THE HARMONIC IMPLICATIONS OF THE NON-HARMONIC TONES IN THE FOUR-PART CHORALES OF JOHANN SEBASTIAN BACH

by
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Liberty University

A Dissertation Presented in Partial Fulfillment
Of the Requirements for the Degree
Doctor of Education

Liberty University
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ABSTRACT

This study sought to identify the harmonic implications of the non-harmonic tones in the four-part chorales of Johann Sebastian Bach and to identify if the implications were modern, extended harmonies. The study examined if non-harmonic tones implied traditional or extended harmonies more often, which non-harmonic tones more frequently implied extended harmonies, and which chords typically preceded implied extended harmonies. The study was a corpus analysis of the four-part chorales. The data collected was organized in and analyzed with frequency charts and a chi-square goodness of fit test and chi-square tests of independence from the chordal analysis conducted by the researcher. Harmonic implications of extended harmonies not only exist in the chorales but are also nearly as plentiful as implications of seventh chords. A single non-harmonic tone is most likely to produce an implication of an extended harmony and triads are most likely to precede an extended harmony.

Descriptors: corpus analysis, non-harmonic tones, harmonic implications, chordal analysis, Bach chorales

Dedication/Acknowledgments Pages

I would like to thank and dedicate this project to my lovely and gracious wife Jennifer, for her patience, understanding, and support through this project. I would like to thank Dr. Mark Angle for his patient guidance through this project. I would like to thank Dr. Amanda Szapkiw, Dr. Samuel Wellman, and Dr. Mark Smith for their valuable input. I would also like to thank James McDonald for his aid in editing the various drafts. I would like to thank Ben Lane for his guidance in the computation of the statistics. I would like to thank Dr. Frank Garlock for the inspiration to do this research.

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CHAPTER ONE: INTRODUCTION

Introduction

This study was a corpus analysis of all 371 of Johann Sebastian Bach's (hereafter Bach's) four-part chorales; it examined the harmonic implications of the non-harmonic tones. This study was completed within the theoretical framework of music theory; specifically it studied if the non-harmonic tones are exclusively non-harmonic or if the non-harmonic tones imply extended harmonies (i.e., ninth, eleventh, and thirteenth chords) when considered as part of the harmony. Music theory provides insight to the language of music much like grammar provides insight to the English language. Foundational to the study of music theory is Bach's music and his compositional practices, especially his four-part chorales (Kostka & Payne, 2009; McHose, 1947). Since harmonic implication of the non-harmonic tones has yet to be studied, a gap exists in the understanding of Bach's compositional practices as applied in music theory research and music theory education. This study begins to fill that gap.

Background

In 1911, Arnold Schoenberg declared, "There are no non-harmonic tones, for harmony means tones sounding together" (Schoenberg, 1911/1978, p. 318). This declaration defied a significant portion of the traditional teaching of harmony since the time of Jean-Philippe Rameau. In 1722, Rameau established the system of chords and inversions which is still the basis of tonal harmony instruction today. Rameau taught that C, E, G and E, G, C, and G, C, E are all the same triad, but in different inversions instead of three different chords (Rameau, 1722/1971, p. 40). This system greatly reduced the number of chords and organized the various types of chords in such a way as to be more readily useable by various musicians, especially composers. This system of chords was complemented by the system to account for notes which are not part of the chord—the non-harmonic tones.

Through the years leading up to Schoenberg, composers gradually moved from considering non-harmonic tones as not being part of the harmony to forming new harmonies with notes which were previously non-harmonic (Piston & DeVoto, 1987, p. 115). An early example is the passing seventh. Composers would write a dominant triad (V) which leads to a tonic triad (I), but would add a passing tone which created the interval of a minor seventh above the root of the V chord. Since this sound strengthened the V chord's push to tonic, composers over time would use the minor seventh simultaneously with the dominant triad, creating the V^7 chord. Later composers would non-harmonically add the interval of a ninth to a V^7 chord, which led to the creation of a V^9 (Buchler, 2006). This practice of creating extended harmonies via non-harmonic tones continued until all notes of a scale were included to create a V^{13} (Piston & DeVoto, 1987; Schoenberg, 1911/1978).

In *Theory of Harmony,* Schoenberg's discussion of the lack of non-harmonic tones leads to his premise that each note of the chromatic scale is of equal importance, an element of atonal or non-tonal music. However, this study focused on tonal music (i.e., music with a tonal center), specifically the 371 four-part chorales of Bach. A corpus analysis was conducted to seek to discover what harmonies are formed by the non-harmonic tones and if perhaps Bach formed extended, modern harmonies, such as ninth, eleventh, and thirteenth chords.

The majority of research in music analysis is qualitative; however, there is an emerging body of quantitative research that uses corpus analysis to analyze music. The corpus analysis methodology has been used in music research since 1943, when Budge first created this methodology based on Thorndike's research of word usage frequency in the English language (Budge, 1943). DeClercq and Temperley (2011) observe a resurgence over the previous decade of the corpus analysis methodology related to the increased usage of computers and empirical methodology in music research. In 2009, Temperley noted the lack of quantitative or empirical studies on the question of how closely certain principles of the common practice period were actually followed by composers. The year prior to this claim, Rohrmeier and Cross (2008) completed a corpus analysis of the Bach chorales. Their research used pitch class sets and studied tonality in the chorales. These studies have statistically analyzed the frequency of chords in various corpora of music, but none have addressed harmonic implication in Bach. This study addresses that gap, while adding to this currently growing body of research.

College level music theory students are inundated with extended harmonies in the modern music they listen to and as such may hear common practice period music differently than their theory instructors. Kosar (2001) views this difference in hearing music as an opportunity for the instructor to facilitate learning based on the student's needs. In the hypothetical discussion that ensues, Kosar facilitates a discussion on the possible analyses of the harmony and the non-harmonic tones on bar one count three of figure 1. In the course of the discussion, the harmonic implications of non-harmonic tones were discussed by viewing the eighth note B as an accented passing tone and the C♯ as a passing tone, thus producing two different harmonic implications (Kosar, 2001).

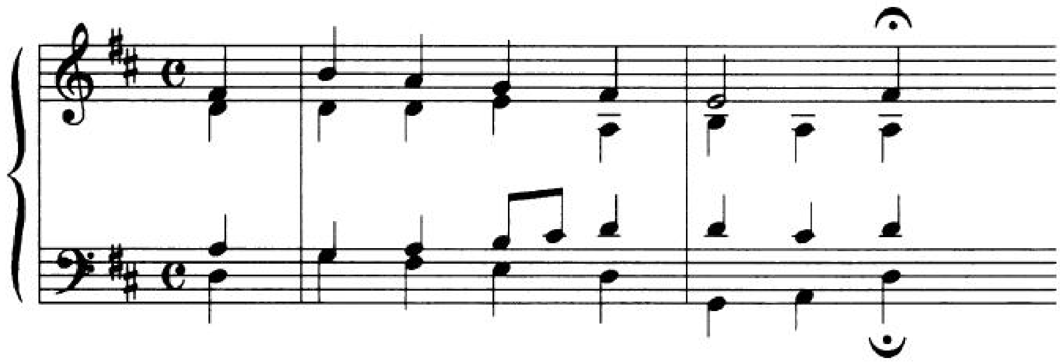

Figure 1. Bach, Chorale 367, mm. 1-2 (Kosar, 2001, p. 113)

In the music theory class, studying music written by significant composers is a form of problem based learning (PBL), since the music studied is that which the student could encounter in lessons, performing groups, and in their own future teaching. PBL began in the medical field as those studying to be nurses or doctors were placed in real life situations and given problems to solve (Jackson, Warelow, & Wells, 2009; Spronken-Smith & Harland, 2009). For music students, a real life problem is understanding the music they are performing. Just as the English language is better understood through a study of grammar, so also the language of music is better understood through a study of music theory. In grammar, the student is able to identify the subject, verb, etc. In music theory, the student identifies the type of harmony and what is not part of the harmony (Roig-Francolí, 2011). With current music widely using extended harmonies such as ninth, eleventh, and thirteenth chords, a problem music students face is identifying harmonic tones and non-harmonic tones, since many non-harmonic tones form extended harmonies (Kosar, 2001). Traditional music theory instruction would tend to reject the notion that Bach used extended harmonies in his four-part chorales; however, today's music theory students could possibly hear extended harmonies in Bach chorales, since they are accustomed to these harmonies in modern music (Turek, 2007, p. 81). If students are in a PBL setting and hear non-harmonic tones as part of the harmony, a music theory instructor may not be open to that possibility due to the long history of the use of non-harmonic tones. This study benefits all collegiate music theory professors, as its findings aid them in answering the proclamation, “But I don’t hear it that way!” (Kosar, 2001).

Problem Statement

Brahms is commonly attributed as having said, “Study Bach, there you'll find everything” (Smith, 1996, p. 1). As modern theory students are presented with the “problem” of the Bach chorales in the PBL curriculum, they may hear modern harmonies that many music theoreticians and, by extension, many music theory teachers do not think Bach used, such as ninth, eleventh, or thirteenth chords. Perhaps “everything” includes harmonies that are more commonly found in modern music. Though much research exists on Bach’s music, and even some by Rubin (1976), which explores the possibility of Bach as a modern composer, no quantitative research exists on the possibility of the non-harmonic tones in the four-part chorales forming extended harmonies. Thus, a corpus analysis of the harmonic implications of the non-harmonic tones needed to be conducted to discover if extended harmonies are present in the non-harmonic tones of the four-part chorales. If Bach did use, or at least imply, extended harmonies, then music education is missing valuable guidance on effective usage of extended harmonies, especially in college level theory, arranging, and composition courses (Shir-Cliff, Jay, & Rauscher, 1965).

Purpose Statement

The purpose of this quantitative corpus analysis study was to test the theory of music that non-harmonic tones are exclusively non-harmonic or that non-harmonic tones imply extended harmonies (i.e., ninth, eleventh, and thirteenth chords) when considered as part of the harmony. This theory was tested via corpus analysis as originally developed by Budge in 1943 and which has more recently become an emerging body of research (Budge, 1943; De Clercq & Temperley, 2011; Rohrmeier & Cross, 2008; Temperley, 2011). One variable was the non-harmonic tones in the four-part chorales of Bach. A non-harmonic tone is a note that does not fit into the harmony. For example, a C major triad consists of the notes C, E, and G. If an A is present in addition to the C, E, and G, it could be considered as a non-harmonic tone, since it is not a C, E, or G. An additional variable is the harmony formed by the non-harmonic tone. The A in the previous example could also be considered as part of the harmony creating an A minor seventh chord, thus the A minor seventh chord would be the harmonic implication of the non-harmonic A with the C major triad (see Figure 2). The purpose of this study was to catalog the harmonic implications of the non-harmonic tones in entire corpus (371) of Bach’s four-part chorales and to create frequency tables from this data and analyze via a chi-square goodness of fit test and chi-square tests of independence to reveal if extended harmonies were hidden within the non-harmonic tones. If harmonies are implied by the non-harmonic tones, are they more frequently triads, seventh chords, or extended harmonies? Are certain extended harmonies more often implied by a certain number of non-harmonic tones than triads or seventh chords? Are certain extended harmonies more frequently preceded by certain harmonies than triads or seventh chords?

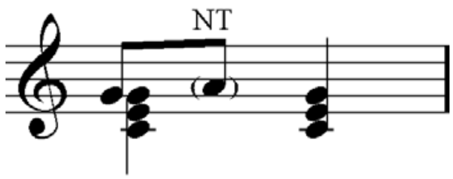

Figure 2. C major triad with an A as a neighbor tone; harmonic implication, A minor seventh first inversion

Significance of the Study

Since extended harmonies such as the ninth, eleventh, and thirteenth chords were formed by non-harmonic tones becoming harmonic tones; since Schoenberg declared that there are no such things as non-harmonic tones; since many music theoreticians think that extended harmonies (especially eleventh and thirteenth) were used by classical composers and beyond; and since the Bach chorales are a microcosm of common practice period harmonic practice, this study sought to see if Bach implied extended harmonies with the non-harmonic tones in his chorales. If he did, it will inform best practices on voice leading with extended harmonies, as well as which non-harmonic tones were typically used to form which extended harmony. Research has not yet been conducted to see if Bach did write with extended harmonies via non-harmonic tones. The results of this study address that gap in the literature on Bach's compositional practices, add to the emerging body of research known as corpus analysis, and greatly benefit collegiate level music theory, composition, and arranging instruction.

Research Questions and Hypotheses

The research questions for this study were:

RQ1: Is there a statistically significant difference in the frequency of occurrence of harmonic implication of extended harmonies (i.e., ninth, eleventh, and thirteenth chords), triads, and seventh chords within the harmonic implications?

RQ2: Is there a statistically significant difference in the proportion of extended harmonic implication versus triad harmonic implications or seventh chord harmonic implications based on the number of non-harmonic tones forming the implication (i.e., a single non-harmonic tone, two non-harmonic tones occurring simultaneously, or three non-harmonic tones occurring simultaneously)?

RQ3: Is there a statistically significant difference in the proportion of extended harmonic implication versus triad harmonic implications or seventh chord harmonic implications based on the type of preceding harmony (i.e., triads, seventh chords, or extended harmonies)?

The following were the null research hypotheses:

$\mathbf{H_{o1}}$: There is no statistically significant difference in the frequency of occurrence of extended harmonies (i.e., ninth, eleventh, and thirteenth chords), triads, and seventh chords within the harmonic implications.

$\mathbf{H_{o2a}}$: There is no statistically significant difference in the proportion of extended harmonic implication versus triad harmonic implications based on the number of non-harmonic tones forming the implication (i.e., a single non-harmonic tone, two non-harmonic tones occurring simultaneously, or three non-harmonic tones occurring simultaneously).

$\mathbf{H_{o2b}}$: There is no statistically significant difference in the proportion of extended harmonic implication versus seventh chord harmonic implications based on the number of non-harmonic tones forming the implication (i.e., a single non-harmonic tone, two non-harmonic tones occurring simultaneously, or three non-harmonic tones occurring simultaneously).

H_{o3a}: There is no statistically significant difference in the proportion of extended harmonic implication versus triad harmonic implications based on the type of preceding harmony (i.e., triads, seventh chords, or extended harmonies).

H_{o3b}: There is no statistically significant difference in the proportion of extended harmonic implication versus seventh chord harmonic implications based on the type of preceding harmony (i.e., triads, seventh chords, or extended harmonies).

Identification of Variables

Several variables were utilized in this study. Each research question sought to discover the number and type of harmonic implications derived from the non-harmonic tones. Non-harmonic tones are notes typically considered not part of the harmony and are classified based on their usage (Kostka & Payne, 2009). Harmonic implication describes the harmony formed when the non-harmonic tones are considered as part of the harmony (Roig-Francolí, 2011). In question one, the variable was the type of chord implied by the non-harmonic tone, whether the implication was a triad, a seventh, or one of the extended harmonies (i.e., ninth, eleventh, or thirteenth chords; Benward & Saker, 2009). Triads, seventh chords, and the extended harmonies are tertian since they are built in thirds. A triad has three notes, a seventh chord has four notes, a ninth chord has five notes, an eleventh chord has six notes, and a thirteenth chord has seven notes (Gauldin, 2004; Kostka & Payne, 2009; Turek, 2007) In question two, the variable which could have a possible effect on the harmonic implications was the number of non-harmonic tones used in the harmonic implication (i.e., a single non-harmonic tone, two non-harmonic tones occurring simultaneously, or three non-harmonic tones occurring simultaneously; Turek, 2007). In question three, the variable which could have a possible effect on the harmonic implications was the type of chord which preceded the harmonic implication (i.e. triads, seventh chords, or extended harmonies; Piston & DeVoto, 1987). A chi-square goodness of fit test compares the nominal data from the frequency charts against theoretical expected frequencies (Howell, 2011; McDonald, 2009a). The theoretical expected frequencies were drawn from a table of chord frequencies found in Quinn's study, "Are Pitch-Class Profiles Really 'Key for Key'?" (2010, p. 155). Quinn's study provided a strong basis for the expected frequencies for research question one, since each new note was considered as a new harmony, thus matching the methodology of this research. A chi-square test of independence compares the nominal data from to variables to see if the difference is significant or possibly a product of chance (McDonald, 2009b).

Definitions

Harmonic implication refers to the typically disregarded harmony formed by considering the tones sounding concurrently with a non-harmonic tone as a new harmony. This term is derived from the process of deducing the underlying or implied harmonies of a melodic line (Roig-Francolí, 2011, p. 275).

Tertian harmony is harmony based on the interval of a third. Though a major triad can be described as having the intervals of a major third and a perfect fifth from the root, it could also be described as a minor third stacked on top of a major third (Piston & DeVoto, 1987, p. 14; Roig-Francolí, 2011, p. 55).

Non-harmonic tones are tones not considered, by many, to be part of the sounding harmony, since many times they are notes outside of the commonly used harmonies of the common practice period (Kostka & Payne, 2009, p. 181; Roig-Francolí, 2011, p. 188).

Passing tones are non-harmonic tones which "pass" from one harmony to another by a stepwise motion in the same direction. Passing tones can move in ascending or descending motion and can sometimes involve more than one note (see figure 3; Kostka & Payne, 2009, p. 183; Turek, 2007, p. 142).

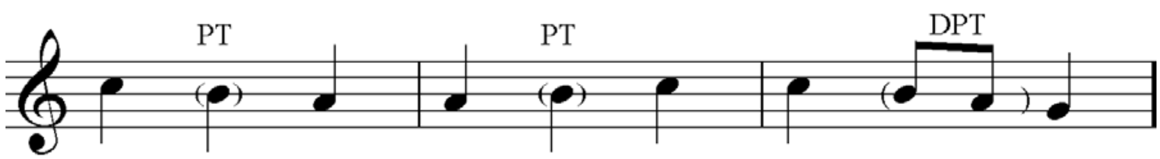

Figure 3. Examples of passing tones

Neighbor tones are non-harmonic tones which move in a stepwise motion to an adjacent note and then return to the starting note. Neighbor tones can move to an upper or lower note (see figure 4; Piston & DeVoto, 1987, p. 118; Turek, 2007, p. 144).

Figure 4. Examples of neighbor tones

Suspensions are formed when a note is "suspended" from a previous harmony and resolved in a downward direction. The type of suspension is indicated by the interval formed between the suspended note as well as the note of resolution and the bass (e.g., 4-3; see figure 5; Kostka & Payne, 2009, pp. 185-186; Turek, 2007, pp. 148-149).

Figure 5. Example of a suspension

A *major triad* consists of a root, a third, and a fifth. The interval formed between the root and third is a major third and the interval formed between the root and the fifth is a perfect fifth (see figure 6; Aldwell, Schachter, & Cadwallader, 2011, pp. 47-48; Piston & DeVoto, 1987, p. 14).

A *minor triad* consists of a root, a third, and a fifth. The interval formed between the root and third is a minor third and the interval formed between the root and the fifth is a perfect fifth (see figure 6; Aldwell, et al., 2011, pp. 47-48; Piston & DeVoto, 1987, p. 14).

A *major-minor seventh chord* consists of a root, a third, a fifth, and a seventh. The interval formed between the root and third is a major third, the interval formed between the root and the fifth is a perfect fifth, and the interval formed between the root and the seventh is a minor seventh. This chord can be formed by adding the interval of a minor seventh to the root of a major triad, thus the name of major-minor seventh chord (see figure 6; Aldwell, et al., 2011, p. 57; Roig-Francolí, 2011, pp. 60-61).

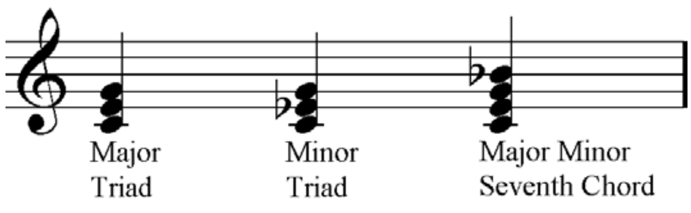

Figure 6. Examples of a major triad, a minor triad, and a major-minor seventh chord

Extended harmonies are harmonies which extend beyond the boundary of the seventh chords by adding additional thirds beyond the seventh (Kostka & Payne, 2009, p. 507; Roig-Francolí, 2011, p. 667).

Ninth chords are formed by adding the interval of a ninth to the root of a seventh chord. This fifth note in the harmony creates the possibility of many different forms of this harmony. Two common types are the V^9 and $V^{\flat 9}$, both based on the major-minor seventh chord: the first with the interval of a major ninth from the root and the second with the interval of a minor ninth from the root (see figure 7; Gauldin, 2004, pp. 631-632; Turek, 2007, pp. 520-521).

Eleventh chords are formed by adding the interval of the eleventh to a ninth chord. The possible variations are greater than the ninth but are slightly limited since the eleventh is a perfect interval and as such has no minor version. Since six notes are possible in this harmony, it can be formed by combining two triads, such as a IV chord over a V chord or a V^{11} (see figure 7; Gauldin, 2004, p. 638; Kostka & Payne, 2009, pp. 450, 507).

Thirteenth chords contain all seven notes of a scale, though arranged in thirds. This harmony is formed by adding the interval of a thirteenth to the root of an eleventh chord. Since the chorales analyzed consisted of four voices, the implication of this harmony would likely be rare; however, it could also served as a possible explanation of some of the vaguer implications if all other explanations had been exhausted (see figure 7; Gauldin, 2004, p. 639; Kostka & Payne, 2009, pp. 450, 507).

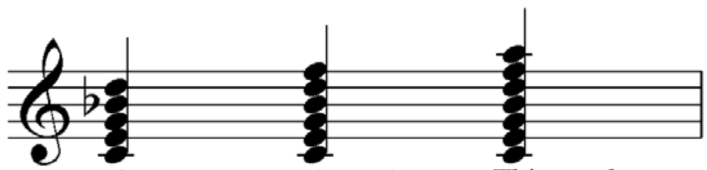

Figure 7. Examples of a ninth chord, an eleventh chord, and a thirteenth chord

Research Summary

A quantitative corpus analysis of the entire corpus (371) of Bach's four-part chorales was conducted by the researcher via chordal analysis with the harmonies, non-harmonic tones, and harmonic implications identified. To verify the researcher's accuracy with coding via chordal analysis, a sample selection of five Bach chorales was coded (analyzed) by the researcher and three additional music theoreticians. The results of the chordal analysis coding were compared via Cohen's Kappa tests. If there was what Landis and Koch (1977) deemed as substantial or greater agreement (Kappa coefficient ≥ 0.61) between the researcher and two of the three additional coders, the researcher was deemed as accurate with chordal analysis coding, and was to have been the sole coder (i.e., analyzer). If there was a less than substantial agreement (Kappa coefficient < 0.61) between the researcher and two of the three additional coders, three coders (the researcher and two additional experts) would have completed the chordal analysis coding of the chorales (Budge, 1943).

A standard chordal analysis was used for the coding of the harmonies. The first step was to identify the harmonies and assign the appropriate label. Once the harmonies were identified, the non-harmonic tone(s) were identified and labeled following standard practice. The non-harmonic tone(s) were then considered with the other tones sounding simultaneously with the non-harmonic tone(s) to ascertain the harmonic implication formed. The data of the harmony implied, together with the type of non-harmonic tone(s) and the preceding harmony were recorded. Frequency charts were created from the data, and tested with a chi-square goodness of fit test and chi-square tests of independence to address the various research questions (Budge, 1943; De Clercq & Temperley, 2011; Howell, 2011; Temperley, 2011). Chi-square would be an appropriate test for this study since the data collected is categorical and not continuous (Gall, Gall, & Borg, 2007). Additionally, chi-square tests will be appropriate since the categories of the various questions are independent and not affected by another category (i.e., a triad is not a seventh chord nor an extended harmony) and since the entire corpus (population) was analyzed, the counts of each group will be well above the minimum requirements of five (Isaac & Michael, 1981). Though the entire corpus of Bach's four-part chorales was researched and the data collected is the entire population of the harmonic implications, as opposed to the typical education research practice of taking a sample of the population, Gall, Gall, and Borg indicate that inferential statistics would sometimes be completed (2007, p. 141). This step permitted the results to be presented in a manner familiar to the educational researcher.

Assumptions and Limitations

Assumptions

This study assumed the following: (a) the four-part chorales cataloged by C. P. E. Bach are J. S. Bach's work. (b) The chorales are accurate and a true presentation of what Bach wrote. (c) Since Bach's music, especially his four-part chorales, codifies the foundation of tonal harmony, it would be appropriate literature on which to perform a corpus analysis. (d) If there was substantial or greater agreement between the coding of two of the three additional analyzers and the researcher of the sample, there would not be a statistically significant difference in the coding (analysis) of the entire corpus (Budge, 1943, pp. 4-5)

Limitations

This study was limited to the four-part chorales of Bach, and as such would have limited application to the balance of Bach's compositional output or the compositions of other composers. The corpus of the four-part chorales of Bach was selected since it is foundational to tonal music of the common practice period (Kostka & Payne, 2009; McHose, 1947). Due to the foundational nature of this corpus of music, the results may have application to other total music; however, that application would likely be limited. Budge (1943), based on the results of the original corpus analysis, observed that Bach’s harmonic vocabulary is more extensive than any of the other composers studied in her corpus. Since this study was researching the presence of extended harmonies, which would be illustrative of an extensive harmonic vocabulary, these results could have limited application to another composer’s works.

The study limited the type of harmonic analysis to a chordal analysis to minimize the vagueness caused by the shifting tonality which is a typical feature of the Bach chorales. The study was limited to tertian harmonies. If other harmonic systems such as secundal, quartal, or quintal had been used, the number of possibilities of harmonic implications would have been so numerous as to become unwieldy and impractical for collecting and analyzing empirical data. Though the method of chordal analysis was used to minimize researcher bias, it was still a possible limitation of this study.

CHAPTER TWO: REVIEW OF THE LITERATURE

Introduction

Bach's compositional practices serve as the foundation for western music theory, specifically the time of music history referred to as the common practice period (Piston & DeVoto, 1987, p. xvi; Turek, 2007, p. 81). The basic harmonic usage of the common practice period was primarily limited to four triads (i.e., major, minor, diminished, and augmented) and five seventh chords (i.e., major-major, major-minor, minor-minor, diminished-minor, and diminished-diminished; Roig-Francolí, 2011, pp. 57, 61). Modern harmonic practice (i.e., both popular music, such as jazz, and recent classical composers, such as Debussy) uses many more chords, including extended harmonies like the ninth, eleventh, and thirteenth (Ligon, 2001; Nadeau, 1979). Though the V^9 and $V^{\flat 9}$ became more common with composers which followed Bach (e.g., Haydn, Beethoven, and Mozart), Bach is not typically thought to write with modern, extended harmonies, especially the eleventh and thirteenth chords. However, several researchers (e.g. Berger, 2007; Butt, 2010a, 2010b; Marshall, 1976; Rubin, 1976) have explored the various aspects of the possibility of Bach as a modern composer. Research has not yet been conducted to see if perhaps Bach did write with extended harmonies via non-harmonic tones. If Bach did imply these harmonies, then he could give guidance for the best practice of extended harmony usage, as he does for other harmonies.

Collegiate music theory instruction is based upon Bach's compositions, especially the four-part chorales. In their book, *Tonal Harmony*, Kostka and Payne use a phrase of a Bach chorale as their first musical example (Kostka & Payne, 2009, p. x). McHose places special emphasis on the music of Bach in his theory text *The Contrapuntal*

Harmonic Technique of the 18th Century and requests that each student have a copy of the Bach chorales to study as a supplement to the text (1947, pp. ix-x). Therefore, discovery of the extended harmonies within the non-harmonic tones of the four-part chorales would greatly benefit college music theory instruction.

This chapter presents the theoretical framework of this research, harmonic analysis. Since this paper was addressed to all educators and not solely music educators, a thorough and systematic presentation of harmonic analysis was conducted. The chapter concludes with a review of the literature pertaining to corpus analysis.

Theoretical Framework

Music theory, specifically harmonic analysis, provided the theoretical framework for this study. Music of ancient times was a single melody, but over the years additional parts were added creating polyphony or many sounds. Since music theoreticians attempt to describe what composers wrote, various theoreticians attempted to describe the vertical aspect of music (i.e., harmony). Some thought that C, E, G was a different chord from E, G, C, and G, C, E (Holtmeier, 2007). However, Rameau (1722/1971) in his *Treatise on Harmony* put forth the idea that these were inversions of the same harmony instead of three different harmonies. This concept of inversion is still taught in music theory courses and is foundational to the study of harmony.

Music of the common practice period is based on tertian harmony (i.e., harmony built in thirds). The letters used in the musical alphabet are A through G used in a loop, so A follows G as the notes continue upward. Moving from one note to the next is referred to as an interval of the second. Skipping a letter would create the interval of a third, such as C to E or E to G. The C major triad is created from two thirds stacked one on top of the other (Aldwell, et al., 2011). Though common practice period harmony is tertian, stacking two thirds for a triad also creates the interval of a fifth between the lowest and highest note (Gauldin, 2004, p. 56). Adding an additional third, B, to C, E, G, creates a seventh chord, since the chord has the interval of the seventh between C and B. Continuing this pattern creates a ninth chord with the notes C, E, G, B, and D, an eleventh chord with the notes C, E, G, B, D, and F, and a thirteenth chord with the notes C, E, G, B, D, F, and A (see figure 8; Roig-Francolí, 2011, p. 667). (Note that all of the notes of the musical alphabet are included in the thirteenth chord).

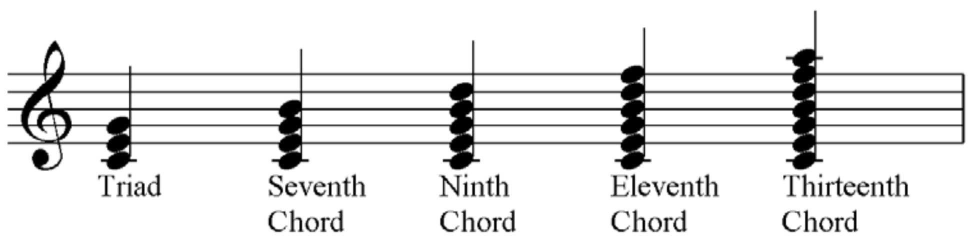

Figure 8. Examples of a triad, a seventh chord, a ninth chord, an eleventh chord, and a thirteenth chord

In addition to size, intervals have a quality. The quality is determined by whether the notes of the interval are diatonic to the major scale. For a note to be diatonic, it must be one of the notes of the scale (Kostka & Payne, 2009, p. 19). The C major scale consists of the notes C, D, E, F, G, A, and B. Therefore E is diatonic and F♯ is not diatonic (also called chromatic; Turek, 2007, p. 26). Diatonic intervals have qualities of either major or perfect depending on their size. Perfect intervals are the intervals of unison (one), fourth, fifth, and the octave (eight). Major intervals are the intervals of second, third, sixth and seventh (Benward & Saker, 2009, p. 56). Qualities of intervals change as the size of the interval changes. If a major interval (C to E) is made smaller by a half step (C to E♭), then it is a minor interval. If a minor or perfect interval is made smaller by a half step, the interval is a diminished interval. If a major or perfect interval is made larger by a half step, the interval is augmented (Turek, 2007, p. 22). Though the size of the interval adjusts by a half step to adjust the quality of the interval, the letter names of the notes do not change, only the accidentals change. In order to change the perfect interval of C to G into a diminished interval, G would be lowered a half step to G♭. On the piano, the note G♭ is also F♯; but if the interval is written as C to F♯; it is no longer a diminished fifth, but an augmented fourth. The perfect interval of C to G can also be changed to a diminished fifth by raising C by a half step to C♯ (Turek, 2007, p. 22).

The type of triad changes according to the quality of intervals in the triad. A major triad consists of a major third and a perfect fifth (C, E, G, see figure 9). A minor triad consists of a minor third and a perfect fifth (C, E♭, G, see figure 9). A diminished triad consists of a minor third and a diminished fifth (C, E♭, G♭, see figure 9). An augmented triad consists of a major third and an augmented fifth (C, E, G♯, see figure 9). Note how the triad's name is drawn from the significant interval. Since a perfect fifth is an open or indistinct interval, the third of the chord gives that triad its quality. When the interval of the fifth changes to diminished or augmented, it becomes the more characteristic interval, thus supplying the name (Piston & DeVoto, 1987, p. 14).

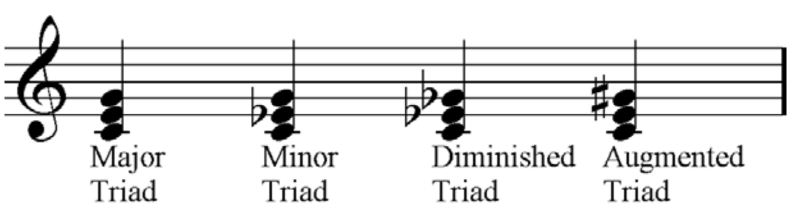

Figure 9. Examples of a major triad, a minor triad, a diminished triad, and an augmented triad

Seventh chords are commonly referred to by the quality of the triad and the quality of the interval of the seventh. A major triad with a major seventh is called a major-major (MM) seventh chord (e.g., C, E, G, and B, see figure 10). If the interval of the seventh is changed to minor, then the major-major seventh chord changes to a major-minor (Mm) seventh chord (e.g., C, E, G, and B♭, see figure 10). In the music of the common practice period, this chord has a function; it helps establish the key by pushing to the tonic harmony. Tonic is a diatonic triad built on the first note of the scale; for the key of C major, this is a C major triad. Diatonically the only seventh chord that is a major-minor seventh is the seventh chord built on the fifth scale degree; thus the major-minor seventh chord is often referred to as the five seven (V^7; Aldwell, et al., 2011, p. 57).

Seventh chords can be built with any combination of triad and seventh qualities, but only five types were typically used in the common practice period (Roig-Francolí, 2011, p. 61). Other than the two types previously mentioned, three other types of seventh chords were typically used. A minor-minor (mm) seventh chord has a minor triad with a minor seventh (e.g., C, E♭, G, and B♭, see figure 10). Two seventh chords are built on the diminished triad but one has a minor seventh (e.g., C, E♭, G♭, and B♭, see figure 10) and the other has a diminished seventh (e.g., C, E♭, G♭, and B♭♭, see figure 10). The diminished-minor (dm) seventh chord is also known as a half diminished seventh chord (ø), and the diminished-diminished (dd) seventh chord is also known as a fully diminished seventh chord (○; Aldwell, et al., 2011, p. 57).

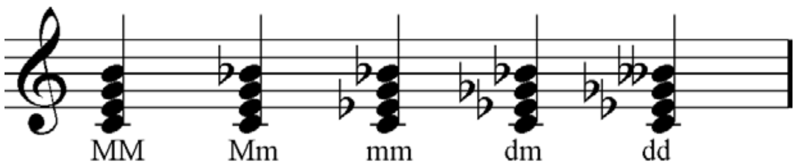

Figure 10. Examples of MM, Mm, mm, dm, and dd seventh chords

Seventh chords, like triads, remain the same chord when they are inverted. The major-minor seventh chord of C, E, G, B♭ is the same chord as E, G, B♭, C, and G, B♭, C, E, and B♭, C, E, G. Instead of stating that these are four different chords, they are referred to as different inversions. Root position is C, E, G, B♭; first inversion is E, G, B♭, C; second inversion is G, B♭, C, E; and third inversion is B♭, C, E, G (see figure 11; Kostka & Payne, 2009, p. 47).

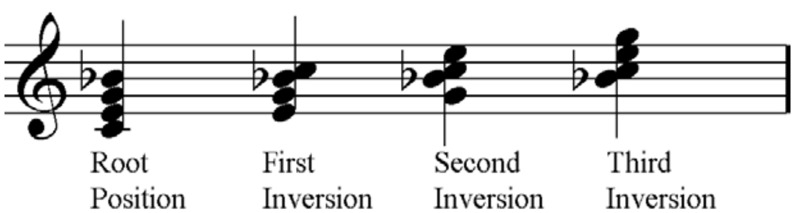

Figure 11. Examples of the inversion of a seventh chord

Problem based learning (PBL) also framed this study, since in collegiate music theory, much of the music studied is real world (i.e., music written by master composers). It is music the student could encounter in their lessons or performing groups. PBL originated in medical education as a means to train doctors and nurses in real world situations they would encounter in their work place. As students encounter these problems, they seek solutions and from that discover knowledge which they tend to retain (Jackson, et al., 2009; Spronken-Smith & Harland, 2009).

Review of the Literature

Types of Common Practice Period Harmonies

Harmonies frequently used in the common practice period were triads and seventh chords. All four forms (major, minor, diminished, and augmented) of the triad were used; however, augmented triads were not very common (Budge, 1943, p. 24). The commonly used triads are diatonic in the major scale. The diatonic chords of the major scale form the following pattern: I (major), ii (minor), iii (minor), IV (major), V (major), vi (minor), vii$^{\circ}$ (diminished; Benward & Saker, 2009, p. 78). The Roman numerals included with each are a means of identifying a chord's place and function within a key. Though this is a common system of harmonic analysis, this study used a chordal analysis to eliminate the ambiguity caused by the frequent modulations in the Bach chorales (Gauldin, 2004, p. 497; Pardo & Birmingham, 2001). The augmented triad is not diatonic to the harmonies formed from the major scale and only occurs on the third scale degree of the harmonic and ascending melodic minor scales; thus it was infrequently used during the common practice period (Kostka & Payne, 2009, p. 63).

With the options of four types of triads and three types of the interval of the seventh, twelve different seventh chords are possible; however, only five versions of seventh chords were typically used in the common practice period (Aldwell, et al., 2011, p. 57). Four of the five types of commonly used seventh chords are diatonic to the major scale. The order of the diatonic seventh chords in the major scale are I^7 (MM), ii^7 (mm), iii^7 (mm), IV^7 (MM), V^7 (Mm), vi^7 (mm), and vii$^{\varnothing 7}$ (dm). The other commonly used seventh chord is diatonic to the harmonic minor on the seventh scale degree as vii$^{\circ 7}$ (dd; Kostka & Payne, 2009, p. 67). Two additional types of seventh chords are diatonic to the harmonic and ascending melodic minor scales: the i^{M7} (mM) and the III^{+A7} (AM) Like the augmented triad, these seventh chords were rarely used in the common practice period (Aldwell, et al., 2011, p. 57). Though these two types, as well as other forms of seventh chords, were not typically used in the common practice period, they are used in modern music such as jazz and may have been implied in common practice period music via the non-harmonic tones.

Types of Non-Harmonic Tones

Non-harmonic tones are notes which do not fit in the sounding harmony. The name of the non-harmonic tone depends on how the non-harmonic tone is approached and resolved (Benward & Saker, 2009, p. 102). A passing tone is approached and left by a step in the same direction. If a C is a passing tone, then it would be approached from a B and resolved to a D or approached from a D and resolved to a B. In harmonic analysis, non-harmonic tones are circled or bracketed and labeled with an abbreviation. A passing tone is labeled PT (see figure 12). A neighbor tone is approached and left by a step in opposite directions, and is labeled NT. If a C is a neighbor tone, then it would be approached from a B and return to the same B or approached from a D and return to the same D (see figure 12; Aldwell, et al., 2011, pp. 376-378).

Figure 12. Examples of passing tones and neighbor tones

Two non-harmonic tones, suspension and retardation, are approached via a common note. A suspension resolves downward by a step. If a C is a suspension, then it would be approached (also called prepared) by a C in a previous harmony, then the C would be repeated or held (also called suspended) into a harmony where it is a non-harmonic tone; finally it is resolved downward by a step to a B. In addition to labeling a suspension with the abbreviation SUS, numbers are also included to indicate the distance of the dissonant note and of the note of resolution from the bass note. The most common suspension numbers are 4-3, 7-6, and 9-8. A 4-3 suspension could occur in the alto voice, but may be more than an octave away from the bass. Instead of labeling this suspension as an 11-10, the interval is reduced to within an octave (e.g., 4-3, see figure 13). The exception to this is the 9-8 suspension which is distinct from the 2-1 suspension and labeled separately (see figure 13). A retardation resolves upward by a step, and is labeled RET. Numbers are also needed for retardation, and the number indicating resolution is larger than the number for the dissonance (see figure 13). If the bass note changes at the point of resolution, the resolution number is measured from the bass note at the point of dissonance (Benward & Saker, 2009, pp. 106-108).

Figure 13. Examples of suspensions and a retardation

A non-harmonic tone which is resolved via common tone is the anticipation and is labeled (ANT). As the name implies, this non-harmonic tone is formed by a note in the next harmony arriving too soon. An anticipation is typically approached by a step, either downward or upward, and resolved by a common tone, anticipating a note in the new harmony (see figure 14; Roig-Francolí, 2011, p. 193).

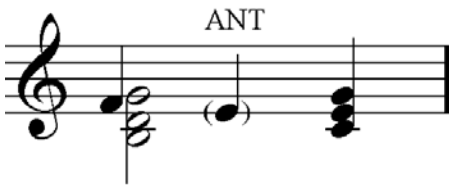

Figure 14. Example of an anticipation

Some non-harmonic tones involve leaps either leading into or away from the non-harmonic tone. The appoggiatura is approached by a leap and resolved by a step, typically in opposite directions. If a C is an appoggiatura and is approached via a leap from a lower note, it would resolve down to a B, or if it is approached via a leap from a higher note, it would resolve up to D. Since the C is dissonant with the sounding harmony, it is labeled APP (see figure 15). The escape tone is approached by a step and resolved by a leap, much like someone attempting an escape from a fenced area. If C is an escape tone and is approached from a B, it would leap to a chord tone lower than C, or if it is approached from a D, it would leap to a chord tone higher than C. The escape tone is labeled as ET (see figure 15; Kostka & Payne, 2009, pp. 197-199). Though infrequent, a non-harmonic tone can be both approached and resolved by a leap. This type of non-harmonic tone is so rare that it is referred to as a free tone (Gauldin, 2004, p. 108; Piston & DeVoto, 1987, p. 131). Since the free tone is not widely considered a non-harmonic tone, the researcher did not use it as a non-harmonic tone.

Figure 15. Examples of an appoggiatura and an escape tone

Thus far, all of the non-harmonic tones involve a single dissonant note, but a couple of non-harmonic tones have two dissonant notes. Passing tones can have two notes that are dissonant. Double passing tones can happen when the melody line passes from the fifth of a chord up to the root or the reverse. In a C major triad (e.g., C, E, G), the melody may move from G up to C (e.g., G to A to B to C). If this happens while the C major triad is still sounding, A and B would both be passing tones. To label a double passing tone, both dissonant tones are circled or bracketed and labeled DPT (see figure 16; Turek, 2007, p. 152). Double passing tones typically created two separate harmonic implications; thus, they were labeled as separate passing tones. The changing tone also has two dissonant notes and is somewhat similar to a neighbor tone. If a changing tone started on a chord tone C, it would move to dissonant notes D then B before returning to C (the dissonant notes may begin with the lower note instead). Like the double passing tones, both dissonant notes of the changing tone are circled or bracketed and labeled CT (see figure 16; Turek, 2007, p. 146). Again, the changing tones were labeled separately when they created two separate harmonic implications.

Figure 16. Examples of double passing tones and changing tones

The pedal tone is distinctly different from the other non-harmonic tones. It is most often found in the bass and sustains a note while various harmonies sound above it. The pedal note is usually either tonic (the first scale degree) or dominant (the fifth scale degree), and the harmonies which sound above the pedal were analyzed separately from the pedal note. The pedal note is labeled with the abbreviation PED and uses a horizontal bracket for the duration of the pedal tone (see figure 17; Benward & Saker, 2009, p. 111; Roig-Francolí, 2011, pp. 203-205).

Figure 17. Example of a pedal tone

Types of Extended Harmonies

Extended harmonies add additional notes, in thirds, to seventh chords, building increasingly taller harmonies. A ninth chord is formed by adding the interval of a ninth to a seventh chord. Two common (in the common practice period) ninth chords are extensions of the V^7. If a major ninth is added to a V^7, it becomes a V^9. If minor ninth is added to a V^7, it becomes a $V^{\flat 9}$ (see figure 18; Turek, 2007, pp. 520-521) Some theoreticians acknowledge that ninth chords can have inversions, but the labeling of inversions in Roman numeral analysis is unwieldy (Kostka & Payne, 2009, p. 451; Schoenberg, 1911/1978, pp. 345-346; Turek, 2007, p. 524). To be considered a ninth chord, several theoreticians state that the seventh must be present (Aldwell, et al., 2011, p. 517; Gauldin, 2004, p. 631; Roig-Francolí, 2011, p. 668). If a ninth is present with a triad but without a seventh, jazz analysis considers this harmony an add9 (or add2 depending on its relation to the bass note; see figure 13; Gauldin, 2004, p. 643). A C major triad with a D would be a C^{add2} or C^{add9}. In four-part writing, one of the five chord tones must be omitted. Since a perfect fifth occurs low in the overtone series, the ear tends to fill in this note even when it is not sounding, thus it can be omitted without significantly changing the sound of the chord (Roig-Francolí, 2011, p. 668).

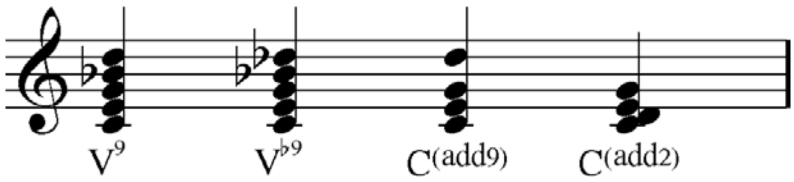

Figure 18. Examples of ninth chords and an add9 and an add2 chord

An eleventh chord is formed by adding an eleventh to a ninth chord. Since the interval of the eleventh is an octave higher than the fourth, its diatonic quality is perfect. Since the perfect form of this interval is a half-step away from the third, the third of the eleventh chord is frequently omitted. A common usage of the eleventh chord is a V^{11}, also known as a four over five ($\frac{IV}{V}$; see figure 19). The $\frac{IV}{V}$ harmony consists of the fifth scale degree in the bass and the major triad built on the fourth scale degree. For the key of C major, the fifth scale degree is G and the four chord is F, A, C (Roig-Francolí, 2011, pp. 670-671). This chord also illustrates why many theoreticians say that extended harmonies can only be in root position, since if F is considered the root, this chord could be analyzed as an F^{add2} or F^{add9} (an F major triad with an added second/ninth). As the ninth can be considered an add2 or add9 when the seventh is not present, so also the eleventh can be considered a sus4 if the seventh or ninth is not present. The label "sus" is taken from the non-harmonic tone, suspension. Of the various suspended intervals, the 4-3 suspension is quite common, so much so that eventually composers and arrangers have come to use the addition of the interval of the fourth as a unique harmony, the sus4 (see figure 19). As with the ninth, the third of the chord is typically omitted; if a sus4 chord was built on C, it would have the notes C, F, G. If the F were a 4-3 suspension, it would move down to an E, but as a C^{sus4} it would not move to E; instead it would be a chord in its own right (Kostka & Payne, 2009, p. 596).

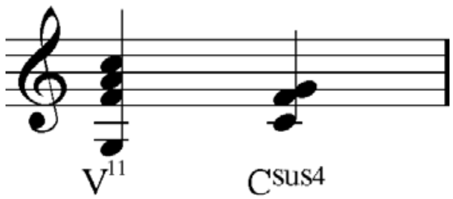

Figure 19. Example of a V^{11} chord ($\frac{IV}{V}$) and a C^{sus4}

A thirteenth chord continues the previous pattern of extending the tertian harmony by adding the interval of a thirteenth to any eleventh chord. A thirteenth chord built on C would have these notes: C, E, G, B, D, F, A (see figure 20). If these notes are rearranged in alphabetical order (C, D, E, F, G, A, B), it is clear that all of the notes of the major scale are present in the thirteenth chord. Since only seven letter names are available in the musical alphabet, the thirteenth chord is the tallest of the extended harmonies. As in the ninth and eleventh chord, if only the thirteenth is present, then the chord could be an add6 chord. A C^{add6} chord has the notes C, E, G, A; however, these are also the notes for an Am^7 chord in first inversion (see figure 20; Gauldin, 2004, p. 642). Thus there would need to be compelling reasons for considering the C as the root instead of the A. Such as, if the C is doubled in the left hand of a piano part or doubled in the bass instruments of an orchestration, then the C could be considered as the root. Since this study analyzed four-part chorales, the reasons for labeling a chord as a thirteenth chord were the inclusion of the seventh (or possibly the ninth or eleventh). Thus C, D, G, A or C, F, G, A or other similar variations could have been considered as a possible thirteenth chord. With the inclusion of all members of the scale in this harmony, it could have become a default analysis for any vague harmonic implication. Instead, less complex harmonic analyses were ruled out before a thirteenth chord was considered.

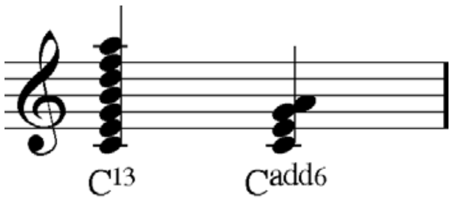

Figure 20. Example of a C^{13} and a C^{add6} chord

Chordal Analysis

Chordal analysis was the data collection method of this corpus analysis. Other corpus analyses have used various methods based on the body of music analyzed or the research questions. For the original corpus analysis, Budge (1943) used a Roman numeral analysis as the method to analyze the scores from the various composers which were in that corpus. De Clercq and Temperley (2011) also used Roman numeral analysis, but were working from recordings instead of printed music. Since Bach routinely modulated in the four-part chorales and since Roman numeral analysis is dependent on the key, the vagaries of the many modulations could have negatively impacted the accuracy of the analysis (Andrew, 2011; Pardo & Birmingham, 2001). With chordal analysis, C, E, G is always a C major triad (Kostka & Payne, 2009, p. 49). With Roman numeral analysis, a C major triad could have several different Roman numeral labels depending on the context (e.g., C major could be the I chord of C major, the V chord of F major, the IV chord of G major, the ♭VI of E major, etc.). The context determines the best choice of Roman numeral. For example, if a C major chord is preceded by a G^7, the G^7 would likely be the V^7 in C major, and the C major triad would likely be a I chord. However, if the rest of the surrounding harmonies indicated F major, then the G^7 becomes a V^7/V (read V^7 of V), and the C major triad becomes a V chord. With chordal analysis, these vagaries are eliminated since a C major triad would always be a C major triad (Aldwell, et al., 2011; Andrew, 2011; Gauldin, 2004; Pardo & Birmingham, 2001; Roig-Francolí, 2011).

Chordal analysis is best suited and commonly used for extended harmonies. Jazz, pop, and other current musicians use the chord symbols of chordal analysis as part of their performances and analysis (Andrew, 2011; Biamonte, 2010; Levine, 1995; Ligon, 2001; Pardo & Birmingham, 2001). Church music frequently includes the chord symbols used in chordal analysis above the piano part to aid pianists who may struggle to play the written accompaniment by allowing them to improvise an accompaniment based on the harmonic progressions of the song. Though extended harmonies are frequently used in the various styles of current music, chord symbols are not standardized; thus the balance of this section will specify which version of these symbols were used in this study (Kostka & Payne, 2009; Turek, 2007).

Chordal analysis symbols describe both the type and quality of the chord as well as the inversion. A C major triad in first inversion is labeled as C/E. The capital C indicates that the triad is major, and the E on the right of the slash is the bass note. Since E is the third of the triad and it is in the bass, the triad is in first inversion. A lower case c indicates that the triad is minor. However, a handwritten C can be difficult to determine whether it is a capital or a lower case C; thus a lower case m was included to clarify that the triad is minor (cm) and a capital M was used for major (CM). For an augmented triad, a plus sign is included with a capital letter (C^{+}). For a diminished triad, a degree sign (○) is included with a lower case letter (c°; Kostka & Payne, 2009, p. 49).

Chordal analysis includes numbers with the letter of the triad to indicate chords larger than a triad. The most common seventh chord, the V^7 (Mm), is expressed by just adding a 7 to a capital letter (C^7). To distinguish the MM and mm seventh chords from the Mm seventh chord, an upper or lower case M is added next to the 7. Thus C^{M7} is a MM seventh chord and c^{m7} is a mm seventh chord. For the two types of diminished seventh chords, a circle (○) and a circle with a slash (ø) are used to indicate fully diminish and half diminished respectively. Thus $c^{\circ 7}$ is a fully diminished seventh chord and $c^{\text{ø}7}$ is a half diminished seventh chord (Kostka & Payne, 2009, p. 49).

According to Budge and confirmed by Quinn, most of the harmonies found in the chorales are the typical harmonies of the common practice period (Budge, 1943; Quinn, 2010). However, when a non-typical harmony was encountered, it was described specifically to clarify its unique qualities. Only five combinations of seventh chords were typically used in the common practice period since they are diatonic in the major and minor scales (Kostka & Payne, 2009; Turek, 1996); however, other combinations were found. When found, both the triad and the seventh were described; for example, if a C augmented triad with a major seventh was found, it was labeled C^{AM7}.

Symbols for extended harmonies are not as standardized as the symbols for triads and seventh chords, but for a study such as this, consistency of labeling was vital to have accurate results. The chord symbols detailed in the following paragraphs were used to aid in obtaining consistent results while labeling the extended harmonies. An additional challenge presented by extended harmonies in the four-part chorale texture was the lack of complete harmonies. A complete ninth chord has five notes, an eleventh chord has six notes, and a thirteenth chord has seven; thus one to three notes were not present in a four-part texture. Though there is little agreement among music theorists in any area of extended harmonies, several state that the seventh of the chord must be present to consider an extended harmony as a ninth, eleventh, or a thirteenth (Aldwell, et al., 2011, p. 517; Gauldin, 2004, p. 631; Roig-Francolí, 2011, p. 668). Quinn (2010) decided to group certain extended harmonies (e.g., the added ninth and the sus4 chord) as well as other incomplete chords together in a category labeled as other. Quinn notes that this other category is non-tertian, since these chords were not completely built in thirds instead featured seconds or other non-tertian intervals. This corpus analysis considered the various extended harmonies described below as tertian, even when notes were omitted causing seemingly non-tertian intervals.

There are three ways of labeling a ninth. If the seventh was not present, it was labeled as an add9 or add2 depending on the distance from the root. When the interval of a sixteenth was present, it was labeled as an add9 instead of an add16. When the seventh was also present, it was labeled with a 9 (Gauldin, 2004). Additional qualifiers were added to indicate the type of ninth present. For example, the dominant ninth harmony has two forms, one with a major ninth and the other with a minor ninth; though both are over a dominant seventh harmony. The dominant ninth chord with the major ninth was labeled with just a 9, and the dominant ninth chord with the minor ninth was labeled with a ♭9 (e.g., C^{9} or $C^{\flat 9}$). For ninths added to other seventh chords, the M9 or m9 was added to the label of the seventh chord. A major ninth added to a C^{M7} chord was labeled C^{M7M9}, and a minor ninth added to a C^{m7} seventh chord was labeled c^{m7m9} (Kostka & Payne, 2009).

An eleventh chord was also labeled based on the presence of a seventh in the harmony. Since the suspension is quite common in the chorales, and since the 4-3 suspension is a common suspension, the presence of a fourth, eleventh, or eighteenth without a seventh was a possibility and was labeled as a sus4 (Quinn, 2010). The sus4 is now a separate harmony (i.e., not followed by the resolution to the third of the chord); however, in the chorales, the interval of the fourth would have likely been followed by a resolution to the third of the chord, especially when it was a part of the sus 4-3 (Quinn, 2010). The presence of a sus4 in the chorales, not followed by a resolution to the third of the harmony, would have been a significant find for harmonic implication. Finding an unresolved sus4 chord would have been significant, since it is a modern harmony and since Bach likely would have resolved the suspended fourth to the third. If the seventh is present, the harmony could be labeled as an eleventh. However, if the voicing of the chord indicated the sus4 instead of an eleventh, the chord was labeled as a 7susP4 (C^{7susP4}). As with the ninth, the various parts of the harmony were specifically identified, and any tones missing were indicated; since two of the six notes were not present, the absence of those notes was noted. The third of the chord is commonly missing since the eleventh is dissonant with the third; thus no additional label was required if the third is missing (Turek, 2007). However, to be clear with the analysis of extended harmonies, any missing notes were indicated. A C eleventh chord, without a third or a fifth, but with a minor seventh, major ninth, and perfect eleventh, was labeled as $C^{11(\text{no } 3,\ 5)}$. This follows the previous assumptions of a minor seventh and major ninth, since the dominant harmony is a frequent harmony and the labels for the major-minor seventh and the major–minor-major ninth harmonies are just a 7 and 9 respectively. Deviations from the minor seventh and major ninth would be labeled following the previously established patterns. A m or d would be used for a lowered interval based on the amount of change, and an A would be used for a raised interval. Thus if a C^{M7} without a fifth but with an eleventh were encountered, the label would be $C^{M7M9P11(no\ 3,5)}$ (Kostka & Payne, 2009).

The thirteenth chord was also labeled based on the presence of the seventh. As before, with the seventh present the chord was labeled as a thirteenth chord, and without a seventh an add6, but either form of this chord was likely to have been rare. A complete thirteenth chord has seven voices; thus in a four-part texture almost half the voices must be omitted. The fifth can readily be omitted due to the interval of the fifth happening low in the overtone series, causing it still to be perceived even when it is not truly sounding (Kostka & Payne, 2009; Schenker, 1906/1954). Either the third or eleventh was typically omitted due to the strong dissonance formed by the minor ninth within the harmony. If both the third and eleventh are omitted, then just four notes remain; however, if one of these two is still present, then the ninth must be omitted. The reason for the required ninth omission is a process of elimination: the root must be present to identify the chord, the seventh must be present to qualify as an thirteenth and not an add6, and, of course, the thirteenth must be present. With the fifth and the third or eleventh already omitted and the other four notes unable to be omitted for the chord to be considered a thirteenth, the ninth is the only chord member available to be omitted (Turek, 2007). Though a possibility, the add6 was also likely to have been rare. For example, a C major chord could have an added sixth which would add an A to the C, E, and G. However, with just these four notes, a better analysis would possibly be an A mm seventh chord over a C. An exception would be a plagal cadence. If the previous chord was found in a cadence and was followed by a G major chord, the better analysis would be a plagal cadence instead of a ii^7 to I (Roig-Francolí, 2011). Though harmonic function was not identified through the chord symbols used in this study, it was considered as an aid in obtaining the best analysis.

Though the various forms of the thirteenth chord were likely to have been rare in the four-part texture of the Bach chorales, if it did appear, it would have had the following labels. If the seventh was present, the chord would receive the label M13 or m13. Additionally, the other parts of the harmony were described as indicated for the ninth and eleventh chords. Missing chord members were labeled with a “no,” the major-minor seventh was labeled with a 7, and the diatonic ninth and eleventh were labeled with a 9 and 11 respectively. All raised notes were labeled with an A and all lowered notes with an m or d, depending on the amount of change. The notes C, E, B, A could be considered as a thirteenth chord and would have been labeled $C^{M7M13(\text{no } 5,9,11)}$ (Kostka & Payne, 2009). However, since a simpler analysis existed, this harmony was labeled as an $Am^{\text{addM2 or } 9}$.

With the complexities and vagaries of extended harmonies in a four-part texture, how was the best label chosen? Many music theorists state that extended harmonies are typically found in root position (Gauldin, 2004; Piston & DeVoto, 1987; Roig-Francolí, 2011; Turek, 2007). This greatly helps direct the analysis by eliminating much of the guess work. What if the root position analysis does not work well in the context? Perhaps with the previous example, an A minor harmony fits the progression better than a C^{13}. The context of the surrounding harmonies provided guidance to the analysis, but the non-harmonic tones in the context were likely to provide better insight into the analysis. Of the four notes in this example, if B is a passing tone, then the better analysis would have been an Am harmony with an add2 implication. If the A was a non-harmonic tone, perhaps a neighbor tone, then a better analysis would have been a C^{M7} with the implication of a $C^{M7,13(no5,9,11)}$. The non-harmonic tones provided the best guidance in the chordal analysis since traditional music theory does not consider these tones as part of the harmony. Thus the harmony without the non-harmonic tone(s) served as the foundation for discerning the harmonic implication. As discussed by Kosar (2001), two notes within a harmony might have equal validity as non-harmonic tones, thus creating two harmonic implications. In a situation such as this, both harmonic implications would be included in the tally. However, this was only used as an exception when two notes truly had equal standing as non-harmonic tones.

Other harmonies were likely to have been encountered other than the previously discussed chords. An example is the augmented sixth chords. The augmented sixth chord is an altered predominant harmony. In a typical western harmonic progression, the V chord is often preceded by a IV or ii chord, but could be preceded by a major II chord, which is often analyzed as a V/V. The V/V is an altered predominant since it changes the minor ii chord into a major triad. The augmented sixth chords also typically precede a V chord and have alterations, specifically the interval of the augmented sixth. These chords are outside of the previously discussed harmonies as the interval of the augmented sixth is not present in any of those harmonies. The symbols commonly used to label these harmonies were also used in this study. There are three common augmented sixth chords and a fourth that is a fairly common alternate spelling of the German augmented sixth chord (Roig-Francolí, 2011).

The first of these harmonies is a three-part harmony known as an Italian augmented sixth chord. This chord is built so that the interval of the augmented sixth resolves out by a half step to the fifth scale degree. A typical voicing of an augmented sixth chord has the lowered sixth scale degree in the bass. The raised fourth scale degree forms the interval of the augmented sixth, and this interval resolves out to the octave as they each move by half step to the fifth scale degree. The third voice is the tonic scale degree, which, in a four-part texture such as the chorales, would be the doubled note. The Italian augmented sixth chord would be labeled as It^{+6}. The other augmented sixths are based on the Italian, but add a fourth voice to the harmony in place of the doubled tonic. One of the doubled tonic notes of the Italian augmented sixth chord moves to the third of the V chord, and the other moves to the fifth of the V (Kostka & Payne, 2009).

The French augmented sixth chord adds the second scale degree to the Italian, creating a chord with the scale degrees flat six, sharp four, one, and two. The interval of the augmented sixth still resolves to the octave of the root of the V chord, the second scale degree is the fifth of the V chord and is typically retained as a common tone, and tonic steps to the third of the V chord. The French augmented sixth chord is labeled Fr^{+6} (Gauldin, 2004).

The third augmented sixth chord is the German augmented sixth which replaces the second scale degree of the French augmented sixth chord with a lowered third scale degree. The lowered third scale degree forms a perfect fifth with the lowered sixth scale degree, and the smoothest resolution of the voicing would have the lowered third scale degree step down to the fifth of the V chord. With the lowered sixth also resolving down by a step, parallel perfect fifths would be formed in the resolution to a V chord. Since parallel perfect fifths are typically avoided, composers would employ other resolutions to avoid the parallel perfect fifths (Aldwell, et al., 2011; Turek, 1996). Some would change to one of the other two augmented sixth chords, and others would resolve to the i^6_4 chord before moving to the V chord (Turek, 1996). These three augmented sixth chords originated in the minor keys, which makes each of the notes of the chords diatonic, except for the raised fourth scale degree. These chords are also found in major, but the sixth scale degree must be lowered a half step to correctly spell each chord. The German augmented sixth chord has an additional alteration; the lowered third scale degree is enharmonically spelled as a raised second scale degree since it now moves up a half step when it resolves to a I^6_4. Some music theorists call this additional alteration a doubly augmented sixth chord (++4, Turek, 2007) or a Swiss sixth (Piston & DeVoto, 1987). The German and Italian augmented sixth chords both sound like V^7 chords, though the Italian is missing the equivalent of the fifth of the V^7, but these harmonies are not spelled like a V^7 (augmented sixth spelling instead of minor seventh) and do not resolve like a V^7; thus they should be labeled as augmented sixth harmonies (Kostka & Payne, 2009).

Additional harmonies exist which would receive separate Roman numeral designations; these harmonies can be described by chordal analysis symbols which have already been described. An example is the Neapolitan sixth chord. Like the augmented sixth chords, this harmony first appeared in minor keys and is a major triad built on the lowered second scale degree. The Neapolitan triad frequently appears in first inversion; thus it is known as the Neapolitan sixth chord and has a Roman numeral label of N^6. In the key of C minor, this harmony is a D♭ major triad over F and would have a chordal analysis label of D♭/F. Another harmony which some music theorists give a separate label to is a fully diminished seventh chord which embellishes a harmony instead of tonicizing it. When a fully or half diminished seventh chord tonicizes a chord (whether on a secondary or a primary level), the root of the diminished seventh chord is a half-step lower than the root of the chord to which it is leading and has no common tones with the chord of resolution. The embellishing diminished seventh chord has a common tone with the chord it embellishes, and the other notes move a step away and/or back to the embellished chord. Since the primary focus of this study was not the functional aspect of harmony (i.e., assigning Roman numerals to the harmonies to determine a chords role or function within the progression of harmonies), embellishing seventh chords would be labeled according to their quality as described previously (Aldwell, et al., 2011; Roig-Francolí, 2011).

Schoenberg and His Denial of the Existence of Non-Harmonic Tones

Non-harmonic tones have existed since the beginning of combining melodies to form harmonies, known as counterpoint. By the sixteenth century, contrapuntal composition was organized into various species, depending on the ratio of notes from the primary melody (also called *cantus firmus*) to the counterpoint. The first species uses a note against note ratio (e.g., for every note in the *cantus firmus* there is one note in the accompanying melody). The second species has two notes in the counterpoint for every one note in the *cantus firmus*. Any dissonant notes in the counterpoint line were passing tones since this was the only type of non-harmonic tone allowed in this species. The third species has four notes in the counterpoint for every one note in the *cantus firmus*. In addition to the passing tone, an additional non-harmonic tone is introduced, the neighbor tone. The fourth species uses syncopation, thus allowing for suspensions as the counterpoint is syncopated to the *cantus firmus*. The fifth species of counterpoint freely combines the four other species. Sixteenth century counterpoint was not limited to just two voices, but voices were added as additional lines of counterpoint, which happened to form harmonies and non-harmonic tones. The primary concern was the intervals each line formed with the *cantus firmus* and with each other (Fux, Mann, & Edmunds, 1725/1965; Roig-Francolí, 2011).

As composers wrote compositions with more voices, music theoreticians sought ways to explain and comprehend the harmonies formed through the combinations of voices. Rameau's theory of inversions, as postulated in his *Treatise on Harmony*, put forth that C, E, G and E, G, C, and G, C, E were not three distinct harmonies, but rather three inversions of the same harmony (Rameau, 1722/1971). This theory, combined with the non-harmonic theory from sixteenth century counterpoint, is the basis of music theory instruction as well as compositional practice during the common practice period.

Just as harmonies were formed by adding additional lines of counterpoint lines to the *cantus firmus*, new harmonies were formed from non-harmonic tones, becoming part of the harmony. The passing seventh exemplifies this phenomenon, for composers would strengthen the function of the V triad by adding, via passing tone, a minor seventh. A V triad would typically have the root of the chord doubled; so if it were a G major triad, the notes would be G, B, D, and G. Composers discovered that if the top G moved down to an F, the interval of the tritone was formed between the third and seventh. This unstable interval has a strong tendency to collapse in by a half step, which resolves to the root and third of the tonic triad. The additional push of the tritone soon became so desirable to use that composers would include the seventh as part of the harmony instead of adding the minor seventh via passing tone. Thus the V^7 chord was born (Aldwell, et al., 2011; Schoenberg, 1911/1978).

The practice of creating new harmonies via non-harmonic tones continued throughout the common practice period until all of the modern extended harmonies became considered as harmonies instead of non-harmonic tones sounding with traditional harmonies. This practice is the basis for Schoenberg's declaration, "There are no non-harmonic tones, for harmony means tones sounding together" (Schoenberg, 1911/1978, p. 318). From this point, Schoenberg reasons away the tonal system in an effort to make all twelve notes of equal importance (Peles, 2010). This study was completed firmly in the system of theory from the common practice period; however, since Bach was ahead of his time in other areas, perhaps he used non-harmonic tones to form or at least imply extended harmonies. Since no research had thoroughly explored this possibility, this study adds to the significant body of knowledge on Bach's music and the harmonic aspect of music theory.

Corpus Analysis of Music

Corpus analysis of music is a quantitative study of a body of music. Budge completed the first corpus analysis of music to investigate the order of presentation of theory concepts in music theory texts. Several music theoreticians favored presenting the concepts based on the frequency of use of the harmonies, since this manner of presentation was used in teaching other languages. Budge followed Thorndike's lead by applying the principles from *Teacher's Word Book of 10,000 Words* in a study of a corpus of common practice period music. Since music theoreticians had interest in presenting music based on the frequency of chord usage, Budge sought to discover which chords were used the most frequently in the eighteenth and nineteenth centuries (Budge, 1943).

Budge's original corpus analysis opened a new quantitative area of music research which several researchers have used in various forms to quantitatively investigate aspects of music theory. One such aspect which was investigated via corpus analysis was chord progressions, specifically which chords typically follow one another. Norman (1945) studied a corpus of works of Bach, Beethoven, and Wagner to discover the frequency by which a certain chord is followed by another chord. As Norman completed his harmonic analysis, he disregarded non-harmonic tones since he considered each new note or notes as a new harmony. This practice of disregarding the non-harmonic tones is verified by a sample analysis of chorale 46 included in the study. Norman's Bach corpus consisted of the first 100 of the 371 Bach four-part chorales. Though he disregarded all non-harmonic tones and analyzed every note as a harmony, he was not studying the harmonic implications of non-harmonic tones. Instead, he just regarded everything as part of the harmony (Norman, 1945, p. 5). By contrast, this study fully acknowledged the existence of non-harmonic tones and sought to discover their harmonic implications which were perhaps extended harmonies. If the implied harmonies were extended harmonies, then college level music theory instruction could be enhanced with the knowledge of how Bach wrote these modern extended harmonies.

Other corpus analyses have followed in Budge's footsteps, in that they follow most if not all of the methodology Budge used. Pinkerton (1956) made an early attempt at automated music composition based on information theory. Using a corpus of thirty-nine children's songs from *The Golden Song Book*, Pinkerton sought mathematical patterns on the melodies of these songs to use as a basis for the BANAL TUNE-MAKER. This research led the way for future research into computer composition. Youngblood used a corpus of twenty songs to test if musical style can be determined using information theory (i.e., the freedom a composer or author has in decisions made during the creation process or the responses a listener or reader has to those choices; 1958, p. 25). Other researchers have eliminated redundancy from the corpus they assembled, but Youngblood saw the redundancy as a feature of the music and as such, should be included in the corpus. Youngblood included several frequency charts of the results as did Budge.

Marshall (1970) sought to discover how Bach composed the four-part chorales by studying a corpus of surviving autographs. Marshall looked at evidence such as corrections and changes to such things as time signatures, note values, and melody lines. A total of 257 corrections were discovered within the corpus of thirty-four chorales (Marshall, 1970). Moore (1992) sought to find patterns in the harmonies of jazz, rock, and pop songs by harmonically analyzing a large corpus of popular music. Though he used a functional harmonic analysis (i.e., Roman numeral analysis instead of chordal analysis) he was not implying that this body of music would follow harmonic function in classical music. The results of the corpus analysis were organized by class, then by pattern, in an extensive table at the end of his report.

More recent research has continued to follow Budge methodology. Temperley and Sleator (1999) designed a computer program to try to determine the meter and harmonic analysis of music based on a preference-rule system. The researchers did not specify the exact corpus used to test the program other than to say that the pieces were from the common practice period and included works by Bach, Beethoven, and Brahms. As with Budge, they presented results and outputs in tables and charts. Though not referencing Budge in their report, Pearce and Wiggins followed several aspects of Budge's methodology in their research with "*Prediction by Partial Match*," a type of Markov model, on various corpora of folk melodies (Pearce & Wiggins, 2004, p. 367). Their goal was to attempt to improve "statistical modeling of monophonic music" (Pearce & Wiggins, 2004, p. 367).

Zipf's law was tested on music by Zanette (2006) to see if it would have similar results as were obtained in an analysis of literature. According to Zipf's law, if all the words of a large piece of literature (corpus) are counted and then ranked, they will have occurrences proportional to their rank (i.e., the most frequently occurring word will occur ten times as often as the tenth ranked word). Zanette has tested Zipf's law on a great amount of western classical music from the various eras, but his article only presents the results of four of those tests Those tests were performed on "Prelude N. 6 in d from the second book of Das Wohltemperierte Klavier, by J. S. Bach; the first movement, Allegro, from the Sonata in C [K. 545] by W. A. Mozart; the second movement, Menuet, from the Suite Bergamasque by C. Debussy; and the first of Three Piano Pieces [Op. 11, N. 1] by A. Schoenberg" (p. 8). As Budge did, Zanette presented the findings via charts (i.e., graphs), but to test Zipf's law, Zipf's formulas were used for computation. In another departure from the typical corpus analysis, a chi-square test was used to compare the four pieces to check for statistical equivalency. Budge used a chi-square test to compare sample analyses to determine that she could be the sole analyzer for the research project; however, the results of the research used descriptive statistics and counts (Budge, 1943).

In an attempt to discover if pitch class set sequences could predict keys, Rohrmeier (2006) used the Bach chorales, both four-part and those containing more than four parts, to test the hypothesis. Not only was Budge's research and methodology referenced several times as support, but the report also included various tables with the results of the research. In 2008, Rohrmeier, with Cross, expanded the previous research in an effort to expand the empirical understanding of tonality. Again the Bach chorales were used and analyzed with pitch class sets (Rohrmeier & Cross, 2008). Temperley (2010) tested six different probabilistic models of rhythm on two corpora; one was the first violin part of the Haydn and Mozart string quartets, and the other was the Essen folksong collection. Throughout the report, Temperley included many aspects of Budge's methodology, such as charts and tables of results.

A recent study investigated the harmonic patterns of rock songs. De Clercq and Temperley listened to popular rock songs to aurally identify the harmonies used to detect changes in usage patterns through the years. The songs were selected from the decades of the 1950's through the 1990's and used the top twenty songs of each decade based on *Rolling Stone* magazine's top 500 list. Each researcher listened to each song and provided a Roman numeral analysis. Since only the audio of the music was available, this duplication of aural analysis allowed for cross checks between the two researchers to ensure accuracy. Though some differences existed in their analyses, they had 92.4% agreement on chromatic relative root (i.e., roots based on the key) and 94.4% agreement on absolute root (i.e., roots based on the note name; De Clercq & Temperley, 2011). Not only did De Clercq and Temperley discover how often each harmony was used in the corpus, they also sought patterns within the harmonies, such as which harmonies most often followed other harmonies, and harmonic trigrams (i.e., groups of three harmonies frequently found together). The researchers also created tables revealing the intervallic distance of chord roots in the progressions. Since the songs were grouped by decade; they also provided a table which showed the change in harmonic usage through the decades (De Clercq & Temperley, 2011).

Other researchers based their research on a corpus of music, though they did not completely follow Budge in methodology. Several researchers used a corpus of music to attempt to train computers to analyze music. An early attempt was completed by Winograd (1968) by writing a program based on the LISP system designed to analyze the harmonies. Several different corpora were tested: some of the Schubert dances, examples from the theory texts of Forte, Hardy and Fish, and the Bach chorales. This early attempt had some success on the simpler Schubert dances, but struggled with the more complex Bach chorales. Ponsford, Wiggins, and Mellish (1999) created a program based on Hidden Markov Models to analyze the harmonic content as well as the structure, phrase lengths, etc. The corpus used was sets of two dances from the seventeenth century, the chaconne and passacaglia. Additional sub-corpora were also created to test more specific aspects of the study. Once the analysis was complete, the researchers proceeded to attempt to generate pieces based on the data acquired. Instead of a harmonic analysis or other analyses of the notes, Beran and Mazzola used RUBATO to analyze the performance data from various performances of “Schumann's ‘Traumerei’ op. 15/7, Webern's second Variation for Piano op. 28/2, Bach's Canon Cancricans from Das Musikalische Opfer BWV 1079 and Schumann's ‘Kuriose Geschichte’ op. 15/2” (p. 48). Though only four songs were analyzed, the corpora was the many different recordings compared in the research (Beran & Mazzola, 1999).

Research into computer analysis of music continued in the twenty-first century. Hoffman and Birmingham (2000) adapted the constraint-satisfaction problem model to first identify cadences, then determine the key, and finally a harmonic analysis for a set of chorales by Samuel Scheidt. The WedelMusic project, a program created by Barthélemy and Bonardi, sought to harmonically analyze music and discover the tonalities from the extracted figure bass. Though only four examples are referenced in the report, a larger corpus seems to have been used in testing their program (Barthélemy & Bonardi, 2001). Pardo and Birmingham (2001) used a corpus of thirty-two pieces drawn from a sampling of Western classical music to test their chordal analysis algorithm, HarmAn. The following year they created a segment labeling algorithm for harmonic analysis and tested it using the Kostka-Payne corpus (i.e., all of the examples used in the Kostka-Payne theory textbook, Tonal Harmony; Pardo & Birmingham, 2002). Conklin (2002) wrote an algorithm to find vertical patterns (i.e., harmonies and progressions) in the Bach chorales. The size of this corpus yielded about 9000 harmonies in the data set; from this data, thirty-two short but significant patterns were discovered.

Temperley completed a sizable amount of quantitative research with corpus analysis in an effort to train computers to analyze music. Temperley (2002) adjusted the key-profile model of key finding by adding principles from the Bayesian cognitive model and tested this refinement with the Kostka-Payne corpus. Temperley utilized many different corpora as he expanded on the Bayesian cognitive model to improve its ability in finding keys and to use the Bayesian cognitive model to detect meter. Some of the corpora used were the Essen Folksong Collection, the piano sonatas of Mozart, Haydn, and Beethoven, and the Temperley corpus (i.e., the first eight measures of all Mozart and

Haydn string quartet movements; Temperley, 2007). Temperley (2009a) wrote a perl-script to draw statistical data from the Kostka-Payne corpus, which served to support traditional music theory principles. In “A unified probabilistic model for polyphonic music analysis,” Temperley continued to refine his expanded probabilistic approach and tested the refinements on the Kostka-Payne corpus (Temperley, 2009b).

Radicioni and Esposito have used two corpora of Bach four-part chorales and the Kostka-Payne corpus in their research. They used these corpora to test various algorithms, such as BREVE, GERAINT, and CarpeDiem. They found the CarpeDiem to be most effective of the three algorithms for harmonic analysis and effective for frequently asked question (FAQ) segmentation and for text recognition as well (Radicioni & Esposito, 2006a, 2006b, 2007, 2009). Huron and Ommen (2006) used the Humdrum Toolkit software to perform an audio analysis of syncopation on a corpus of recordings. Their corpus was a sampling of popular American music from 1890-1939. Nielsen (2009) performed various statistical analyses on a corpus of Mozart’s music to test for entropy. The corpus contained 571 of Mozart’s works which includes the majority of his compositional output. Entropy was analyzed based on a combination of category of the work, year the work was composed, and the length of the work. Quinn (2010) created an algorithm to attempt to find the keys in the Bach chorales via pitch class sets.

Some computer music research has also been conducted to train computers to compose music. CHORAL was written by Ebcioğlu with Backtracking Specification Language and contained a set of 350 rules to guide the program to create chorales in the style of Bach. Though Ebcioğlu did not use Bach’s four-part chorales directly to train the program, the chorales as well as “traditional theoretical treatises” guided the creation and programing of the CHORAL program (Ebcioğlu, 1988, p. 47). Though absolute rules were formed from this research, Ebcioğlu soon realized that each chorale could have its own set of rules; thus the rules were organized into a hierarchical order to allow the program to break certain rules in order to adhere to others. Though a subjective output such as a chorale is difficult to rate objectively, the system seemed to be effective in creating chorales (Ebcioğlu, 1990).

Meyerson (2001) attempted to create chorale harmonizations with the program Tlearn using a back-propagation network. Due to limitations of the network, only the soprano and bass lines were used, and the chorales used to train the program were limited to chorales in major keys and without apparent key changes; thus only seventeen chorales were used. Allan (2002) employed Hidden Markov Models combined with the Viterbi algorithm and had the program learn from a corpus of Bach chorales in order to have the computer attempt to compose a chorale in the style of Bach. Hidden Markov Models make predictions regarding underlying or hidden items in the music from music events which are observed. Allan indicated that the output from the computer was successful and further indicated that the program would have had similar success even if the chorales had not been used to train the computer. Allan continued this research with Williams (2005) and obtained similar results. Though Allan postulated that the results would have been similar in his previous work without the use of the Bach chorales in training the algorithm, the researchers decided to include the training with the Bach chorales.

Jorgensen and Madsen (2002) had little success in using a generic algorithm to create a chorale harmonization in the style of Bach. In describing their lack of success, they referred to their best outcome as sounding like it was the result of “a drunk improvising organist whose nose is itching all the time” (Jorgensen & Madsen, 2002, p. 24). CPUBach, a program created by Hanlon and Ledlie, had good success with the output of the program. The researchers acknowledged that appreciation for the chorales created by the program was subjective, but they saw consistently good results though a wide variety of chorale phrases. Though they did not use the Bach chorales to train CPUBach, they recommended this training as a part of future study (Hanlon & Ledlie, 2002).

Bach chorales have served as the corpus for several different research studies. Winograd (1968) included some Bach chorales as one of the corpora that was used to test the harmonic analysis program written using LISP. Marshall (1970) used an unusual corpus to search for evidence of Bach’s compositional process of the four-part chorales: surviving autographs of the chorales. Though not having the program directly train with the Bach chorales, Ebcioğlu used them, as well as theory texts, to create the rules which his CHORAL program followed in creating chorales (Ebcioğlu, 1988, 1990). Manzara, Witten, and James (1992) used a set of one hundred Bach chorale melodies in the Chorale Casino Computer Program. The goal of their research was to test the level of entropy (i.e., the amount of randomness present in music) in the chorale melodies, by having the participants place a wager on their guess of each subsequent note in a chorale melody. Though the program has one hundred chorale melodies, the first round used chorale No. 151 and the second used No. 61. Not only were just two chorale melodies used, but the study only had fifteen participants. Though the program had a large corpus at its disposal, the study took more of a qualitative approach to the problem.

Bach chorales have served as corpora in recent studies as well. Meyerson (2001) used seventeen major key chorales, without apparent modulation, to train the back-propagation network program Tlearn to harmonize chorales in the style of Bach. Using pitch class set sequences, Rohrmeier (2006) attempted to find key implications in the Bach chorales. Rohrmeier and Cross continued Rohrmeier's previous work with the Bach chorale corpus, as they sought to find the "statistical properties of tonal harmony" (Rohrmeier & Cross, 2008, p. 619) Radicioni and Esposito (2006b, 2007, 2009) used sets of Bach chorales as they attempted to train computers to perform harmonic analysis via the GERAINT and CarpeDiem algorithms. Quinn (2010) used the majority of the 371 chorales from the Riemenschneider edition in the key finding study; however, duplicate chorales were eliminated since Quinn thought the duplication of harmonies would negatively influence the outcome of the research. In proposing a possible grammar of western music, Tymoczko uses a corpus containing both 70 Bach chorales and 56 Mozart piano sonata movements (Tymoczko, 2010). Tymoczko (2011) compares root-motion theories, scale-degree theories, and function theories via thirty of Bach's four-part major key chorales. In a preliminary study of the possibility of Schenker's theory serving as a theory of perception, Temperley (2011) references several corpora: the Kostka-Payne corpus, 98 Bach chorales, and the first eight measures of all Mozart and Haydn string quartet movements. To quantitatively test this theory, Temperley recommends additional "corpus analyses and music perception experiments" (Temperley, 2011, p. 166).

Corpus Analysis Contrasted With Content Analysis

Educators probably have noticed the similarity between corpus analysis and content analysis. However since corpus analysis is used in music theory research, a crucial distinction must be addressed. In music theory research, the researcher typically writes the report and is the sole researcher/analyzer. The recent issues of three prominent music theory journals illustrate this difference. In recent issues of the *Journal of Music Theory*, specifically Volume 51 Number 1 (2007) though Volume 57 Number 1 (2013), there are fifty-one content articles (i.e., analysis of music or discussion of theories of music, not reviews or editorial articles) and all fifty-one articles were by a single author who worked alone on the research and on writing the article. Whether the author was completing some form of music analysis or developing some aspect of music theory, each one worked alone on all the research and/or analysis. An example is Rusch's (2012) investigation as to why Schubert crossed out and did not include a section of *Drei Klavierstücke*, though he had included it in an earlier manuscript. Rusch references and builds upon the work of other music theoreticians, but completed the research and analysis on her own. Another example is Roeder's (2011) analysis of three pieces of Arvo Pärt by using an analytic technique known as mathematical formalism. A third example is an article in which Hartt (2010), who built on the theories of Fuller, provides a more nuanced discussion of the sonorities in Machaut's three-voice motets.

Another journal, *Music Analysis*, has fifty-eight articles (Volume 26 Issue 1-2 [2007] through Volume 32 Issue 1 [2013]) in which the author worked alone on the music analysis or music theory discussion. For example, Chua (2007) demonstrated how Stravinsky was rioting within the music of the *Rite of Spring*. Snarrenberg's (2012) analysis of Brahms' first set of six songs identified Brahms's understanding of the poetry through the details included in the music. Lowe (2011) used a corpus analysis of forty-three recordings of Sibelius' Fifth Symphony to further the eighty year discussion of whether there are two movements or just one in this symphony. This journal also had two articles with two authors/researchers. Juslin and Lindström (2010) completed a qualitative study of the perception of emotion in music, and Huovinen and Tenkanen (2007) created an algorithm to attempt to complete pitch class analysis of music. These two articles fell outside of the norm for single researcher/author, since one followed a typical qualitative methodology and the other was developing a computer program for analysis of music.

A third music theory journal, *Music Theory Spectrum*, further illustrates the practice within music theory research of the author solely completing the music analysis and/or research. From Volume 29 Number 1 (2007) through Volume 35 Number 1 (2013), seventy-five articles have a single author/researcher. For example, Ng (2012) builds on the work of Cone, Hepokoski and Darcy, and Rothestein and Temperly to demonstrate how phrase rhythm can be used to determine form in classical music. Forrest (2010) used Schenker's theory of prolongation, developed for analyzing tonal music, to analyze three tonally vague choral pieces of Britten. Benadon (2009) analyzed recordings of jazz from the 1920's to study two types of rhythmic transformations: the flux and the shift. Within these issues, four articles have two authors each. Robinson and Hatten (2012), one a professor of philosophy and one a professor of music, put forth the concept that music could have a persona, such that some music is able to convey complex emotion. Roeder and Tenzer (2012), one a music theoretician and one an ethnomusicologist, worked together to analyze the non-Western Balinese piece *Gabor*. Woolhouse and Cross (2010) worked together to apply the mathematically complex interval cycles analysis to Krumhansl and Kessler's tonal hierarchies theory. Lombardi and Wester (2008) combined the various tone series in Boulez's *Structures 1a* into a hypercube known as a tesseract. Two of the four previous articles involve highly complex music analysis, another applies philosophical research to music, and the fourth is an analysis of non-Western music, thus outside of the typical practice of sole analyzer for articles in this journal.

Corpus analyses have also been completed solely by the researcher who wrote the research report. Lowe (2011) was the sole analyzer of the patterns of tempo in forty-three different performances of Sibelius' Fifth Symphony. Biamonte analyzed a small corpus to discover "triadic modal and pentatonic patterns in rock music" (Biamonte, 2010, p. 95) Väisälä (2009) used Schenkerian analysis to analyze the Bach Inventions. Franck tested Schenker's claim that invertible counterpoint at the twelfth is "a fallacious concept" by performing Schenkerian analysis on a corpus on Bach's music (Franck, 2010, p. 121). Temperley (2008) analyzed a corpus of music of Mozart, Beethoven, Chopin, and Mendelssohn to find hypermetrical transitions.

Some corpus analyses were completed by more than one researcher when the research fell outside of the typical research that would be accomplished individually. Several corpus analyses involved the researchers creating a computer program to either analyze music or attempt to write music (Allan & I.Williams, 2005; Beran & Mazzola, 1999; Radicioni & Esposito, 2006a, 2006b, 2007, 2009). A notable corpus analysis completed by two human analyzers was "a corpus analysis of rock harmony" (De Clercq & Temperley, 2011, p. 47). The researchers did not have printed music to analyze; instead they completed their analysis by ear. Since aural analysis made room for error, they decided to each aurally analyze the corpus and compare the results. This present analysis of the Bach chorales was done with printed music, thus eliminating the accuracy concerns inherent in aural analysis.

Though the vast majority of music theory research is completed by a single researcher, the researcher is not an island. Previous examples have indicated that the researcher built upon previous research, but occasionally researchers propose a theory or complete an analysis that is not accepted by other music theoreticians. Thus music theory journals publish response articles to present opposing viewpoints. A case of strong reaction to an article is exemplified in *Music Theory Spectrum's* Volume 33 Number 2 issue when Taruskin's (2011a) article "Catching up with Rimsky-Korsakov" garnered eight difference response articles (Agawu, 2011; Gjerdingen, 2011; M. Kielian-Gilbert, 2011; Rogers, 2011; D. Tymoczko, 2011; van den Toorn, 2011; Whittall, 2011; Zbikowski, 2011). Taruskin (2011b) was permitted to respond to those who reacted to his article. This dialog demonstrates how music theory research, though primarily based on the work of individuals, does have accountability and dialog via response articles.

Individual research in music theory is not a recent innovation. Rameau (1722/1971) formulated the theory of chord inversions in the late 17th and early 18th centuries. He formulated this theory on his own and it serves as a foundational part of music theory to this day (Beach, 1974). Rameau also refined his position on extended harmonies and how they were formed due to the dialog he had with some of his fellow music theoreticians, demonstrating that dialog in music theory research has a long history as well (Martin, 2012). Schoenberg (1911/1978) and Schenker (1906/1954) were contemporaries in the early 20th century, but their theories on music were significantly different. Schoenberg was working to move music away from a tonal center, while Schenker was developing a new system of analysis for tonal music of the common practice period (Matthew, 2011). These two theoreticians had public disputes on the purposes of music theory and what music theory can address (Peles, 2010). In more recent years, several music theoreticians have continued the practice of working individually. Forte (1979), Piston (1987), Turek (1996, 2007), Gauldin (2004), Spencer (2004), and Roig-Francoli (2011) are a few examples of theoreticians who have written music theory texts on their own. Others, such as Kostka and Payne (2009) and Aldwell, Schachter, and Cadwallader (2011), have chosen to work together on music theory texts.

The fiftieth anniversary issue of the *Journal of Music Theory* had an analysis symposium on "*Das alte Jahr vergangen ist*" which is a good illustration of several of the previous points on the process of music research. Three music theoreticians presented three separate articles with three different perspectives on the nebulous tonalities of this chorale prelude. Renwick (2006), though discussing the tonalities, bases the discussion primarily on the chorale tune and its ambiguities. Kielian-Gilbert (2006) analyzes the chorale prelude within a broader historical perspective of settings of "*Das alte Jahr vergangen ist*" and other chorale tunes by composers such as Hensel and Berg. Temperley (2006) uses a key analysis to attempt to discover the tonalities of this chorale. Since the tonal centers are not clearly defined, all three authors use chord symbols when referencing particular harmonies. The authors took different paths in their analysis of this work, but the different perspectives served to provide fresh insights. If these researchers had consolidated their ideas into a single article, perhaps some of those insights would have been lost in the process of blending their ideas into a single article.

Summary

Since this dissertation was written for an audience of the entire educational community and not solely music educators, a thorough presentation of harmony and chordal analysis was presented. This presentation sought to clarify and codify the theoretical framework in which this study was conducted. This chapter concluded with a summary of various corpus analyses to demonstrate the validity of this research methodology and that it was a good fit for this research. The methodology used in this study adds to the growing body of quantitative research known as corpus analysis and addresses the gap in the literature by identifying the harmonic implications of the non-harmonic tones in the Bach four-part chorales. By addressing this gap, this study adds to the vast body of knowledge on Bach's music and compositional practices, thus aiding college level music theory, arranging, and composition instruction. A case was made for following the common practice in music theory research for the researcher to write the report and serve as the sole analyzer.

CHAPTER THREE: METHODOLOGY

Introduction

This study was a corpus analysis of all 371 of the four-part chorales of Johann Sebastian Bach, but with a specific focus on the harmonic implications of the non-harmonic tones. Chordal analysis was used as the method of coding (analyzing) the harmonies. Harmonic data was collected, organized, and analyzed via frequency charts, a chi-square goodness of fit test, and chi-square tests of independence. This chapter will discuss the methodology of corpus analysis, the sample, procedures for chordal analysis, and data collection and analysis.

Design

A quantitative corpus analysis was conducted on the four-part chorales of Bach to discover the harmonic implications of the non-harmonic tones. Typically music analysis is qualitative in that the elements of the music are described and discussed (Butt, 2010b; Guck, 2006; Ockelford, 2005), but there is a body of research in music which quantitatively analyzes music. The qualitative approach to music analysis tends to focus on a single work and discuss the many facets of that work (Marianne Kielian-Gilbert, 2006; Renwick, 2006; Temperley, 2006); however, the quantitative approach of corpus analysis focuses on a large body of music and discovers relationships and trends and verifies, or at times contradicts, the anecdotal findings of the qualitative research. Budge (1943), in the original corpus analysis, demonstrated the frequency of chord usage for a variety of composers from the 18th and 19th centuries. The findings mostly confirmed that tonic and dominant were frequently used, but some chords were not used as frequently as expected.

Research Questions and Hypotheses

The research questions for this study were:

RQ1: Is there a statistically significant difference in the frequency of occurrence of harmonic implication of extended harmonies (i.e., ninth, eleventh, and thirteenth chords), triads, and seventh chords within the harmonic implications?

RQ2: Is there a statistically significant difference in the proportion of extended harmonic implication versus triad harmonic implications or seventh chord harmonic implications based on the number of non-harmonic tones forming the implication (i.e., a single non-harmonic tone, two non-harmonic tones occurring simultaneously, or three non-harmonic tones occurring simultaneously)?

RQ3: Is there a statistically significant difference in the proportion of extended harmonic implication versus triad harmonic implications or seventh chord harmonic implications based on the type of preceding harmony (i.e., triads, seventh chords, or extended harmonies)?

The following were the null research hypotheses:

H_{o1}: There is no statistically significant difference in the frequency of occurrence of extended harmonies (i.e., ninth, eleventh, and thirteenth chords), triads, and seventh chords within the harmonic implications.

H_{o2a}: There is no statistically significant difference in the proportion of extended harmonic implication versus triad harmonic implications based on the number of non-harmonic tones forming the implication (i.e., a single non-harmonic tone, two non-harmonic tones occurring simultaneously, or three non-harmonic tones occurring simultaneously).

H_{o2b}: There is no statistically significant difference in the proportion of extended harmonic implication versus seventh chord harmonic implications based on the number of non-harmonic tones forming the implication (i.e., a single non-harmonic tone, two non-harmonic tones occurring simultaneously, or three non-harmonic tones occurring simultaneously).

H_{o3a}: There is no statistically significant difference in the proportion of extended harmonic implication versus triad harmonic implications based on the type of preceding harmony (i.e., triads, seventh chords, or extended harmonies).

H_{o3b}: There is no statistically significant difference in the proportion of extended harmonic implication versus seventh chord harmonic implications based on the type of preceding harmony (i.e., triads, seventh chords, or extended harmonies).

Sample Choice

Bach's four-part chorales (Bach, Kirnberger, Bach, & Schubert, 1990) were used for this corpus analysis since they laid the foundation for the tonal harmonic practices of the common practice period and are widely used in theory texts both current and past (Aldwell, et al., 2011; Benward & Saker, 2009; Gauldin, 2004; Kostka & Payne, 2009; McHose, 1947; Piston & DeVoto, 1987; Roig-Francolí, 2011; Turek, 1996, 2007). The sample was the entire collection (371) of four-part chorales as collected and published by his son, C. P. E. Bach (Bach, et al., 1990; David, Mendel, & Wolff, 1998). Studying the complete collection follows the practice of studying a corpus of literature to gain a more complete perspective instead of relying on anecdotal evidence (De Clercq & Temperley, 2011). The complete set of 371 chorales contains some duplicate chorales. Quinn (2010) chose to remove the duplicate chorales from the corpus in his research from concerns that the duplicates might distort the results, since the study was addressing chord progressions and distribution. However, Allan (2002) viewed Bach's practice of setting the same chorale melody multiple times as justification of keeping all duplicate chorales in the corpus. Youngblood (1958) saw redundancy as a feature of the music and as such, should be included in the corpus. This study used the entire set of 371 chorales, including duplicates, with the reasoning that if Bach thought the music worthy of reuse with a different lyric, then that music was worth being counted a second time.

Methodology and Instrumentation

The coding method was chordal analysis, which guided the completion of the instrument. Chordal analysis is validated by three means: its usage in performance of current music, its usage in research, and its usage in music theory texts. The chord symbols of chordal analysis are used in current music. Church music, jazz, and other popular forms of music include chord symbols above the written music as an aid to the performers. In some situations, the chord symbols are just above the piano part, aiding the accompanist in improvising a different accompaniment. Other times, only a melody line is given with chords symbols above, thus the accompaniment must be improvised from the chord symbols (Kostka & Payne, 2009). Chord symbols are more than an aid to accompanists; they are a harmonic analysis tool (Benward & Saker, 2009; Turek, 2007). Analysis of tonal harmony is completed in one of two ways, either through Roman numeral analysis or chordal analysis (Kostka & Payne, 2009). Roman numerals are dependent on the key; and with the vague tonalities in the Bach chorales, this method may not be as reliable. Chordal analysis avoids the vagaries of the shifting tonalities since it is not dependent upon the key (Andrew, 2011; Pardo & Birmingham, 2001).

Instead of being dependent on the key, this form of analysis consists of identifying each chord by the letter name of the root. For example, a triad containing the notes C, E, and G would be labeled a CM indicating a C major triad. A triad with the notes C, E♭, and G would be labeled cm indicating a C minor triad. A seventh chord with the notes C, E, G, and B♭ would be labeled C^7 indicating a major-minor seventh chord. Since there are variations in chord labeling systems, a table of chord symbols (see Appendix A, *Chord Symbols*) was used to identify the meaning of each label. Any chords encountered outside of those listed in the table were labeled by identifying the quality of the triad and the quality of any additional intervals (i.e., seventh, ninth, eleventh, and/or thirteenth).

Not only is chordal analysis used in music performance and analysis, it is also used in music research. Pardo and Birmingham (2001, 2002) used chordal analysis as the labeling output for the results of their analysis program, citing advantages such as lack of dependence on the key. Esposito and Radicioni (2009) used a form of chordal analysis to identify the harmonies analyzed by the CarpeDiem program they wrote. Biamonte (2010) used chordal analysis to label the extended harmonies in a corpus of rock music. Andrews (2011) advocated and used chordal analysis with the program he created to analyze jazz chord sequences. Though there is variation in the specific labels used in chordal analysis, the method of chordal analysis is further validated via its inclusion and usage in music theory texts (Benward & Saker, 2009; Levine, 1995; Spencer, 2004; Steinke & Harder, 2010; Turek, 2007). Chord symbols were also used to identify the harmonies implied by the non-harmonic tones.

As part of the chordal analysis, the non-harmonic tones were identified and labeled (i.e., passing tone, neighbor tone, suspension, etc.). A table of non-harmonic tones was used to identify and clarify the labeling of non-harmonic tones (see Appendix B, *Non-Harmonic Tones*). Though these labels are more standardized in music theory texts than chord symbols, the table served as a legend for the meaning of the labels and will aid in replication (Aldwell, et al., 2011; Gauldin, 2004; Kostka & Payne, 2009; Piston & DeVoto, 1987; Roig-Francolí, 2011). These two appendices and the Excel file were the instruments used to collect the data from the method of chordal analysis.

The scoring was tallied in the following manner. Each chorale was analyzed by identifying all of the harmonies and non-harmonic tones, then each harmonic implication. Each different harmonic implication found was recorded in an Excel workbook with the location (i.e., the chorale number, the measure number within the chorale, and the portion of the beat within the measure) and type of harmonic implication (i.e., M, m, Mm7, etc.). Then each non-harmonic tone, which created the harmonic implication, was also recorded. To address research question two, scoring was based on the number of non-harmonic tones occurring, instead of the types. Since too many small categories, based on the type of non-harmonic tones sounding, were found, then all instances of one, two, or three non-harmonic tones occurring simultaneously were tallied together. The patterns, which emerged in the data of the multiple non-harmonic tones (i.e., passing tones and neighbor tones frequently occur together), were indicated via the tables in Appendix D. In addition, the harmony which precedes the implied harmony was recorded, even if it was another implied harmony. Frequency charts were created, via Excel PivotTables, to begin to address each research question (see Appendices C-E).

Procedures

The researcher began by seeking IRB approval before beginning any research.

The IRB determined that this research qualified as exempt.

The methodology used was corpus analysis that is a form of music theory research. In music theory research, the researcher typically writes the report and is the sole researcher/analyzer. Of the one hundred and eighty-seven recent (2007-2013) articles from three prominent music theory journals (*Journal of Music Theory*, *Music Analysis*, and *Music Theory Spectrum*), only six articles had multiple authors (see "Corpus analysis contrasted with content analysis"). Each of these six articles was outside of the norm for music theory research, thus a second researcher was warranted. This study closely adhered to corpus analysis methodology allowing the researcher to serve as both the author and the sole analyzer. However, since this researcher was not yet established in the music theory research community, and since Budge (1943) set a precedent in the original corpus analysis, a random sample of five Bach chorales was analyzed by the researcher and three qualified music theoreticians to verify that the researcher could follow the common practice in music theory research and serve as the sole analyzer. The experts who participated in this verification are collegiate music theory instructors, arrangers, and/or music editors who have sufficient experience with chordal analysis to be considered expert with this method. The first has taught undergraduate level music theory, choral arranging, and is the music editor for a music publishing company. Among the duties as music editor are completing and/or checking the chordal analysis on all of the choral music published. The second theoretician has taught undergraduate music theory, is a published arranger, and is a recording engineer who uses chordal analysis in all three capacities. The third has taught undergraduate and graduate music theory for many years and is a published arranger and composer. The experts used the tables of chord symbols and non-harmonic tones to clarify the specific chord symbols non-harmonic tone labels they were to use while completing the analysis. Their training consisted of their reading chapter two of this report and the researcher addressing any questions they had, to help them have confidence with this methodology. An honorarium was given to each of the experts as gratitude for their assistance.

Budge (1943), Winograd (1968), and Radicioni and Esposito (2006a) each commented on the great amount of training required to become proficient in music analysis; thus, Budge and seven music analysis experts analyzed four songs from the corpus. Budge used chi-square tests to show that there was no significant difference in the analyses, giving validity to the results that she would find as sole analyzer (Budge, 1943). Since the original corpus analysis, Cohen's Kappa test has been developed and was used to test for inter-rater reliability between the researcher and each of the additional raters of the sample corpus (Cohen, 1968).

The data was collected for the corpus analysis in the following manner. Each of Bach's four-part chorales was first analyzed by the researcher with chord symbols to ascertain the type of each of the harmonies. The non-harmonic tones were each identified and labeled with the appropriate chord symbol. Once the non-harmonic tones were identified in a chorale, then that note, combined with the other notes still sounding, was analyzed to see what harmony is formed (i.e., implied). The implied harmony was labeled with a chord symbol from the table used to identify the other harmonies in the chorale (see Appendix A, *Chord Symbols*).

The chordal analysis data was recorded in a chart in an Excel worksheet. The data recorded included the implied harmony, the non-harmonic tone(s) creating the implied harmony, the preceding harmony, the chorale in which the implied harmony was found, and the chorale number, measure number, and beat within the measure where the implied harmony was found.

Data Analysis

The preliminary data analysis was Cohen's Kappa tests to check for inter-rater reliability between the researcher and the three additional music theoreticians who performed a chordal analysis of the sample corpus. The data collected from chordal analysis was nominal data and each rater/analyzer was compared with the researcher independently. Cohen's Kappa was well suited for this comparison for it takes into account the possibility that some of the agreement of the raters could be caused by chance (Ary, Jacobs, Razavieh, & Sorensen, 2010). Landis and Koch (1977) established a scale of strength to provide consistency in evaluating the Kappa coefficient. They designated the range of 0.61-0.81 as substantial agreement and 0.81-1.00 as almost perfect agreement. The researcher had substantial agreement with two of the three chordal analysis experts, since the score of the Kappa test was above .61 (Landis & Koch, 1977),–rater one: ($K = .72$), rater two: ($K = .75$), and rater three: ($K = .16$). Rater three used a slightly different segmentation than the researcher, causing a substantial difference in the Kappa test.

Following the methodology of corpus analysis, the results were presented in frequency charts (Budge, 1943; De Clercq & Temperley, 2011; Norman, 1945; Temperley, 2011). Charts were created to begin the process of analyzing the five null hypotheses. For example, to address null hypothesis one (There is no statistically significant difference in the frequency of occurrence of extended harmonies [i.e., ninth, eleventh, and thirteenth chords), triads, and seventh chords within the harmonic implications.]), an Excel PivotTable was created that listed the frequency count for each type of harmonic implication. From these charts the totals for the various categories were determined and used in the chi-square goodness of fit and chi-square test of independence statistical calculations.

A chi-square goodness of fit test was used for research question one, since it only had one nominal variable (McDonald, 2009a). A chi-square goodness of fit test compares the nominal data from the frequency charts against theoretical expected frequencies (Howell, 2011; McDonald, 2009a). The theoretical expected frequencies were drawn from a table of chord frequencies found in Quinn's study, "Are Pitch-Class Profiles Really 'Key for Key'?" (2010, p. 155). Quinn's study provided a strong basis for the expected frequencies for research question one, since each new note was considered as a new harmony, thus matching the methodology of this research. Budge (1943) also provided a possible quantitative source for theoretical expected frequencies. However, the sample of Bach's music was not solely the four-part chorales and 2.22% of the total results were not specified in the frequency chart. Quinn's data is complete, making it the better choice (see Table 1, *Chart of Chord Category Totals from Quinn*).

Table 1

Chart of Chord Category Totals from Quinn.

Chord Category	Counts	Percentages
Triads	19,422	57.16%
Seventh Chords	6,916	20.35%
Other*	7,640	22.49%
Total Implications	33,978	

*Quinn (2010) used the term other to refer to extended harmonies.

To compute the expected frequencies for research question one, the percentages gleaned from Quinn's study were applied to the total harmonic implications found, which was 10,313. Thus the expected frequencies were 5,894.964 triads, 2,099.144 seventh chords, and 2,318.892 extended harmonies. The results of the frequency chart revealed that there were harmonic implications of 1,687 triads, 4,492 seventh chords, and 4,134 extended harmonies, thus it would seem that they are different from the expected frequencies. What if these differences were from chance? This is the strength of the chi-square goodness of fit test, since it compares the observed frequencies with the expected frequencies, to determine if the difference is statistically significant or just a product of chance (Howell, 2011).

To calculate chi-square, first the observed frequency was subtracted from the expected frequency. For question one, the expected 5,894.964 triads were subtracted from observed 1,687 triads with a result of -4,207.964 (residual). This number was squared (17,706,960.829) then divided by the expected frequency (5,894.964) for a chi-square of 3,003.744 (X^2 = 3,003.744) for the triads. This procedure was repeated for the other two categories revealing a chi-square of 2,727.665 (X^2 = 2,727.665) for seventh chords, and a chi-square of 1,420.772 (X^2 = 1,420.772) for extended harmonies. The chi-squares were summed for a total chi-square of 7,152.18 (X^2 = 7,152.18). This result was compared against a sample distribution table of critical values (Howell, 2011; National Institute of Standards and Technology, n.d.).

To complete the comparison with the critical value, the degree of freedom (*df*) was calculated. The degree of freedom for chi-square goodness of fit is *k* - 1, where *k* is the number of distinct categories in the chi-square calculation (Howell, 2011). The degree of freedom for chi-square test of independence is (*r* - 1)*(*c* - 1), where *r* is the number of rows and c is the number of columns in the contingency table (Healey, 2010). For research question one, the degree of freedom was two (*df* = 2), since this chi-square goodness of fit test had three separate categories. With the degree of freedom calculated, the distribution table of critical values was consulted (National Institute of Standards and Technology, n.d.). For two degrees of freedom at the lowest alpha level (α = .001) the critical value is 13.816 (X^2.001(2) = 13.816; National Institute of Standards and Technology, n.d.). The critical value indicates that if the null hypothesis were true, then the chi-square value would be equal to or greater than 13.816 only .001% of the time ($X^2 \geq 13.816$). Since the chi-square was 7,152.18 (X^2 = 7,152.18), the null hypothesis was rejected, indicating that the observed and expected frequencies were statistically significantly different and not a result of chance (Howell, 2011).

The effect size of the chi-square is sometimes reported with the results of the chi-square test. Morgan, Reichert, and Harrison “strongly encourage” the reporting of effect size (2002, p. 37). However, Howell disagrees since many times an effect size does not have a “meaningful interpretation” (2013, p. 166). Effect size *w*, developed by Cohen, has a standardized system for determining if the effect size is small (.10), medium (.30), or large (.50), thus effect size *w* will be reported with the results of the goodness of fit test (Cohen, 1988). Though effect size is important in reporting chi-square results, the effect size *w*, can only be considered a correlation for 2 x 2 contingency tables. Since the contingency tables for question one was 3 x 1, the effect size cannot be considered a correlation (Volker, 2006). For this example, the effect size was calculated by dividing X^2 (7,152.18) by the number of chords (10,313) and taking the square root of that result. The result was an effect size of .83, which according to Cohen, is a very large effect size. So not only was the null hypothesis soundly rejected with the low *p* value and high X^2 value, but the effect size was also quite large.

Cramer's *V* was reported with the chi-square tests of independence. Cramer's *V* is calculated similarly to Cohen's *w*, except the number of chords is multiplied by (k - 1) where *k* is either the number of columns or rows depending on which one is smaller (Healey, 2010, p. 292). For the four null hypotheses of research questions two and three, the contingency tables were each 3 x 2. Thus the number of chords was multiplied by one each time, causing each result of the Cramer's *V* to equal Cohen's *w*.

The results of the chi-square were reported in this manner: X^2 (2, N = 10,313) = 7,152.18, $p < .001$, $w = .83$. The various components of those results are as follows: chi-square (degree of freedom, number of observations) observed chi-square value, significance level, and effect size (Morgan, et al., 2002, p. 36). Since the data presented in this study was the complete population drawn from the entire corpus of Bach's four-part chorales, the differences found within that data are true differences and needed no inference of a larger whole (Gall, et al., 2007, p. 142). However, the chi-square tests provide a means of presenting the results in a manner familiar to educational researchers.

Research questions two and three used chi-square test of independence, since the proportions of the harmonic implications of extended harmonies verses the harmonic implications of triads and seventh chords were compared (McDonald, 2009b). The computations for chi-square test of independence were similar to the chi-square goodness of fit. The contingency table was 3 x 2 for each null hypothesis, since the three categories of the extended harmonies were compared with the same three categories of triads or seventh chords. The expected frequencies for each cell of the contingency table was computed by multiplying the total for that row by the total for that column and dividing by the overall all total (Howell, 2011). Once the expected frequencies were completed the balance of the calculations are very similar to that of the chi-square goodness of fit (Howell, 2013; McDonald, 2009b).

CHAPTER FOUR: RESULTS

Introduction

This study was a corpus analysis of all 371 of the four-part chorales of Johann Sebastian Bach, but with a specific focus on the harmonic implications of the non-harmonic tones. This chapter presents the results of the corpus analysis of all 371 of the four-part chorales of Bach. Presented first are the results of the Cohen's Kappa tests performed to confirm if the author of the report could serve as the sole researcher. As is typical of corpus analysis, each research question was addressed initially by a frequency chart to facilitate the chi-square goodness of fit test and chi-square tests of independence (Budge, 1943; De Clercq & Temperley, 2011; Gall, et al., 2007; Howell, 2011; Isaac & Michael, 1981; National Institute of Standards and Technology, n.d.; Norman, 1945; Quinn, 2010; Temperley, 2011).

Confirmation of Sole Researcher

To confirm that the author of this report could serve as the sole researcher, a sample of five chorales was analyzed by the researcher and three additional chordal analysis experts. The sample was randomly chosen by use of the random number generator on www.random.org. The numbers generated were 183, 288, 227, 123, and 100. This sample of chorales contained 415 chords, including harmonic implications. The results of the analysis of the three chordal analysis experts were compared individually with the researcher via Cohen's Kappa test. Two of the three experts had a score above .61 (Landis & Koch, 1977), which signified substantial agreement–rater one: (K = .72), rater two: (K = .75), and rater three: (K = .16). Rater three used a slightly different segmentation than the researcher, causing a substantial difference in the Kappa test. Temperley (2009b) describes segmentation as the portion of the beat(s) which is considered as part of the harmony. This study only considered the notes sounding with the non-harmonic tone(s) as part of the harmonic implication; however, this chordal analysis expert used a slightly larger segmentation. The larger segmentation still had many of the same harmonies as the researcher, but this expert found some chords that the researcher did not. These distinct chords created additional categories in the Kappa test which the researcher did not have and despite the agreement in other categories, the Kappa score was very low. Since two of the three chordal analysis experts had substantial agreement with the researcher, the researcher was able to serve as the sole analyzer.

Research Question 1

RQ1: Is there a statistically significant difference in the frequency of occurrence of extended harmonies (i.e., ninth, eleventh, and thirteenth chords), triads, and seventh chords within the harmonic implications?

H_{o1}: There is no statistically significant difference in the frequency of occurrence of harmonic implication of extended harmonies (i.e., ninth, eleventh, and thirteenth chords), triads, and seventh chords within the harmonic implications.

H_{o1} was evaluated with a chi-square goodness of fit test to ascertain if there was a statistically significant difference between the number of occurrences of extended harmonies, triads, and sevenths within the harmonic implications and theoretical expected frequencies (see Table 1: *Frequency Chart of Observed* n, *Expected* n, *and Residual by Chord Category*). The results of the chi-square test were statistically significant. Seventh chords were implied the most frequently with 4,492 occurrences, extended harmonies were implied second most frequently with 4,134 occurrences, and triads were implied the least frequently with 1,687 occurrences, X^2 (2, N = 10,313) = 7,152.18, $p <$ .001, w = .83. The researcher rejected the null hypothesis. (Note: Following this methodology, no thirteenth chords were found in the harmonic implications, since simpler analyses were always available.)

Table 2

Frequency Chart of Observed n*, Expected* n*, and Residual by Chord Category*

Chord Category	Observed *n*	Expected *n*	Residual
Triads	1,687	5,894.964	-4,207.964
Seventh Chords	4,492	2,099.144	2,392.856
Extended Harmonies	4,134	2,318.892	1,815.108
Total Implications	10,313		

Note: See Appendix C *Detailed Harmonic Implication Counts* for the complete results.

Research Question 2

RQ2: Is there a statistically significant difference in the proportion of extended harmonic implication versus triad harmonic implications or seventh chord harmonic implications based on the number of non-harmonic tones forming the implication (i.e., a single non-harmonic tone, two non-harmonic tones occurring simultaneously, or three non-harmonic tones occurring simultaneously)?

H_{o2a}: There is no statistically significant difference in the proportion of extended harmonic implication versus triad harmonic implications based on the number of non-harmonic tones forming the implication (i.e., a single non-harmonic tone, two non-harmonic tones occurring simultaneously, or three non-harmonic tones occurring simultaneously).

H_{o2a} was evaluated via a chi-square test of independence to ascertain if there was a statistically significant difference in the proportion of extended harmonic implications versus triad harmonic implications based on the number non-harmonic tones creating the implications. The results of the chi-square test were significant. Table 3 indicates the cross tabulations for extended harmonies and triads, X^2 (2, N = 5,821) = 398.11, p < .001, V = .26. The researcher rejected the null hypothesis.

Table 3

Cross Tabulation Table of Extended Harmonies and Triads by the Type of Non-Harmonic Tones

Non-harmonic Tones	Extended Harmonies	Triads
Single Non-harmonic Tone	3,091 (74.77%)	1,183 (70.12%)
Two Non-harmonic Tones	989 (23.92%)	278 (16.48%)
Three Non-harmonic Tones	54 (1.31%)	226 (13.40%)

Note: See Appendix D, *Harmonic Implications and Non-Harmonic Tones* for the complete results.

H_{o2b}: There is no statistically significant difference in the proportion of extended harmonic implication versus seventh chord harmonic implications based on the number of non-harmonic tones forming the implication (i.e., a single non-harmonic tone, two non-harmonic tones occurring simultaneously, or three non-harmonic tones occurring simultaneously).

H_{o2b} was evaluated via a chi-square test of independence to ascertain if there was a statistically significant difference in the proportion of extended harmonic implications versus triad harmonic implications based on the number non-harmonic tones creating the implications. The results of the chi-square test were significant. Table 4 indicates the cross tabulations for extended harmonies and seventh chords, X^2 (2, N = 8,626) = 126.42, p < .001, V = .12. The researcher rejected the null hypothesis.

Table 4

Cross Tabulation Table of Extended Harmonies and Seventh Chords by the Type of Non-Harmonic Tones

Non-harmonic Tones	Extended Harmonies	Seventh Chords
Single Non-harmonic Tone	3,091 (74.77%)	3,252 (72.40%)
Two Non-harmonic Tones	989 (23.92%)	978 (21.77%)
Three Non-harmonic Tones	54 (1.31%)	262 (5.83%)

Note: See Appendix D, *Harmonic Implications and Non-Harmonic Tones* for the complete results.

Research Question 3

RQ3: Is there a statistically significant difference in the proportion of extended harmonic implication versus triad harmonic implications or seventh chord harmonic implications based on the type of preceding harmony (i.e., triads, seventh chords, or extended harmonies)?

H_{o3a}: There is no statistically significant difference in the proportion of extended harmonic implication versus triad harmonic implications based on the type of preceding harmony (i.e., triads, seventh chords, or extended harmonies).

H_{o3a} was evaluated via a chi-square test of independence to ascertain if there was a statistically significant difference in the proportion of extended harmonic implications versus triad harmonic implications based on the type of preceding harmony. The results of the chi-square test were significant. Table 5 indicates the cross tabulations for extended harmonies and triads, X^2 (2, N = 5,821) = 77.19, p < .001, V = .12. The researcher rejected the null hypothesis.

Table 5

Cross Tabulation Table of Extended Harmonies and Triads by the Type of Preceding Harmony

Chord Category	Extended Harmonies	Triads
Triads	2,861 (69.21%)	1,166 (69.21%)
Seventh Chords	841 (20.34%)	451 (26.73%)
Extended Harmonies	432 (10.45%)	70 (4.15%)

Note: See Appendix E, *Harmonic Implications and Preceding Harmonies* for the complete results.

H_{o3b}: There is no statistically significant difference in the proportion of extended harmonic implication versus seventh chord harmonic implications based on the type of preceding harmony (i.e., triads, seventh chords, or extended harmonies).

H_{o3b} was evaluated via a chi-square test of independence to ascertain if there was a statistically significant difference in the proportion of extended harmonic implications versus seventh chord harmonic implications based on the type of preceding harmony. The results of the chi-square test were significant. Table 6 indicates the cross tabulations for extended harmonies and seventh chords, X^2 (2, N = 8,626) = 380.76, p < .001, V = .21. The researcher rejected the null hypothesis.

Table 6

Cross Tabulation Table of Extended Harmonies and Triads by the Type of Preceding Harmony

Chord Category	Extended Harmonies	Seventh Chords
Triads	2,861 (69.21%)	3,881 (86.40%)
Seventh Chords	841 (20.34%)	446 (9.93%)
Extended Harmonies	432 (10.45%)	165 (3.67%)

Note: See Appendix E, *Harmonic Implications and Preceding Harmonies* for the complete results.

Conclusion

This chapter reported the results of the corpus analysis of all 371 of the four-part chorales of Bach. The results of the comparison of the researcher to the three chordal analysis experts were presented. The results of the research were obtained through frequency charts which were evaluated by a chi-square goodness of fit test and chi-square tests of independence. The presentation of the results was organized by research question.

CHAPTER FIVE: DISCUSSION

Introduction

This study was a corpus analysis of all 371 of the four-part chorales of Johann Sebastian Bach, but with a specific focus on the harmonic implications of the non-harmonic tones. This chapter will reiterate the research questions and hypotheses, review the methodology used, review the results from the previous chapter, and identify the steps taken to minimize the limitations of this research. This chapter will also discuss the results of the research including implications for practice and theory, as well as recommendations for additional research.

Research Questions and Hypotheses

The research questions for this study were:

RQ1: Is there a statistically significant difference in the frequency of occurrence of harmonic implication of extended harmonies (i.e., ninth, eleventh, and thirteenth chords), triads, and seventh chords within the harmonic implications?

RQ2: Is there a statistically significant difference in the proportion of extended harmonic implication versus triad harmonic implications or seventh chord harmonic implications based on the number of non-harmonic tones forming the implication (i.e., a single non-harmonic tone, two non-harmonic tones occurring simultaneously, or three non-harmonic tones occurring simultaneously)?

RQ3: Is there a statistically significant difference in the proportion of extended harmonic implication versus triad harmonic implications or seventh chord harmonic implications based on the type of preceding harmony (i.e., triads, seventh chords, or extended harmonies)?

The following were the null research hypotheses:

H_{o1}: There is no statistically significant difference in the frequency of occurrence of extended harmonies (i.e., ninth, eleventh, and thirteenth chords), triads, and seventh chords within the harmonic implications.

H_{o2a}: There is no statistically significant difference in the proportion of extended harmonic implication versus triad harmonic implications based on the number of non-harmonic tones forming the implication (i.e., a single non-harmonic tone, two non-harmonic tones occurring simultaneously, or three non-harmonic tones occurring simultaneously).

H_{o2b}: There is no statistically significant difference in the proportion of extended harmonic implication versus seventh chord harmonic implications based on the number of non-harmonic tones forming the implication (i.e., a single non-harmonic tone, two non-harmonic tones occurring simultaneously, or three non-harmonic tones occurring simultaneously).

H_{o3a}: There is no statistically significant difference in the proportion of extended harmonic implication versus triad harmonic implications based on the type of preceding harmony (i.e., triads, seventh chords, or extended harmonies).

H_{o3b}: There is no statistically significant difference in the proportion of extended harmonic implication versus seventh chord harmonic implications based on the type of preceding harmony (i.e., triads, seventh chords, or extended harmonies).

Review of Methodology

The methodology of this study was corpus analysis. The study began with a five chorale sample that was analyzed by the researcher and three additional chordal analysis experts to determine if the researcher could follow the common practice of music theory research and serve as both the author and sole analyzer (see "Corpus analysis contrasted with content analysis"). Since the results of the Cohen's Kappa test showed that two of the three chordal analysis experts had a greater than substantial agreement with the researcher (Landis & Koch, 1977), the researcher served as the sole analyzer as well as the author of this report.

A corpus analysis was conducted on the four-part chorales of Bach via chordal analysis to ascertain the harmonic implications of the non-harmonic tones. To find the non-harmonic tones, a standard chordal analysis was conducted where the chords were identified and labeled with chord symbols. Then any notes which did not fit the harmony were labeled based on the type of non-harmonic tone. Once the chords and non-harmonic tones were identified, the non-harmonic tones were then considered as part of the harmony to determine the harmonic implication formed by the non-harmonic tones. As indicated previously, all simpler analyses (e.g., triads, sevenths, ninths) were eliminated before more complex analyses (e.g., elevenths and thirteenths) were considered. Within the extended harmonies, inversions of less complex extended harmonies were considered before more complex harmonies were considered (Aldwell, et al., 2011; Benward & Saker, 2009; Gauldin, 2004; Kostka & Payne, 2009; Piston & DeVoto, 1987; Roig-Francolí, 2011).

Once a chorale was completely analyzed, including the harmonic implications, the type of harmonic implication was recorded in an Excel spreadsheet. To address the research questions, the non-harmonic tone(s) and the type of the preceding harmony were recorded with the harmonic implications. The data also included location data to facilitate copying the correct data for the repeated chorales. This manner of data collection with this instrument allowed for the creation of frequency charts via Excel's PivotTable function.

Another benefit of the sample comparison with the other chordal analysis experts was the realization that some of the differences were due to small errors in the chordal analysis (i.e., the researcher missing that a note was no longer sounding at the point of the harmonic implication). To ensure accuracy in the analysis, the researcher set up additional checks during the process of entering the data in Excel. As each chorale was entered, each chord entered was re-analyzed to make sure that all data entered in the Excel spreadsheet was accurate. Additionally, the process of entering the data in Excel began after the researcher had analyzed over 200 chorales, so consistency of analysis was scrutinized and maintained during data entry. With the complexity of extended harmonies and the additional vagaries of missing chord members, the researcher had settled on specific analyses for specific situations and checked that those analyses were consistently used as the data was entered.

Regarding repetition in the chorales, entire chorales which were repeated were recorded twice in the data. However, repeat signs within the chorales were disregarded except for the few used in the context of a first and second ending. Frequently the first and second endings contained different harmonies, and in some editions of the chorales, first and second endings were not used. Instead the chorale was written out completely.

Once all of the data was checked and recorded, frequency count charts were created via Excel's PivotTable function. Three charts were created to address the three research questions. The first chart featured counts of the types of implied harmonies to determine which type was implied most frequently (see Appendix C *Detailed Harmonic Implication Counts*). A second set of charts was formed from the combination of the implied harmony and the non-harmonic tone(s) which formed the harmonic implication (see Appendix D, *Harmonic Implications and Non-Harmonic Tones*). A third set of charts was formed from the combination of the implied harmony and the harmony which preceded the implied harmony (see Appendix E, *Harmonic Implications and Preceding Harmonies*). From these charts, totals were calculated which were used to perform a chi-square goodness of fit test and chi-square tests of independence to determine if the observed differences were statistically significant and not a product of chance. A chi-square goodness of fit test compares the nominal data from the frequency charts against theoretical expected frequencies (Howell, 2011; McDonald, 2009a). The theoretical expected frequencies were drawn from a table of chord frequencies found in Quinn's study, "Are Pitch-Class Profiles Really 'Key for Key'?" (2010, p. 155). Quinn's study provided a strong basis for the expected frequencies for research question one, since each new note was considered as a new harmony, thus matching the methodology of this research.

Review of Results

The results of the comparison with the other three chordal analysis experts had one that, according to the Kappa test, had only slight agreement (K =. 16; Landis & Koch, 1977). This slight agreement stemmed from a difference of segmentation. This chordal analysis expert used a slightly larger segmentation. The larger segmentation still had many of the same harmonies as the researcher, but this expert found some chords that the researcher did not. These distinct chords created additional categories in the Kappa test which the researcher did not have and despite the agreement in other categories, the Kappa score was very low. The other two chordal analysis experts had significant agreement with the researcher–rater one: (K = .72) and rater two: (K = .75).

The results of the first research question showed that frequencies of the harmonies implied by non-harmonic tones differs statistically significantly from expected frequencies, X^2 (2, N = 10,313) = 7,152.18, p < .001, w = .83. The researcher rejected the null hypothesis. The results of the first null hypothesis of the second research question demonstrated that the proportion extended harmonic implication versus triad harmonic implications based on the number of non-harmonic tones forming the implication were statistically significantly different, X^2 (2, N = 5,821) = 398.11, p < .001, V = .26. The researcher rejected the null hypothesis. The results of the second null hypothesis of the second research question demonstrated that the proportion of extended harmonic implication versus seventh chord harmonic implications based on the number of non-harmonic tones forming the implication were statistically significantly different, X^2 (2, N = 8,626) = 126.42, p < .001, V = .12. The researcher rejected the null hypothesis. The results of the first null hypothesis of the third research question indicated that the proportion of extended harmonic implication versus triad harmonic implications based on the type of preceding harmony were statistically significantly different , X^2 (2, N = 5,821) = 77.19, p < .001, V = .12. The researcher rejected the null hypothesis. The results of the second null hypothesis of the third research question indicated that the proportion of extended harmonic implication versus seventh chord harmonic implications based on the type of preceding harmony were statistically significantly different , X^2 (2, N = 8,626) = 380.76, p < .001, V = .21. The researcher rejected the null hypothesis.

Research Limitations

The limitation of application of this research to other corpora of music was minimized by selecting a corpus of music that is foundational to the tonal music of the common practice period, the four-part chorales of Bach (Kostka & Payne, 2009; McHose, 1947). The functional harmonic vagueness of the chorales which occurs as a result of the rapidly shifting tonalities was minimized through the usage of chordal analysis instead of Roman numeral analysis, which depends on the key for accurate labeling. Chordal analysis is independent of the key since labels are applied to the harmonies based solely on the notes present in the harmony (i.e., a C major chord is always a C major chord to matter what key it is in; Andrew, 2011; Pardo & Birmingham, 2001).

The study was limited to tertian harmonies. If other harmonic systems such as secundal, quartal, or quintal had been used, the number of possibilities of harmonic implications would have been so numerous as to become unwieldy and impractical for collecting and analyzing empirical data. This follows the pattern of other researchers limiting the analytical method used to a single type, such as pitch-class sets (Rohrmeier & Cross, 2008) or Roman numerals (De Clercq & Temperley, 2011).

The limitation of researcher bias was addressed in the specifics of methodology laid out in the literature review. Since this study sought to discover if extended harmonies were implied by the non-harmonic tones, the researcher was hopeful to find extended harmonies. Otherwise the researcher would not have had an interest in completing the study. However, the desire to find extended harmonies could have led the researcher to mislabel a simpler harmony (triads or seventh chords) as an extended harmony. Thus precautions were added into the methodology to prevent such occurrences. The methodology required, and the researcher adhered to, the practice of completing a standard harmonic analysis by labeling the harmonies and non-harmonic tones (Aldwell, et al., 2011; Kostka & Payne, 2009; Levine, 1995; Piston & DeVoto, 1987; Roig-Francolí, 2011; Spencer, 2004; Turek, 1996, 2007). Then the researcher analyzed the harmonic implications. In the process of determining the harmonic implications, the methodology stipulated that all simpler analyses must be eliminated, before extended harmonies could be considered. Thus the researchers eliminated the possibility of the harmonic implication being a triad or seventh, before any extended harmonies were considered. The effectiveness of researcher bias minimization was demonstrated in the outcome of the comparison with the three other chordal analysis experts. If the researcher had been biased by the desire to find extended harmonies, then the outcome of the Cohen's Kappa tests would not have had substantial agreement with two of the three chordal analysis experts (Cohen, 1968).

The possibility of human error was minimized via two methods. First, a sample of five chorales was analyzed by the researcher and three other chordal analysis experts (Budge, 1943). The success of this check, demonstrated by two of the three raters having greater than substantial agreement with the researcher, showed that substantial agreement was maintained even with a few errors in the analysis. Second, since a few errors were discovered during the comparison of the sample chorales the researcher followed a series of double checks through the process of analysis and data entry, which caused each harmonic implication and the surrounding harmonic material to be analyzed twice. Once during the analysis and a second time as the data was entered into Excel. Errors were corrected through this process, thus minimizing this limitation.

Discussion

In 1911, Arnold Schoenberg declared, “There are no non-harmonic tones, for harmony means tones sounding together” (Schoenberg, 1911/1978, p. 318). This was part of the impetus that led to a departure from tonality in Schoenberg's and other's music; however, no research had been conducted on tonal music to see what harmonies were implied by the non-harmonic tones. Quinn (2010) furthered this notion when defining what would be considered as a chord for his research; he confirmed Schoenberg’s declaration by agreeing that chords are the sum of the notes sounding at a given time and as such, there are no non-harmonic tones. The only research completed on harmonic implication was performed on various atonal pieces in an attempt to discover the harmonic implications within non-tonal music (John Roeder, 1989). In addition to proposing a new area of research in tonal music (e.g., harmonic implication), the methodology of this research adds to the emerging body of research, corpus analysis (Benadon, 2009; Budge, 1943; De Clercq & Temperley, 2011; Lowe, 2011; Marshall, 1970; Moore, 1992; Norman, 1945; Pearce & Wiggins, 2004; Pinkerton, 1956; Rohrmeier, 2006; Rohrmeier & Cross, 2008; Temperley, 2008, 2010; Temperley & Sleator, 1999; Väisälä, 2009; Zanette, 2006). Bach's four-part chorales were the basis of this research since that corpus of music is foundational to the music and music theory of the common practice period (Aldwell, et al., 2011; Benward & Saker, 2009; Gauldin, 2004; Kostka & Payne, 2009; McHose, 1947; Piston & DeVoto, 1987; Roig-Francolí, 2011; Turek, 1996, 2007).

The term harmonic implication is an intentional choice, for this researcher has no desire or intention to undermine the theory of the music of the common practice period. The chords found in the harmonic implications were not necessarily functional harmonies. However, many of the harmonic implications either strengthen the function of the harmony (i.e., the 1,324 passing tones which formed major-minor seventh chords) or were functional themselves. Other than these functional harmonies, many of the implied harmonies were not functional, but they were chosen by Bach as part of the sound. Though these harmonies are implied, they are sounds present in the chorales and as such, are sounds that Bach intended to be heard in the chorales.

The results of the first research question showed that the seventh chords were implied the most frequently of the three groups of chords (i.e., triads, sevenths, and extended harmonies), with 4,492 occurrences (43.56%). Of the seventh chords, Bach implied the five common types (i.e., major-major, major-minor, minor-minor, diminished-minor, and diminished-diminished) the most frequently, but he also implied the augmented-major seventh and the minor-major seventh chords. These two seventh chords are diatonic in the harmonic minor and melodic minor ascending scales; the augmented-major seventh is diatonic as the III^{+M7} chord and the minor-major seventh chord is diatonic as the i^{M7} (Aldwell, et al., 2011; Kostka & Payne, 2009). Bach only implied chords which are diatonic to the major or minor scales and did not use any other possible combinations (e.g., diminished-major). The major-minor seventh chord was implied the most frequently with 1,731 occurrences (see Appendix C *Detailed Harmonic Implication Counts*), which aligns with other corpus analyses that studied the non-implied usage of chords (Budge, 1943; Quinn, 2010; Rohrmeier & Cross, 2008).

The results of the first research question also demonstrated that extended harmonies had only 358 fewer implications than seventh chords (a 1:1.0866 ratio of extended harmonies to seventh chords) and 2,447 more implications than triads (a 1:2.4505 ratio of triads to extended harmonies). The extended harmonies were implied 4,134 times and 74% (3,076) of those implications were some form of a ninth chord (including 430 which also had a fourth/eleventh). Though this study analyzed the corpus from the perspective of tertian harmony, the harmony which included both the ninth/second and the eleventh/fourth, also included the root and fifth; and as such is a type of secundal harmony. Persichetti, in his chapter on secundal harmony in *Twentieth-Century Harmony,* includes three $M^{susP4addM2}$ chords in a music excerpt featuring secundal harmony (1961, p. 125). Bach implied the $M^{susP4addM2}$ chord 23 times and the $M^{susP4addM9}$ chord 255 times in the chorales. Though the second and ninth are commonly separated in non-harmonic tone analysis, the sound of these two chords is quite similar, so it could be considered that Bach used this specific modern, secundal sound 278 times.

As discussed previously, the four-part chorales only have four notes sounding at a time, where a complete thirteenth chord would have seven notes. For a thirteenth chord to be present in a four-part chorale, almost half the notes would be omitted. For the purpose of minimizing researcher bias, the research methodology required that all simpler analyses be eliminated before more complex analyses could be considered. Thus no thirteenth chords were found in the harmonic implications. In the Well-Tempered Clavier II, Prelude No. 9, Piston points out that Bach implied a I^{13} (Piston & DeVoto, 1987, p. 391), thus Bach did imply thirteenth chords in other contexts when he had more than four notes. As will be recommended later, additional research could be done with the chorales to see if thirteenths are also implied by a larger segmentation or by considering more complex options for the analysis of the extended harmonies.

The results of the second research question demonstrated that the single passing tone not only was the most used with 5,001 occurrences (48.48% of all implications; see Table 7: *Frequency Chart of the Top Three Non-Harmonic Tones which Implied Extended Harmonies*), but also was the non-harmonic tone which most often implied extended harmonies with 1,514 extended harmonies implied (36.62% of extended harmony implications). The single passing tone implied thirty-one distinct extended harmonies; however, it implied the addM9 harmony the most with 635 occurrences. Of those 635 occurrences, 363 implied a M^{addM9} and 272 implied the m^{addM9}. Of all the added ninth harmonies, 476 had the ninth in the bass which also implies an eleventh chord. However, following the methodology, the simpler analysis was recorded. The third and fourth most implied of those 635 occurrences also featured the major ninth, with 191 implications of the M^{MM9} chord and 133 implications of the m^{mM9} chord.

Table 7

Frequency Chart of the Top Three Non-Harmonic Tones which Implied Extended Harmonies

Non-harmonic Tone	Count	Frequency
PT	1,514	36.62%
PT, PT	666	16.11%
sus 4-3	659	15.94%

Note: See Appendix D, *Harmonic Implications and Non-Harmonic Tones* for the complete results.

The results of the second research question also showed that two passing tones occurring simultaneously had the second most frequent occurrence of harmonic implication of extended harmonies with 666 implications (16.11%; see Table 7: *Frequency Chart of the Top Three Non-harmonic Tones which Implied Extended Harmonies*). Two passing tones occurring simultaneously implied twenty-seven distinct extended harmonies. The two extended harmonies most frequently implied by two passing tones occurring simultaneously were the $M^{susP4addM9}$ with 165 occurrences and the $m^{susP4addM9}$ with 104 occurrences. The two passing tones occurring simultaneously implied harmonies which featured the interval of the major ninth 587 times. Two passing tones occurring simultaneously was by far the most frequent of the group of two non-harmonic tones occurring simultaneously. The second most frequent set of two non-harmonic tones occurring simultaneously involved a passing tone occurring simultaneously with a neighbor tone, which had 86 occurrences (54 with the passing tone lower than the neighbor tone and 32 with the neighbor tone lower than the passing tone). The third most frequent occurrence was two neighbor tones occurring simultaneously with 65 occurrences.

The results of the second question also revealed that the sus 4-3 non-harmonic tone had the third most frequent number of occurrences with 659 instances (15.94%; see Table 7: *Frequency Chart of the Top Three Non-harmonic Tones which Implied Extended Harmonies*). The previous non-harmonic tone(s) have each implied a wide variety of harmonies. The sus 4-3 has the distinction of the greatest number of implications of a single extended harmony. The sus 4-3 implied the M^{susP4} 528 times. The second greatest number of implications of an extended harmony by a non-harmonic tone was the single passing tone which implied a M^{addM9} harmony 363 times. The large number of implications of the M^{susP4} chord was due to the strong link that this non-harmonic tone has with the sus4 harmony (Kostka & Payne, 2009, p. 596). Further the sus 4-3 almost exclusively implied the interval of the perfect fourth, 524 of the 528 occurrences (99.24%) The other four harmonic implications of the sus 4-3 were various ninth chords in inversion.

The results of the second question additionally revealed that within the group of three non-harmonic tones occurring simultaneously, three passing tones had the greatest number of extended harmonies implied with 23 occurrences (0.56%). Though three passing tones had the greatest frequency of extended harmonic implication, this was just a fraction (10.22%) of the total 225 implications of all types of harmonies (e.g., triads, sevenths, and extended harmonies). The bulk of these implications (201 or 89.33%) were triads and seventh chords. This result is logical, since the majority of the harmonies used in Bach's music are triads and seventh chords (Budge, 1943; Quinn, 2010). In the four-part texture of the chorales, three non-harmonic tones are only one note away from being considered as a new, non-implied harmony.

The results of the third research question demonstrated that the major triad was the most frequent harmony to precede an extended harmony with 1,778 occurrences (43.01%). The minor triad was the second most frequent with 998 occurrences (24.14%). The major-minor seventh chord was third most frequent with 383 occurrences (9.26%; see Table 8: *Frequency Chart of the Top Three Chord Types which Preceded Extended Harmonies*). The most frequent of the extended harmonies to precede another extended harmony was the M^{addM9}; however, it was sixth most frequent with 92 occurrences (2.23%). Though not a result of any research question, two augmented sixth chords were implied by non-harmonic tones. An Italian augmented sixth was implied by a neighbor tone, and a French augmented sixth was implied by three passing tones sounding simultaneously.

Table 8

Frequency Chart of the Top Three Chord Types which Preceded Extended Harmonies

Chord Type	Count	Frequency
Major	1,778	43.01%
Minor	998	24.14%
Major-minor Seventh	383	9.26%

Note: See Appendix E, *Harmonic Implications and Preceding Harmonies* for the complete results.

Several researchers have proposed the possibility that Bach was ahead of his time and used various compositional techniques typically thought of as modern innovations (Berger, 2007; Butt, 2010a, 2010b; Marshall, 1976; Rubin, 1976). Though seventh chords were implied more frequently than extended harmonies, there were only 358 more sevenths than extended harmonies within the harmonic implications (a 1:1.0866 ratio of extended harmonies to seventh chords). Bach frequently added the ninth/second and fourth/eleventh to the root and fifth of a chord. Many times he created this sound on a weak portion of a beat typically by the means of two voices moving in passing tones. However in chorale 108 on the down beat of bar ten, Bach has the root and fifth of an f minor chord sounding together with a perfect fourth and a major ninth. This sound lasts for a full beat before resolving to the f minor harmony. This one example, combined with the 419 times that this harmonic implication occurred in various forms, demonstrates that Bach favored this sound in the chorales. This specific sound is a small percentage (4.06%) of the total number of harmonic implications (10,313), but it is a type of modern harmony specifically mentioned by Persichetti in *Twentieth-Century Harmony* when he discussed secundal harmonies (1961, p. 125). Thus Bach was ahead of his time in his usage of modern harmonies.

Implications for Practice

Though Schoenberg was pushing music toward a lack of tonality with his statement; “There are no non-harmonic tones, for harmony means tones sounding together” (Schoenberg, 1911/1978, p. 318), he opened the door to a different approach in the teaching of music theory, harmonic implication. Music educators, especially music theory professors, may react to the very premise of non-harmonic tones forming harmonies, since the definition and even the term itself states that these tones are not part of the harmony (Roig-Francolí, 2011). How can music educators even entertain the idea of looking for harmonies where there are not supposed to be any harmonies?

If music educators are closed to the concept of harmonic implication as presented in this paper, how will they react when one of their theory students has an analysis different from theirs? Will they just shut down the student’s interest in exploring the various options in the analysis, by stating that a note is non-harmonic and as such could never be considered as part of the harmony? Instead this research should help them seize the opportunity to use the “problem” of harmonic implication to further learning. As Kosar (2001) advocates a problem based learning approach in the music theory classroom, he demonstrates how the exploration of the various facets of the “problem” of harmonic implication adds great depth to the instruction. Unfortunately a music theory professor who is unwilling to consider the possibility that non-harmonic tones have harmonic implication would stifle the student’s ability to discover how to apply reasoning skills to discern what would be the best analysis. Even if a music theory professor is hesitant to fully advocate the concept of harmonic implication, hopefully they teach that the M^{m7} (V^7) chord was formed by the addition of a minor seventh passing tone to a major triad (Aldwell, et al., 2011; Roig-Francolí, 2011; Shir-Cliff, et al., 1965). If they consider that to be true, perhaps they are open to the idea that V^9 chords were also created via non-harmonic tones (Kostka & Payne, 2009; Piston & DeVoto, 1987; Turek, 2007). When they consider where a G^{sus} chord's name came from perhaps they would be open to considering other possibilities of harmonic implication (Levine, 1995; Ligon, 2001). However, if the music theory teacher is open to these new ideas, the discussions created from evaluating the various aspects of the harmonic implications can aid not only in their student's understanding of the music but also enhance their ability to reason (Major, 2007).

With openness to the findings of this research, music theory and composition instructors have a new concept to aid them in instruction on extended harmonies. For now, they can look to composers such a Bach not only for the best practices on the usage of traditional common practice period harmonies (Kostka & Payne, 2009; Shir-Cliff, et al., 1965), but also modern extended harmonies via the non-harmonic tones. They would not only be alerted to good voice leading practices for extended harmonies, but also could discover sounds and harmonies not widely thought to exist in the work of composers such as Bach. This research is just the first step in a new journey of discovery of harmonic implications. Perhaps other composers, such as Mozart or Haydn, used extended harmonies in their music; this research sets the music educator and music researcher on a new path of discovery.

Since many of the harmonic implications found in this study enhanced the functionality of the harmonic progression, this concept of harmonic implications could improve music performance practice as well. Music students study music theory for the same reason that an author or public speaker studies grammar. Music students need to be proficient in the language of music to have success as a performer (Shir-Cliff, et al., 1965). For example, if they understand that tonal music is functional and how the functional aspect of harmony moves music forward, then they will be more skilled in creating engaging and interesting music performances (Ward, 2004). However, if they have little idea of the meaning of the music they are playing or singing, how can they convey any meaning to their audience? Thus an analysis of the music to be performed can greatly aid and enhance a student's performance skill (Major, 2007). If the student is following the traditional method of distinguishing the sounds based on whether or not they can be considered as part of the harmony, they are missing out on knowledge which would better inform their playing (Shir-Cliff, et al., 1965).

An example of how harmonic implication can enhance a student's or any musician's performance practice was discovered by the researcher while teaching a music theory course. One of the analytical assignments was a portion of Haydn's *Sonata, H. XVI:34*. At the end of measure 49 a D♯, which was not part of the a minor harmony, was labeled as an appoggiatura (Turek, 1996, p. 106). In the past, the researcher would have been satisfied with following the practice of labeling and forgetting the non-harmonic tones. However, the harmonic implication was discussed and since the note D♯ appears with an A, the harmonic implication is that of a d♯o chord. The next harmony is an E major harmony. Without considering harmonic implication, the progression is a very typical i_4^6 to V progression. However, when the harmonic implication is added, the progression has a much stronger push to the V chord; since the progression is now a i_4^6 chord pushing through a vii^{o6}/V to a V chord. The D♯ has much greater significance now that the harmonic implication is identified, and the musician can enhance the movement of the music through their awareness of this functional harmonic implication. The D♯ had been there since Haydn wrote it, and possibly musicians would have intuitively sensed the forward motion and have naturally added the extra push in their interpretation. With harmonic implication, they would understand why the music has an extra push with the D♯ non-harmonic tone. Just as one can speak fairly well and not understand grammar, many musicians are content with playing by ear or instinct, instead of seeking out the full harmonic meaning of the music. Thus application of this research in music performance practice can enhance the interpretation of the music (Major, 2007; Ward, 2004).

Theoretical Implications

The concept of harmonic implications could aid music theory research. Instead of being dismissive of non-harmonic tones in music theory research, the harmonic implications could be considered as part of the analysis process and perhaps new insights of functionality may be revealed. In the fiftieth anniversary edition of the *Journal of Music Theory*, an analysis symposium was conducted by several notable music theoreticians on the chorale prelude, "*Das alte Jahr vergangen ist.*" Each brought a different perceptive to the challenge of determining the tonal areas of this work (Marianne Kielian-Gilbert, 2006; Renwick, 2006; Temperley, 2006). In this chorale prelude, the harmonic implications might have given guidance to the key area as it did in the Haydn sonata referenced in the previous section (Turek, 1996). Thus if the harmonic implications of the non-harmonic tones were checked, then the tonal areas might possibly had become a bit clearer.

As will be demonstrated in the following section, much more research is required to discover the full impact of this theoretical concept. But perhaps other composers used non-typical harmonies within their harmonic implications. Perhaps harmonies previously thought to be rare, are more plentiful as harmonic implications. As this study discovered, Bach seemed to favor the interval of the ninth as he used it 3,076 times within the 4,134 extended harmonic implications or 74% of the extended harmonic implications. Perhaps this abundant use of the ninth in the chorales, helped to encourage its acceptance as a separate harmony. Exciting discoveries such as this may be in the harmonic implications of other composers, thus much more research is needed.

Further Study

This corpus analysis of the four-part chorales of Bach was a preliminary look at the concept of harmonic implication in tonal music. Thus much more research could be done to explore this concept more fully. Additional quantitative research could be conducted on a smaller corpus by performing a functional analysis with Roman numerals. This recommended research could focus on how many implications are functional (i.e., V, V^7, vii°, $vii^{ø7}$, or $vii^{\circ 7}$) compared with nonfunctional implications (i.e., all the other harmonies, sometimes referred to as color chords; Turek, 1996). A specific type of implication, such as the susP4addM9, could be researched throughout the chorales by looking at voicing, inversions, and other characteristics to see if certain characteristics are more indicative of that implication. Quantitative research could be conducted with a larger segmentation, following the prevailing harmonic rhythm instead of limiting the implications to only the notes sounding with the non-harmonic tone(s).

Harmonic implication usage patterns could be discovered through the chorales. Bach seemed to use certain non-harmonic patterns with some frequency, such as variations on a V chord at a cadence. This implication began with a 4-3 suspension then changed to a major-minor seventh by a passing tone. Sometimes the 4-3 suspension would resolve to the major triad before one of the doubled roots would move down to the seventh (see chorale 228, measure 6, beat 2), but other times the passing tone moved simultaneously with the resolution of the 4-3 suspension (see chorale 4, measure 2, beat 2).

Harmonic implication could be studied in other composers' works. Perhaps Haydn or Mozart favored certain harmonic implications and wrote with harmonies that would not typically be associated with their style. For example, an F^{+M7} chord or a $c^{\#o7sus4}$ chord would not be something that musicians might think of as a harmonies Haydn would use, but they are harmonic implications in his *Sonata, H. XV:37* movement II (Turek, 1996, p. 80). Additionally, other composer's usage of harmonies within the harmonic implications could be compared with Bach's usage of triads, seventh chords, and extended harmonies.

Qualitative research into harmonic implications could also be conducted. Certain chorales which have many harmonic implications could be researched from a qualitative perspective to discover details which are not immediately evident in quantitative research. Qualitative research may also be better suited for looking at the functional aspect of the harmonic implications of the chorales (i.e., Roman numeral analysis), since the reasoning behind the decisions made during the Roman numeral analysis would give insights into the functionality of some of the harmonic implications. Qualitative research could explore the various harmonies implied in certain situations (e.g., all the occurrences of the M^{addM9} and the voicing, non-harmonic tone(s) used, and other pertinent details).

Since this research was quantitative and designed to minimize researcher bias, it was determined that the less complex harmonies would be eliminated as possibilities before more complex harmonies were considered. With the possibility of extended harmonies appearing in inversions, this research found many ninths and elevenths but no thirteenth chords. However, some implications could possibly also be considered as thirteenth chords, thus qualitative research could explore those possibilities. Qualitative research would be appropriate for exploring the option of a larger segmentation, for it facilitates a deeper discussion of this approach.

Much educational research could be done with the concept of harmonic implications. Experimental research could be designed to test the effect of using the concepts of harmonic implication during music theory instruction on extended harmonies. A randomly assigned control group could receive instruction on extended harmonies without using the concept of harmonic implication and the randomly assigned experimental group would receive instruction on harmonic implication. Since an experiment of this nature would likely have a small sample size, a pretest-posttest design is recommended (Gall, et al., 2007). If this design is not feasible, perhaps a one-group pretest-posttest design would be feasible using a Likert scale based survey of the student's perceived level of understanding of extended harmonies and their usage. The students could be instructed on the usage of extended harmonies, and then given a survey that would check their perceived level of understanding of extended harmonies and their usage. Then the students would be instructed on the concepts of harmonic implications and a second survey administered to see if the student's perceived level of understanding was affected (Gall, et al., 2007).

Music perception research is another area of research that could perform future studies on the concept of harmonic implication. For as Kosar (2001) pointed out, not every student will perceive the music the same way. Some students would likely be more inclined to hear harmonic implications, perhaps from other music they listen to, while other students would perceive the common practice period harmonies and identify additional notes as non-harmonic. Scholars adept in research on music perception would have many research avenues to pursue with the new music theory concept of harmonic implications.

Conclusion

This study was a corpus analysis of all 371 of the four-part chorales of Johann Sebastian Bach, but with a specific focus on the harmonic implications of the non-harmonic tones. The research discovered that Bach did imply modern, extended harmonies within the non-harmonic tones, though slightly less frequently than seventh chords were implied. This research also discovered that a single non-harmonic tone implied the most extended harmonies and that triads preceded an extended harmony the most frequently.

This research can enhance music theory instruction by adding the concept of harmonic implication, perhaps as an introduction to extended harmonies. Since extended harmonies were accepted into common practice after they were introduced via the non-harmonic tones (Piston & DeVoto, 1987, p. 115), harmonic implication could be a method of introduction of the theoretical concept of extended harmonies. This method of introduction has the advantage of focusing on voice leading so that the complex extended harmonies are written in a manner that makes them more accessible to perform, since Bach is looked to for the best practices in voice leading.

This research further enhances music theory instruction, for it shows which harmonic implications Bach used most often in the four-part chorales (see Appendixes C-D). These harmonies are more characteristic of the chorales, and as such may be characteristic of additional common practice period music. Thus if students would seek to write in a common practice period style, they could use the implications revealed by this research study. However, if a student wanted to write current music with extended harmonies as a primary feature, this research also informs which non-harmonic tones more frequently imply certain extended harmonies, thus what voice leading will help the extended harmony be performed the best. This research can enhance music performance when musicians become aware that non-harmonic tones can imply harmonies which help to move the music along via their harmonic function (Major, 2007; Shir-Cliff, et al., 1965; Ward, 2004).

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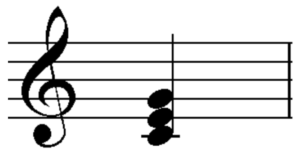
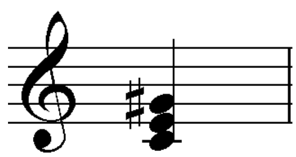
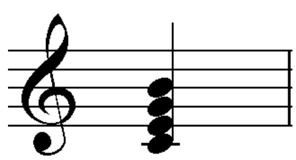
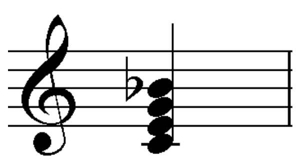
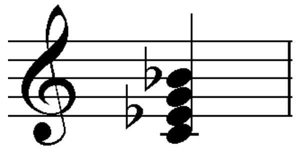
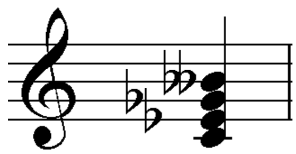
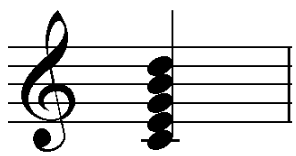
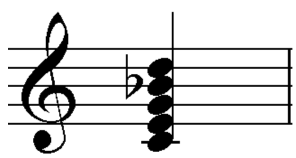
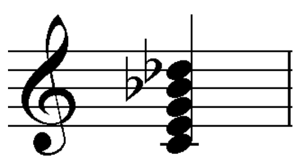
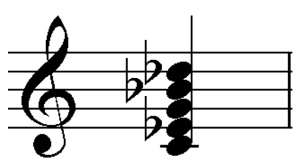
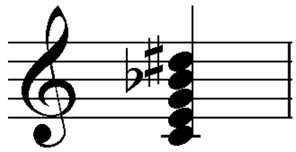
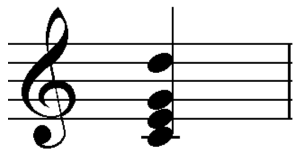
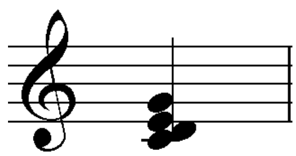
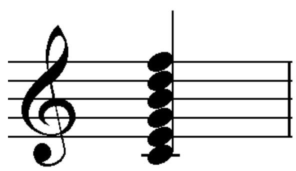
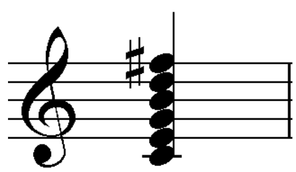
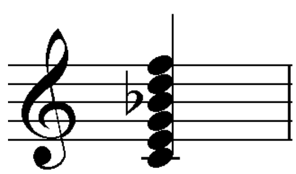
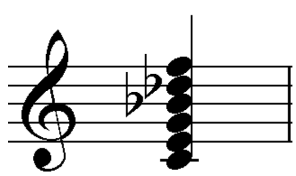
APPENDIX A: CHORD SYMBOLS

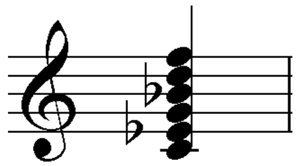
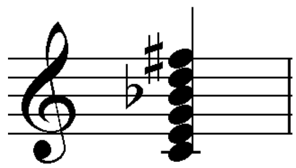
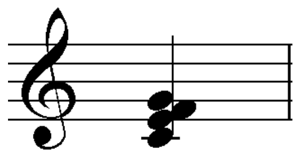
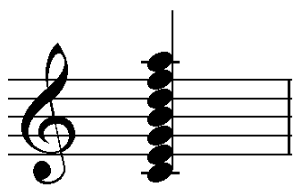
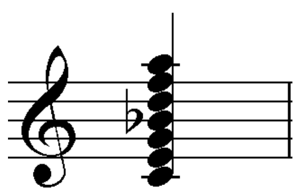
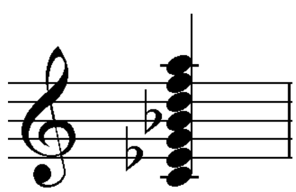
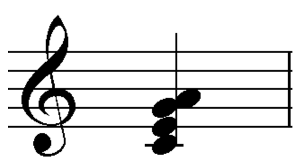
Table 9

Chord Symbols used for the Choral Analysis

Type of Chord	Symbol	Chord on the Staff	Qualities of the Chord
Triad	CM		Major triad
Triad	cm		Minor triad
Triad	c°		Diminished triad
Triad	C^{+}		Augmented triad
Seventh chord	C^{M7}		MM seventh chord
Seventh chord	C^{7}		Mm seventh chord

Type of Chord	Symbol	Chord on the Staff	Qualities of the Chord
Seventh chord	C^{m7}		mm seventh chord
Seventh chord	$C^{ø7}$		dm seventh chord
Seventh chord	$C^{\circ 7}$		dd seventh chord
Ninth chord	C^{M7M9}		MMM ninth chord
Ninth chord	C^{9}		MmM ninth chord
Ninth chord	$C^{\flat 9}$		Mmm ninth chord
Ninth chord	C^{m7m9}		mmm ninth chord

Type of Chord	Symbol	Chord on the Staff	Qualities of the Chord
Ninth chord	C^{7A9}		MmA ninth chord
Ninth chord	C^{addM9}		M triad with a M ninth
Ninth chord	C^{addM2}		M triad with a M second
Eleventh chord	$C^{M7M9P11}$		MMMP eleventh chord
Eleventh chord	$C^{M7M9A11}$		MMMA eleventh chord
Eleventh chord	C^{11}		MmMP eleventh chord
Eleventh chord	$C^{11\,(\flat 9)}$		MmmP eleventh chord

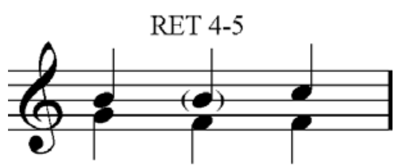
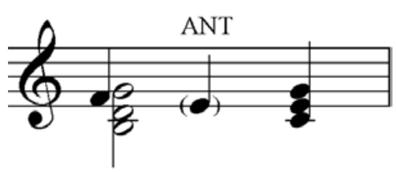
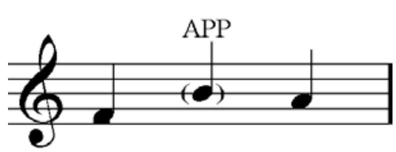
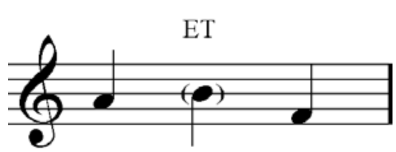
Type of Chord	Symbol	Chord on the Staff	Qualities of the Chord
Eleventh chord	$C^{m7M9P11}$		mmMP eleventh chord
Eleventh chord	C^{9A11}		MmMA eleventh chord
Eleventh chord	C^{susP4}		M triad with a P fourth
Thirteenth chord	$C^{M7M9P11M13}$		MMMPM thirteenth chord
Thirteenth chord	C^{13}		MmMPM thirteenth chord
Thirteenth chord	$C^{m7M9P11M13}$		mmMPM thirteenth chord
Thirteenth chord	C^{addM6}		M triad with a M sixth

M=major, m=minor, A=augmented, d=diminished, P=perfect

APPENDIX B: NON-HARMONIC TONES

Table 10

Non-Harmonic Tones and Their Labels

Type of non-harmonic tone	Label	Non-harmonic tone on the staff
Passing tone	PT	
Neighbor tone	NT	
Suspension	SUS 4-3	
Retardation	RET 4-5	
Anticipation	ANT	
Appoggiatura	APP	
Escape tone	ET	

Type of non-harmonic tone	Label	Non-harmonic tone on the staff
Double passing tones	DPT	
Changing tone	CT	
Pedal tone	PED	

APPENDIX C: DETAILED HARMONIC IMPLICATION COUNTS

Table 11

Counts of Harmonic Implications Organized by Chord Type

Chord Type	Count
Triads	
A	72
d	488
M	524
m	603
Triads Total	1,687
Sevenths	
A^{M7}	64
d^{d7}	124
d^{m7}	465
M^{M7}	823
M^{m7}	1,731
m^{M7}	52
m^{m7}	1,233
Sevenths Total	4,492
Extended Harmonies	
A^{addM9}	16
A^{MM9}	25
A^{susP4}	2
d^{addm2}	3
d^{addM9}	3
d^{addm9}	57
d^{d7P11}	2
$d^{d7susP4}$	4

Chord Type	Count
d^{dm9}	7
$d^{m7susP4}$	3
d^{mm9}	92
d^{susP4}	3
M^{addA2}	2
M^{addA9}	2
M^{addM2}	124
m^{addM2}	77
m^{addm2}	3
M^{addM9}	817
M^{addm9}	6
m^{addM9}	591
m^{addm9}	21
M^{m7P11}	2
m^{m7P11}	2
$M^{M7susA4}$	1
$M^{m7susA4}$	1
$m^{m7susA4}$	2
$M^{m7susd4}$	2
$M^{M7susP4}$	12
$M^{m7susP4}$	141
$m^{M7susP4}$	1
$m^{m7susP4}$	37
M^{MA9}	7
M^{MM9}	309
M^{mM9}	137
M^{mm9}	2

Chord Type	Count
m^{MM9}	7
m^{mM9}	258
m^{mm9}	80
M^{susA4}	11
$M^{susA4addM9}$	9
$m^{susd4addm9}$	1
M^{susP4}	707
m^{susP4}	125
$M^{susP4addM2}$	23
$m^{susP4addM2}$	6
$M^{susP4addM9}$	255
$m^{susP4addM9}$	135
$m^{susP4addm9}$	1
Extended Harmonies Total	4,134
Subtotal of Harmonic Implications	10,313
Augmented Sixth Chords	
Fr^{+6}	1
It^{+6}	1
Augmented Sixth Total	2
Total Harmonic Implications	10,315

APPENDIX D: HARMONIC IMPLICATIONS AND NON-HARMONIC TONES

Table 12

Counts of Triad Implications Formed by a Single Non-Harmonic Tone

	ANT	APP	ET	NT	PT	ret 3-4	ret 5-6	sus 4-3
A	6	8	1	11	36	1		
d	3	1	10	64	324			
M	15	7	9	52	214		1	3
m	23	4	9	84	286		1	1
Grand Total	47	20	29	211	860	1	2	4

Table 12 (Continued)

	sus 6-5	sus 7-6	sus 9-8	Grand Total
A	3			66
d				402
M	2			303
m	2	1	1	412
Grand Total	7	1	1	1,183

Table 13

Counts of Seventh Chord Implications Formed by a Single Non-Harmonic Tone

	ANT	APP	ET	NT	PT	sus 2-3	sus 4-3	sus 5-4
AM7	3	1		4	36			
dd7	3	5		55	9			
dm7	7	3	2	28	135	4		4
MM7	30	8	1	41	514			2
Mm7	46	53	5	18	1,324	3		1
mM7	12	7		22	2			

	ANT	APP	ET	NT	PT	sus 2-3	sus 4-3	sus 5-4
mm7	20	18	6	50	607	1	1	4
Grand Total	121	95	14	218	2,627	8	1	11

Table 13 (Continued)

	sus 7-6	sus 9-8	Grand Total
AM7			44
dd7	5		77
dm7	39	2	224
MM7	27	1	624
Mm7	1		1,451
mM7			43
mm7	82		789
Grand Total	154	3	3,252

Table 14

Counts of Extended Harmonic Implications Formed by a Single Non-Harmonic Tone

	ANT	APP	CT	ET	NT	PT	ret 4-5	ret 5-6
A^{addM9}				1		8		
A^{MM9}						17		
A^{susP4}						2		
d^{addm2}					1			
d^{addM9}						1		
d^{addm9}				3	1	15		
$d^{d7susP4}$					2	2		
d^{dm9}					3			
$d^{m7susP4}$						3		
d^{mm9}					9	62		
d^{susP4}					1	2		

	ANT	APP	CT	ET	NT	PT	ret 4-5	ret 5-6
M^{addA2}					2			
M^{addA9}					2			
M^{addM2}	2	8			28	27		
m^{addM2}	2				14	34		
m^{addm2}					1	1		
M^{addM9}	12	13		2	86	363		1
M^{addm9}	1					2		
m^{addM9}	7				36	272		
m^{addm9}				1	1	13		
M^{m7P11}	1							
$M^{m7susA4}$					1			
$m^{m7susA4}$					2			
$M^{M7susP4}$	3				1	1		
$M^{m7susP4}$	8				4	30		
$m^{m7susP4}$	1				1	10		
M^{MA9}					4			
M^{MM9}	2	1			13	191		
M^{mM9}		3			12	69		
M^{mm9}						2		
m^{mM9}					25	133		
m^{mm9}						51		
M^{susA4}						11		
$M^{susA4addM9}$						1		
M^{susP4}	7	3		12	18	113	1	2
m^{susP4}				1	5	42		2
$M^{susP4addM2}$								
$m^{susP4addM2}$						2		
$M^{susP4addM9}$			1		2	23		

	ANT	APP	CT	ET	NT	PT	ret 4-5	ret 5-6
$m^{susP4addM9}$	1					11		
Grand Total	47	28	1	20	275	1,514	1	5

Table 14 (Continued)

	ret 7-8	sus 2-1	sus 2-3	sus 4-3	sus 7-6	sus 9-8	Grand Total
A^{addM9}							9
A^{MM9}							17
A^{susP4}							2
d^{addm2}							1
d^{addM9}							1
d^{addm9}						3	22
$d^{d7susP4}$							4
d^{dm9}							3
$d^{m7susP4}$							3
d^{mm9}							71
d^{susP4}							3
M^{addA2}							2
M^{addA9}							2
M^{addM2}		24	9				98
m^{addM2}		15	4	1			70
m^{addm2}							2
M^{addM9}	1		31	2	6	207	724
M^{addm9}						1	4
m^{addM9}			5			203	523
m^{addm9}							15
M^{m7P11}							1
$M^{m7susA4}$							1
$m^{m7susA4}$							2

	ret 7-8	sus 2-1	sus 2-3	sus 4-3	sus 7-6	sus 9-8	Grand Total
$M^{M7susP4}$				1			6
$M^{m7susP4}$				58	2		102
$m^{m7susP4}$				2			14
M^{MA9}							4
M^{MM9}						5	212
M^{mM9}						1	85
M^{mm9}							2
m^{mM9}				1		3	162
m^{mm9}						1	52
M^{susA4}							11
$M^{susA4addM9}$							1
M^{susP4}			3	528	2	3	692
m^{susP4}			2	65		1	118
$M^{susP4addM2}$			1				1
$m^{susP4addM2}$							2
$M^{susP4addM9}$			6		1		33
$m^{susP4addM9}$				1		1	14
Grand Total	1	39	61	659	11	429	3,091

Table 15

Counts of Triad Implications Formed by Two Non-Harmonic Tones

	ANT, ANT	ANT, APP	ANT, ET	ANT, PT	APP, ANT	APP, APP
A						
d				1		2
M	2		2			2
m	1	1	1		2	
Grand Total	3	1	3	1	2	4

Table 15 (Continued)

	APP, ET	APP, PT	ET, ANT	ET, NT	ET, PT	NT, ANT	NT, APP
A							
d		1					1
M	1	1	2	1	1	4	2
m							4
Grand Total	1	2	2	1	1	4	7

Table 15 (Continued)

	NT, APP	NT, ET	NT, NT	NT, PT	NT, sus 4-3	NT, sus 9-8
A			3			
d	1		2			
M	2	1	10	9	1	1
m	4	1	12	3	3	
Grand Total	7	2	27	12	4	1

Table 15 (Continued)

	NT, susP4	NT, susP4	PT, ANT	PT, APP	PT, ET	PT, NT	PT, PT
A						1	1
d					1	1	37
M	1	1	1	3	1	10	42
m			2	2	1	6	78
Grand Total	1	1	3	5	3	18	158

Table 15 (Continued)

	PT, sus 4-3	PT, sus 9-8	sus 4-3, NT	sus 4-3, sus 6-5	Grand Total
A					5
d					46
M	1		5	1	105

	PT, sus 4-3	PT, sus 9-8	sus 4-3, NT	sus 4-3, sus 6-5	Grand Total
m		1	4		122
Grand Total	1	1	9	1	278

Table 16

Counts of Seventh Chord Implications Formed by Two Non-Harmonic Tones

	ANT, ANT	ANT, ET	ANT, NT	ANT, PT	AP, PT	APP, ANT
AM7	1					
dd7						
dm7	1					
MM7	6					
Mm7	1	1	2	2		1
mM7	6					
mm7	1			1	1	2
Grand Total	16	1	2	3	1	3

Table 16 (Continued)

	APP, APP	APP, ET	APP, NT	APP, PT	ET, ET	ET, NT	ET, PT
AM7							
dd7	1		2	2			
dm7	2		3	1			
MM7				2			2
Mm7			2	1	1	1	
mM7							
mm7	1	2	1	1	2		
Grand Total	4	2	8	7	3	1	2

Table 16 (Continued)

	ET, PT	NT, ANT	NT, APP	NT, ET	NT, NT	NT, PT	NT, sus 7-6
AM7					2		1
dd7			1	1	6	6	
dm7		3		1	15	5	
MM7	2		1	1	4	4	
Mm7		29	1		13	6	
mM7					2		
mm7		3		1	29	6	
Grand Total	2	35	3	4	71	27	1

Table 16 (Continued)

	NT, sus 9-8	PT, ANT	PT, APP	PT, ET	PT, NT	PT, PT
AM7	1		1		7	6
dd7			2		8	12
dm7		1	2	3	10	166
MM7				2	7	140
Mm7		3	2	3	4	85
mM7						1
mm7	3	3	1	4	13	256
Grand Total	4	7	8	12	49	666

Table 16 (Continued)

	PT, sus 2-1	PT, sus 4-3	PT, sus 9-8	sus 2-3, PT	sus 2-3, sus 6-5
AM7					
dd7					
dm7			2	1	1
MM7			18		
Mm7		1			

	PT, sus 2-1	PT, sus 4-3	PT, sus 9-8	sus 2-3, PT	sus 2-3, sus 6-5
mM7					
mm7	1	2	11		
Grand Total	1	3	31	1	1

Table 16 (Continued)

	sus 7-6, APP	Grand Total
AM7		19
dd7		41
dm7		217
MM7		187
Mm7	1	160
mM7		9
mm7		345
Grand Total	1	978

Table 17

Counts of Extended Harmonic Implications Formed by Two Non-Harmonic Tones

	ANT, ANT	ANT, PT	APP, ANT	APP, APP	APP, NT	APP, PT
A^{addM9}						2
A^{MM9}						
d^{addm2}						
d^{addm9}					1	
d^{d7P11}						
d^{dm9}						
d^{mm9}					1	
M^{addM2}				1		2
m^{addM2}						
m^{addm2}						1

	ANT, ANT	ANT, PT	APP, ANT	APP, APP	APP, NT	APP, PT
M^{addM9}					1	
M^{addm9}						
m^{addM9}						
m^{addm9}						
M^{m7P11}						
m^{m7P11}						
$M^{M7susA4}$						
$M^{m7susd4}$						
$M^{M7susP4}$						
$M^{m7susP4}$		2	4			
$m^{M7susP4}$						
$m^{m7susP4}$			2			
M^{MA9}						
M^{MM9}					1	
M^{mM9}				1		
m^{MM9}	1			1		
m^{mM9}		1				
m^{mm9}						
$M^{susA4addM9}$						
$m^{susd4addm9}$						
M^{susP4}						2
m^{susP4}						
$M^{susP4addM2}$						
$m^{susP4addM2}$						
$M^{susP4addM9}$				1		
$m^{susP4addM9}$						
$m^{susP4addm9}$						
Grand Total	1	3	6	4	4	7

Table 17 (Continued)

	APP, sus 4-3	ET, ET	ET, NT	ET, PT	NT, ANT	NT, APP
A^{addM9}						
A^{MM9}						
d^{addm2}				1		
d^{addm9}						
d^{d7P11}						
d^{dm9}						
d^{mm9}						1
M^{addM2}	1					
m^{addM2}	1					
m^{addm2}						
M^{addM9}			1		2	
M^{addm9}						
m^{addM9}					1	
m^{addm9}						
M^{m7P11}						
m^{m7P11}						
$M^{M7susA4}$						
$M^{m7susd4}$						
$M^{M7susP4}$						1
$M^{m7susP4}$						
$m^{M7susP4}$						
$m^{m7susP4}$						
M^{MA9}						
M^{MM9}						
M^{mM9}						1
m^{MM9}						
m^{mM9}						1

	APP, sus 4-3	ET, ET	ET, NT	ET, PT	NT, ANT	NT, APP
m$^{\text{mm9}}$						
M$^{\text{susA4addM9}}$						
m$^{\text{susd4addm9}}$						
M$^{\text{susP4}}$						
m$^{\text{susP4}}$						
M$^{\text{susP4addM2}}$						
m$^{\text{susP4addM2}}$						
M$^{\text{susP4addM9}}$		1				
m$^{\text{susP4addM9}}$				1		
m$^{\text{susP4addm9}}$						
Grand Total	2	1	1	2	3	4

Table 17 (Continued)

	NT, ET	NT, NT	NT, PT	NT, sus 4-3	NT, sus 6-5	NT, sus 7-6
A$^{\text{addM9}}$			1			
A$^{\text{MM9}}$						
d$^{\text{addm2}}$						
d$^{\text{addm9}}$		1	1			
d$^{\text{d7P11}}$						
d$^{\text{dm9}}$		1				
d$^{\text{mm9}}$		1	1			
M$^{\text{addM2}}$		2	1	4		
m$^{\text{addM2}}$				1		
m$^{\text{addm2}}$						
M$^{\text{addM9}}$		4	1	1		1
M$^{\text{addm9}}$		1				
m$^{\text{addM9}}$						
m$^{\text{addm9}}$		1				

	NT, ET	NT, NT	NT, PT	NT, sus 4-3	NT, sus 6-5	NT, sus 7-6
M^{m7P11}		1				
m^{m7P11}		1				
$M^{M7susA4}$						
$M^{m7susd4}$			1			
$M^{M7susP4}$						
$M^{m7susP4}$		1		4		
$m^{M7susP4}$		1				
$m^{m7susP4}$		1				
M^{MA9}			2			
M^{MM9}		15	2		2	
M^{mM9}		5	1			
m^{MM9}		5				
m^{mM9}		7	8			
m^{mm9}	1					
$M^{susA4addM9}$		1				
$m^{susd4addm9}$			1			
M^{susP4}		1				
m^{susP4}						
$M^{susP4addM2}$			1			
$m^{susP4addM2}$						
$M^{susP4addM9}$		13	9	2		
$m^{susP4addM9}$	1	2	2			
$m^{susP4addm9}$						
Grand Total	2	65	32	12	2	1

Table 17 (Continued)

	NT, sus 9-8	PT, ANT	PT, APP	PT, ET	PT, NT	PT, PT
A^{addM9}						4

	NT, sus 9-8	PT, ANT	PT, APP	PT, ET	PT, NT	PT, PT
A^{MM9}						3
d^{addm2}						
d^{addm9}					1	13
d^{d7P11}					2	
d^{dm9}						1
d^{mm9}			1		3	11
M^{addM2}		2	1	1		6
m^{addM2}						2
m^{addm2}						
M^{addM9}		2	3	1	2	64
M^{addm9}				1		
m^{addM9}		2			4	58
m^{addm9}				1		2
M^{m7P11}						
m^{m7P11}						1
$M^{M7susA4}$						1
$M^{m7susd4}$						1
$M^{M7susP4}$		2				2
$M^{m7susP4}$	1	23				4
$m^{M7susP4}$						
$m^{m7susP4}$		2			1	13
M^{MA9}						
M^{MM9}		1	2		6	64
M^{mM9}			1		9	30
m^{MM9}						
m^{mM9}			1		11	61
m^{mm9}					3	19
$M^{susA4addM9}$		1				6

	NT, sus 9-8	PT, ANT	PT, APP	PT, ET	PT, NT	PT, PT
$m^{susd4addm9}$						
M^{susP4}			2	1	1	7
m^{susP4}						4
$M^{susP4addM2}$		2		1		16
$m^{susP4addM2}$						4
$M^{susP4addM9}$		7			6	165
$m^{susP4addM9}$		2	1	1	5	104
$m^{susP4addm9}$						
Grand Total	1	46	12	7	54	666

Table 17 (Continued)

	PT, sus 4-3	PT, sus 7-6	PT, sus 9-8	ret 5-6, sus 9-8	sus 2-1, NT
A^{addM9}					
A^{MM9}					
d^{addm2}					
d^{addm9}					
d^{d7P11}					
d^{dm9}					
d^{mm9}					
M^{addM2}	4				
m^{addM2}	3				
m^{addm2}					
M^{addM9}	1		1	1	
M^{addm9}					
m^{addM9}					
m^{addm9}					
M^{m7P11}					
m^{m7P11}					

	PT, sus 4-3	PT, sus 7-6	PT, sus 9-8	ret 5-6, sus 9-8	sus 2-1, NT
$M^{M7susA4}$					
$M^{m7susd4}$					
$M^{M7susP4}$					
$M^{m7susP4}$					
$m^{M7susP4}$					
$m^{m7susP4}$	2				
M^{MA9}					
M^{MM9}					
M^{mM9}					2
m^{MM9}					
m^{mM9}	1		1		
m^{mm9}			1		
$M^{susA4addM9}$					
$m^{susd4addm9}$					
M^{susP4}					
m^{susP4}					
$M^{susP4addM2}$	1				
$m^{susP4addM2}$					
$M^{susP4addM9}$	5				
$m^{susP4addM9}$					
$m^{susP4addm9}$		1			
Grand Total	17	1	3	1	2

Table 17 (Continued)

	sus 2-1, PT	sus 2-1, sus 4-3	sus 2-3, ANT	sus 4-3, APP
A^{addM9}				
A^{MM9}				
d^{addm2}				

	sus 2-1, PT	sus 2-1, sus 4-3	sus 2-3, ANT	sus 4-3, APP
d^{addm9}				
d^{d7P11}				
d^{dm9}				
d^{mm9}				
M^{addM2}				
m^{addM2}				
m^{addm2}				
M^{addM9}				1
M^{addm9}				
m^{addM9}			1	
m^{addm9}				
M^{m7P11}				
m^{m7P11}				
$M^{M7susA4}$				
$M^{m7susd4}$				
$M^{M7susP4}$				
$M^{m7susP4}$				
$m^{M7susP4}$				
$m^{m7susP4}$	1			
M^{MA9}				
M^{MM9}				
M^{mM9}				
m^{MM9}				
m^{mM9}				
m^{mm9}				
$M^{susA4addM9}$				
$m^{susd4addm9}$				
M^{susP4}				

	sus 2-1, PT	sus 2-1, sus 4-3	sus 2-3, ANT	sus 4-3, APP
m^{susP4}				
$M^{susP4addM2}$		1		
$m^{susP4addM2}$				
$M^{susP4addM9}$				
$m^{susP4addM9}$				
$m^{susP4addm9}$				
Grand Total	1	1	1	1

Table 17 (Continued)

	sus 4-3, NT	sus 4-3, PT	sus 4-3, sus 7-6	sus 4-3, sus 9-8
A^{addM9}				
A^{MM9}				
d^{addm2}				
d^{addm9}				
d^{d7P11}				
d^{dm9}				
d^{mm9}				
M^{addM2}				
m^{addM2}				
m^{addm2}				
M^{addM9}		1		
M^{addm9}				
m^{addM9}				
m^{addm9}				
M^{m7P11}				
m^{m7P11}				
$M^{M7susA4}$				
$M^{m7susd4}$				

	sus 4-3, NT	sus 4-3, PT	sus 4-3, sus 7-6	sus 4-3, sus 9-8
$M^{M7susP4}$				
$M^{m7susP4}$				
$m^{M7susP4}$				
$m^{m7susP4}$	1			
M^{MA9}				
M^{MM9}				
M^{mM9}				
m^{MM9}				
m^{mM9}				
m^{mm9}				
$M^{susA4addM9}$				
$m^{susd4addm9}$				
M^{susP4}	1			
m^{susP4}				
$M^{susP4addM2}$				
$m^{susP4addM2}$				
$M^{susP4addM9}$	4	1	1	1
$m^{susP4addM9}$				
$m^{susP4addm9}$				
Grand Total	6	2	1	1

Table 17 (Continued)

	sus 7-6, sus 9-8	sus 9-8, APP	sus 9-8, PT	sus 9-8, sus 4-3
A^{addM9}				
A^{MM9}				
d^{addm2}				
d^{addm9}				
d^{d7P11}				

	sus 7-6, sus 9-8	sus 9-8, APP	sus 9-8, PT	sus 9-8, sus 4-3
d^{dm9}				
d^{mm9}				
M^{addM2}				
m^{addM2}				
m^{addm2}				
M^{addM9}				
M^{addm9}				
m^{addM9}				
m^{addm9}				
M^{m7P11}				
m^{m7P11}				
$M^{M7susA4}$				
$M^{m7susd4}$				
$M^{M7susP4}$				
$M^{m7susP4}$				
$m^{M7susP4}$				
$m^{m7susP4}$				
M^{MA9}		1		
M^{MM9}	2			
M^{mM9}				
m^{MM9}				
m^{mM9}				
m^{mm9}	1			
$M^{susA4addM9}$				
$m^{susd4addm9}$				
M^{susP4}				
m^{susP4}				
$M^{susP4addM2}$				

	sus 7-6, sus 9-8	sus 9-8, APP	sus 9-8, PT	sus 9-8, sus 4-3
$m^{susP4addM2}$				
$M^{susP4addM9}$			1	5
$m^{susP4addM9}$				2
$m^{susP4addm9}$				
Grand Total	3	1	1	7

Table 17 (Continued)

	sus 9-8, sus 7-6	Grand Total
A^{addM9}		7
A^{MM9}		3
d^{addm2}		1
d^{addm9}		17
d^{d7P11}		2
d^{dm9}		2
d^{mm9}		19
M^{addM2}		25
m^{addM2}		7
m^{addm2}		1
M^{addM9}		88
M^{addm9}		2
m^{addM9}		66
m^{addm9}		4
M^{m7P11}		1
m^{m7P11}		2
$M^{M7susA4}$		1
$M^{m7susd4}$		2
$M^{M7susP4}$		5
$M^{m7susP4}$		39

	sus 9-8, sus 7-6	Grand Total
$m^{M7susP4}$		1
$m^{m7susP4}$		23
M^{MA9}		3
M^{MM9}	1	96
M^{mM9}		50
m^{MM9}		7
m^{mM9}		92
m^{mm9}		25
$M^{susA4addM9}$		8
$m^{susd4addm9}$		1
M^{susP4}		15
m^{susP4}		4
$M^{susP4addM2}$		22
$m^{susP4addM2}$		4
$M^{susP4addM9}$		222
$m^{susP4addM9}$		121
$m^{susP4addm9}$		1
Grand Total	1	989

Table 18

Counts of Triad Implications Formed by Three Non-Harmonic Tones

	ANT, ANT, ANT	ANT, ET, ET	ANT, ET, PT	APP, APP, NT
A				
d				1
M				
m	1	1	1	
Grand Total	1	1	1	1

Table 18 (Continued)

	APP, APP, PT	APP, ET, NT	APP, ET, PT	APP, NT, ANT
A				
d	1			
M		1	1	1
m				
Grand Total	1	1	1	1

Table 18 (Continued)

	APP, NT, APP	APP, NT, ET	APP, PT, APP	APP, PT, ET
A			1	
d				
M	1	2		1
m				
Grand Total	1	2	1	1

Table 18 (Continued)

	APP, PT, PT	ET, ANT, ANT	ET, ANT, ET	ET, ANT, NT
A				
d				
M	2		1	4
m		2		
Grand Total	2	2	1	4

Table 18 (Continued)

	ET, APP, ANT	ET, ET, ANT	ET, ET, ET	ET, ET, NT
A				
d				
M	1	3	1	

	ET, APP, ANT	ET, ET, ANT	ET, ET, ET	ET, ET, NT
m				1
Grand Total	1	3	1	1

Table 18 (Continued)

	ET, ET, PT	ET, NT, ANT	ET, NT, PT	ET, PT, NT
A				
d				
M		6	1	1
m	1			1
Grand Total	1	6	1	2

Table 18 (Continued)

	ET, PT, PT	NT, ANT, ET	NT, ANT, NT	NT, ANT, PT
A				
d				
M	4	2		2
m	1		1	
Grand Total	5	2	1	2

Table 18 (Continued)

	NT, APP, ET	NT, APP, NT	NT, ET, ET	NT, ET, NT
A				
d	1			1
M			1	
m		2		
Grand Total	1	2	1	1

Table 18 (Continued)

	NT, NT, ANT	NT, NT, ET	NT, NT, NT	NT, PT, ANT
A				
d				
M	1		1	1
m		3		
Grand Total	1	3	1	1

Table 18 (Continued)

	NT, PT, ET	NT, PT, NT	NT, PT, PT	NT, sus 9-8, NT
A				
d	2			
M			2	
m		1	1	1
Grand Total	2	1	3	1

Table 18 (Continued)

	PT, ANT, ET	PT, ANT, NT	PT, ANT, PT	PT, APP, ET
A				
d		1		4
M	1	2		
m			3	
Grand Total	1	3	3	4

Table 18 (Continued)

	PT, APP, NT	PT, APP, PT	PT, ET, ET	PT, ET, NT
A				
d	1			
M	1	3	2	1

	PT, APP, NT	PT, APP, PT	PT, ET, ET	PT, ET, NT
m		4		
Grand Total	2	7	2	1

Table 18 (Continued)

	PT, ET, PT	PT, NT, ANT	PT, NT, ET	PT, NT, PT
A				
d	2		2	
M	3	5	1	8
m	1	1		3
Grand Total	6	6	3	11

Table 18 (Continued)

	PT, PT, ANT	PT, PT, ET	PT, PT, NT	PT, PT, PT
A				
d		7		17
M			2	45
m	1	1	1	36
Grand Total	1	8	3	98

Table 18 (Continued)

	sus 4-3, PT, NT	Grand Total
A		1
d		40
M	1	116
m		69
Grand Total	1	226

Table 19

Counts of Seventh Chord Implications Formed by Three Non-Harmonic Tones

	ANT, ANT, ANT	ANT, PT, PT	APP, APP, APP	APP, APP, PT
AM7				
dd7				
dm7				1
MM7				1
Mm7	1	5	1	
mm7	1			
Grand Total	2	5	1	2

Table 19 (Continued)

	APP, NT, ANT	APP, NT, APP	APP, NT, PT	ET, PT, PT
AM7				
dd7				
dm7	1			
MM7				1
Mm7		1	2	
mm7	2			
Grand Total	3	1	2	1

Table 19 (Continued)

	NT, ANT, NT	NT, APP, ANT	NT, NT, ANT	NT, NT, NT
AM7				
dd7				
dm7		1	2	
MM7				
Mm7	1			4

	NT, ANT, NT	NT, APP, ANT	NT, NT, ANT	NT, NT, NT
mm7		1	1	1
Grand Total	1	2	3	5

Table 19 (Continued)

	NT, NT, sus 4-3	NT, PT, ANT	NT, PT, NT	NT, PT, PT
AM7				
dd7				
dm7				1
MM7				
Mm7		2	2	2
mm7	1		2	2
Grand Total	1	2	4	5

Table 19 (Continued)

	PT, ANT, ANT	PT, ANT, APP	PT, ANT, ET	PT, ANT, NT
AM7				
dd7				
dm7				1
MM7				
Mm7		1	3	8
mm7	1			3
Grand Total	1	1	3	12

Table 19 (Continued)

	PT, ANT, PT	PT, APP, ANT	PT, APP, APP	PT, APP, PT
AM7				
dd7			1	2
dm7				

	PT, ANT, PT	PT, APP, ANT	PT, APP, APP	PT, APP, PT
MM7				
Mm7	9	1		
mm7				
Grand Total	9	1	1	2

Table 19 (Continued)

	PT, ET, PT	PT, NT, ANT	PT, NT, APP	PT, NT, NT
AM7				
dd7				
dm7		2		2
MM7				
Mm7	1			4
mm7		1	2	1
Grand Total	1	3	2	7

Table 19 (Continued)

	PT, NT, PT	PT, PT, ANT	PT, PT, APP	PT, PT, ET	PT, PT, NT
AM7					
dd7	1				
dm7	1	1			1
MM7		1		1	1
Mm7	9	16	3		14
mm7	14	3		2	6
Grand Total	25	21	3	3	22

Table 19 (Continued)

	PT, PT, PT	PT, PT, sus 9-8	PT, sus 9-8, PT	Grand Total
AM7	1			1

	PT, PT, PT	PT, PT, sus 9-8	PT, sus 9-8, PT	Grand Total
dd7	2			6
dm7	10			24
MM7	7			12
Mm7	29		1	120
mm7	54	1		99
Grand Total	103	1	1	262

Table 20

Counts of Extended Harmonic Implications Formed by Three Non-Harmonic Tones

	APP, APP, APP	APP, APP, PT	APP, PT, PT	ET, PT, PT
A^{MM9}				
d^{addm2}				
d^{addM9}				
d^{addm9}				
d^{dm9}		1		
d^{mm9}				
M^{addM2}				
M^{addM9}	1			
m^{addM9}				
m^{addm9}				
$M^{M7susP4}$				
M^{MM9}			1	
M^{mM9}				
m^{mM9}				1
m^{mm9}				
m^{susP4}				
Grand Total	1	1	1	1

Table 20 (Continued)

	NT, APP, APP	NT, APP, PT	NT, ET, ET	NT, NT, NT
A^{MM9}				
d^{addm2}			1	
d^{addM9}		1		
d^{addm9}				5
d^{dm9}				1
d^{mm9}				
M^{addM2}				
M^{addM9}				
m^{addM9}				
m^{addm9}				
$M^{M7susP4}$				
M^{MM9}				
M^{mM9}				
m^{mM9}	1			
m^{mm9}	1			
m^{susP4}				
Grand Total	2	1	1	6

Table 20 (Continued)

	NT, NT, PT	PT, APP, PT	PT, NT, PT	PT, PT, NT
A^{MM9}			1	3
d^{addm2}				
d^{addM9}				
d^{addm9}		4	4	1
d^{dm9}				
d^{mm9}				1
M^{addM2}				

	NT, NT, PT	PT, APP, PT	PT, NT, PT	PT, PT, NT
M^{addM9}	1			
m^{addM9}				
m^{addm9}				
$M^{M7susP4}$				
M^{MM9}				
M^{mM9}				
m^{mM9}				
m^{mm9}			1	
m^{susP4}				
Grand Total	1	4	6	5

Table 20 (Continued)

	PT, PT, PT	PT, PT, sus 9-8	Grand Total
A^{MM9}	1		5
d^{addm2}			1
d^{addM9}	1		2
d^{addm9}	4		18
d^{dm9}			2
d^{mm9}	1		2
M^{addM2}	1		1
M^{addM9}	3		5
m^{addM9}	1	1	2
m^{addm9}	2		2
$M^{M7susP4}$	1		1
M^{MM9}			1
M^{mM9}	2		2
m^{mM9}	2		4
m^{mm9}	1		3

	PT, PT, PT	PT, PT, sus 9-8	Grand Total
m^{susP4}	3		3
Grand Total	23	1	54

APPENDIX E: HARMONIC IMPLICATIONS AND PRECEDING HARMONIES

Table 21

Counts of Chord Types which Preceded Triad Implications

	m	M	mm7	Mm7	dm7	d	MM7	dd7	maddM9
A	18	16	2	18	8	4	1	2	
d	219	115	80	33	14	2	10		9
M	258	95	56	35	27	11	5	11	9
m	93	282	27	46	28	52	32	16	3
Grand Total	588	508	165	132	77	69	48	29	21

Table 21 (Continued)

	MsusP4	MaddM9	mmM9	MmM9	msusP4	Mm7susP4	maddM2
A	1					2	
d			1		1		1
M	11	4		1			
m	9	10	2	1	1		
Grand Total	21	14	3	2	2	2	1

Table 21 (Continued)

	MsusP4addM9	MMM9	MaddM2	A	mmm9	Grand Total
A						72
d	1		1	1		488
M					1	524
m		1				603
Grand Total	1	1	1	1	1	1,687

Table 22

Counts of Chord Types which Preceded Seventh Chord Implications

	M	m	mm7	Mm7	d	dm7	MM7	maddM9
AM7	14	9	2	15	6	1	1	8
dd7	26	39	14	12	4	26	2	
dm7	229	120	60	11	12	2	15	6
MM7	610	117	28	15	9	2	4	19
Mm7	1,425	129	39	3	13	31	8	3
mM7	16	23		5	1		1	4
mm7	428	580	15	54	65	11	29	11
Grand Total	2,748	1,017	158	115	110	73	60	51

Table 22 (Continued)

	MaddM9	MsusP4	dd7	A	msusP4	MMM9	mmM9	MaddM2
AM7		1	4	1				
dd7								
dm7	5		1	1	1			
MM7	10	2					2	
Mm7	8	31	32	4	1	1		2
mM7					1			
mm7	23	5	1		2	3	1	1
Grand Total	46	39	38	6	5	4	3	3

Table 22 (Continued)

	MmM9	MMA9	maddM2	mm7susP4	mM7	msusP4addM9
AM7				2		
dd7					1	
dm7		2				
MM7	1		2		1	

	MmM9	MMA9	maddM2	mm7susP4	mM7	msusP4addM9
Mm7						
mM7						
mm7	1					2
Grand Total	2	2	2	2	2	2

Table 22 (Continued)

	mmm9	MsusP4addM9	Mm7susd4	ddm9	Grand Total
AM7					64
dd7					124
dm7					465
MM7		1			823
Mm7			1		1,731
mM7				1	52
mm7	1				1,233
Grand Total	1	1	1	1	4,492

Table 23

Counts of Chord Types which Preceded Extended Harmony Implications

	M	m	M^{m7}	m^{m7}	M^{M7}	M^{addM9}	d	d^{m7}	M^{MM9}
A^{addM9}	3	1	2		2	1	1		
A^{MM9}	10	1	6	1			1	2	
A^{susP4}	1	1							
d^{addm2}		1					2		
d^{addM9}		2					1		
d^{addm9}	31	9	1	5			10		
d^{d7P11}	2								
$d^{d7susP4}$		1							
d^{dm9}	2	1						2	

	M	m	M^{m7}	m^{m7}	M^{M7}	M^{addM9}	d	d^{m7}	M^{MM9}
$d^{m7susP4}$	1							1	
d^{mm9}	60	17			1		2	8	
d^{susP4}		1							
M^{addA2}	1	1							
M^{addA9}	1			1					
M^{addM2}	71	7	15	7	1	1	1	3	1
m^{addM2}	8	44	13		2		1		
m^{addm2}		3							
M^{addM9}	505	46	111	43	16	34	18	10	4
M^{addm9}	4		1		1				
m^{addM9}	83	292	101	9	7	4	12	7	
m^{addm9}	2	13	2		1	1	1		
M^{m7P11}		1	1						
m^{m7P11}	2								
$M^{M7susA4}$	1								
$M^{m7susA4}$			1						
$m^{m7susA4}$				2					
$M^{m7susd4}$	1					1			
$M^{M7susP4}$	3	2			4				
$M^{m7susP4}$	38	24	21	5	9	7		1	7
$m^{M7susP4}$									
$m^{m7susP4}$	11	7	1	6	1			4	
M^{MA9}		2			4		1		
M^{MM9}	253	26	8	8	4	1	6	1	
M^{mM9}	52	52	22	2		4			
M^{mm9}	1	1							
m^{MM9}		6					1		
m^{mM9}	58	146	2	37	2	2	3	2	2

	M	m	M^{m7}	m^{m7}	M^{M7}	M^{addM9}	d	d^{m7}	M^{MM9}
m^{mm9}	35	19	3	4	5	1	7	2	2
M^{susA4}	11								
$M^{susA4addM9}$	7								
$m^{susd4addm9}$		1							
M^{susP4}	273	105	41	58	37	13	8	26	55
m^{susP4}	19	45	23	6	1	4	2	5	3
$M^{susP4addM2}$	19		1			1			
$m^{susP4addM2}$		3						2	
$M^{susP4addM9}$	206	2	6	6	4	14		1	
$m^{susP4addM9}$	3	115	1	2	1	3	1	2	
$m^{susP4addm9}$					1				
Grand Total	1,778	998	383	202	104	92	79	79	74

Table 23 (Continued)

	M^{susP4}	d^{d7}	m^{addM9}	m^{mM9}	M^{mM9}	d^{mm9}	M^{addM2}	$M^{m7susP4}$
A^{addM9}					1			
A^{MM9}						3		
A^{susP4}								
d^{addm2}								
d^{addM9}								
d^{addm9}								
d^{d7P11}								
$d^{d7susP4}$		3						
d^{dm9}		2						
$d^{m7susP4}$		1						
d^{mm9}		2	1			1		
d^{susP4}		1		1				
M^{addA2}								

	M^{susP4}	d^{d7}	m^{addM9}	m^{mM9}	M^{mM9}	d^{mm9}	M^{addM2}	$M^{m7susP4}$
M^{addA9}								
M^{addM2}	9				1		1	6
m^{addM2}	4	2	1	1				
m^{addm2}								
M^{addM9}	16			1	5		1	1
M^{addm9}								
m^{addM9}	1	22	32	3	3			
m^{addm9}					1			
M^{m7P11}								
m^{m7P11}								
$M^{M7susA4}$								
$M^{m7susA4}$								
$m^{m7susA4}$								
$M^{m7susd4}$								
$M^{M7susP4}$	1		1		1			
$M^{m7susP4}$	11		5	5				1
$m^{M7susP4}$						1		
$m^{m7susP4}$	2		4				1	
M^{MA9}								
M^{MM9}							1	
M^{mM9}	1			1	1		2	
M^{mm9}								
m^{MM9}								
m^{mM9}	1		1				2	
m^{mm9}			1				1	
M^{susA4}								
$M^{susA4addM9}$								
$m^{susd4addm9}$								

	M^{susP4}	d^{d7}	m^{addM9}	m^{mM9}	M^{mM9}	d^{mm9}	M^{addM2}	$M^{m7susP4}$
M^{susP4}	7	19	7	28	13	6		
m^{susP4}		6		2	4	2	2	
$M^{susP4addM2}$	1							
$m^{susP4addM2}$			1					
$M^{susP4addM9}$	14	1						
$m^{susP4addM9}$		1	5					
$m^{susP4addm9}$								
Grand Total	68	60	59	42	30	13	11	8

Table 23 (Continued)

	m^{M7}	m^{susP4}	A	$m^{susP4addM9}$	m^{addM2}	A^{M7}	$M^{susP4addM9}$	m^{mm9}
A^{addM9}			5					
A^{MM9}					1			
A^{susP4}								
d^{addm2}								
d^{addM9}								
d^{addm9}	1							
d^{d7P11}								
$d^{d7susP4}$								
d^{dm9}								
$d^{m7susP4}$								
d^{mm9}								
d^{susP4}								
M^{addA2}								
M^{addA9}								
M^{addM2}								
m^{addM2}		1						
m^{addm2}								

	m^{M7}	m^{susP4}	A	$m^{susP4addM9}$	m^{addM2}	A^{M7}	$M^{susP4addM9}$	m^{mm9}
M^{addM9}		1			1			2
M^{addm9}								
m^{addM9}		2	1	3	2	2		
m^{addm9}								
M^{m7P11}								
m^{m7P11}								
$M^{M7susA4}$								
$M^{m7susA4}$								
$m^{m7susA4}$								
$M^{m7susd4}$								
$M^{M7susP4}$								
$M^{m7susP4}$	6					1		
$m^{M7susP4}$								
$m^{m7susP4}$								
M^{MA9}								
M^{MM9}								1
M^{mM9}								
M^{mm9}								
m^{MM9}								
m^{mM9}								
m^{mm9}								
M^{susA4}								
$M^{susA4addM9}$	1					1		
$m^{susd4addm9}$								
M^{susP4}		1		2	1		4	
m^{susP4}		1						
$M^{susP4addM2}$						1		
$m^{susP4addM2}$								

	m^{M7}	m^{susP4}	A	$m^{susP4addM9}$	m^{addM2}	A^{M7}	$M^{susP4addM9}$	m^{mm9}
$M^{susP4addM9}$		1						
$m^{susP4addM9}$								
$m^{susP4addm9}$								
Grand Total	8	7	6	5	5	5	4	3

Table 23 (Continued)

	M^{MA9}	A^{addM9}	d^{addm9}	$M^{M7susA4}$	$M^{susP4addM2}$	$m^{m7susP4}$
A^{addM9}						
A^{MM9}						
A^{susP4}						
d^{addm2}						
d^{addM9}						
d^{addm9}						
d^{d7P11}						
$d^{d7susP4}$						
d^{dm9}						
$d^{m7susP4}$						
d^{mm9}						
d^{susP4}						
M^{addA2}						
M^{addA9}						
M^{addM2}						
m^{addM2}						
m^{addm2}						
M^{addM9}				1		1
M^{addm9}						
m^{addM9}	2	2				
m^{addm9}						

	M^{MA9}	A^{addM9}	d^{addm9}	$M^{M7susA4}$	$M^{susP4addM2}$	$m^{m7susP4}$
M^{m7P11}						
m^{m7P11}						
$M^{M7susA4}$						
$M^{m7susA4}$						
$m^{m7susA4}$						
$M^{m7susd4}$						
$M^{M7susP4}$						
$M^{m7susP4}$						
$m^{M7susP4}$						
$m^{m7susP4}$						
M^{MA9}						
M^{MM9}						
M^{mM9}						
M^{mm9}						
m^{MM9}						
m^{mM9}						
m^{mm9}						
M^{susA4}						
$M^{susA4addM9}$						
$m^{susd4addm9}$						
M^{susP4}			2		1	
m^{susP4}						
$M^{susP4addM2}$						
$m^{susP4addM2}$						
$M^{susP4addM9}$						
$m^{susP4addM9}$						
$m^{susP4addm9}$						
Grand Total	2	2	2	1	1	1

Table 23 (Continued)

	$d^{d7susP4}$	A^{MM9}	Grand Total
A^{addM9}			16
A^{MM9}			25
A^{susP4}			2
d^{addm2}			3
d^{addM9}			3
d^{addm9}			57
d^{d7P11}			2
$d^{d7susP4}$			4
d^{dm9}			7
$d^{m7susP4}$			3
d^{mm9}			92
d^{susP4}			3
M^{addA2}			2
M^{addA9}			2
M^{addM2}			124
m^{addM2}			77
m^{addm2}			3
M^{addM9}			817
M^{addm9}			6
m^{addM9}		1	591
m^{addm9}			21
M^{m7P11}			2
m^{m7P11}			2
$M^{M7susA4}$			1
$M^{m7susA4}$			1
$m^{m7susA4}$			2
$M^{m7susd4}$			2

	$d^{d7susP4}$	A^{MM9}	Grand Total
$M^{M7susP4}$			12
$M^{m7susP4}$			141
$m^{M7susP4}$			1
$m^{m7susP4}$			37
M^{MA9}			7
M^{MM9}			309
M^{mM9}			137
M^{mm9}			2
m^{MM9}			7
m^{mM9}			258
m^{mm9}			80
M^{susA4}			11
$M^{susA4addM9}$			9
$m^{susd4addm9}$			1
M^{susP4}			707
m^{susP4}			125
$M^{susP4addM2}$			23
$m^{susP4addM2}$			6
$M^{susP4addM9}$			255
$m^{susP4addM9}$	1		135
$m^{susP4addm9}$			1
Grand Total	1	1	4,134