothesis was tested again while controlling for years teaching geometry. Again, the hypothesis failed to be rejected with the underlying statistics demonstrating only the smallest of changes, Wilk’s $\Lambda = .926$, $F(4, 56) = 1.119$, $p = .357$.

Limitations

Sample size. One limitation from this study is that the number of Christian school participants was just below the 32 recommended to conclusively demonstrate differences (if present). As the number of participants in a study increases, individual responses become less likely to influence the overall results, and the variation among averaged responses decreases. Thus it is possible that a difference between two groups that is statistically significant with sufficient responses fails to be statistically significant with fewer responses.

To address this situation, an additional Christian school response using extreme responses was created, and the MANOVA was recalculated to see if an additional response could possibly change the overall conclusion. Among actual responses, the Christian schools had higher observed averages on all four of the aspects. This simulated Christian school survey was thus created using the highest possible scores on each of the four aspects: Concept = 4.00; approach = 4.00, usage = 4.00, and practices = 5.00. Recalculating the MANOVA using this additional, simulated data showed that it would be impossible for another survey to create a statistically significant difference, Wilk's $\Lambda = .912$, $F(4, 58) = 1.405$, $p = .244$.

This simulated survey shows that, despite this study receiving fewer responses than the statistical literature recommends, the results as calculated from the actual data are still quite robust. Consequently, the calculations from the actual responses that demonstrated no statistically significant difference between the two types of schools are still reasonably dependable.

Extrapolating findings. Some remaining limitations to this study are those common to research in general. For instance, as noted in Chapter 1, this study is not actually comparing public school and Christian school geometry teachers; this study is instead comparing those teachers who choose to complete a survey.

A similar limitation is that projecting these results onto other types of schools is not valid. For example, nearly half of all students in private, religious schools attend schools that are part of the National Catholic Educational Association (NCES, 2008). This is by far the largest private school association of any type, religious or not. While Catholic institutions and the conservative Christian institutions have many philosophical similarities, the differences between the two groups prohibit the conclusion that public school geometry teachers and Catholic school geometry teachers also have similar perspectives on the place of proofs in the geometry classroom.

Furthermore, it is important not to project these results onto all public schools as well. Not only was this study limited to Florida and Georgia, but the public school districts that consented to participate in the study were among the more politically conservative districts in the two states. This is pertinent in that this study defined *Christian school* by investigating "conservative Christian" schools. Though the terms *political conservative* and *Christian conservative* equivocate on the word *conservative*, studies have shown that those with the strongest religious beliefs are more likely to be members of the more conservative Republican party (Newport, 2011). By definition, teachers in a Christian school have strong enough religious beliefs to seek ministry within a wholly Christian atmosphere. Thus, it is likely that the Christian teachers and the public school teachers that actually participated in the study already shared common views in many areas outside of the geometry classroom. This limitation in particular is perhaps that most susceptible to criticism of the overall conclusion that there is no difference between the two types of geometry teachers.

Relevance of Results

The main conclusion from this study is that public school and Christian school geometry teachers do not view the place of proof in the classroom differently. Any expectations for finding a difference in the teachers' views would have been based upon the concept that public and Christian school teachers in general have foundational philosophical differences and that those philosophical differences translate into pedagogic differences. Accordingly, the failure to find a pedagogic difference leads to more questions.

Theoretical framework. The theoretical framework that guided this study was that of Ernest's (1989) contention that a teacher's beliefs influence that teacher's practices. Ernest uses the term *practices* as a description of the entirety of a teacher's classroom procedures; typically, this would be used to describe the four areas of teaching proof. However, Truelove uses the term *practices* to describe the amount of instructional time devoted to teaching proof. To avoid confusing the use of the word *practices*, this study has used the term *aspects* for the four areas of teaching proof and reserved *practices* for use as defined by Truelove.

In this case, the beliefs in question are the underlying philosophical differences that the literature demonstrates exist between the public school and Christian school settings. Most fundamentally, this difference concerns the nature of truth. Theoretically, the public schools consider truth to be a socially constructed collection of principles that are subject to change as that society changes; on the other hand, Christian schools picture truth as a timeless set of facts that have either been handed down as direct revelation from God or have been deduced from that revelation.

The conclusion that public school teachers and Christian school teachers do not exhibit different aspects (Ernest's practices) concerning the teaching of geometry proofs can be explained in two different ways. First, it is possible that the strength of the association between beliefs and practices posited by Ernest is minimal at best. An argument in favor of this suggestion is that some of the public school teachers chafed at the limited amount of emphasis on proofs necessitated by increased testing requirements. One teacher stated bluntly, "Today, less time is given to formal proofs than before due to increased testing!" Another concurred that "The importance of proofs is dwindling because of the EOC [end of course exam]."

Another teacher's remark was more subtle: "Perspectives in teaching do not alter a teacher's requirement to teach within the framework of district and state curriculum guides." Such comments suggest that some of these teachers would have changed the amount of time spent teaching proof if permitted. Such changes could have affected responses in the *practices* aspect to the point of demonstrating some statistically significant difference. This suggests a minimization of the association given by Ernest in that the teachers' beliefs might not affect their practices simply because they are not given the choice of allowing their beliefs to strictly govern their practices.

Additionally, a comment made by one of the public school teachers hints at another argument that the association between belief and practice might not be strong. Question 13 of the survey asks if one believes that "Proof should only be covered as a single unit in geometry." This teacher commented beside the question, "No, but that's how I prefer to teach them." This comment suggests that the teacher was distinguishing between some personal ideal of proof and pedagogic necessity. Thus, some beliefs may not influence practices in that the teachers may judge that implementing those beliefs may lead to subject matter beyond the students' current capabilities.

However, there is also reason to believe that there is a strong association between beliefs and practices. For one, there is little philosophical support for suggestion that the association between beliefs and practices is minimal. A foundational facet of Dewey's educational philosophy was that "education is the fundamental method of social progress and reform" (1959, p. 30). After renouncing changing society by changing laws as "transitory and futile," he further remarked that "education is a process of coming to share in the social consciousness; and that the *adjustment of individual activity* [i.e., practices] *on the basis of this social consciousness* [i.e., beliefs] is the only sure method of social reconstruction" (Dewey, 1959, p. 30, emphasis added). The National Education Association (NEA) suggests a similar sentiment: "We believe public education is the cornerstone of our republic" (NEA, 2013). In other words, changes in the actions of our society begin with changes to the minds of those engaged in that society.

The Christian perspective would be similar. An oft-quoted biblical passage is Proverbs 4:23 which states, "Keep thy heart with all diligence; for out of it are the issues of life." The word *heart* in this verse does not mean the literal chest organ, but rather the focus of one's will and intellect (Strong, n.d.). Additionally, the word *issues* does not refer to the colloquial usage of the synonym *events*. Instead, it means "to exit" or "to go forth." Literally, then, this verse reads that out of one's intellect proceeds that person's very life. Additionally, Proverbs 23:7 states plainly that "As a man thinketh in his heart, so is he." These verses essentially restate Ernest's theory: What one believes will cause what one practices.

Comparable beliefs. Thus, both public and Christian schools' underlying philosophies promote, even rest upon, the concept of beliefs influencing practices in a fundamental way. Therefore, a more plausible explanation for failing to find a difference in the aspects of teaching geometry is that the underlying differences in beliefs between Christian and public schools are not as strong as the histories of the two educational systems would suggest.

There are certain obvious differences in beliefs. For example, being innately religious in nature, Christian schools overtly teach biblical principles—often incorporating formal curricula concerning the Bible. This is in stark contrast to the well-publicized prohibitions on public schools' displays of even the most basic Christian tenets. Note as well that one of the most famous trials of the twentieth century (*Scopes v. The State of Tennessee*) addressed the appropriateness of teaching the biblical account of creation versus the naturalistic account of evolution in the public school science curriculum.

Yet, the above examples present dichotomous choices. Christian schools choose to teach the Bible as foundational truth; public schools generally omit the teaching of all religion. Similarly, Christian schools often teach the origin of the universe as a direct, creative act of God; public schools teach origins from a purely naturalistic standpoint. In these cases, there is a clear biblical teaching with which an educational community must make a decision to accept or reject.

In other areas, though, there is more of a continuum of ideas. For example, there has been a great deal of discussion about the teaching of reading, in particular the amount of emphasis placed upon phonics (Rasinsky, Rupley, & Nichols, 2008). Yet, even the most ardent proponents of phonics must admit that some words (such as *colonel*) cannot be learned phonetically while those who favor the whole word technique cannot ignore that words beginning with the letter *b* tend to start with a particular sound. A mathematical example of this continuum would be the intense argument that took place over the New Math of the 1970s in which educators differed over the amount of rigor needed at the different levels (Latterell, 2005).

In these areas then, where broad movements react continuously rather than discretely, it is possible that there is little difference between Christian and public school perspectives. Simply stated, there is philosophical "room" with which to nuance one's stance, and beliefs that are often moderately different may drift closer at times. However, in actual practice it may be that whatever differences are present (no matter how small) could become magnified in rhetoric as the schools compete for enrollment and the subsequent monies that follow that enrollment (Carr, 2006).

Dichotomy of proof methods. In many ways, this study's investigation concerning how proofs are taught provides another mathematical example of gauging decisions that are made upon a wide range of incremental responses. Yet this is an oversimplification for only the aspects of concept, usage, and practices can be answered solely on a continuous scale as these are concerned with the amount of time and number of techniques devoted to proof. The aspect of approach however is more dichotomous in nature as the participants express a preference for either inductive or deductive reasoning.

Studying the amount of time devoted to the two types of reasoning gives a way to relate them in a continuous manner. As Pedemonte (2007) noted, students can better visualize a general case by examining specific instances. Exploring a problem inductively allows the students to more effectively picture how the mechanics of the general, deductive case operate. One Christian school teacher expressed support for this technique by using inductive methods so that students "can see specific examples that lead them to a conjecture, then also show them the truth deductively."

In contrast, a dichotomous perspective of inductive and deductive reasoning exists in their differing abilities to be used for mathematically-acceptable proofs. One cannot *prove* a geometric concept in the fullest sense of the term through inductive techniques; concepts can only be strongly suggested by these methods. Absolute proof is instead reserved solely for deductive means. For the present study, the approach aspect of teaching proofs was designed to measure a teacher's preference for inductive or deductive proof when teaching geometric concepts, but not to measure whether a teacher defines *proof* in the general sense as inductive or deductive.

One teacher's notations on the survey indicated that defining *proof* as inductive or deductive at the outset of the survey would have produced different responses from that teacher. For example, on question 7, this teacher inserted “formal” parenthetically into the question: “Proof is fundamental to the (formal) study of geometry.” Apparently, an “informal” study would have elicited a different response. Several teachers emphasize this point by explicitly identifying different types of geometry courses. One teacher stated, “At my present school, we have two different geometry courses: Regular and informal. In the informal geometry course, proofs are not taught at all! In regular geometry they are not emphasized and are not tested on the district's subject area exam.” One could conclude then that there is some confusion as to what constitutes geometry as a distinct mathematical discipline in the first place, and further that there is confusion as to what constitutes proof.

That there would be some confusion is not surprising when recounting Pedemonte’s (2007) other finding that some interpret *proof* as *argumentation*. Using this interpretation, the judgment of what is true (such as one’s guilt in a court proceeding) is determined by whoever provides the more convincing evidence. Even this very study relied on determining the existence of a difference between the populations of public and Christian school teachers by gauging the likelihood of particular observed differences in the selected samples. If the observed differences in the samples had been larger than what would be likely to occur due to random variation, then a difference in the populations would have been considered to be present and public and Christian school teachers would have been “proven” to be different from each other. The actual observed differences in the samples were small enough to be considered plausible, random differences in populations that would have been identical; thus, this study concludes that there are no differences in the two populations.

Considering the close arithmetic proximity to pure mathematics and statistics, that mathematics teachers would experience confusion as to what constitutes proof is not unexpected. The same teachers that studied statistics which uses inductive arguments would have also studied some pure mathematics which relies upon deductive means. In essence, if differences are easier to spot when using discrete rather than continuous choices, then presenting teachers with the stark alternatives of defining geometric proof as inductive or deductive could provide an avenue for discerning differences in beliefs.

Extrapolation of existing research. When considering education in a more universal manner, projecting the lack of an observable difference between public and Christian school geometry teachers onto all public and Christian school teachers has a key benefit regarding the general literature. Though the vast bulk of research into this field has been done in the public school systems, those findings can be reasonably projected onto Christian education. In the past, only philosophical conjecture permitted projecting findings in the general educational literature (usually conducted in the public school systems) to Christian education. Though this study is admittedly quite limited in scope, the similarities in the two types of teachers as presented in this study provide some empirical evidence in favor of this conjecture.

The reasonableness of projecting findings from public education onto Christian education is significant in that Boerema's (2011) key lament was that there is a paucity of research into Christian educational practices. One possible explanation for this is from Boerema's observation of a Christian educator's opinion that Christian educators in general prefer philosophical discussions about Christian education over the expense of original, quantitative research into the field. While research that is uniquely targeting these Christian practices is still rare, it is possible to use the general education literature as a tentative measure of how Christian school teachers educate and Christian school students learn.

Implications

Methodological implications. When reviewing the methodology implemented in this study, the first area in need of improvement concerns the homogeneity of the public school districts that elected to participate, especially in light of the political similarities between those public schools districts and Christians in general. While this study utilized sampling techniques that selected districts of all different political and demographic strata, districts that could have provided additional political viewpoints simply did not choose to grant permission for inclusion.

Part of the difficulty in obtaining different types of districts is that the politically conservative districts that granted approval were small, rural districts containing very few schools and having correspondingly small administrative departments. The relevant administrators in these small districts were comparatively easy to contact and, though usually not the actual superintendents, had the bureaucratic autonomy to grant permission for inclusion. These individuals usually required only a few of the details about the study and knew further that granting permission would inconvenience only a very few teachers.

Conversely, attaining permission at the larger, highly urban districts was much more difficult. With many more schools to oversee, these districts had correspondingly larger administrative departments. The size of these districts also makes them prized targets for research because just one such district can offer access to thousands of students or hundreds of teachers; thus, these districts have been continually petitioned with requests for research. To handle these numerous requests, these districts often had stringent application processes for research or had simply closed the district to additional requests for research beyond those already in process. Several districts compromised on the number of requests by granting permission for research only to those who were already employed by the district itself. Thus, future attempts to include a broad range of districts will need to allow for a much longer application process for the larger districts and perhaps make the application more palatable to the district by limiting the request to just a few schools.

Besides ensuring that the participants in the study were sufficiently diverse, another methodological consideration from this study is to ensure that the philosophical roots that underlie the participants' views are sufficiently diverse as well. In this study, the teaching of geometry as a logical system of thought was admittedly one area that was likely to have public and Christian school teachers share similar views. It was hoped that finding a difference in this area could be used to generalize differences in nearly all academic areas.

But teaching geometry is perhaps the most constructivist of traditional, Christian educational practices. Thus, any differences that might be present when comparing traditional educators to more constructivist educators would be minimized by examining geometry teachers—possibly to the point at which these differences would be undetectable. Other subjects though could provide a better avenue for exploring the existence of such differences.

Therefore, a more appropriate approach to identifying differences would be first to rank academic areas by the strength of their shared philosophical roots. For example, this current topic of geometry would be at the one end sharing similar philosophical roots and the teaching of universal origins would be at the other; in between would be the remaining academic subjects suitably ranked. Then, a topic could be selected from the middle of the list to identify if there is a difference at that point. The results of such a study would identify where to explore further for any differences. Gradually dividing this list in half over and over again as studies identify any differences could eventually be used to pinpoint where public and Christian school philosophies ultimately result in an academic difference.

Practical implications. Because Truelove's (2004) original study has already provided a glimpse into public school teachers' perspectives, the practical implications from this study are mainly directed toward Christian education. Perhaps the most important implication for Christian education concerns Hoeksema's original question: Are Christian schools truly distinct? Apparently, they are not as distinct as Christian educators would like. Though this charge should be tempered by the limitations given above, the desire among Christian educators that biblical philosophy would filter down into all aspects of the school does not appear to be taking place.

Another implication for Christian schools is that their students are not receiving extra training (relative to their public school counterparts) in deductive reasoning. This is important in that the Christian faith is a deductive system of thought (Knight, 2006). As mentioned earlier, all Christian tenets can ultimately be deduced from certain absolutes that are given in the Bible. Even the Roman Catholic Church (which has vastly different beliefs from the conservative, fundamental Christians in this study) ultimately grounds papal authority in an interpretation of Matthew 16:18 (Hiers, 1985).

Some Christian school geometry teachers already recognize the use of geometry and proofs as a tool for a deeper understanding of the deductive nature of the Bible and Christian principles. One teacher wrote, "I desire to teach the students to approach their Bible study . . . deductively. God's work is coherent; it supports itself and has a very deductive approach. This is the only thing that will last for all eternity."

Conversely, the separation of church and state that prohibits the public schools' endorsement of any one religion precludes any appeal to a transcendent authority. Thus, there are no absolutes or axioms from which a deductive epistemology can emerge. In such a situation, the only remaining possibility for determining truth can be through inductive means. Considering that the vast majority of society has attended public education, it is reasonable to conclude that the vast majority of society would thus favor inductive argument over deductive as the final arbiter of discovering truth.

This creates a difficulty for the Christian school students in that they ostensibly subscribe to a deductive worldview but cannot effectively utilize deductive reasoning while living within a largely inductive society. In other words, when a Christian makes a claim that is ultimately based upon some biblical teaching and that Christian is challenged to "prove" that claim, there would be confusion on the means of providing a satisfactory "proof." One Christian school teacher noted, "As it is, anyone with an opinion, no matter how unfounded, can solicit a worldwide audience via the internet. It is imperative that we teach our students to think critically, not only for the responsible presentation of their own ideas, but also for the evaluation of the ideas of others." Though there is ambiguity about the "we" this quote is addressing, the directive is clear. All parties involved in fruitful discussion have the responsibility to understand both their own positions and the premises and logic that form other positions.

Consider, for instance, the intense cultural debate that is presently ongoing concerning gay marriage. Speaking broadly, for the government to legalize gay marriage is for the citizenry (because the government of the United States is "of the people") to declare its blessing on homosexual practices in certain cases. However, the heart of the Christian argument against permitting such a practice is that homosexuality is immoral ("wrong") in *all* cases. It is understandable then that those who are in favor of gay marriage would question why Christians would claim that homosexuality is wrong. After all, if the absolute immorality of homosexuality is groundless, then all other arguments against the practice are based purely upon one's preference; and in a democratic republic, the preferences of the majority prevail as long as they do not violate other fundamental rights. In fact, the argument for legalizing gay marriage is typically that to prohibit the practice is itself violating some individuals' fundamental rights. Thus, the burden of proof is often placed upon those opposed to gay marriage to demonstrate why they practice is "wrong."

Though Christian thought is inherently deductive in structure, Christian opposition to gay marriage often resorts to some scientific (i.e., inductive) finding in order to craft an appealing argument. One noted example is the Family Research Council's publication that summarizes data linking homosexuality to pedophilia (Sprigg & Dailey, 2004). In short, this particular argument against homosexuality runs as follows: Pedophilia is evil, and homosexuality leads to pedophilia; *ergo*, homosexuality leads to evil. However, even if one grants that there is a positive correlation between homosexuality and pedophilia, this inductive approach does not address the underlying morality. In other words, the immorality of homosexuality is contingent upon the immorality of pedophilia; and if the immorality of pedophilia were ever to be rejected, then so would the resultant immorality of homosexuality.

This example illustrates the difficulty in projecting Christian principles that are deduced from transcendent premises onto a society that only accepts "principles" that are induced from some scientific observation. A Christian appeal to Scripture makes use of a deductive method of argument that runs counter to the methods currently accepted by the majority of the citizenry. Such appeals are further undermined if Christian students have not been trained to think effectively in a deductive manner in the first place.

Recommendations

Several prospects for future study have already been addressed. Among these would be distinguishing how teachers define *proof* as inductive, deductive, or both depending upon context. Further, there could be a lengthy effort to distinguish at what point Christian and secular philosophies become evident in the classroom. The most obvious recommendation in the context of this particular study would be to replicate the study after correcting for the homogeneity of school districts. Doing so could perhaps demonstrate that a difference in public and Christian school geometry teachers was present all along.

There is also a hint among the data that one possible difference might actually exist at the administrative levels instead of at the pedagogic levels. Quotes from several public schools teachers expressed dismay at the current levels of emphasis on proof; in particular, there was blame leveled at the constraints dictated by standardized testing requirements. Such a charge posits a belief that the public school administrators are (understandably) more concerned with teaching material that most conforms to state testing criteria; and, if the criteria do not emphasize proof elements, then the administrators are not concerned with proof either.

On the other hand, there is another indication that Christian school administrators actually are at the other end of the spectrum and desire much more emphasis on proofs.

The slight evidence for this statement is the marginally significant difference in the types of textbooks used by the geometry teachers. Though teachers often give input onto textbook decisions, those decisions are usually made at the administrative level. That Christian school geometry texts have more of a deductive approach suggests that the administrators are expecting a more deductive approach in the classroom.

These thoughts provide a conjecture of the continuum of geometry education that mimics quite well the geometric paradigms described by Houdement and Kuzniak (1999). Public school administrators are at the one end favoring the more easily-tested inductive concepts (Geometry I); Christian school administrators are at the other end favoring deductive emphasis (Geometry III); and the teachers from both types of schools sit somewhat unhappily in the middle (Geometry II), tempering the administrative positions in the classroom.

Such a conjecture provides two immediate avenues for future study. One possibility would be to examine the beliefs and practices of the administrators at the schools. Such individuals are more removed from the day-to-day instruction of students than are teachers, and thus are more likely to avoid conflating philosophical and pedagogical practices. In a way, these administrators would be able to provide a more "pure" philosophy of education.

The other avenue of study from the previous thoughts would be to distinguish what teachers actually do in the classroom from what teachers *want* to do in the classroom. In the context of geometry education, Truelove's survey already forms a foundation for such a study. Two versions could be developed—one reflecting what teachers actually do in the classroom, and the other stating what teachers wish that they could do without administrative interference.

Perhaps an even more fundamental question that needs addressed is to determine what teachers perceive as the need to learn geometry in the first place. Students' familiar question of "Why do I need to learn this stuff?" stretches across all academic disciplines.

But within the field of geometry and proof, the number of possible interpretations of the meaning of proof suggests that there are as many responses to the students' question as there are teachers available to answer it.

One teacher has already developed two answers the students' question: "No boss is going to ask you to do a proof, but many of them need you to present a logical argument for something," and "Your job will never depend on a two-column proof, but it may depend on solving a problem from start to finish logically." This teacher addresses the need to study proofs from a practical standpoint, but others confine the need to study proof to a self-contained study of mathematics. Another teacher wrote simply, "I feel that geometry proofs are foundational to future mathematical learning."

Both teachers convey concepts that are true within different conceptualizations of the role of proof. And many different ways to categorize these conceptualizations have already been created: The four arguments espoused by González and Herbst (2006); the adaptation of authentic mathematics presented by Weiss, Herbst, and Chen (2009); and the geometric paradigms of Houdement and Kuzniak (1999). However, outside of Truelove's survey, little has been done to quantify teachers' beliefs within any of these categorizations. The development of new survey instruments within the framework of one of these categorizations should be explored.

Conclusion

It was mentioned at the outset of this research that the constructivist nature of mathematical proof would provide moderately common ground for public school and Christian school teachers. Had a difference been shown in this area, it would have been plausible to project a difference in virtually all educational disciplines. But the lack of a difference in this area does not mean that a difference does not exist somewhere. At the very least, there is a difference in the dichotomous areas of recognition of religion and accounts of the universe's origin.

Some might however claim that there is a clean distinction between purely "religious" and purely "academic" areas of study. Hoeksema's (1992) original concern about Christian schools being "truly distinct" is oriented toward this topic: Does the Christian faith saturate the entirety of the Christian school curricula or is it isolated to the "purely religious" areas? Unfortunately, this study was unable to answer the question definitively in either direction. Because of the constructivist nature of proof and proof construction, it is plausible for even the most traditional of Christian school teachers to share beliefs about geometry proof with their public school counterparts. Yet for those who desire the Christian school to exhibit a marked distinction from the public school in all areas, the failure to find a statistically significant difference between the two types of geometry teachers should serve notice that Christian education does not presently offer this distinction.

This strikes at the heart of Hoeksema's question concerning the distinctiveness of the Christian school. Is the Christian school nothing more than a public school with a layer of Bible thrown in? While this question obviously minimizes how important that layer of Bible might be, Christian educators need to ask themselves if their approach to education is helping further the popular notion that life can be segregated into distinctly sacred and secular areas. It is hoped that this look into the geometry classroom has provided sufficient information to generate the discussion necessary to allow Christian educators to explore this question more deeply.

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APPENDIX A

Survey of Geometry Perspectives

DIRECTIONS: Please respond to the following statements about proofs in geometry courses. Mark the response that **best** describes you at this time.

SCALE: Each statement describes me :

4 = very well   3 = usually   2 = somewhat   1 = not at all

1. I believe that proof is a key concept in the study of geometry.	4	3	2	1
2. I believe that students should have experience with inductive reasoning in the study of proof.	4	3	2	1
3. I believe that inductive reasoning has little value in learning to construct proofs.	4	3	2	1
4. I believe that homework plays an integral part in learning proofs.	4	3	2	1
5. In a formal study of geometry, inductive reasoning has limited value.	4	3	2	1
6. I believe that a variety of learning situations should be provided for students to learn proof.	4	3	2	1
7. Proof is fundamental to the study of geometry.	4	3	2	1
8. An inductive approach to proof should be studied in connection with a deductive approach to proof.	4	3	2	1
9. Students should be provided with opportunities to work on proof in class activities.	4	3	2	1
10. Proofs should be integrated throughout a geometry course instead of covered only in a single unit.	4	3	2	1
11. Work with inductive reasoning in courses prior to geometry is necessary for the study of proof in a geometry course.	4	3	2	1
12. Students only need to experience deductive reasoning to construct proofs in a geometry class.	4	3	2	1
13. Proof should only be covered as a single unit in geometry.	4	3	2	1
14. Without proof, geometry would not be a stand alone course.	4	3	2	1
15. Proof should be taught using a formal, deductive approach.	4	3	2	1
16. Proof should be taught with a blend of inductive and deductive approaches.	4	3	2	1

17.	As part of my instructional practice in teaching proof, I believe that students should be required to complete proofs on tests.	4	3	2	1
18.	For proof instruction, I believe that it is important for a teacher to demonstrate proofs in class.	4	3	2	1
19.	I believe that the most important issue covered in geometry is proof.	4	3	2	1
20.	Teachers should not waste class time teaching proofs.	4	3	2	1
21.	I believe that students only need a general overview of proof in geometry.	4	3	2	1
22.	Proof should be only one of many topics covered in geometry.	4	3	2	1
23.	Teaching proof should be the primary focus of a geometry course.	4	3	2	1
24.	Percentage of instructional time per week that you provide examples of worked proofs to in a geometry class. A) 0 to 19 B) 20 to 39 C) 40 to 59 D) 60-79 E) 80 to 100				
25.	Percentage of instructional time per week that you work examples of proof in a geometry class. A) 0 to 19 B) 20 to 39 C) 40 to 59 D) 60-79 E) 80 to 100				
26.	Percentage of instructional time per week that you provide teacher-guided class-interactive examples of proof. A) 0 to 19 B) 20 to 39 C) 40 to 59 D) 60-79 E) 80 to 100				
27.	Percentage of instructional time per week that students work in groups to solve proofs. A) 0 to 19 B) 20 to 39 C) 40 to 59 D) 60-79 E) 80 to 100				
28.	Percentage of instructional time per week that students work independently in class to solve proofs. A) 0 to 19 B) 20 to 39 C) 40 to 59 D) 60-79 E) 80 to 100				
29.	Percentage of homework problems per week that students work on proofs. A) 0 to 19 B) 20 to 39 C) 40 to 59 D) 60-79 E) 80 to 100				
30.	Percentage of questions per exam that students are required to solve proofs. A) 0 to 19 B) 20 to 39 C) 40 to 59 D) 60-79 E) 80 to 100				
31.	Which best describes your pre-service training in geometry? Informal (investigation/exploration) Formal (technical/rigorous)				

32.	Which do you prefer teaching? Algebra Geometry
33.	Which best describes how often you teach geometry? Frequently Infrequently
34.	Which best describes the approach of the geometry texts you most currently have used? Inductive Deductive Discovery
35.	Which best describes your support of NCTM standards? Weak Moderate Strong
36.	In what year did you receive your initial license to teach mathematics? ______________________________
37.	How many years have you taught geometry? ______________________________
38.	What was your major in college? ______________________________
Comments:	______________________________ ______________________________ ______________________________ ______________________________ ______________________________ ______________________________ ______________________________

APPENDIX B

Permission to Use the Survey of Geometry Perspectives

Benjamin-

Thanks for sending the information I requested. I have finally been able to review the documents you sent me about your proposal. I am impressed with the work you have done to this point and the plan you have outlined for completion of your program. Additionally, I have communicated with Dr. Tierce about your dissertation and spoke with a colleague who did her doctoral study at Liberty (so that I could get a better understanding of the process at LU).

With granting approval for the use of my survey in your dissertation, I have a couple of minor requests. I would like a copy of the data set you collect so that I could combine it with the data I have (and my intention would be that any potential publications coming out the joint data would be co-authored). Also, I would ask for you to seek my permission for any other use of the survey beyond the completion of your dissertation (presentations, publications, additional research, etc).

Now that I have a better understanding about the nature of my involvement for serving on your dissertation committee, I see no problem with honoring your request. I wanted to be sure that I would be able to serve effectively. I consider your invitation of being asked to serve in that capacity an honor.

As you continue your work, I will be happy to assist you in any way that I can. I look forward to seeing the completion of your dissertation and future scholarly work. In a few months, it will be honor to call you Dr. Lane!

Jim

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APPENDIX C

School Cover Letter

Survey of Geometry Perspectives
Benjamin Lane
Liberty University
Department of Education

Dear Administrator,

Let me first thank you for your agreement to assist me in a study of geometry teachers' perspectives on geometry proofs. Let me next remind you of the details of your involvement.

Enclosed are individual packets to distribute to your geometry teachers. Each packet contains the following: (1) a cover letter introducing the study, (2) an Informed Consent document that outlines the risks, benefits, and methods of teacher participation (3) the Survey of Geometry Perspectives, (4) a postage-paid return envelope, and (5) a five-dollar Starbucks coffee gift card as an incentive for completing the survey. The incentive is merely a small token of gratitude for participation. However, each teacher's participation remains voluntary throughout the study, and receiving the incentive does not commit the teacher to participation.

Other than distributing one packet to each geometry teacher, I only ask that you remind each teacher in a few days about completing the survey.

If you have any questions, feel free to contact me (Benjamin Lane, 850-384-3298, bclane@liberty.edu) or my university advisor (Dr. Kenneth Tierce, 940-441-2378, krtierce@liberty.edu).

If you have any questions or concerns regarding this study and would like to talk to someone other than the researchers, **you are encouraged** to contact the Institutional Review Board, Dr. Fernando Garzon, Chair, 1971 University Blvd, Suite 1582, Lynchburg, VA 24502 or email at fgarzon@liberty.edu.

I again thank you for your cooperation. While I am obviously grateful for your help toward me completing my degree, I also thank you for helping the educational community to better understand teacher perspectives and practices.

Sincerely,

[Signature]

Benjamin C. Lane

APPENDIX D

Teacher Cover Letter

Dear Geometry Teacher,

I am a graduate student at Liberty University working on my doctoral degree in curriculum and instruction. I am conducting a research study into the perspectives of geometry teachers concerning proofs in the geometry class—in particular, how Christian school and public school teachers may differ on these perspectives. Because of your position as a geometry teacher, you are invited to participate in this study by completing the included Survey of Geometry Perspectives.

In addition to this opening letter, this packet of materials contains the following:

(1) an informed consent document that outlines the risks, benefits, and procedures for participation;
(2) the Survey of Geometry Perspectives;
(3) a stamped return envelope for returning the completed survey; and
(4) a $5 Starbucks Coffee gift card.

The gift card is an incentive for your participation and is yours to keep regardless of your participation in this study.

Any questions that you might have concerning this study should be addressed in the included informed consent document. If you still have any questions, that document has information for contacting the researchers involved in this study.

While I obviously desire your cooperation so that I might complete my degree, I also desire to help the educational community to better understand teacher perspectives and practices, especially in the field of mathematics education. I serve as a geometry teacher myself and can appreciate the struggles that you encounter as you teach this difficult subject.

I thank you for your time and wish you well in the rest of your school year.

Sincerely,

[Signature]

Benjamin C. Lane

APPENDIX E

Teacher Informed Consent Form

CONSENT FORM
Survey of Geometry Attitudes
Benjamin Lane
Liberty University
Department of Education

Dear Geometry Teacher,

I am a graduate student at Liberty University working on my doctoral degree in curriculum and instruction. I am studying geometry teachers' perspectives concerning the teaching of proofs in the geometry class. Because of your position as a geometry teacher, you are invited to participate in this study by completing the included Survey of Geometry Perspectives. This document outlines the risks, benefits, and procedures for participation in this study. I ask that you read this letter and ask any questions you may have before agreeing to participate.

This study is being conducted by Benjamin Lane, a doctoral student in the Liberty University Department of Education, under the direction of Dr. Kenneth Tierce.

Background Information:

The purpose of this study is to examine the beliefs and practices of geometry teachers from different types of schools. Specifically, this is to determine how teachers from private and public schools may hold differing perspectives on the role of proofs in the geometry class. This is an expansion of research first conducted by Truelove (2004).

Procedures:

If you agree to participate in this study, you need only to complete the included Survey of Geometry Perspectives and return it using the enclosed postage-paid return envelope. This survey takes approximately 5-10 minutes to complete.

Risks and Benefits of being in the Study:

There are no known risks associated with completing the survey beyond a possible breach of confidentiality. To minimize this risk, all possible procedures are being used to safeguard the anonymity of your response. The "Confidentiality" section below outlines these procedures.

The chief benefit to participation is your direct involvement in the development of a better understanding of educational practices. While much research has been done concerning perspectives on geometry proofs, most of that research has focused on student perspectives on learning proofs or teacher perspectives on how students learn proofs. Little has been done to study teacher perspectives on *teaching* proofs. This study is designed to improve the educational community's understanding in this area.

Compensation:

A five-dollar Starbucks coffee gift card has been enclosed as a small token of gratitude for your participation. However, you may keep the gift card whether or not you actually complete the survey. Though I value your response, your participation in this study is completely voluntary.

Confidentiality:

The records of this study will be kept private. In any sort of report we might publish, we will not include any information that will make it possible to identify a participant. Research records will be stored securely and only researchers will have access to the records.

None of the researchers associated with the study have been given the names of any participants. This survey packet was mailed to your school, and your school's administration has been asked to deliver this survey to the geometry teachers. Furthermore, there are no identifying marks on the survey or on the return envelope. Even if the researchers knew the names of participants, there is no way to link any participant to any particular completed survey. All received data will be stored electronically in an encrypted format, and all paper documents will be kept in a locked safe.

If the data should be compromised, your school's administration will be contacted so that they may inform you of the breach. However, the lack of identifying marks on the survey ensure that anyone illicitly obtaining the documents will not be able to discern any private information.

Voluntary Nature of the Study:

Participation in this study is voluntary. Your decision whether or not to participate will not affect your current or future relations with your employer or with Liberty University. If you decide to participate, you are free to not answer any question or withdraw at any time without affecting those relationships.

Contacts and Questions:

The researcher conducting this study is Benjamin Lane, working under the direction of Dr. Kenneth Tierce as faculty advisor. You may ask either of these individuals any questions you have now. If you have questions later, **you are encouraged** to contact them at

Benjamin Lane	Dr. Kenneth Tierce
bclane@liberty.edu	krtierce@liberty.edu

If you have any questions or concerns regarding this study and would like to talk to someone other than the researchers, **you are encouraged** to contact the Institutional Review Board, Dr. Fernando Garzon, Chair, 1971 University Blvd, Suite 1582, Lynchburg, VA 24502 or email at fgarzon@liberty.edu.

Statement of Consent:

(NOTE: Your completion and return of the Survey of Geometry Perspectives will be taken as your agreement to the following statement.)

I have read and understood this consent form. I have asked questions and have received answers. I consent to participate in the study.

Reference:

Truelove, J. E. (2004). *Geometry teachers' conceptions of proof.* (Doctoral dissertation). Retrieved from ProQuest Dissertations and Theses.

IRB Code Numbers: 1452.021913 (exempt)

IRB Expiration Date: May 31, 2013

APPENDIX F

IRB Approval

Dear Benjamin,

The Liberty University Institutional Review Board has reviewed your application in accordance with the Office for Human Research Protections (OHRP) and Food and Drug Administration (FDA) regulations and finds your study to be exempt from further IRB review. This means you may begin your research with the data safeguarding methods mentioned in your approved application, and that no further IRB oversight is required.

Your study falls under exemption category 46.101 (b)(2), which identifies specific situations in which human participants research is exempt from the policy set forth in 45 CFR 46:

> (2) Research involving the use of educational tests (cognitive, diagnostic, aptitude, achievement), survey procedures, interview procedures or observation of public behavior, unless:
> (i) information obtained is recorded in such a manner that human subjects can be identified, directly or through identifiers linked to the subjects; and (ii) any disclosure of the human subjects' responses outside the research could reasonably place the subjects at risk of criminal or civil liability or be damaging to the subjects' financial standing, employability, or reputation.

Please note that this exemption only applies to your current research application, and that any changes to your protocol must be reported to the Liberty IRB for verification of continued exemption status. You may report these changes by submitting a change in protocol form or a new application to the IRB and referencing the above IRB Exemption number.

If you have any questions about this exemption, or need assistance in determining whether possible changes to your protocol would change your exemption status, please email us at irb@liberty.edu.

Sincerely,

Fernando Garzon, Psy.D.
Professor, IRB Chair
Counseling
(434) 592-4054

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APPENDIX G

Comments from Public School Geometry Teachers

For the past two years I have taught "informal geometry," a class that is being phased out. This is a lower-level geometry class that is not rigorous enough to prepare students for the Florida geometry end of course exam. The main way that this class was "dumbed down" was through the removal of almost all proofs. Proofs have all but been removed from my lower level "informal geometry" class. Our book provides very little information on proofs and derives no postulates or theorems (Glencoe-McGraw-Hill Geometry Concepts and Applications). P.S., I notice the same type of book "Concepts and Applications" used by a Christian school student.

Prior to teaching I spent 10 years in construction where geometry was used daily.

More important than just learning proofs in geometry, students are learning how to justify their statements and reasoning—logical reasoning. Regardless of a Christian school or public school, geometric concepts remain the same.

The importance of proofs is dwindling because of the EOC. Students will not have to write a proof, so in a non-honors course, public teachers where I am basically don't see the point to stress it to students. When I took geometry honors in high school, proofs were a big deal and pushed in every chapter. They pushed me to think outside of the box and trained my mind to use logic.

I teach standard geometry. There is an honors geometry that delivers more instruction using/requiring proofs. The student capability right now precludes the use of proofs in instruction and testing. I do believe that proofs are a definite requirement for understanding geometry on a 95-100% level yet we require a less rigorous final requirement.

At my present school, we have two different geometry courses: regular and informal. In the informal geometry course, proofs are not taught at all! In regular geometry they are not emphasized and are not tested on the district's subject area exam.

I believe the most important part of doing proofs in geometry is sketching the picture or figure and marking the congruence, etc. as you go along.

Perspectives in teaching do not alter a teacher's requirement to teach within the framework of district and state curriculum guides and teacher evaluation based on student success on a state exam that does not include proofs.

Today, less time is given to formal proofs than before due to increased testing! Not enough class time—however, I believe "proof" (informal and formal) is a very important part of geometry.

For questions 24-30, since proofs are integrated throughout the course, the amount of proofs done per week varies with the topics being covered. When working with triangles, proofs can be up to 80% of what we spend time on both in and out of class. However, we spend less than 20% of our time on proofs during studies of volume and surface area.

I have previously taught gifted geometry but currently teach trigonometry and calculus—both of which rely heavily of proofs, so I feel that geometry proofs are foundational to future mathematical learning.

Teachers generally think of the "two-column" variety when "proof" is mentioned. I prefer flow-chart or paragraph proofs as these work well with the content in my grade level.

[Question 32:] I like both [smiley face; teaching geometry and algebra].

In Georgia, there has been a significant decrease in the teaching and practicing of proofs. I find this very sad! We expect our students to go on and be successful in upper-level math classes but don't take the time to teach them how to think! [comment on Question 17:] Do not teach proofs anymore...no longer part of the curriculum [frowny face].

I haven't taught Geometry for several years now. Georgia has moved to a more integrated curriculum. However, when I did teach Geometry, I preferred to teach proofs at the end of the semester as a single unit. This was a good review of everything before the EOCT. [Question 13:] No; but that's how I prefer to teach them. [Practices questions:] Only taught proofs in the congruent and similar triangles units and the quadrilaterals unit.

Although I think proofs are important, I believe or (*sic*) current integrated curriculum is designed so that we cover LOTS of material but do not get to go into great detail with any material. There is no time to "teach" proofs properly, and they are not assessed on most state/national tests.

[Question 17:] Often I fill in some parts and ask them to fill in others.

APPENDIX H

Comments from Christian School Geometry Teachers

I teach two 10th grade geometry classes using the Saxon textbook. My students are exceptional students with learning disabilities (including ADHD, Asbergers, autism, vision and memory deficits). Tests include one-two proofs which are fill in the blanks. God bless you in your research and career and home!

Early years I taught rigid proofs according to the Christian textbooks provided. I still use the same (updated) books but try to focus more on investigation and understanding the entire process and reasoning behind the proofs—not just rote operations.

I do not believe geometry is a stand-alone course.

In my experience two or three chapters deal with proofs as the main focus, and then they are sprinkled throughout the rest of the book (MacDougall-Littell books).

I love geometry because I believe it trains the students to think why. I desire to teach the students to approach their Bible study both deductively, God's work is coherent it supports itself and has a very deductive approach; this is the only thing that will last for all eternity, also inductively, so students can see specific examples that lead them to a conjecture, then also show them the truth deductively.

I use a blend of paragraph, flow chart, and formal proofs. I stress the logic more than the structure. "No boss is going to ask you to do a proof, but many of them need you to present a logical argument for something." "Your job will never depend on a two-column proof, but it may depend on solving a problem from start to finish logically." These are two quotes I use on "why I need to know proofs."

There are some chapters where proofs are 80% of the material (e.g. [congruent triangles]); other chapters have none (right [triangles], area, volume). I believe proofs are very important but not the only important thing. My students do best with a structure, formal approach to proofs. I've taught for 42 years, geometry for at least 1/2.

Not a lot of experience yet, but maybe this was of help!

Typically cover proofs in chapters 2 and 3 (out of 12), and don't use them the rest of the year.

The skills of thinking and drawing rational, logical conclusions are sorely lacking. Not to require proofs as part of the curriculum would further erode the academic rigor of our courses. The thinking skills honed through geometric proofs are needed for analyzing literature, identifying cause and effect relationships in science, developing plausible arguments for persuasive speeches and writing, and apologetics. Geometry could be taught as a "stand alone course," for there is plenty of additional material that is not covered extensively which falls under the umbrella of Geometry. Doing so, however, would diminish the education of our students unless a separate course in logic, with an emphasis on proofs and a prerequisite of Geometry completion, were to be developed and required. This could not be in place of another Mathematics course, but rather in addition. As it is, anyone with an opinion, no matter how unfounded, can solicit a worldwide audience via the internet. It is imperative that we teach our students to think critically, not only for the responsible presentation of their own ideas, but also for the evaluation of the ideas of others. Proofs are an integral component for teaching critical thinking skills. [comment on #32]: Both: this is like asking me which of my children I like best–I enjoy them both [teaching geometry and algebra].

The amount of good text books has been steadily going down over the years. I have taught the last 6 years with books I myself am not real pleased with. Older texts from the late 80's and early 90's were much better and teaching concepts and incorp. [incorporating?] proofs. I do a good bit out of the book and most materials used to support lesson is not from the text I teach from.

Proofs are the most difficult concept to teach, but the most important for application in life (thinking through a process and justifying your choices).