NUMBER LINE ESTIMATION: THE USE OF NUMBER LINE MAGNITUDE ESTIMATION TO DETECT THE PRESENCE OF MATH DISABILITY IN POSTSECONDARY STUDENTS

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Number Line Estimation: The Use of Number Line Magnitude Estimation to Detect the Presence of Math Disability in Postsecondary Students

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ABSTRACT

This study arose from an interest in the possible presence of mathematics disabilities among students enrolled in the developmental math program at a large university in the Mid-Atlantic region. Research in mathematics learning disabilities (MLD) has included a focus on the construct of working memory and number sense. A component of number sense is the formation of a mental number line. This study looked at the mental representations of the number line in postsecondary developmental math students. It was found that the overall representation was linear, linear for three of four academic levels, and there were linear representations based upon gender. The presence of increased error rates on number line estimations between 23 and 39 needs to be explored.

Key Words: developmental math, learning disabilities, mathematics disabilities, mathematics learning disabilities (MLD), number line, number sense, postsecondary, working memory

Acknowledgements/Dedication

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List of Abbreviations

Absolute Estimation Error percentage (ABE%): a calculation that measures the percentage error of a given estimation on the number line estimation instrument

ACT: a standardized test that assesses high school students' general educational development and their ability to complete college-level work.

Analysis of Variance (ANOVA): a statistical method for comparing the means of two or more groups to identify the presence or absence of a statistically significant difference

Fisher’s Least Squares Difference (LSD): a statistical analysis that measures differences between group relationships

Intelligence Quotient (IQ): a standardized measure of overall intelligence

Low achieving students (LA)

Learning Disabilities (LD)

Mathematics Disabilities (MD)

MD-10: A cutoff criteria based on students scoring ten percent or below on a standardized mathematics achievement test

MD-11-25: A cutoff criteria based on students scoring between eleven to twenty-five percent on a standardized mathematics achievement test

Mathematics Learning Disability (MLD)

Math Learning Disabilities/Reading Disabilities (MLD/RD): Comorbid math and reading disabilities more commonly seen as MD/RD

Mean Absolute Error Percentage ($M_{ABE\%}$) is the average of the ABE% for a participant or group of participants

Professional Analytic Software (PASW): a statistical program developed by SPSS and now owned by IBM

Reading Disabilities (RD)

Stanford Achievement Test Series 9 (SAT-9): the math component of this standardized test used to baseline students in mathematics learning disability studies

SAT Reasoning Test: a standardized test for college admissions in the United States

Typically achieving students (TA)

Test of Early Math Ability (TEMA): another standardized test used to baseline students in mathematics disability research

CHAPTER ONE: INTRODUCTION

Introduction

To be able to understand and apply the various concepts involved with mathematics is a critical element in the twenty first century. From basic consumer math to applied calculus mathematics is involved in every aspect of modern life. While there are certainly core concepts and skill sets within mathematics, there is also a broad range of domains in which these core concepts are applied. Failure to master the core of mathematics increases the challenges faced within the various domains (Geary, 2004; M. M. Murphy, M. M. M. Mazzocco, L. B. Hanich, & M. C. Early, 2007b). The ability to understand and apply mathematical concepts has a direct effect on both academic and employment success (Geary, 2000; Mazzocco & Thompson, 2005).

While there have been significant steps taken in the research of learning disabilities (LD) related to reading disabilities (RD), research in mathematics disabilities (MD) is still developing (Geary, 1993; Geary, Hamson, & Hoard, 2000; Gersten, Jordan, & Flojo, 2005; Mazzocco, & Myers, 2003). There is a broad diversity among specific math deficits. The ability to provide individualized instruction is not readily available in mainstream classrooms (Wadlington & Wadlington, 2008). Mazzocco & Thompson (2005) stress that "It is important to identify risk for MLD, because—like poor reading achievement—poor math achievement is a risk factor for negative outcomes in both childhood and adulthood" (p. 142). With all that is involved, and at stake, it is important for educators to understand mathematics disability. Mathematics disability needs to be defined, current research understood, screening criteria established, and effective instructional interventions applied.

Most children can learn the core concepts of math when given a robust learning environment and effective instruction. However, there are children who struggle with the core concepts even when the environment and instruction are focused on effective learning. Three to six percent of the student population is challenged with mathematics learning disabilities (MLD). MLD is broadly defined as a consistent score below the 35^{th} percentile in mathematics achievement tests while possessing and average or above intelligence quotient (Gersten, Jordan, & Flojo, 2005).

Without effective intervention the segment of the student population that is affected by MLD can face continued challenges in learning core mathematical concepts and applying those concepts in the various domains of math. Studies have shown that resistance to intervention past the second grade can lead to prolonged difficulties with math (Jordan & Hanich, 2003).

One of the key concepts in learning mathematics is number sense. While there is a broad range of operational definitions, number sense can generally be described as:

> (a) fluency in estimating and judging magnitude, (b) ability to recognize unreasonable results, (c) flexibility when mentally computing, (d) ability to move among different representations and to use the most appropriate representation. (Kalchman, Moss, & Case, 2001, p. 2)

Dehaene (2001) stated hypothesis is "that number sense qualifies as a biologically determined category of knowledge. I propose that the foundations of arithmetic lie in our ability to mentally represent and manipulate numerosities on a mental ‘number line’ an analogical representation of number” (p. 17).

The importance of developing the linear representation of the number line, and thus an accurate concept of magnitude, cannot be overstated. Booth and Siegler (2008) stated that “representations of numerical magnitude are both correlationally and causally related to arithmetic learning” (p. 1016), and that “numerical magnitude representations are not only positively related to a variety of types of numerical knowledge but also predictive of success in acquiring new numerical information, in particular, answers to arithmetic problems” (p. 1027).

Several studies have been conducted that sought to investigate the presence and structure of mental number lines (Booth & Siegler, 2006, 2008; Geary, Hoard, Nugent, & Byrd-Craven, 2008; Laski & Siegler, 2007; Siegler & Booth, 2004). The underlying view of these studies is that children who do not transition from a logarithmic mental representation of the number line to a linear mental representation of the number line continue to have difficulties with number sense, specifically in magnitude estimations, this leads to challenges in several domains of mathematics.

What is the importance of number line estimation in relationship to mathematics learning disabilities (MLD)? Testing for MLD has largely settled on two assessment measures (a) the IQ-discrepancy measure, and (b) standardized mathematics test score cutoff criteria. However, both these methodologies are inadequate in themselves to diagnose the presence of MLD (Geary, 2005; Murphy, et al., 2007b). Siegler and Booth (2004) found significant correlations between percentage error on number line estimations and performance on the mathematics section of the Stanford Achievement

Test Series, SAT-9. Given the relationship between number line estimation and performance on achievement tests, the use of number line estimation could serve as a component of detecting the presence of mathematics learning disabilities (MLD).

Currently the studies of number line estimation (Booth & Siegler, 2006, 2008; Geary, et al., 2008; Laski & Siegler, 2007; Siegler & Booth, 2004) focused on early childhood education. What has not occurred is the application of this methodology to postsecondary students who display correlates of MLD. Specifically, students involved with remediation in developmental math programs. Postsecondary remediation provides the opportunity to resolve instructional inequalities present in primary and secondary education. Postsecondary remediation also provides functional competency in economic and political settings while preparing the student for successful negotiation of college coursework Bahr (2008),

McGlaughlin, Knoop, and Holliday (2005) and Sullivan (2005) stated that research based interventions for teaching postsecondary students with mathematics learning disabilities (MLD) were lacking. However, methodologies proven effective in early childhood education can prove effective in teaching postsecondary students with MLD. This presents the possibility that detection of MLD in postsecondary students could lead to appropriate instructional interventions. Most developmental math students are placed in the program based on a cutoff criterion on standardized and/or admissions tests. However, the criteria vary widely in both four year and two year institutions (Bahr, 2008; Hadden, 2000). The current study was an effort to explore the feasibility of using number line estimation measures to detect the presence of mathematics disability in postsecondary students enrolled in the developmental math program at a major university in the mid-Atlantic.

Problem Statement

The problem was the presence of significant challenges in the mathematical competencies of postsecondary students enrolled in the developmental math program at a large university in the Mid-Atlantic region. These challenges may or may not be related to the presence of math learning disability but few studies exist to sufficiently determine this criterion. The ability to detect and clearly define the particular challenges faced by the students would not only improve instructional practice but lead to effective interventions. The particular problem addressed by this study was the presence of a logarithmic representation of the mental number line in students participating in the developmental math program. Researchers have found that students with MLD have a difficult time in making the transition from using the mental logarithmic number line to using the learned linear number line (Booth & Siegler, 2008; Geary, et al., 2008; Siegler & Booth, 2004; Siegler & Opfer, 2003).

Purpose

The implications of a math disability have profound consequences throughout an individual's lifespan. Large-scale studies estimated that five to ten percent of students would face a mathematics deficit (Geary, et al., 2008). The majority of research conducted on math learning disabilities (MLD) has occurred in the primary grades. However, McGlaughlin, Knoop, and Holliday (2005) discovered that deficits in post secondary students mirrored those of elementary and secondary levels. This study's purpose was to use number line estimation to examine the potential presence or absence of MLD in postsecondary students enrolled in a developmental math program.

Hypotheses

Based upon research indicating that the development of a linear representation of the mental number line is crucial to the acquisition of many mathematical skills, this study looked at the mental representation of the number line in developmental math students. In addition, linear representations were examined based upon gender, academic level, and prior enrollment in developmental math. Finally, the mean absolute estimation error percentages ($M_{ABE\%}$) were compared based upon gender, academic level, and prior enrollment in developmental math.

Null Hypothesis 1: The representation of the mental number line of developmental math students approximates a linear line as determined by analyzing the statistical significance of the model and the R^2 correlational statistic.

Null Hypothesis 2: The representation of the mental number line of developmental math students approximates a linear line as determined by analyzing the statistical significance of the model and the R^2 correlational statistic based upon gender.

Null Hypothesis 3: The representation of the mental number line of developmental math students approximates a linear line as determined by analyzing the statistical significance of the model and the R^2 correlational statistic based upon academic level.

Null Hypothesis 4: The representation of the mental number line of developmental math students approximates a linear line as determined by analyzing the statistical significance of the model and the R^2 correlational statistic based upon prior enrollment in developmental math.

Null Hypothesis 5: The mean absolute error percentage will not be statistically different by gender.

Null Hypothesis 6: The mean absolute error percentage will not be statistically different by academic level.

Null Hypothesis 7: The mean absolute error percentage will not be statistically different by prior enrollment in developmental math.

Definitions

Absolute Error Percentage – A calculated estimation error percentage based upon the absolute value of the difference between the number and the estimation, divided by the scale of the instrument.

Figure 1. Equation for Computation of ABE%

$$ABE\% = \left| \frac{Number - Estimation}{Scale} \right|$$

Developmental Math – One or more courses in mathematics, normally beginning algebra and intermediate algebra, designed to prepare students for required math classes in their major.

Dyscalculia – A neurologically based disorder that affects an individual's ability to solve mathematical problems.

Math Learning Disabilities – A score at or lower than the thirty-fifth percentile on a mathematics achievement test with a low average or higher IQ score and the continuation of this condition over successive school years (Geary, 2004; Gersten, et al., 2005; Mazzocco & Myers, 2003).

Mean Absolute Error Percentage ($M_{ABE\%}$) is the average of the absolute error percentage for a participant or group of participants.

Mental Number Line – A mental representation of the standard number line used for comparisons of magnitude and numerical estimation (Booth & Siegler, 2008; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Geary, et al., 2008; Laski & Siegler, 2007).

Number Sense – A theoretical construct that defines the ability to count, recognize number patterns, comparisons of magnitude, estimation skills, and numerical transformation (Berch, 2005).

Working Memory – A theoretical construct that represents the ability of the brain to hold information in memory and process the verbal and spatial aspects of that information simultaneously.

CHAPTER TWO: REVIEW OF THE LITERATURE

Theoretical Background

Generally speaking, there are three main traditions within the study of intellectual development: (a) the empiricist, (b) the rationalist, and (c) the historic-cultural tradition (Case, 1987). The tradition that bears most upon this present study is the rationalist. The rationalist tradition is mainly based upon Kant's reaction to British empiricism. The premise of Kant's writings is that order is imposed upon information received by existing structures within the learner and not from the order existing in the data itself. Those who have accepted this view propose that the study of development should be guided by the explanation of these inherent structures (Case, 1987).

One of the most influential rationalist/constructivist researchers into cognitive development was Jean Piaget. Knight and Sutton (2004) stated, "Piaget's work provided a useful framework for understanding how children and adolescents grow and change in how they think about their world and solve problems" (p. 48). Case (1993) stated that "one of Piaget's most important suggestions was that, at several different points in their growth, children acquire new systems of cognitive operations (structures) that radically alter the form of learning of which they are capable" (p.219).

During Piaget's work with Theodore Simon, co-author of the Binet-Simon intelligence scale, he became focused on the types and differences of errors that children made on reading tests. This research led Piaget to construct a theory of cognitive development built upon four main stages of development and the sub-stage processes of assimilation, accommodation and equilibrium. While Piaget's stages of development are associated with chronological age, the completion of one stage and movement to the next does not happen automatically and development can be hindered at a particular stage (Dunn, 2005).

Over the years Piaget's work came under increased scrutiny, and during the 1970s the classical Piagetian theories of stage development proved to be inadequate for totally explaining cognitive development (Morra, 2008). "It was the early 1970s when the first neo-Piagetian theories were published as a new integration of Piagetian concepts with ideas originating from Human Information Processing and other classical psychological frameworks" (Morra, 2008, p. 1). The emergence of the neo-Piagetians was influenced by a desire to expand on Piaget's basic concepts and incorporate new research in cognitive development (Knight & Sutton, 2004).

One of the most important challenges to Piaget's epistemological structure of development was the discovery that children possess basic linguistic and enumerative structures from birth (Case & Sowder, 1990). However, neo-Piagetians preserve a number of Piaget's central concepts:

> In particular, they preserve notions that (a) children's knowledge is not just passively received but is actively constructed via a set of internal epistemic operations; (b) these operations are organized, that is, they possess an internal structure; (c) different levels of structural organization can be identified that transcend any particular cognitive domain; (d) structures at higher levels are assembled via the coordination of lower level structures; and (e) there is an age related "upper boundary" to the level of structure children can assemble at any

> age, even under optimal environmental circumstances. (Case & Sowder, 1990, p. 82)

One neo-Piagetian, Robbie Case, has provided a rich theoretical background for the concepts of number sense and the mental representation of the number line (Morra, 2008; Okamoto & Case, 1996). Case's four major stages of development are: (a) sensimotor, (b) interrelational, (c) dimensional, and (d) vectorial (or abstract) (Case, 1987; Morra, 2008). Okamoto and Case were able to show that number sense increases in complexity at each stage of development (Okamoto & Case, 1996).

Morra (2008) also stated that "Case accounts for the developmental progression within each of the four major stages by postulating a sequence of four recurring substages", and that, "the development of the substages is very much a function of the number of mental elements of a particular task that can be represented simultaneously. This complexity is defined by the number of 'basic units of thought' the child is able to control" (p.192).

Math Learning Disabilities

There is no question on the importance of having the ability to solve mathematical problems. The ability to understand and apply mathematical concepts has a direct effect on both academic and employment success (Geary, 2000; Mazzocco & Thompson, 2005).

> Although the history of mathematical learning disability (MLD) is relatively short, any depiction of its history requires delving into the histories of medicine (particularly neurology), developmental psychology, cognitive science, mathematics education, special education, and even law. Indeed, each of these

> fields has contributed to the foundations of contemporary MLD research, including the areas of identification, diagnosis, and treatment of math disabilities. These multiple sources of information and the perspectives associated with each of these fields have given rise to the multidisciplinary field of MLD research and practice that exists today. (Gersten, Clarke, & Mazzocco, 2007. p.7)

While there have been significant steps taken in the research of learning disabilities (LD) related to reading disabilities (RD), research in mathematics disabilities (MD) is still developing (Geary, 1993; Geary, Hamson, & Hoard, 2000; Gersten, Jordan, & Flojo, 2005; Mazzocco, & Myers, 2003). There is a broad diversity among specific math deficits. The ability to provide individualized instruction is not readily available in mainstream classrooms (Wadlington & Wadlington, 2008). Mazzocco & Thompson (2005) stress that "It is important to identify risk for MLD, because—like poor reading achievement—poor math achievement is a risk factor for negative outcomes in both childhood and adulthood" (p. 142).

Early work in defining mathematical disabilities focused on three cognitive aspects: (a) a procedural deficit, (b) a memory retrieval deficit, and (c) a visual spatial, also known as visuospatial, deficit (Geary, 1993). Research in mathematics learning disability (MLD) has generally followed this cognitive pattern over the past fifteen years. Recent research into math disabilities/difficulties (MD) has centered on cognitive deficits and neurological factors (Bryant, Bryant, & Hammill, 2000; Geary, 2004; Osmon, Smerz, Braun, & Plambeck, 2006; Schuchardt, Maehler, & Hasselhorn, 2008; Seethaler & Fuchs, 2006). Generally speaking, mathematics disability " is likely best understood in terms of the relations between different cognitive processes and the impact that a deficit in one area has on the other areas and on mathematics achievement" (Mabbott & Bisanz, 2008, p. 17).

The memory retrieval deficit is distinguished by difficulties in retrieving math facts from memory and variations in answer retrieval times. Procedural deficits are associated with difficulties with computation strategies and problems in acquiring math algorithms. A visual spatial deficit involves problems with placing numbers and understanding visual representations of numbers (Mazzocco & Myers, 2003).

With all that is involved and all that is at stake it is important for educators to understand mathematics disability. Mathematics disability needs to be defined, current research needs to be understood, screening criteria need to be established, and effective instructional interventions need to be applied.

Mathematics is a broad and complex field of study. Geary (2004) states that mathematics disability "can result from deficits in the ability to represent or process information in one or all of the many mathematical domains (e.g., geometry) or in one or a set of individual competencies within each domain" (p. 4).

Most children can learn the core concepts of math when given a robust learning environment and effective instruction. However, there are children who struggle with the core concepts even when the environment and instruction are focused on effective learning. Three to six percent of the student population is challenged with mathematics learning disabilities (MLD). MLD is broadly defined as a consistent score below the 35^{th} percentile in mathematics achievement tests while possessing and average or above intelligence quotient (Gersten, et al., 2005).

Without effective intervention the segment of the student population that is affected by MLD can face continued challenges in learning core mathematical concepts and applying those concepts in the various domains of math. Studies have shown that resistance to intervention past the second grade can lead to prolonged difficulties with math (Jordan & Hanich, 2003).

Difficulties in math can result from a number of factors that can include poor instruction, socioeconomic factors, or cognitive factors (Mazzocco, 2005). Mazzocco calls for a broad definition of students who have difficulties with math and a more specific definition, math disabilities, for those who are challenged by biologically based deficits. Generally speaking, mathematics learning disability (MLD) " is likely best understood in terms of the relations between different cognitive processes and the impact that a deficit in one area has on the other areas and on mathematics achievement" (Mabbott & Bisanz, 2008, p. 17).   Geary (1993) states that "from a cognitive perspective, the lower order deficits of MD children potentially reside in five component skills: procedural, memory retrieval, conceptual, working memory, and speed of processing (especially counting speed)" (p. 348). Of these five areas two appear to be prevalent in mathematics disability research. They are counting skills and memory retrieval (Geary, 1993; Geary 2004; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Gersten, Jordan, & Flojo, 2005; Mabbott & Bisanz, 2008; Mazzocco & Myers, 2003; Wadlington & Wadlington, 2008).

Counting Skills

Research into preschool counting skills has settled onto two different theories of counting acquisition. One theory states that counting is inherent the states that counting is inductively learned through experience. No matter if counting skills are inherent or inductive counting is a related skill to solving addition problems (Geary, 1993).

Geary (2004) states that “many children with MLD, independent of their reading achievement levels or IQ, have a poor conceptual understanding of some aspects of counting” (p. 6). It is unclear if deficits in counting continue beyond the second grade. While this may not be the case, the early counting deficits lead to difficulties in solving math problems that require the use of counting strategies (Geary, 2004). Gersten, Jordan, & Flojo (2005) point out that “maturity and efficiency of counting strategies are valid predictors of students’ ability to profit from traditional math mathematics instruction” (p. 295).

Errors in counting can have an effect on the development of math skills. Geary, Hoard, Byrd-Craven, Nugent, & Numtee (2007) state that “poor skill at detecting counting errors may compromise ability to correct these errors and thus result in more errors in situations in which counting is used to solve arithmetic problems” (p. 1344).

Memory Retrieval

Two important factors are at work in the functioning of memory storage (a) the speed with which numbers can be counted, and (b) the quantity of numbers held in working memory (Geary, 1993). “Working memory is the ability to hold a mental representation of information in mind while simultaneously engaging in other mental processes” (Geary, et al., 2007, p. 1345). Problems with the speed of counting and narrower working memory can lead to a failure to adequately place math facts in long-term memory (Geary 1993).

Not only do math facts need to be effectively retrieved from long-term memory, students must be confident that these facts are correct. Geary, Hamson, & Hoard (2000) state that “the use of retrieval-based processes is moderated by a confidence criterion that represents an internal standard against which the child gauges confidence in the correctness of the retrieved answer” (p. 239).

Gersten, Jordan & Flojo (2005) describe the importance of memory retrieval by stating that “failure to instantly retrieve a basic combination, such as 8 + 7, often makes discussions of the mathematical concepts involved in algebraic equations more challenging” (p. 294). Gersten, Jordan & Flojo point to the fact that “the ability to store this information in memory and easily retrieve it helps students build both procedural and conceptual knowledge of abstract mathematical principles, such as commutativity and the associative law” (p. 295). Finally Miller & Hudson state that “the ability to memorize mathematical information and quickly retrieve the information helps students as they progress through the hierarchical mathematics curriculum (i.e., sequence of skills that become increasingly complex; each skill builds on previous skill)” (p. 53).

Screening for MLD

To a great degree the screening methodology for learning disabilities has been based on a discrepancy between IQ scores and achievement. However, this methodology has come under increasing scrutiny in recent years. The absence of a discrepancy does not necessarily indicate the absence of LD or MD in particular (Mazzocco & Myers, 2003). The developing field of neuroscience is providing screening methods for mathematics disability. This methodology is in conjunction with the study of dyscalculia. (Katzir & Pare'-Blagoev, 2006). Studies that use the performance measures of dyscalculia research have shown that five to eight percent of school age children present some type of mathematics disability (Geary, 2004).

There are two main assessment measures for identifying children at risk of math disabilities (a) the IQ-discrepancy measure, and (b) standardized mathematics test score cutoff criteria. The IQ-discrepancy measure generally looks at the presence of scores below the 20th to 25th percentile on mathematics achievement tests with average and above IQ for grade level and age. The standardized mathematics test score cutoff varies from a very strict 15% to a very broad 35% (Geary, 2004, 2005; Mabbott & Bisanz, 2008; Murphy, et al., 2007b; Schuchardt, et al., 2008). Murphy et al. (2007b) take issue with the use of the IQ-discrepancy measure stating that:

> In essence, not only do IQ-discrepancy definitions lack discriminant validity but they also leave unspecified at which point a discrepancy becomes significant, and they do not account for changes over time in the stability and interpretability of discrepancy scores. (p. 459)

Concerning standardized test cutoff score criteria Geary (2005) stated:

> The road to the development of assessment measures specifically for mathematical disabilities (MD) perforce runs through existing standardized achievement tests. These tests, however, should only be viewed as initial screening measures—that is, as a means to identify children who might have a cognitive disability that interferes with mathematical learning.

Compared to RD researchers in MLD have yet to develop a criterion based set of diagnostic measures (Mazzocco & Myers, 2003).

Mazzocco (2005) stated “there is much variability in how mathematics difficulties are defined and measured, and even in the terms used to refer to them” (p. 318). This may be because mathematics is not a unified subject area but spreads over several domains. (Geary, 2004) noted:

> The breadth and complexity of the field of mathematics make the identification and study of the cognitive phenotypes that define mathematics learning disabilities (MLD) is a formidable endeavor. In theory, a learning disability can result from deficits in the ability to represent or process information in one or all of the many mathematical domains (e.g., geometry) or in one or a set of individual competencies within each domain. (p. 4).

Gersten, Jordan, and Flojo (2005) stated that because “tests are based on many different types of items, specific deficits might be masked. That is, children might perform at an average level in some areas of mathematics but have deficits in others” (p. 294).

Screening should encompass several combinations of test items and take developmental issues into consideration (Mazzocco, 2005). Screening must balance between sensitivity and specificity and include a sufficient level of difficulty so that refinement in MD subtypes can be detected (Fuchs, et al., 2007). Clearly defining mathematics disability can have a significant effect on screening. Using a broad criterion in definitions and measurements can lead to two different outcomes. Studies may eventually converge on standard definitions and methodologies or they would diverge in such a fashion that application and generalization of research would be impossible. Divergence would prevent a standardization of screening definitions (Murphy, et al., 2007b).

On method of screening uses cutoff levels on mathematics achievement tests. One of the results of using too lenient range is to mix severe cases of MD with mild cases or even students merely experiencing math difficulties (Geary, et al., 2007). Geary et al. found that "children identified with a stringent versus a lenient cutoff criterion differ in important ways and should not be conflated" (p. 1355).

Establishing a criterion for cutoff points may not be as straight forward as it seems if we fail to take developmental aspects into consideration. Development of skills in mathematics is a cumulative process that continues beyond formal education. Developmental differences may exist for MD subtypes over time and screening methods should take into account a pattern of disability that exists over time (Mazzocco & Myers, 2003). The differences in general and specific mathematics learning disabilities can vary over time. It is important to hold a longitudinal view when diagnosing MLD in early elementary students (Jordan & Hanich, 2003).

Gender and Math Difficulties

While gender differences seem to exist during particular phases of education, the overall differences in mathematics performance are being equalized as a result of current educational practices in the United States (Ding, Song, & Richardson, 2006). Mundia (2010) found that both female and male students with math difficulties struggled on particular tests. However, Mundia also found that females in coeducational classes possessed a high level of self esteem and confidence. Mundia called upon educators and parents to work to remove stereotypes surrounding gender differences in mathematical performance.

Cognitive Development and MLD

Between five and eight percent of children attending schools have some type of cognitive or memory deficit that affects their ability to learn methods and concepts involving math (Geary, 2004; Geary, et al., 2008; Mabbott & Bisanz, 2008; Murphy, et al., 2007b). One of the challenges in studying and identifying the various characteristics of MLD is the wide range of domains (algebra, trigonometry, geometry, etc.) and the various skills within each domain (Geary, 2004). Compared to the wealth of research in reading disabilities (RD), research into the core deficits of MLD have not produced a set of readily identifiable deficits (Mazzocco & Myers, 2003).

MLD then is difficult to diagnose and is broad in its scope of possible manifestations. While the study of MLD has developed over the past three decades, it was a seminal article by Geary (1993) that began the integration of MLD research and research in the cognitive sciences (Berch, 2008; Gersten, et al., 2005).

Geary (1993) stated that "despite the lack of systematic research into the MD area, extant cognitive and neuropsychological studies of mathematical achievement and mathematical disorder provide valuable insights into the specific deficits that might underlie mathematical disabilities" (p. 345). Geary sought to provide an integration of cognitive and neuropsychological studies involving the visible discrepancies between typically achieving (TA) students and students with math disabilities (MD). Children with MD show two primary numerical deficits; immature counting strategies and difficulty with fact retrieval from long term memory. The procedural errors associated with counting strategies could be influenced by a poor working memory capacity but appear to be a developmental delay. However, the retrieval of mathematical facts from long term memory is more fundamental and does not disappear with developmental growth (Geary, 1993).

Geary also stated that “from a cognitive perspective, the lower order deficits of MD children potentially reside in five cognitive component skills: procedural, memory retrieval, conceptual, working memory, and speed of processing (especially counting speed)” (Geary, 1993, p. 348). Procedural and memory retrieval skills have a direct affect on pencil and paper tests. The combined effect of all five can negatively affect performance on achievement and ability tests.

In reference to neuropsychological studies, Geary (1993) stated “the neurological studies in fact suggest three relatively distinct types of basic lower order mathematical deficit: fact retrieval, procedural, and spatial representation” (p. 354). Fact retrieval and procedural errors are related to anarithmetria, a deficit associated with damage to the posterior regions of the left hemisphere. Difficulties in spatial representation are associated with damage to the posterior regions of the right hemisphere and are referred to as spatial acalculia (Geary, 1993). “Specific problems associated with spatial acalculia include the misalignment of numbers in multicolumn arithmetic problems, number omissions, number rotation, misreading arithmetical operation signs, and difficulties with place value and decimals” (Geary, 1993, p. 352).

Additional research into math disabilities/difficulties (MD) has centered on cognitive deficits and neurological factors (Bryant, et al., 2000; Geary, 2004; Osmon, et al., 2006; Schuchardt, et al., 2008; Seethaler & Fuchs, 2006). Generally speaking, mathematics disability “ is likely best understood in terms of the relations between different cognitive processes and the impact that a deficit in one area has on the other areas and on mathematics achievement" (Mabbott & Bisanz, 2008, p. 17).

Murphy, Mazzocco, Hanich, and Early (2007b) demonstrated how the cognitive profile of students with mathematics learning disabilities (MLD) could vary depending upon the cutoff criterion used in relation to achievement tests. The use of a cutoff percentage on the mathematics portion of achievement tests can be problematic but it serves as standard methodology for defining MLD. When setting a high cutoff percentage, say thirty-five to forty-five percent, the goal may be to increase sample size. However, this can lead to problems with heterogeneity and the inclusion of children simply facing a developmental delay (Murphy, et al., 2007b).

Murphy et al. (2007) studied three groups of students using cutoff scores of ten percent, eleven to twenty-five percent, and above twenty-five percent on the *Test of Early Math Ability* (TEMA). The groups were delineated as mathematics learning disabilities students at ten percent or below (MLD-10), mathematics learning disabilities students at eleven to twenty-five percent (MLD-11-25), and typically achieving students above twenty-five percent (TA). The study led to three major implications. First, there were group differences attributable to global cognitive deficits between the MLD groups and the TA group. However, global cognitive deficits did not account for any differences between the MLD-10 and MLD-11-25 groups. Murphy et al. (2007) stated that "a global deficit alone does not account for the differences between the two groups" (p. 475). Second, "the differences between the two MLD groups do not appear to arise as a function of consistent deficits in a single specific math related skill measured in this study" (Murphy et al., 2007, p. 475). Finally, Murphy et al. (2007) did find that it was possible to distinguish between differences in improvement of math related skills over time. "The growth trajectories of the two MLD groups appear to diverge by third grade, despite evidence of continued growth in math skills through third grade" (Murphy, et al., 2007b, p. 475).

Developmental Math

The practice of developmental math instruction at postsecondary institutions must move beyond remediation and effectively develop the student's ability to understand the concepts and procedures of mathematics (Kinney, 2001). Kinney states that "pedagogy in developmental mathematics must be informed by theory and research that specifically addresses the learning process" (p. 10).

The majority of students in developmental math programs are not successful in remediating skills deficiencies. However, those that experience successful remediation have overall outcomes that mirror students not needing developmental intervention (Bahr, 2008). Lesik (2007) confirmed that "the risk of leaving college among students who participate in developmental mathematics programs was significantly lower than for equivalent students who did not participate in such programs" (p.583).

Initially, developmental math programs sought to provide the necessary bridge between remediation, review, and reengagement with math so that students could complete the required mathematics components of their major. Recently, the focus has shifted to the impact of developmental math in helping students apply mathematical reasoning and problem solving processes to other academic domains (Johnson & Kuennen, 2004). Johnson and Kuennen (2004) found that students who had taken the prerequisite developmental math courses did significantly better in a college microeconomics class than those who were currently taking developmental math, and those who were concurrently taking developmental math did significantly better than those who had delayed taking the required prerequisite.

One of the pivotal requirements of successful matriculation for students in postsecondary education is the proper placement of the student in classes in which they have an opportunity to succeed. Proper placement is especially important for those students who require developmental instruction (Jacobson, 2006). Jacobson points out that placement standards alone, do not guarantee that a student will enroll. However, the use of placement exams increased the overall probability that a student would enroll in a developmental math course.

Donovan and Wheland (2008) point to several factors that emphasize the need for proper placement in developmental courses: (a) the United States continues to lag in mathematics literacy among developed nations, (b) there is a high correlation between failure to complete post secondary education and preparedness, (c) the cost of tuition is escalating, and (d) There is an increase in the post secondary student population requiring developmental education, and this is especially true for developmental mathematics.

The normal procedure for developmental placement is performance on one cognitive exam. However, there are numerous factors that lead to successful completion of a developmental course or courses. The process could be enhanced if other factors were taken into consideration (Boylan, 2009).

The postsecondary institution where this study was conducted employs three cognitive test thresholds for placement in the developmental math program. The students enrolled in the developmental math course meet the following criteria:

- A score of less than 450 on the math component of the SAT Reasoning Test
- A score of less than 15 on the math component of the ACT
- A score of less than 23 on the Math Assessment Test Part One (*Faculty/Adjunct Handbook*, 2009)

The university administers the Math Assessment Test to place students at various levels in the developmental math program.

Standardized test cutoff score criteria has a relationship to mathematics disabilities (MD), and the development of possible screening measures for MD. However, standardized testing should be viewed as one component of an overall assessment strategy (Geary, 2005). In order to develop a more comprehensive understanding of the challenges faced by developmental math students we should broaden our understanding of those specific challenges and the potential interventions associated with specific developmental deficits. This would require looking at other measures of cognitive challenges. One area that is of increasing interest is working memory. Holmes, Adams and Hamilton (2008) stated that:

> Unlike other performance indicators, such as measures of IQ, working memory assessments are independent of knowledge acquired through school and home. They measure different underlying constructs from other indicators of performance, and are relatively independent of background factors such as preschool education and socioeconomic factors. (p. 273)

Working Memory

Cognitive development in children involves multiple cognitive processes such as procedural knowledge, concept acquisition, and working memory. One way of looking at mathematical learning disabilities (MLD) is to understand it in terms of the impact that a deficit in one process has on other related processes (Mabbott & Bisanz, 2008).

One area of process research that has been the focus of leading researchers in the field of MLD is the connection between MLD and the construct of working memory (Andersson, 2007; Berch, 2008; Geary, 1993, 2005; Geary, et al., 2000; Geary, et al., 2007; Gersten, et al., 2007; Jordan, Kaplan, Locuniak, & Ramineni, 2007; Mazzocco & Kover, 2007). "Working memory is the ability to hold a mental representation of information in mind while simultaneously engaging in other mental processes" (Geary, et al., 2008, p. 279).

The reason that there is a growing interest in the functions of working memory, and its affect on MLD, is that recent research has shown that both the central executive and each of the two subcomponents of working memory play an important role in the development of mathematical knowledge (Andersson, 2007; Andersson & Lyxell, 2007; D'Amico & Guarnera, 2005; Geary, et al., 2008; Holmes, et al., 2008; Mabbott & Bisanz, 2008; Rasmussen & Bisanz, 2005; Schuchardt, et al., 2008). Rasmussen and Bisanz (2005) stated that, "working memory is implicated in academic performances, including reading comprehension and mathematics in both children and adults" (p. 139).

Working memory should be distinguished from short-term memory. The distinguishing factor between working memory and short term memory is the ability to process the information being stored and to move that information to and from long term memory. Short term memory is a brief storage system that decays rapidly and has no processing component (Andersson, 2007).

Andersson (2007) found that developmental growth in typically achieving children brought an increase in the capacity and function of working memory. Mainly in the ability to process and store verbal information concurrently and the ability to shift information to and from long term memory. The ability to easily retrieve information from long term memory has implications for the ability to calculate using known math facts while the concurrent processing of verbal information increases accuracy on word problems (Andersson, 2007). Andersson and Lyxell (2007) found that children with mathematics disabilities (MD) faced challenges in working memory related to concurrent processing and the storage of information.

Not only do math facts need to be effectively retrieved from long-term memory, students must be confident that these facts are correct. Geary, Hamson, & Hoard (2000) stated that "the use of retrieval-based processes is moderated by a confidence criterion that represents an internal standard against which the child gauges confidence in the correctness of the retrieved answer" (p. 239).

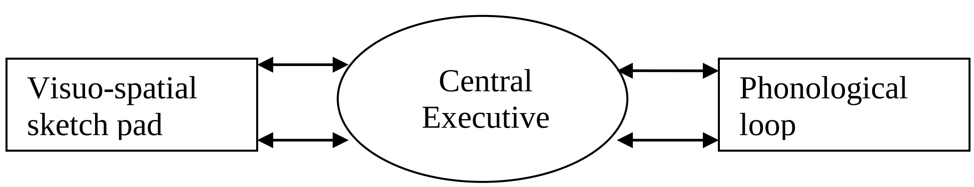
Gersten, Jordan & Flojo (2005) describe the importance of memory retrieval by stating that "failure to instantly retrieve a basic combination, such as 8 + 7, often makes discussions of the mathematical concepts involved in algebraic equations more challenging" (p. 294). Gersten, Jordan & Flojo point to the fact that "the ability to store this information in memory and easily retrieve it helps students build both procedural and conceptual knowledge of abstract mathematical principles, such as commutativity and the associative law" (p. 295). Finally Miller & Hudson stated that "the ability to memorize mathematical information and quickly retrieve the information helps students as they progress through the hierarchical mathematics curriculum (i.e., sequence of skills that become increasingly complex; each skill builds on previous skill)" (p. 53).

"Working memory capacity (that is, the capacity to hold various pieces of information simultaneously and to use them for further processing) is a critical feature of several models of human cognition, and it is widely recognized that it affects performance on many tasks" (Morra, 2008, p. 3) . Case (1987) stated that "the variable that determines the maximum rate at which within-stage progress can take place is the size of the child's working memory, which is seen as growing in response to both maturational and experiential variables" (p. 572).

The Development of the Baddeley Model of Working Memory

In the early 1970s Allen Baddeley and Graham Hitch began a three year research project to investigate the relationship between short-term and long-term memory. Using students as participants they discovered that a unitary model of working memory was insufficient to describe the processing of short-term memory tasks. This led to the three part model of working memory as shown in Figure 1 (Baddeley, 2006; Baddeley & Hitch, 1974).

Figure 2. Baddeley Three Component Model of Working Memory



The three component model developed by Baddeley (Baddeley, 1986; Baddeley & Hitch, 1974), is composed of:

> The central executive a domain-general limited capacity system responsible for inhibition, planning, switching attention, and monitoring the processing of temporarily stored information, controls two slave systems: the phonological loop and the visuospatial sketchpad. These systems, both domain specific and limited in capacity, are responsible for the temporary maintenance and manipulation of verbal and visuospatial information respectively. (Holmes, et al., 2008, pp. 272-273)

Rasmussen and Bisanz (2005) state that “numerous brain imaging and neuropsychological studies have supported the three-component model of working memory proposed by Baddeley and colleagues” (p. 138), and that “working memory is implicated in academic performances, including reading comprehension and mathematics in both children and adults” (p. 139). The three component model is the primary theory used by MLD researchers today (Berch, 2008; Geary, et al., 2008; Holmes, et al., 2008; Rasmussen & Bisanz, 2005; Schuchardt, et al., 2008).

The Central Executive

Current research on tasks associated with the function of the central executive have shown that it operates in three major domains; concurrent processing, inhibition control, and shifting to and from the phonological loop and visuo-spatial sketchpad. These functions have been shown to account for performance differences on written computation and problem solving. One of the key elements in the central executive’s tasks is to bring relevant information from long term memory while simultaneously inhibiting irrelevant information (Andersson, 2007).

The central executive is significantly involved in mathematical tasks in both children and adults. The central executive does not contain memory storage capabilities, storage is provided by the phonological loop and the visuo-spatial sketchpad. Specific deficits in children with mathematics disabilities have been linked to the processing of numerical and visuo-spatial information by the central executive (Andersson & Lyxell, 2007).

Geary et al. (2008) stated that higher scores on central executive task measures were related to the use and formation of a linear representation of the mental number line. Geary et al. also pointed the inhibitory control aspect of the central executive. While intelligence quotient (IQ) was related to number line development in second grade, longitudinally, number line performance was related to central executive function. Geary et al. hypothesized that the inhibitory functions of the central executive contributed to the suppression, or lack of suppression, of the natural number-magnitude representation in contrast to the use of the learned linear representation.

The Visuo-spatial Sketchpad and Phonological Loop

The two subcomponents of working memory, the phonological loop and visuo-spatial sketch pad, provide domain specific storage capacities. The phonological loop is comprised of a memory store and repetitive vocalization process. The visuo-spatial sketchpad also contains a memory store for visual information along with an ability to dynamically represent visual information (Schuchardt, et al., 2008)

D'Amico & Guarnera (2005) found that the deficits faced in children who perform poorly on math related tasks stemmed from deficits in maitaining and manipulating numerical information in working memory. Specifcally they found deficits in the visuospatial sketch pad related to numerical magnitude and ordinal arrangement of numbers. This was of particular intrest to the present study.

Not surprisingly, Simmons and Singleton (2008) reported that studies focused on the phonological loop found significant relationships to deficits in the phonological loop and the solution of word problems. However, the studies also found that the process of addition was hindred as well as the successful storage of numerical facts.

Geary et al. (2007) found that the phonological loop and visuo-spatial sketchpads contributed to specific deficits while the central executive contributed to an overall deficit. Overall the deficits centered on numerical processing, number line estimations, and the retrieval of addition facts.

The Impact of Working Memory on Mathematical Learning

Research in MLD has shown that children with moderate to severe mathematics learning difficulties have trouble completing tasks related to the functioning of working memory. Working memory deficits are found in the majority of children who learning difficulties. Working memory deficits are especially prevalent in mathematics and reading disabilities. Working memory deficits are distinct from other functional measures, such as IQ in that they are not influenced by previous knowledge (Holmes et al. 2008). The working memory deficit affects students on three fronts: (a) the ability to inhibit distractions and inefficient strategies in the central executive, (b) the ability to hold visual representations of concepts in the visuospatial sketchpad, and (c) the ability to integrate verbal information concurrently with operations in both the central executive and the visuospatial sketchpad (Andersson & Lyxell, 2007; Booth & Siegler, 2008;

D'Amico & Guarnera, 2005; Geary, et al., 2008; Holmes, et al., 2008; Mabbott & Bisanz, 2008; Rasmussen & Bisanz, 2005; Schuchardt, et al., 2008).

Number Sense

When deficits are present in working memory the development of number sense is affected. Number sense is a holistic construct of mathematical relationships and Number sense is directly linked to higher order thinking skills in solving mathematical problems (Gersten et al., 2005).

While there is a broad range of operational definitions, number sense can generally be described as:

> (a) fluency in estimating and judging magnitude, (b) ability to recognize unreasonable results, (c) flexibility when mentally computing, (d) ability to move among different representations and to use the most appropriate representation. (Kalchman, et al., 2001, p. 2)

Dehaene (2001) stated hypothesis is “that number sense qualifies as a biologically determined category of knowledge. I propose that the foundations of arithmetic lie in our ability to mentally represent and manipulate numerosities on a mental ‘number line’ an analogical representation of number” (p. 17).

One of the components of number sense is the formation of a mental number line and its relationship to accurately judging the differences in number magnitude (Booth & Siegler, 2008; D'Amico & Guarnera, 2005; Geary, et al., 2007; Geary, et al., 2008; Gersten, et al., 2005; Laski & Siegler, 2007). “Children’s understanding of numerical magnitudes is closely related to their general math achievement, estimation skills, and arithmetic proficiency” (Laski & Siegler, 2007, p. 1723).

The Mental Number Line and the Importance of Magnitude

Children possess a mental number line that is logarithmic in scale. This means that the difference between two and three appears to be greater than the difference between 89 and 90. Upon entering school children, begin to make a transformation from the mental logarithmic model to the linear model where magnitude is equal across all numbers (Booth & Siegler, 2008; Geary, et al., 2007; Geary, et al., 2008; Laski & Siegler, 2007). The importance of developing the linear representation of the number line, and thus an accurate concept of magnitude, cannot be overstated. Booth and Siegler (2008) stated that "representations of numerical magnitude are both correlationally and causally related to arithmetic learning" (p. 1016), and that "numerical magnitude representations are not only positively related to a variety of types of numerical knowledge but also predictive of success in acquiring new numerical information, in particular, answers to arithmetic problems" (p. 1027). The mental representation of the number line is intricately involved in mathematical domains such as geometry and algebra. Failure to accurately develop a correct mental representation of the number line has consequences through secondary education and beyond (Geary et al. 2008). Researchers have found that children with MLD have a difficult time in making the transition from using the mental logarithmic number line to using the learned linear number line.

Several studies have been conducted that sought to investigate the presence and structure of mental number lines (Booth & Siegler, 2006, 2008; Geary, et al., 2008; Laski & Siegler, 2007; Siegler & Booth, 2004). The underlying view of these studies is that children who do not transition from a logarithmic mental representation of the number line to a linear mental representation of the number line continue to have difficulties with number sense, specifically in magnitude estimations, this leads to challenges in several domains of mathematics.

What is the importance of number line estimation in relationship to mathematics learning disabilities (MLD)? Testing for MLD has largely settled on two assessment measures (a) the IQ-discrepancy measure, and (b) standardized mathematics test score cutoff criteria. However, both these methodologies are inadequate in themselves to diagnose the presence of MLD (Geary, 2005; Murphy, et al., 2007b). Siegler and Booth (2004) found significant correlations between percentage error on number line estimations and performance on the mathematics section of the Stanford Achievement Test Series, SAT-9. Given the relationship between number line estimation and performance on achievement tests, the use of number line estimation could serve as a component of detecting the presence of mathematics learning disabilities (MLD).

Recent findings seem to indicate that typically achieving (TA) children move from using the logarithmic mental number line to the learned linear number line by at least the fourth grade (Geary, et al., 2008; Laski & Siegler, 2007). Researchers believe that the logarithmic number line is held in the visuospatial sketchpad automatically until the central executive reaches a point of maturity where it can override this naturally occurring number line with the linear learned number line. Geary et al. (2008) stated that:

> The visuospatial sketchpad is of interest, because the parietal areas associated with number and magnitude processing are situated near brain regions that support aspects of visuospatial processing and because damage to these parietal

regions disrupts the ability to form spatial representations and to imagine a mental number line. (p.280)

This relates to the ability of the central executive to inhibit distracting or irrelevant information from affecting the operations of the phonological loop and the visuospatial sketchpad (Andersson & Lyxell, 2007; Geary, et al., 2008; Laski & Siegler, 2007; Schuchardt, et al., 2008). Children with MLD take longer to make this transition and many enter third grade still using the logarithmic line (Geary, et al., 2008).

Case understood that the development of number sense involved the creation of a mental number line. Around six years of age children begin to understand positional relationships on the number line and use that to make judgments concerning overall quantity. Children also use the number line to reference the increase or decrease of quantity. By mentally mapping number words with their location on the mental number line, children develop the concept of cardinality (Morra, 2008).

Currently the studies of number line estimation (Booth & Siegler, 2006, 2008; Geary, et al., 2008; Laski & Siegler, 2007; Siegler & Booth, 2004) focused on early childhood education. What has not occurred is the application of this methodology to postsecondary students who display correlates of MLD. Specifically, students involved with remediation in developmental math programs. Postsecondary remediation provides the opportunity to resolve instructional inequalities present in primary and secondary education. Postsecondary remediation also provides functional competency in economic and political settings while preparing the student for successful negotiation of college coursework Bahr (2008), Being able to retrieve a linear model of the number line and its accurate magnitude representations facilitates learning by limiting erroneous estimations and increasing the probability of a correct answer (Augustyniak, Murphy, & Phillips, 2005). Researchers have found that students with MLD have a difficult time in making the transition from using the mental logarithmic number line to using the learned linear number line (Booth & Siegler, 2008; Geary, et al., 2008; Siegler & Booth, 2004; Siegler & Opfer, 2003).

Summary

Mathematics learning disability affects five to eight percent of the student population, yet students who encounter difficulty in learning math may comprise a larger percentage of the student population (Mazzocco, 2005). The very nature of math and its broad conceptual base challenge researchers seeking to identify MLD's combined deficits Geary (2004). Currently there is no specific diagnostic system for identifying MLD. Testing for MLD has largely settled on two assessment measures (a) the IQ-discrepancy measure, and (b) standardized mathematics test score cutoff criteria. However, both these methodologies are inadequate in themselves to diagnose the presence of MLD (Geary, 2005; Murphy, et al., 2007b).

While there have been significant steps taken in the research of learning disabilities (LD) related to reading disabilities (RD), research in mathematics disabilities (MD) is still developing (Geary, 1993; Geary, Hamson, & Hoard, 2000; Gersten, Jordan, & Flojo, 2005; Mazzocco, & Myers, 2003). There is a broad diversity among specific math deficits. The ability to provide individualized instruction is not readily available in mainstream classrooms (Wadlington & Wadlington, 2008). Mazzocco & Thompson

(2005) stress that “It is important to identify risk for MLD, because—like poor reading achievement—poor math achievement is a risk factor for negative outcomes in both childhood and adulthood” (p. 142). With all that is involved, and at stake, it is important for educators to understand mathematics disability. Mathematics disability needs to be defined, current research understood, screening criteria established, and effective instructional interventions applied.

Most children can learn the core concepts of math when given a robust learning environment and effective instruction. However, there are children who struggle with the core concepts even when the environment and instruction are focused on effective learning

Within the last two decades, there has been an increasing focus on the relationship between MLD and research in cognition, specifically in the neurological aspects of cognition. This focus has largely settled around the construct of working memory (Andersson, 2007; Andersson & Lyxell, 2007; Berch, 2008; Bull, Espy, & Wiebe, 2008; Geary, 1993; Geary, et al., 2007; Geary, et al., 2008; Mabbott & Bisanz, 2008; Mazzocco & Kover, 2007; Murphy, et al., 2007b; Rasmussen & Bisanz, 2005; Schuchardt, et al., 2008; Swanson & Beebe-Frankenberger, 2004; van Garderen, 2006). The preeminent theoretical construct of working memory used by researchers in the field of MLD is the three-component model developed by Baddeley and his colleagues (Baddeley, 1986; Baddeley & Hitch, 1974). Recently Repovs and Baddeley (2006) have refined the three-component model to include a fourth component called the episodic buffer. This component handles the facilitation of transfer of information between the central executive, the phonological loop, and the visuospatial sketchpad. To date there has not been a body of research in MLD that incorporates the episodic buffer.

One area of research, the relationship between number magnitude and working memory, has caught the attention of David C. Geary (Geary, et al., 2007; Geary, et al., 2008). Geary is a seminal researcher in the connection between MLD and working memory (Geary, 1993). This has led to further exploration of the relationship between numerical magnitude representations and MLD by other researchers in the field (Booth & Siegler, 2008; D'Amico & Guarnera, 2005; Holmes, et al., 2008; Laski & Siegler, 2007).

Interventions for numerical magnitude deficits rely on accurate representations of the linear number line. These representations establish the linear model in long-term memory. As the central executive subsystem of working memory matures, the naturally occurring logarithmic representation of magnitude present in the visuospatial sketchpad is replaced by the more accurate learned linear representation (Augustyniak, et al., 2005; Booth & Siegler, 2008; Geary, et al., 2007; Geary, et al., 2008; Laski & Siegler, 2007).

McGlaughlin, Knoop, and Holliday (2005) and Sullivan (2005) stated that research based interventions for teaching postsecondary students with mathematics learning disabilities (MLD) were lacking. However, methodologies proven effective in early childhood education can prove effective in teaching postsecondary students with MLD. This presents the possibility that detection of MLD in postsecondary students could lead to appropriate instructional interventions. Most developmental math students are placed in the program based on a cutoff criterion on standardized and/or admissions tests. However, the criteria vary widely in both four year and two year institutions (Bahr, 2008; Hadden, 2000)

CHAPTER THREE: METHODOLOGY

The purpose of this study was to investigate the possible existence of math learning disabilities in postsecondary students enrolled in the developmental math program at a large university in the Mid-Atlantic region. The particular problem addressed by this study was the presence of a logarithmic representation of the mental number line in students participating in the developmental math program. Researchers have found that students with MLD have a difficult time in making the transition from using the mental logarithmic number line to using the learned linear number line (Booth & Siegler, 2008; Geary, et al., 2008; Siegler & Booth, 2004; Siegler & Opfer, 2003).

The implications of a math disability have profound consequences throughout an individual's life span. Large-scale studies estimated that five to ten percent of students would face a mathematics deficit (Geary, et al., 2008). The majority of research conducted on math learning disabilities (MLD) has occurred in the primary grades. It has not been evident that math disabilities persist beyond secondary education. However, McGlaughlin, Knoop, and Holliday (2005) discovered that deficits in post secondary students mirrored those of elementary and secondary levels. This study's purpose is to examine the potential presence or absence of MLD in postsecondary students enrolled in a developmental math program.

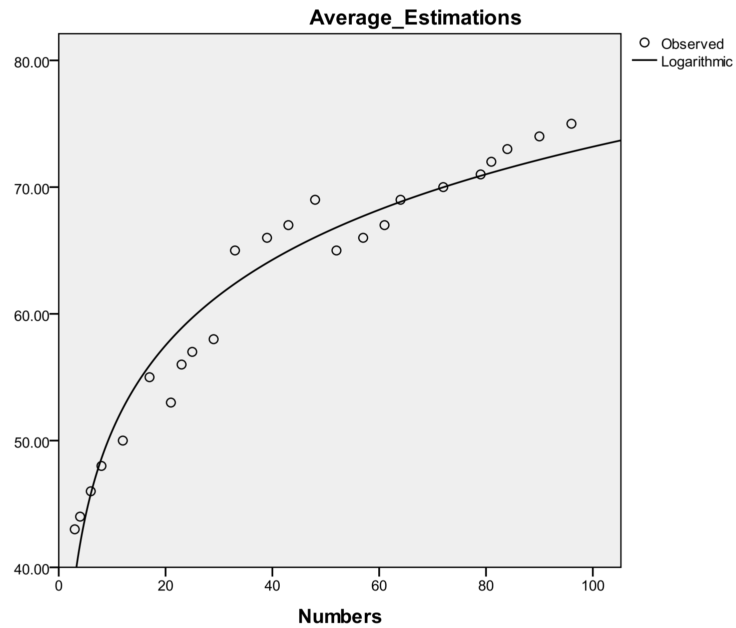
Overview of the Study

This study developed out of a review of research focusing on number magnitude and particularly the representation of magnitude in a construct called the mental number line. Magnitude estimation relates to general math skills, the ability to estimate, and arithmetic competence (Laski & Siegler, 2007). The number line is an essential part of counting and coordinate systems in algebra and geometry. The development of a correct mental image of the number line affects mathematical learning throughout the range of schooling (Geary, et al., 2008).

In a foundational study Siegler and Opfer (2003) found that children and adults possess a mental image of the number line. The number line varies from a logarithmic model to a linear representation. In 2004 Siegler and Booth replicated the experiment of Siegler and Opfer (2003) and found that, given normal cognitive development, the mental number line progresses from a logarithmic to a linear representation during the transition from kindergarten to second grade (Siegler & Booth, 2004). Geary et al. (2008) also duplicated the methodology used by Siegler and Opfer (2003) and Siegler and Booth (2004). In a longitudinal study over first and second grade, Geary et al. found that a logarithmic representation of the mental number line was employed by MLD children more often than their low achieving (LA) and typically achieving (TA) peers. Geary et al. recognized the potential of logarithmic representation of the mental number line to indicate the presence of math disability and/or difficulties.

The problem addressed in this study sought to discover if post secondary students in a developmental math introductory algebra class have a logarithmic mental representation of the number line (See Figure 3).

Figure 3. Example of Logarithmic Line



The presence of a logarithmic line would be a preliminary indicator of math disability and could lead to further study of other indicators and appropriate interventions. The null hypothesis was that the representation of the mental number line of developmental math students approximates a linear line.

Design of the Study

This was a quasi-experimental study. “Quasi-experimental designs lack randomization but employ other strategies to provide some control over extraneous variables. They are used, for instance, when intact classrooms are used as the experimental and control groups” (Ary, Jacobs, & Sorensen, 2010, p. 326).

The study employed a design used by Booth and Siegler (2008), Geary et al. (2008), Siegler and Booth (2004), and Siegler and Opfer (2003). The design centers on the use of estimation skills when marking the location of a given number on a blank number line bounded by 0 on one end and 100 on the other end. The students were given a number between 1 and 100 and are asked to place a mark where they believed the number should go. The results of the estimations were used to determine whether a logarithmic or linear line provides the best fit for the estimations. Also of interest were results based upon gender, academic level, and prior enrollment in developmental math. Comparisons were made for number line representations and absolute estimation error percentages.

Research Questions and Null Hypotheses

Based upon research indicating that the development of a linear representation of the mental number line is crucial to the acquisition of many mathematical skills, this study looked at the mental representation of the number line in developmental math students. In addition, linear representations were examined based upon gender, academic level, and prior enrollment in developmental math. Finally, the mean absolute estimation error percentages ($M_{ABE\%}$) were compared based upon gender, academic level, and prior enrollment in developmental math.

> Null Hypothesis 1: The representation of the mental number line of developmental math students approximates a linear line as determined by analyzing the statistical significance of the model and the R^2 correlational statistic.
>
> Null Hypothesis 2: The representation of the mental number line of developmental math students approximates a linear line as determined by analyzing the statistical significance of the model and the R^2 correlational statistic based upon gender.

Null Hypothesis 3: The representation of the mental number line of developmental math students approximates a linear line as determined by analyzing the statistical significance of the model and the R^2 correlational statistic based upon academic level.

Null Hypothesis 4: The representation of the mental number line of developmental math students approximates a linear line as determined by analyzing the statistical significance of the model and the R^2 correlational statistic based upon prior enrollment in developmental math.

Null Hypothesis 5: The mean absolute error percentage will not be statistically different by gender.

Null Hypothesis 6: The mean absolute error percentage will not be statistically different by academic level.

Null Hypothesis 7: The mean absolute error percentage will not be statistically different by prior enrollment in developmental math.

Data Gathering Methods

Data was gathered from the developmental math classes and the liberal arts math classes during the fall semester. The fall semester was chosen to minimize the number of students who may be taking the developmental class again. This situation can occur if the students do not achieve a grade of C or higher in the developmental class.

Students were asked to provide the following demographic data for the study and to indicate consent (see Appendix A): (a) female/male, (b) academic level (freshman, sophomore, junior, senior), (c) repeating course (yes/no). In compliance with Institutional Review Board approval, completion of the demographic questions and participation in the study implied consent by the participants. Participation was voluntary and the study was explained to the participants prior to participation.

The researcher administered the instrument to students in residential classes. The collection occurred within the first two weeks of the semester and was conducted at the beginning of each class session. Students who chose to participate received a brief description of the study and instructions on how to complete the instrument.

Instrumentation

The instrument that was used was first developed by Siegler and Opfer (2003). Booth and Siegler (2008) used a variation of the methodology, and Siegler and Booth (2004) employed the same instrument and methodology in their study. Geary et al. (2008) employed a slightly modified version in testing first graders and transferred the instrument to a computer-based application used to measure the responses of second graders.

The instrument consisted of 48 sheets of paper, two sets of 24, each with a 23-centimeter line printed across the middle of the page, with 0 at the left end and 100 at the right end. A number between 0 and 100 was printed at the top of each page (see Appendix C). Participants are required to mark an estimated location of the numerical value on the number line. In keeping with the methodology of Siegler and Booth (2004), the following procedure was followed:

> To improve our ability to discriminate between linear and logarithmic estimation patterns, numbers below 30 were oversampled, with 10 numbers between 0 and 30 and 14 numbers between 30 and 100. The 24 numbers presented were 3, 4, 6, 8, 12, 17, 21, 23, 25, 29, 33, 39, 43, 48, 52, 57, 61, 64, 72, 79, 81, 84, 90, and 96.

> Within each set of 24 number line sheets, the pages were ordered randomly. (p. 432)

The pages were randomly ordered and bound at the top. A cover page was included to collect the demographic data (see Appendix B). Students used a pen provided by the researcher to indicate their responses.

Siegler and Booth (2004) found that the accuracy of this instrument was comparable to that used by Siegler and Opfer (2003). Booth and Siegler (2008) employed computer generated representations of the instrument. Booth and Siegler also employed a technique that increased the frequency of numbers below 30 and stated "we slightly oversampled the numbers at the low end of the 0 – 100 range by including four numbers from each decade below 30 and two numbers from each successive decade" (p. 1020). Booth and Siegler used three sets of randomly generated numbers. Booth and Siegler found that their study both replicated and extended the findings of previous studies that employed this instrumentation.

Geary et al. (2008) was one of the first longitudinal studies that assessed the linearity of the mental number line, and its corresponding effects on the development of mathematical ability, over first and second grade. Geary et al. replicated the study of Siegler and Booth (2004) but modified the delivery method to computer based during the second grade. Geary et al. stated, "For all three of the groups assessed in the current study, number line performance was consistent with theoretical predictions and with previous empirical studies" (p. 293).

Given its ability to be replicated over time and contexts, the instrument presented itself as a reliable tool for measuring the mental number line representation in early elementary students. Given that the construct in question is a number line, the use of number lines to estimate magnitude is valid. This was believed to be the first study to replicate Booth and Opfer's (2003) instrument in a post secondary population.

Population and Sampling Procedures

The students enrolled in the developmental math course at the large university in the Mid-Atlantic region met the following criteria:

- A score of less than 450 on the math component of the SAT Reasoning Test
- A score of less than 15 on the math component of the ACT
- A score of less than 23 on the Math Assessment Test Part One (*Faculty/Adjunct Handbook*, 2009)

The university administers the Math Assessment Test to place students at various levels in the developmental math program.

In the fall of 2010 there were 588 students in 25 classes of the introductory course for developmental math. The average developmental math class size was 24. Prior to the beginning of the fall semester instructors in the developmental math introductory course were queried on their willingness to participate. Three developmental math instructors expressed a willingness to conduct the experiment in their classrooms. Seven sections of developmental math classes were used in the experiment. There were a total of 136 participants, out of which came 123 valid instruments. Validation was based upon completion of the survey and the absence of non-standard markings. There were two participants who chose to mark 50 on all pages of the instrument and were not included in the valid instruments. There were eleven participants who chose not to complete the instrument.

Required sample size was computed based on an ANOVA fixed effects omnibus one-way hypothesis. For an medium effect size of .25, an α level of .05, and a power (1-β probability of error) of .95, the sample size should be at least 210 participants (Faul, 2007). By sampling each number twice, in accordance with Siegler and Booth (2004), the total N value for the instrument was 246, exceeding the recommended sample size. There were 123 total participants with 73 female participants and 50 male participants as shown in Table 1.

Table 1

Participants by Gender

	Frequency	Percent
Female	73	59.3
Male	50	40.7
Total	123	100.0

There were 72 freshman, 37 sophomores, 13 juniors, and one senior participating in the study as shown in Table 2.

Table 2

Participants by Academic Level

	Frequency	Percent
Freshmen	72	58.5
Sophomore	37	30.1
Junior	13	10.6
Senior	1	0.8
Total	123	100.0

Twenty-eight participants had prior participation in developmental math and 95 participants had no prior participation in developmental math as shown in Table 3.

Table 3

Participants by Prior Participation in Developmental Math

	Frequency	Percent
Yes	28	22.8
No	95	77.2
Total	123	100.0

Data Analysis Procedures

Two analyses were used to detect the presence of difficulty in number line estimation, absolute error percentages and curve estimation. Both of these measures have been shown to indicate the presence of challenges in working memory (Geary, et al., 2008; Siegler & Booth, 2004).

A measure of absolute error was calculated by subtracting the estimated quantity (the number at the top of the page) from the estimate and dividing the result by one hundred. The result was an absolute error percentage (see Figure 4).

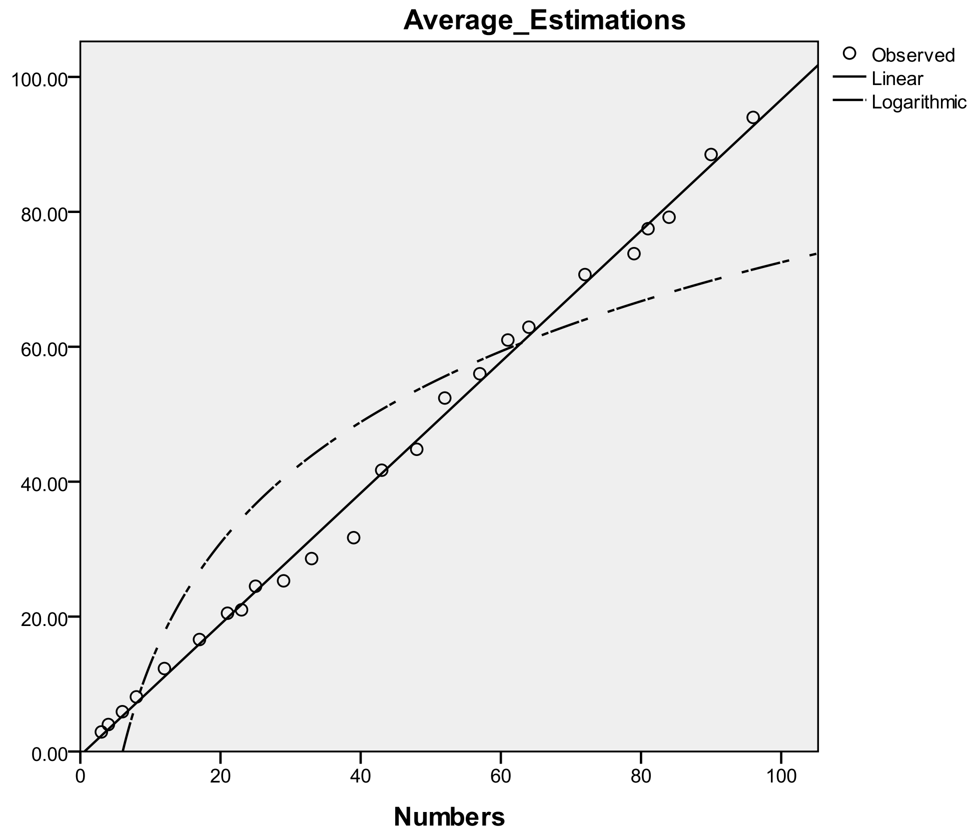
Figure 4. Equation for Computation of ABE%

$$ABE\% = \left| \frac{Number - Estimation}{Scale} \right|$$

Curve estimation is determined by analyzing the statistical significance of the model and the R^2 correlational statistic. In discussing the use of the curve estimation function of PASW Graduate Pack 18 (SPSS, 2009) the program tutorial points out that a significance value of the F statistic below .05 means that the variation explained by the model is not due to chance. The PASW Graduate Pack 18 tutorial stated that “the R Square statistic is a better measure of the strength of relationship The R Square statistic is a measure of the strength of association between the observed and model-predicted values of the dependent variable” (¶ 4).   In describing the process of curve estimation (see Figure 5) PASW Graduate Pack 18 (SPSS, 2009) stated “view a scatter plot of your data; if the plot resembles a mathematical function you recognize, fit your data to that type of model. For example, if your data resemble an exponential function, use an exponential model” (PASW Tutorial, Curve Estimation Models, ¶ 1).

Siegler and Booth (2004) stated “relative to a linear representation of numbers, a logarithmic representation exaggerates the distance between the magnitudes of numbers at the low end of the range and minimizes the distance between magnitudes of numbers in the middle and upper ends of the range” (p.429).

Figure 5. Example of Linear and Logarithmic Lines



For Hypothesis 1: The representation of the mental number line of developmental math students approximates a linear line as determined by analyzing the statistical significance of the model and the R^2 correlational statistic. The analysis included an examination of curve fit using the curve estimation function of PASW Graduate Pack 18 (SPSS, 2009).

For Hypothesis 2: The representation of the mental number line of developmental math students approximates a linear line as determined by analyzing the statistical significance of the model and the R^2 correlational statistic based upon gender. The analysis included an examination of curve fit using the curve estimation function of PASW Graduate Pack 18 (SPSS, 2009).

For Hypothesis 3: The representation of the mental number line of developmental math students approximates a linear line as determined by analyzing the statistical significance of the model and the R^2 correlational statistic based upon academic level. The analysis included an examination of curve fit using the curve estimation function of PASW Graduate Pack 18 (SPSS, 2009).

For Hypothesis 4: The representation of the mental number line of developmental math students approximates a linear line as determined by analyzing the statistical significance of the model and the R^2 correlational statistic based upon prior enrollment in developmental math. The analysis included an examination of curve fit using the curve estimation function of PASW Graduate Pack 18 (SPSS, 2009).

For Hypothesis 5: The mean absolute error percentage will not be statistically different by gender. Analysis of variance (ANOVA) was conducted using PASW Graduate Pack 18 (SPSS, 2009).

For Hypothesis 6: The mean absolute error percentage will not be statistically different by academic level. Analysis of variance (ANOVA) was conducted using PASW Graduate Pack 18 (SPSS, 2009).

For Hypothesis 7: The mean absolute error percentage will not be statistically different by prior enrollment in developmental math. Analysis of variance (ANOVA) was conducted using PASW Graduate Pack 18 (SPSS, 2009).

CHAPTER FOUR: RESULTS/FINDINGS

The purpose of this study was to investigate the possible existence of math learning disabilities in postsecondary students enrolled in the developmental math program at a large university in the Mid-Atlantic region. The particular problem addressed by this study was the presence, or absence, of a logarithmic representation of the mental number line in students participating in the developmental math program. Additional investigations were done to determine if there were differences based upon gender, academic level, and prior enrollment in developmental math.

Testing the Hypotheses

This was a causal comparative and correlational study. The study employed a design used by Booth and Siegler (2008), Geary et al. (2008), Siegler and Booth (2004), and Siegler and Opfer (2003). The design centered on the use of estimation skills when marking the location of a given number on a blank number line bounded by 0 on one end and 100 on the other end. The students were given a number between 1 and 100 and asked to place a mark where they believed the number should go. The following null hypotheses were investigated in this study:

Hypothesis 1: The representation of the mental number line of developmental math students approximates a linear line as determined by analyzing the statistical significance of the model and the R^2 correlational statistic.

Hypothesis 2: The representation of the mental number line of developmental math students approximates a linear line as determined by analyzing the statistical significance of the model and the R^2 correlational statistic based upon gender.

Hypothesis 3: The representation of the mental number line of developmental math students approximates a linear line as determined by analyzing the statistical significance of the model and the R^2 correlational statistic based upon academic level.

For Hypothesis 4: The representation of the mental number line of developmental math students approximates a linear line as determined by analyzing the statistical significance of the model and the R^2 correlational statistic based upon prior enrollment in developmental math.

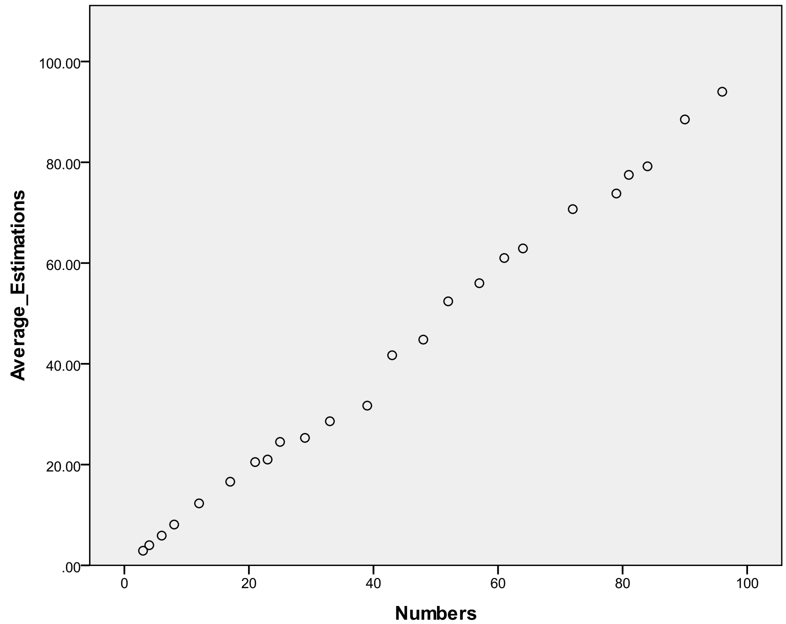
Hypothesis 5: The mean absolute error percentage will not be statistically different by gender.

Hypothesis 6: The mean absolute error percentage will not be statistically different by academic level.

Hypothesis 7: The mean absolute error percentage will not be statistically different by prior enrollment in developmental math.

For Hypotheses 1 – 4 a curve fit analysis was used to determine the strongest model. In describing the process of curve estimation PASW Graduate Pack 18 (SPSS, 2009) stated "view a scatter plot of your data; if the plot resembles a mathematical function you recognize, fit your data to that type of model. For example, if your data resemble an exponential function, use an exponential model" (PASW Tutorial, Curve Estimation Models, ¶ 1). This is demonstrated in Figure 6. The analysis also includes analyzing the statistical significance of the model and the R^2 correlational statistic (SPSS, 2009).

Figure 6. Scatter Plot of Average Overall Number Line Estimations



Hypothesis 1

For Hypothesis 1 the independent variable was the given number at the top of each page, the dependent variable was the numerical estimation value of the corresponding mark on the number line below. Results were screened for nonparticipation or based upon completion of the survey and the absence of non-standard markings.

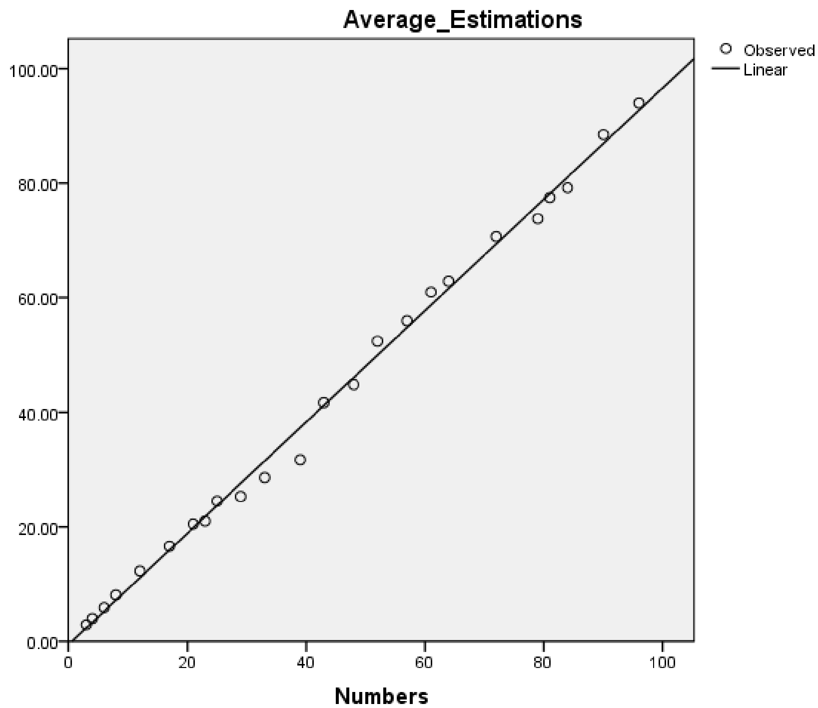
The significance value of the F statistic was below α = .05 (*p* < .001). This means that the variation explained by the model was not due to chance. The R Square statistic (R^2=.996, p < .001) is a measure of the strength of association between the observed and model-predicted values of the dependent variable as shown in Table 4. The best fit model was linear (see Figure 7).

Table 4

Equation fit for Overall Average Estimations for Each Number

	Model Summary					Parameter Estimates	
Equation	R Square	F	df1	df2	Sig.	Constant	b1
Linear	.996	5087.991	1	22	.000	-.580	.972

Figure 7. Linear Model Fit for Overall Average Estimations



When examining the representation of the mental number line of the participants in this study as determined by analyzing the statistical significance of the model, and based upon number line estimation, it was found that the representation was linear.

Therefore the null hypothesis that the representation of the mental number line of developmental math students approximates a linear line as determined by analyzing the statistical significance of the model and the R^2 correlational statistic was not rejected.

Hypothesis 2

For Hypothesis 2 the independent variable was the gender of the participants, the dependent variable was the numerical estimation value of the mark on the number line. When examining the representation of the mental number line as determined by analyzing the statistical significance of the model and the R^2 correlational statistic, based upon estimations by gender, it was found that the representation was linear.

The significance value of the F statistic was below $\alpha = .05$ ($p < .001$) for both female and male participants. This means that the variation explained by the model was not due to chance. The R Square statistic (R^2=.996, p < .001, female), (R^2=.995, p < .001, male) as shown in Table 5, is a measure of the strength of association between the observed and model-predicted values of the dependent variable. The best fit curve models were linear (see Figure 8 and Figure 9).

Table 5

Equation fit for Average Estimations for Each Number by Gender

	Model Summary					Parameter Estimates	
Equation	R Square	F	df1	df2	Sig.	Constant	b1
Linear							
Female	.996	5131.707	1	22	.000	.025	.958
Male	.995	4342.832	1	22	.000	-1.504	.993

Figure 8. Linear Model Fit for Female Participants

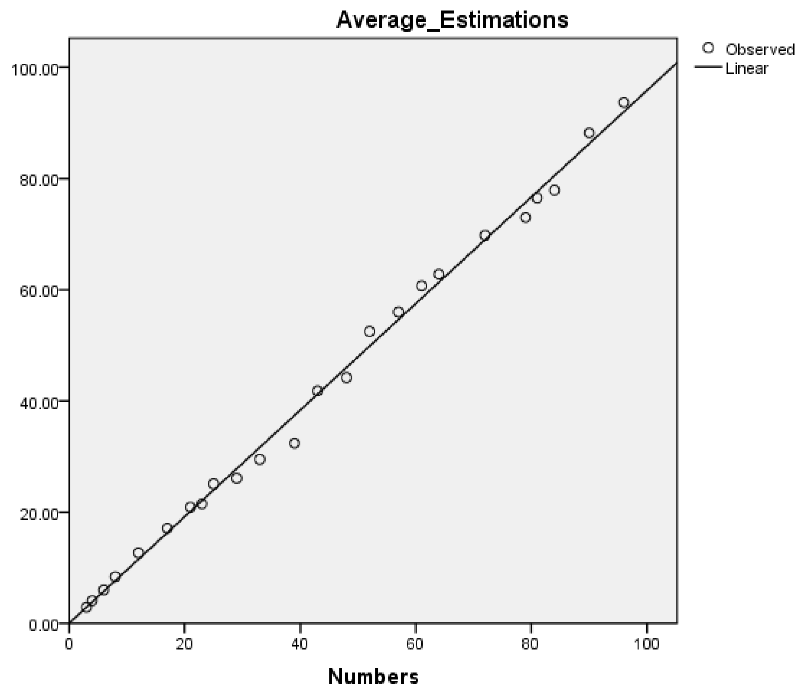
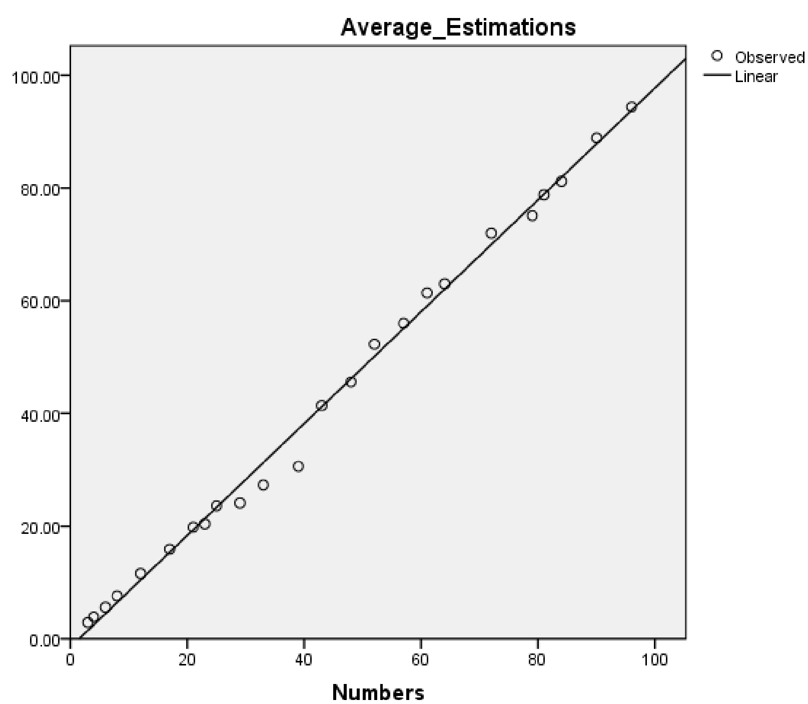


Figure 9. Linear Model Fit for Male Participants



Based upon the strength of the linear models, (R^2=.996, p < .001, female) and (R^2=.995, p < .001, male), visual inspection of the plots of the overall average estimations for the participants, and the corresponding linear equation curve, it was evident that a linear relationship existed. Therefore the null hypothesis that the representation of the mental number line of developmental math students approximates a linear line as determined by analyzing the statistical significance of the model and the R^2 correlational statistic based upon gender was not rejected.

Hypothesis 3

For Hypothesis 3 the independent variable was the academic level of the participants, the dependent variable was the numerical estimation value of the mark on the number line. When examining the representation of the mental number line as determined by analyzing the statistical significance of the model and the R^2 correlational statistic, based upon estimations by academic level, it was found that the representation was linear, with the exception of the single senior participant whose estimations was best represented by a cubic line.

The significance value of the F statistic was below $\alpha = .05$ ($p < .001$) for the freshman, sophomore, and junior academic levels of the participants. This means that the variation explained by the model was not due to chance. The R Square statistic (R^2= .996, p < .001,Freshmen), (R^2=.995, p < .001, Sophomore), (R^2= .993, p < .001, Junior) as shown in Table 6, is a measure of the strength of association between the observed and model-predicted values of the dependent variable. The best fit models were linear for the Freshmen, Sophomore, and Junior participants (see Figure 10, Figure 11, and Figure 12).

Table 6

Equation fit for Average Estimations for Each Number by Academic Level

	Model Summary					Parameter Estimates	
Equation	R Square	F	df1	df2	Sig.	Constant	b1
Linear							
Freshman	.996	5838.711	1	22	.000	- .439	.969
Sophomore	.995	4207.174	1	22	.000	- .646	.980
Junior	.993	3343.973	1	22	.000	- .982	.966

Figure 10. Linear Model Fit for Freshmen Participants

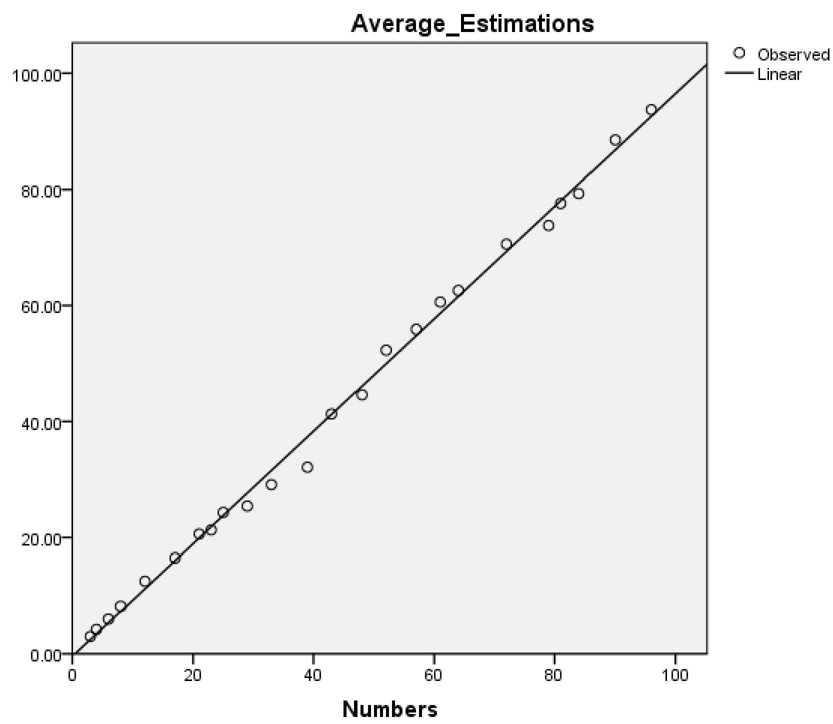


Figure 11. Linear Model Fit for Sophomore Participants

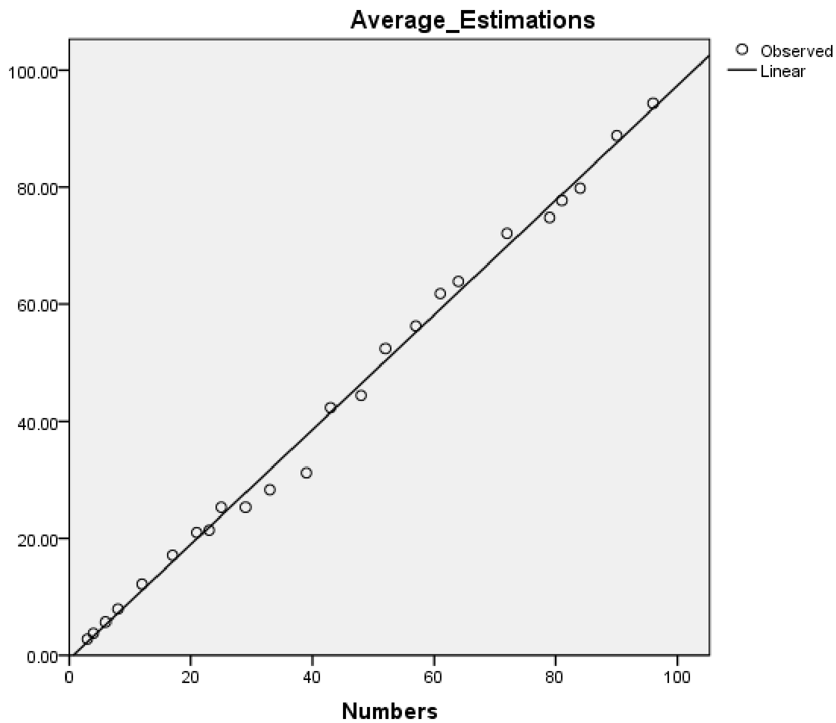
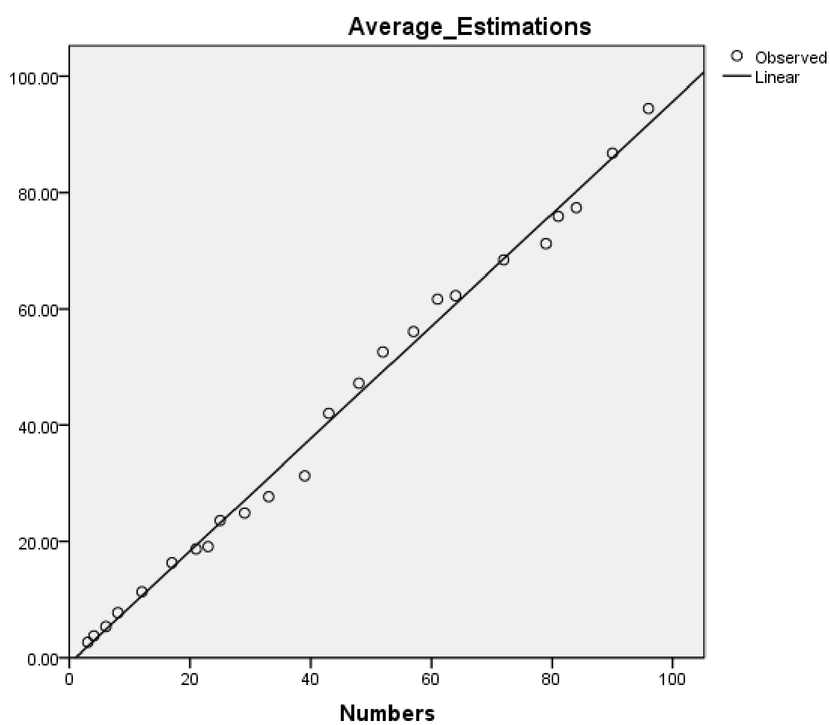


Figure 12. Linear Model Fit for Junior Participants



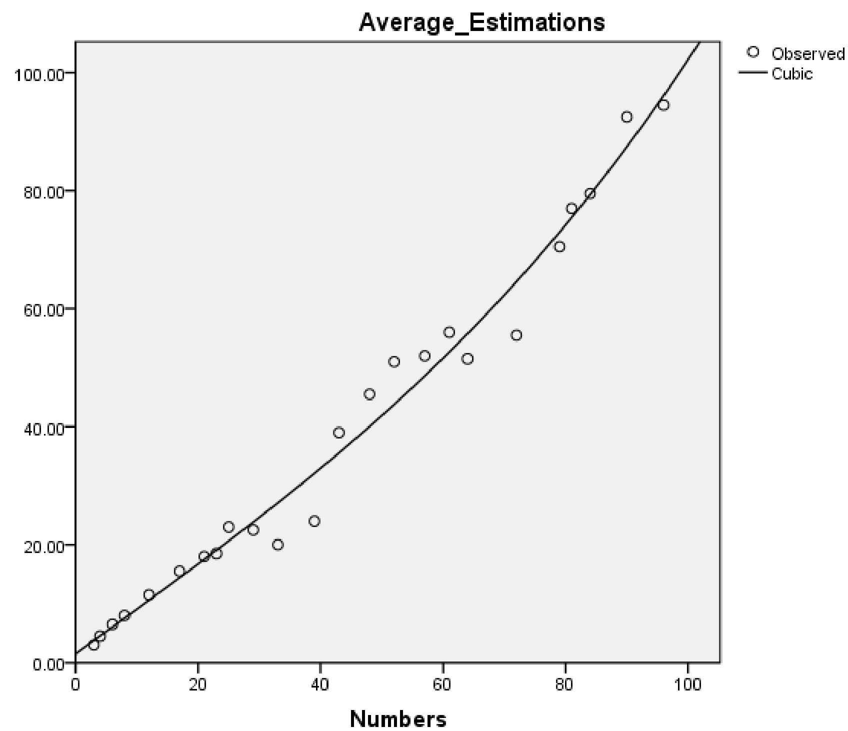
The senior academic level was represented by one participant. However, the strongest curve fit for this participant was cubic (R^2= .980, p < .001, Senior) as shown in Table 7. The best representation of the model was cubic as shown in Figure 13.

Table 7

Equation fit for Average Estimations for Senior Academic Level

	Model Summary					Parameter Estimates			
Equation	R Square	F	df1	df2	Sig.	Constant	b1	b2	b3
Cubic	.980	323.623	1	22	.000	1 .485	.958	-.001	3.094 E -5
Linear	.970	715.513	1	22	.000	-2.385	.993		

Figure 13. Cubic Model Fit for Senior Participant



Based upon the strength of the models, visual inspection of the plots of the overall average estimations for the participants, and the corresponding curves; it was evident that a linear relationship exists. The null hypothesis that the representation of the mental number line of developmental math students approximates a linear line as determined by analyzing the statistical significance of the model and the R^2 correlational statistic based upon academic level was not rejected.

Hypothesis 4

For Hypothesis 4 the independent variable was the presence, or absence, of prior participation in developmental math by the participants, the dependent variable was the numerical estimation value of the mark on the number line. When examining the representation of the mental number line as determined by analyzing the statistical significance of the model and the R^2 correlational statistic, based upon estimations by prior participation, it was found that the representation was linear

The significance value of the F statistic was below $\alpha = .05$ ($p < .001$) for the participants with no prior participation in developmental math, and the participants with prior participation in developmental math. This means that the variation explained by the model was not due to chance. The R Square statistic ($R^2 = .996$, $p < .001$, No Prior), ($R^2 = .995$, $p < .001$, Prior) as shown in Table 8, is a measure of the strength of association between the observed and model-predicted values of the dependent variable.

Table 8

Equation fit for Prior Participation in Developmental Math

	Model Summary					Parameter Estimates	
Equation	R Square	F	df1	df2	Sig.	Constant	b1
Linear							
No Prior	.996	5239.126	1	22	.000	- .025	.966
Prior	.995	4287.343	1	22	.000	-1.336	.993

Figure 14 and Figure 15 demonstrate the appropriateness of a linear model.

Figure 14. Linear Model Fit for Participants with No Prior Developmental Math

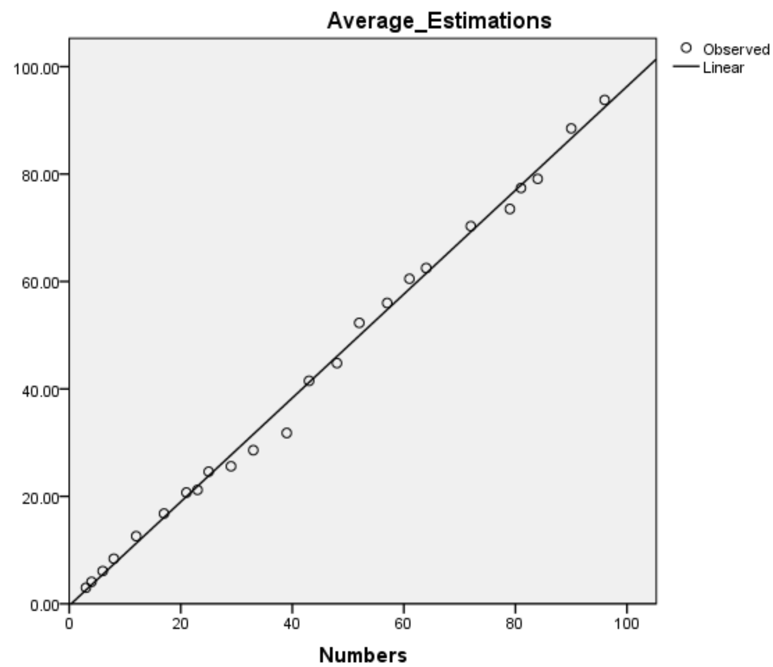
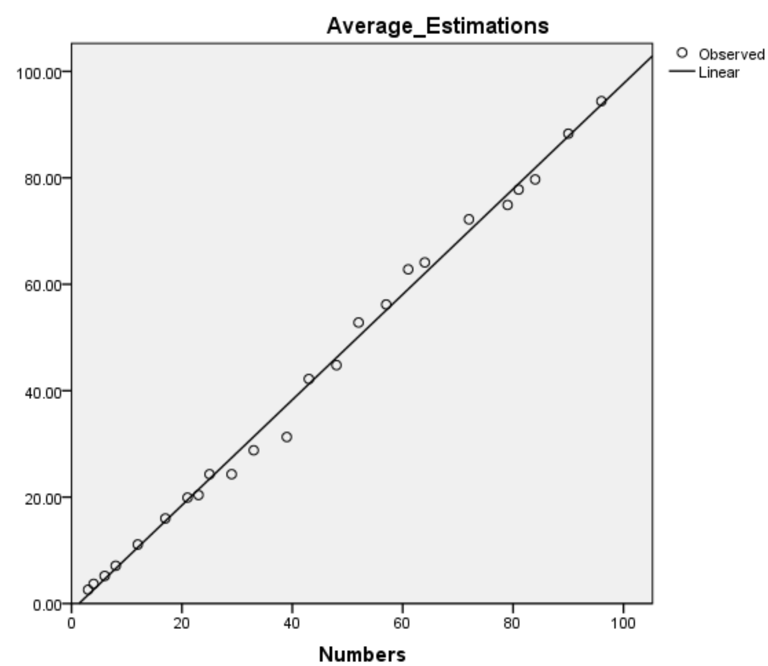


Figure 15. Linear Model Fit for Participants with Prior Developmental Math



Based upon the strength of the linear models, (R^2=.996, P , .001, No Prior) and (R^2=.995, $p < .001$, Prior), visual inspection of the plots of the overall average estimations for the participants, and the corresponding linear equation curve, it was evident that a linear relationship existed. The null hypothesis that the representation of the mental number line of developmental math students approximates a linear line as determined by analyzing the statistical significance of the model and the R^2 correlational statistic based upon prior involvement in developmental math was not rejected.

Hypothesis 5

For Hypothesis 5, the mean absolute error percentage will not be statistically different by gender; the relationships were analyzed using analysis of variance (ANOVA) with an = .05. Although the group sizes were similar the Levene Statistic to test of homogeneity of variance was used to insure stability in the ANOVA model. The null hypothesis of the Levene test is that homogeneity of variance exists. The hypothesis was tested at $\alpha = .10$. Also, a robust ANOVA, Brown-Forsythe was computed. The Brown-Forsythe ANOVA does not require homogeneity of variance.

The mean absolute error percentage by gender was 4.1% for female participants (M = .041, SD = .0123), and 3.8% for male participants (M = .038, SD = .0115) as shown in Table 9. A visual analysis of the means plot did not appear to show a significant difference between the participant groups (see Figure 16).

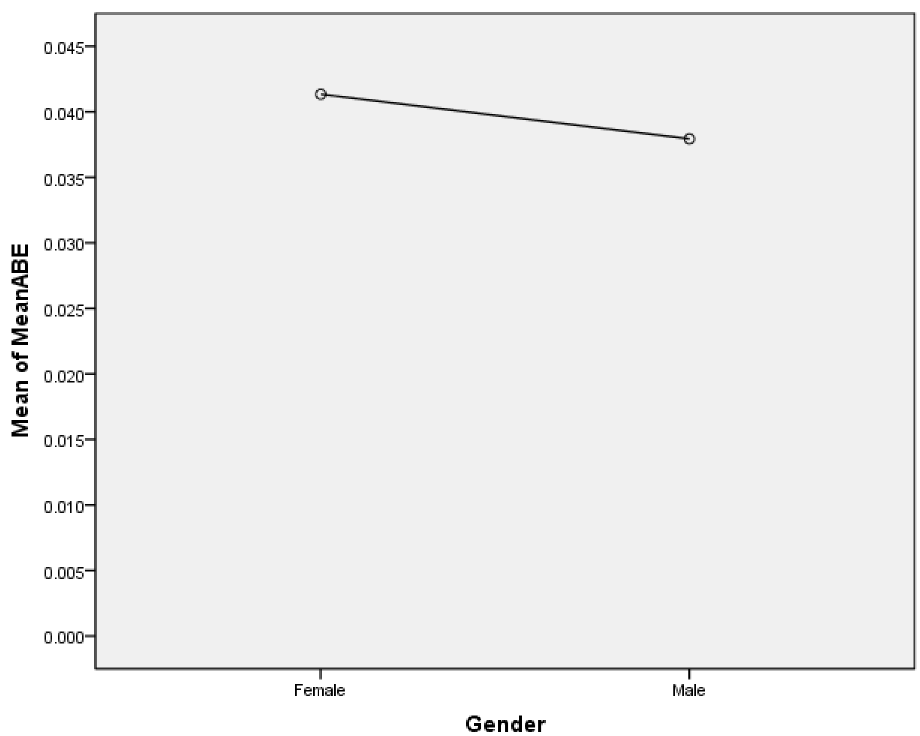
Table 9

Descriptive Statistics of Mean Absolute Error Percentage by Gender

Gender	N	M (SD)	95% CI LL	95% CI UL
Female	73	.041 (.0123)	.0384	.0442
Male	50	.038 (.0115)	.0347	.0412

Note. CI = confidence interval; LL = lower limit, UL = upper limit

Figure 16. Means Plot of ABE% by Gender



Since the Levene Statistic (.253, p = .616) failed to reject the assumption of homogeneity of variance at α = .10 (see Table 10) ANOVA was calculated.

Table 10

Test of Homogeneity of Variance by Gender

Levene Statistic	df1	df2	Sig.
.253	1	121	.616

The null hypothesis, the mean absolute error percentage will not be statistically different by gender, was tested at the p = .05 level. The null hypothesis was not rejected (F = 2.390, p = .125) as shown in Table 11.

Table 11

Analysis of Variance on Mean Absolute Error Percentage by Gender

	Sum of Squares	df	Mean Square	*F*	Sig.
Between Groups	.000	1	.000	2.390	.125
Within Groups	.017	121	.000		
Total	.018	122			

Brown-Forsythe ANOVA supported the failure to reject at α = .05 (Statistic = 2.452, p = .120) as shown in Table 12.

Table 12

Brown-Forsythe Analysis of Variance by Gender

	Statistic[a]	df1	df2	Sig.
Brown-Forsythe	2.452	1	109.959	.120

a. Asymptotically F distributed

Hypothesis 6

For Hypothesis 6, the mean absolute error percentage will not be statistically different by academic level; the relationships were analyzed using analysis of variance (ANOVA). The group sizes differed so the Levene Statistic to test of homogeneity of variance was used to insure stability in the ANOVA model. Also, a robust ANOVA, Brown-Forsythe was computed. Since there was only one senior participant the ANOVA relationship was not computed for this participant. Robust tests of equality of means cannot be performed for participant levels of 1 or less (SPSS, 2009). The mean absolute error percentage ($M_{ABE\%}$), as shown in Table 13, for Freshmen was 3.9% (M = .039, SD = .0121), $M_{ABE\%}$ for Sophomores was 4.0% (M = .040, SD = .0120), $M_{ABE\%}$ for Juniors was 4.4% (M = .044, SD = .0122).

Table 13

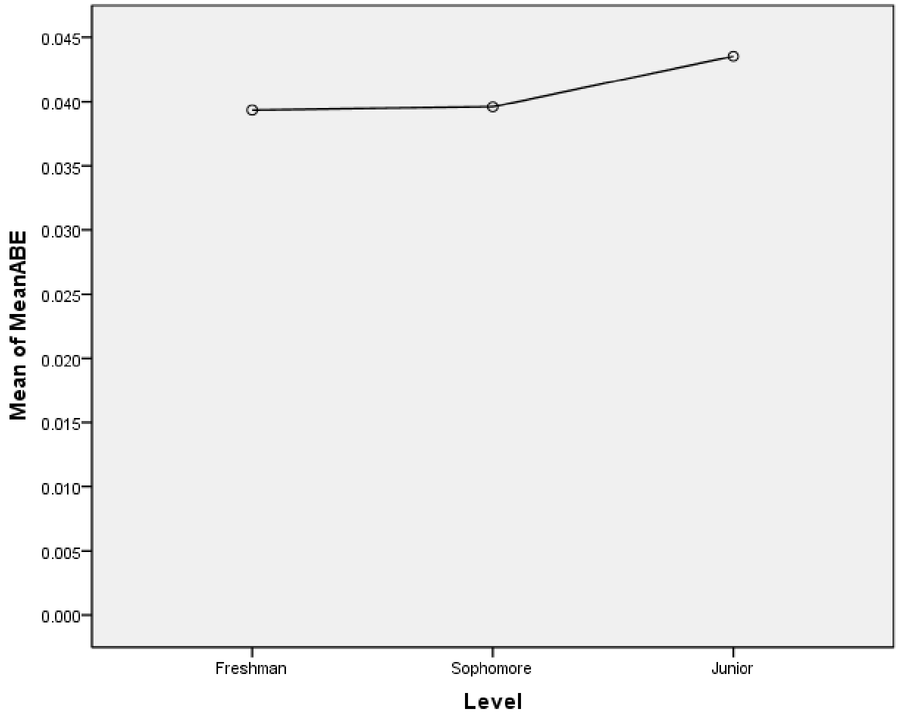
Descriptive Statistics of Mean Absolute Error Percentage by Academic Level

			95% CI	
Gender	N	M (SD)	LL	UL
Freshmen	72	.039 (.0121)	.0365	.0422
Sophomore	37	.040 (.0120)	.0356	.0436
Junior	13	.044 (.0122)	.0377	.0420

Note. CI = confidence interval; LL = lower limit, UL = upper limit

Visual inspection of the means plot indicates little variation in the $M_{ABE\%}$ (see Figure 17).

Figure 17. Means Plot of ABE% by Academic Level



Since the Levene Statistic (.158, p = .854) failed to reject the assumption of homogeneity of variance at α = .10 (see Table 14) ANOVA was calculated.

Table 14

Test of Homogeneity of Variance by Academic Level

Levene Statistic	df1	df2	Sig.
.158	2	119	.854

The null hypothesis, the mean absolute error percentage will not be statistically different by academic level, was tested at the p = .05 level. The null hypothesis was not rejected (F = .674, p = .512) as shown in Table 15.

Table 15

Analysis of Variance on Mean Absolute Error Percentage by Academic Level

	Sum of Squares	df	Mean Square	*F*	Sig.
Between Groups	.000	2	.000	.674	.512
Within Groups	.017	119	.000		
Total	.018	121			

Brown-Forsythe ANOVA supported the failure to reject (Statistic = 2.452. $p = .516$) see Table 16.

Table 16

Brown-Forsythe Analysis of Variance by Academic Level

	Statistic[a]	df1	df2	Sig.
Brown-Forsythe	2.452	1	109.959	.120

a. Asymptotically F distributed

Hypothesis 7

For Hypothesis 7, the mean absolute error percentage will not be statistically different by prior enrollment in developmental math; the relationships were analyzed using analysis of variance (ANOVA). The Levene Statistic to test of homogeneity of variance was used to insure stability in the ANOVA model. Also, a robust ANOVA, Brown-Forsythe was computed.

The $M_{ABE\%}$ for Prior Participation was 4.1% (M = .041, SD = .0130), the $M_{ABE\%}$ for No Prior Participation was 4.0% (M = .040, SD = .0118) as shown in Table 17.

Table 17

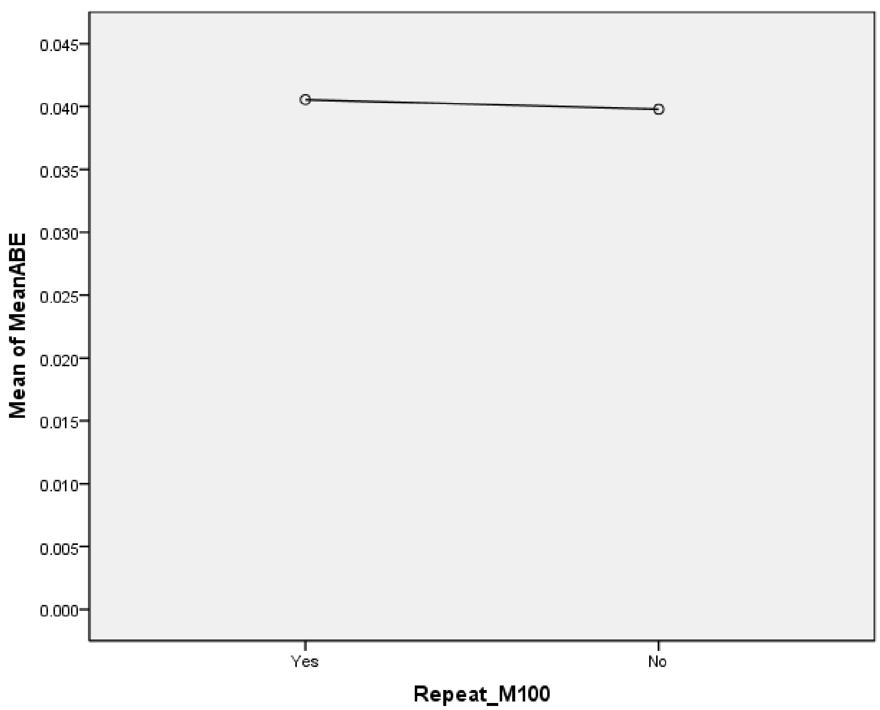
Descriptive Statistics of Mean Absolute Error Percentage by Prior Participation in Developmental Math

Prior Participation	N	M (SD)	95% CI LL	95% CI UL
Yes	28	.041 (.0130)	.0355	.0456
No	95	.040 (.0118)	.0374	.0422

Note. CI = confidence interval; LL = lower limit, UL = upper limit

A visual analysis of the means plot did not show a significant difference between the participant groups (see Figure 18).

Figure 18. Means Plot of ABE% by Prior Participation in Developmental Math



Since the Levene Statistic failed to reject the assumption of homogeneity of variance at α = .10 (.327, p = .568) as shown in Table 18, ANOVA was calculated.

Table 18

Test of Homogeneity of Variance by Prior Participation in Developmental Math

Levene Statistic	df1	df2	Sig.
.327	1	121	.568

The null hypothesis, the mean absolute error percentage will not be statistically different by prior enrollment in developmental math, was tested at the = .05 level. The null hypothesis was not rejected (*F* = .086, *p* = .770) as shown in Table 19.

Table 19

Analysis of Variance on Mean Absolute Error Percentage by Prior Participation in Developmental Math

	Sum of Squares	df	Mean Square	*F*	Sig.
Between Groups	.000	1	.000	.086	.770
Within Groups	.018	121	.000		
Total	.018	122			

Brown-Forsythe ANOVA supported the failure to reject (Statistic = 2.452, *p* = .782) see Table 20.

Table 20

Brown-Forsythe Analysis of Variance by Prior Participation in Developmental Math

	Statistic[a]	df1	df2	Sig.
Brown-Forsythe	2.452	1	109.959	.120

a. Asymptotically F distributed

CHAPTER FIVE: SUMMARY AND DISCUSSION

Summary

What is the importance of number line estimation in relationship to mathematics learning disabilities (MLD)? Testing for MLD has largely settled on two assessment measures (a) the IQ-discrepancy measure, and (b) standardized mathematics test score cutoff criteria. However, both these methodologies are inadequate in themselves to diagnose the presence of MLD (Geary, 2005; Murphy, et al., 2007b).

Recently, researchers in MLD have incorporated measures related to the constructs of number sense and working memory. This allows for a broad spectrum of indicators. Murphy, Mazzocco, Hanich, and Early (2007b) demonstrated how the cognitive profile of students with mathematics learning disabilities (MLD) could vary depending upon the cutoff criterion used in relation to achievement tests. The use of a cutoff percentage on the mathematics portion of achievement tests can be problematic but it serves as standard methodology for defining MLD. When setting a high cutoff percentage, say thirty-five to forty-five percent, the goal may be to increase sample size. However, this can lead to problems with heterogeneity and the inclusion of children simply facing a developmental delay (Murphy, et al., 2007b).

It must be noted that there is a difference between students who have significant cognitive challenges with math and those who have math learning difficulties. Difficulties in math can result from a number of factors that can include poor instruction, socioeconomic factors, or cognitive factors (Mazzocco, 2005). Mazzocco states:

> I propose that the term *math difficulties* be used to refer to a broader group of children that includes children with or without math disability—and that *math disability* be reserved to refer to a presumed biologically based set of math difficulties, even if that basis is not yet fully understood at the neurobiological or genetic level. (p. 321)

Generally speaking, mathematics learning disability (MLD) " is likely best understood in terms of the relations between different cognitive processes and the impact that a deficit in one area has on the other areas and on mathematics achievement" (Mabbott & Bisanz, 2008, p. 17).

The majority of research conducted on math learning disabilities (MLD) has occurred in the primary grades. It has not been evident that math disabilities persist beyond secondary education. However, McGlaughlin, Knoop, and Holliday (2005) discovered that deficits in post secondary students mirrored those of elementary and secondary levels.

This study's purpose was to use number line estimation to examine the potential presence or absence of MLD in postsecondary students enrolled in a developmental math program. This was a causal comparative and correlational study.

The students enrolled in the developmental math course at the large university in the Mid-Atlantic region met the following criteria:

- A score of less than 450 on the math component of the SAT Reasoning Test
- A score of less than 15 on the math component of the ACT
- A score of less than 23 on the Math Assessment Test Part One (*Faculty/Adjunct Handbook*, 2009)

Siegler and Booth (2004) found significant correlations between percentage error on number line estimations and performance on the mathematics section of the Stanford Achievement Test Series, SAT-9. Given the relationship between number line estimation and performance on achievement tests, the use of number line estimation could serve as a component of detecting the presence of mathematics learning disabilities (MLD).

It was not evident in this study that a link existed between number line estimation and the low performance on standardized math achievement tests required for entry into post secondary education. Without comparison to a typically achieving population in the same academic environment we do not know if the differences in mean estimation errors are significant enough to take into account.

The study employed a design used by Booth and Siegler (2008), Geary et al. (2008), Siegler and Booth (2004), and Siegler and Opfer (2003). The design centered on the use of estimation skills when marking the location of a given number on a blank number line bounded by 0 on one end and 100 on the other end. The students were given a number between 1 and 100 and asked to place a mark where they believed the number should go. The results of the estimations were used to determine whether a logarithmic or linear line provided the best fit for the estimations. In addition, linear representations were examined based upon gender, academic level, and prior enrollment in developmental math. Finally, the mean absolute estimation error percentages ($M_{ABE\%}$) were compared based upon gender, academic level, and prior enrollment in developmental math.

The instrument was administered during the first week of class in the fall semester. Seven sections of developmental math classes were used in the experiment.

There were a total of 136 participants, out of which came 123 valid instruments. Validation was based upon completion of the survey and the absence of non-standard markings.

For Hypotheses 1 through 4 the representation of the mental number line of developmental math students was examined to see if it approximated a logarithmic line as determined by analyzing the statistical significance of the model and the R^2 correlational statistic. The analysis employed an examination of curve fit using the curve estimation function of PASW Graduate Pack 18 (SPSS, 2009).

Curve estimation is determined by analyzing the statistical significance of the linear model and the R^2 correlational statistic. In discussing the use of the curve estimation function of PASW Graduate Pack 18 (SPSS, 2009) the program tutorial points out that a significance value of the F statistic below .05 means that the variation explained by the model is not due to chance. The PASW Graduate Pack 18 tutorial goes on to stated that "the R Square statistic is a better measure of the strength of relationship The R Square statistic is a measure of the strength of association between the observed and model-predicted values of the dependent variable" (¶ 4).

The statistical package generates a line from the data. The data line is then compared to a model of a line with predicted values. The analysis then determines if these two lines are significantly close to each other.

Hypothesis 1, the representation of the mental number line of developmental math students approximates a linear line as determined by analyzing the statistical significance of the model and the R^2 correlational statistic was not rejected. The linear model was found to be significant at the $p < .001$ level. There was an associative power of $R^2 = .996$ for the overall linear model.

The results indicate that the overall model for the number line estimation errors is linear (see Figure 7). The absence of a logarithmic model indicates that the mental representation of the number line for these participants is linear. This would also seem to indicate that there is no problem with transferring the mental line to the visuo-spatial sketchpad. What cannot be determined from this, and requires further study, is if this linear representation makes any difference in the mathematical skills of the participants in developmental math. Developmental math, at the institution studied, involves pre-algebra and beginning algebra.

Hypothesis 2, the representation of the mental number line of developmental math students approximates a linear line as determined by analyzing the statistical significance of the model and the R^2 correlational statistic based upon gender was not rejected. The linear model was found to be significant at the $p < .001$ level for both female and male participants. There was an associative power of $R^2 = .996$ for the linear model associated with the female participants and $R^2 = .995$ for the linear model associated with the male participants.

Both of the data based models based on the gender of the participants were linear. This was not unexpected as recent research indicates that differences in gender based performance vary over developmental timeframes.

> Given the inconsistent findings regarding the nature and timing of the gender differences in math, there is a reason to cast doubt on whether there continue to be

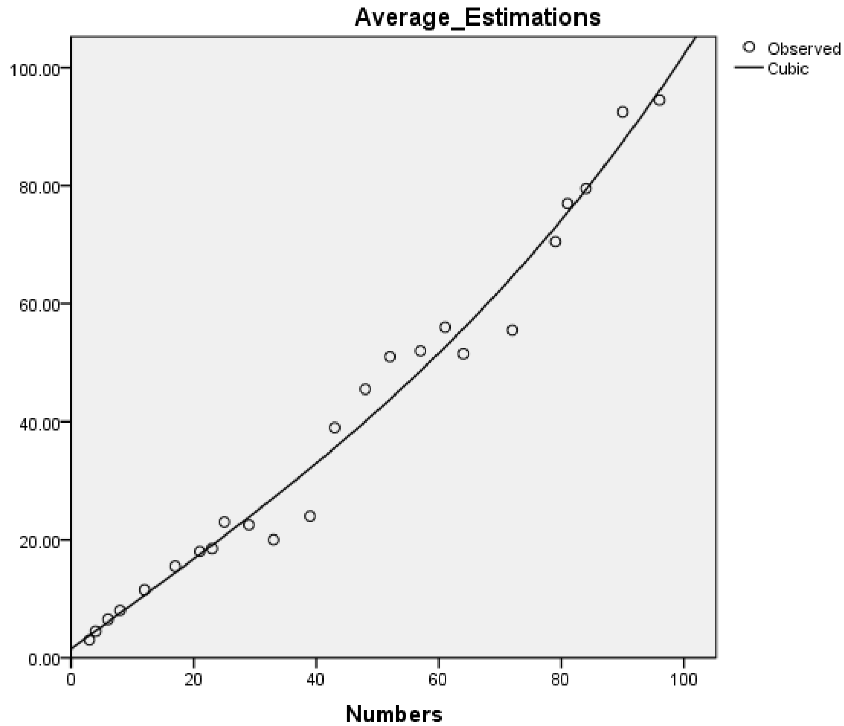
gender differences in mathematics performance as claimed by previous studies, especially in the current educational context in the United States.

(Ding, Song, & Richardson, 2006, p. 282)

Hypothesis 3, the representation of the mental number line of developmental math students approximates a linear line as determined by analyzing the statistical significance of the model and the R^2 correlational statistic based upon academic level was not rejected. The linear model was found to be significant at the $p < .001$ level for freshmen, sophomore, and junior participants. There was an associative power of $R^2 = .996$ for the linear model associated with the freshman academic level participants, $R^2 = .995$ for the linear model associated with the sophomore academic level participants, and $R^2 = .993$ for the linear model associated with the junior academic level participants. The model for the senior academic level participant proved to be cubic at a significance level of $p < .001$. The cubic model had an association of $R^2 = .980$ as compared with an $R^2 = .970$ for a linear model.

Three of the data based models were linear and one was cubic. The results would seem to indicate that delay in completion of the introductory developmental math course does not relate to the presence of a logarithmic mental number line. The cubic line was somewhat significant in that prior studies had not indicated that this model was a possibility (see Figure 19). Since this was an anonymous study there is no way to further evaluate the related aspects of working memory and number sense in this particular participant.

Figure 19. Senior Participant with No Prior Involvement in Developmental Math



Hypothesis 4, the representation of the mental number line of developmental math students approximates a linear line as determined by analyzing the statistical significance of the model and the R^2 correlational statistic based upon prior enrollment in developmental math was not rejected. The linear model was found to be significant at the $p < .001$ level for participants who had no prior enrollment in developmental math as well as those participants who had prior enrollment in developmental math. There was an associative power of $R^2 = .996$ for the linear model associated with the participants who had no prior enrollment and $R^2 = .995$ for the linear model associated with the participants who had prior enrollment in developmental math as shown in Table 21.

Both data based models that involved prior participation in developmental math were linear. This would seem to indicate that a linear representation of the mental number line does not correlate directly with success in developmental math since 22.8% of the participants had taken developmental math at least once prior to this study.

Table 21

Model Summaries of Equation fit for Average Estimations

	Model Summary					Parameter Estimates			
Equation	R Square	F	df1	df2	Sig.	Constant	b1	b2	b3
Linear									
Overall	.996	5087.991	1	22	.000	-.580	.972		
Female	.996	5131.707	1	22	.000	.025	.958		
Male	.995	4342.832	1	22	.000	-1.504	.993		
Freshmen	.996	5838.711	1	22	.000	- .439	.969		
Sophomore	.995	4207.174	1	22	.000	- .646	.980		
Junior	.993	3343.973	1	22	.000	- .982	.966		
No Prior	.996	5239.126	1	22	.000	- .025	.966		
Prior	.995	4287.343	1	22	.000	-1.336	.993		
Cubic									
Senior	.980	323.623	1	22	.000	1 .485	.958	-.001	3.094 E -5

For Hypothesis 5, the mean absolute error percentage ($M_{ABE\%}$) will not be statistically different by gender; the relationships were analyzed using analysis of variance (ANOVA). The null hypothesis, the mean absolute error percentage does not differ by gender, was tested at the α = .05 level. The null hypothesis that the mean absolute error percentage will not be statistically different by gender was not rejected at α = .05 (F = 2.390, p = .125) see Table 11.

Hypothesis 6, the mean absolute error percentage will not be statistically different by academic level; the relationships were analyzed using analysis of variance (ANOVA). Since there was only one senior participant the ANOVA relationship was not computed for this participant. Robust tests of equality of means cannot be performed for participant levels of 1 or less (SPSS, 2009). Since there were more than two groups post hoc tests were performed to check for possible between group differences. The null hypothesis, the mean absolute error percentage will not be statistically different by academic level, was tested at the $\alpha = .05$ level. The null hypothesis was not rejected ($F = .674$, p = .512) as shown in Table 15.

For Hypothesis 7, the mean absolute error percentage will not be statistically different by prior enrollment in developmental math; the relationships were analyzed using analysis of variance (ANOVA). The null hypothesis, the mean absolute error percentage will not be statistically different by prior enrollment in developmental math, was tested at the $\alpha = .05$ level. The null hypothesis was not rejected ($F = .086$, $p = .770$) as shown in Table 19.

Discussion

In hypotheses one through four all of the models proved to be linear with the exception of the senior academic level. The linearity of the model for Hypothesis 1 was not expected but could be a result of the spectrum of academic levels examined. Forty-one of the 123 participants came from the sophomore through senior level (see Appendix A). Maturity and exposure to other aspects of mathematics in various disciplines could have increased the estimation skills of this particular segment of the population. However, this study found that there were positive increases in mean Absolute

Estimation Errors (ABE) over the range of numbers 21 to 43 as academic level increased. This would seem to indicate that the linearity came from the freshmen participants.

The cubic model present for the senior participant in Hypothesis 3 was definitely unexpected. However, with an N = 1, this would prove problematic to generalize on all seniors taking developmental math at this particular institution. It does raise possibilities for further research since Johnson and Kuennen (2004) advised “that students needing mathematics remediation take the course in their first semester and that the importance of developmental courses to other disciplines be stressed” (p. 24). The senior had not taken developmental math prior to this point. The model was not logarithmic but cubic (see Figure 19).

To delay instructional interventions is problematic. However, there was a segment of the population that was repeating this course in developmental math. Out of the 123 participants 22.8 % were repeating the course 9see Appendix A). Table 24 shows 24 sophomores and 4 juniors were repeating the course. Table 23 shows 10 female and 18 male participants were repeating the course.

In Hypothesis 2 and Hypothesis 5 it was found that there was no difference between male and female participants. Ding, Song and Richardson (2006) stated “there is a reason to cast doubt on whether there continue to be gender differences in mathematics performance as claimed by previous studies, especially in the current educational context in the United States” (p. 282). Mundia (2010) found that both genders experienced common mathematical deficiencies. Mundia stated that, in classes where females were not repeating the course, “females had better mathematical skills than their male counterparts. The girls’ high confidence and self-esteem in coeducation classes and in previously male-regarded subjects needs to be encouraged and supported by both teachers and parents to break gender stereotypes" (p. 155).

In Hypothesis 6 it was found that no differences existed based upon academic level. Johnson and Kuennen (2004) found that delaying enrollment in developmental math affected performance in other academic domains. Johnson and Kuennen stressed that students should be encouraged to enroll early and that the costs of delayed enrollment be pointed out.

In Hypothesis 4 and Hypothesis 7 it was found that no differences existed based upon prior enrollment in developmental math. Approximately 23% of the participants were repeating developmental math as shown in Table 22.

Table 22

Participants Repeating Developmental Math

	Frequency	Percent
Yes	28	22.8
No	95	77.2

Out of those who were repeating approximately 64% were male as shown in Table 23.

Table 23

Gender of participants repeating Developmental Math

	Frequency	Percent
Female	10	35.7
Male	18	64.3

This is in line with Mundia (2010) who found that the majority of repeaters were male. Mundia stated that “repetition of a class or grade was neither therapeutic nor advantageous unless the root causes of poor performance in a student were identified and addressed through counseling and remedial teaching to break the vicious circle of repeated failure” (p. 155).

While it cannot be determined specifically from the demographic information, 14% of the repeaters were at the Junior academic level (see Table 24). This would indicate the possibility of having repeated the course more than once.

Table 24

Academic Level of Participants Repeating Developmental Math

	Frequency	Percent
Sophomore	24	85.7
Junior	4	14.3

The importance of developing the linear representation of the number line, and thus an accurate concept of magnitude, cannot be overstated. Booth and Siegler (2008) stated that “representations of numerical magnitude are both correlationally and causally related to arithmetic learning” (p. 1016), and that “numerical magnitude representations are not only positively related to a variety of types of numerical knowledge but also predictive of success in acquiring new numerical information, in particular, answers to arithmetic problems” (p. 1027). Geary et al. (2008) pointed to the fact that an accurate representation of the number line has implications in several mathematical domains and can impact mathematics learning into adulthood.

One of the most interesting findings of this study was unexpected. There appeared to be significant absolute errors of estimation (ABE) in the range of numbers between 23 and 39 as shown in Table 25. This occurred for all four academic levels and for both genders.

Table 25

Relationship to Range and Increased ABE%

Number	Freshmen ABE%	Sophomore ABE%	Junior ABE%	Senior ABE%
23	1.7%	1.6%	3.9%	4.5%
25	0.7%	0.3%	1.4%	2.0%
29	3.6%	3.7%	4.1%	6.5%
33	3.9%	4.7%	5.3%	13.0%
39	6.9%	7.8%	7.7%	15.0%
Overall ABE %	3.9%	4.0%	4.4%	5.1%

Not only was there significant ABE% in the values 29, 33, and 39, these errors tended to increase with academic level. Comparatively, Booth and Siegler (2006) found that accuracy increased with grade level from kindergarten to third grade. The mean absolute error percentage in the third grade was ten percent. Geary et al. (2008) found that typically achieving second graders had a mean absolute error percentage of six percent, low achieving second grade students had a mean absolute error percentage of nine percent, and second grade students with mathematics learning disability had a mean absolute error percentage of fifteen percent.

It is yet to be determined if this finding was significant or not. Additional research could be conducted that measures computational accuracy over this range of numbers. Comparisons could be made with other ranges of numbers and students not enrolled in developmental math.

In reference to number sense; a theoretical construct that defines the ability to count, recognize number patterns, comparisons of magnitude, estimation skills, and numerical transformation (Berch, 2005), it would appear that the participants in this study demonstrated an ability to estimate magnitude. Whether this particular skill is transferred to performance in the mathematical domain of algebra is yet to be seen.

One of the limitations of this study was the absence of number line estimations from typically achieving students enrolled in the beginning liberal arts math course at the institution. Attempts were made to secure this participation, however, the researcher was unable convince the respective department chair of the low threshold of intrusion that this experiment would have on the classes in question.

Recommendations for Further Study

The absence of logarithmic representations of the mental number line in the population studied presents new challenges. Future research could specifically focus on measures of working memory that address the specific constructs of the central executive, phonological loop, and visuo-spatial sketchpad. Booth and Siegler (2008) not only employed number line estimation they included measures of short term memory and mental addition. Booth and Siegler found that "representations of numerical magnitude are both correlationally and causally related to arithmetic learning" (p. 1016). Geary et al. (2008) included a specific battery of tests for the individual components of working memory.

Post Secondary Screening for MLD

In respect to the population being studied, when the information from performance on the mathematics sections of the SAT and/or ACT becomes available those students scoring below the 25th percentile could be queried for participation in working memory research. Screening techniques for MLD at the post secondary level should be unobtrusive and effective. Screening should encompass several combinations of test items and take developmental issues into consideration (Mazzocco, 2005). Screening must balance between sensitivity and specificity and include a sufficient level of difficulty so that refinement in MLD subtypes can be detected (Fuchs, et al., 2007).

Clearly defining post secondary mathematics disability can have a significant effect on screening. Using a broad criterion in definitions and measurements can lead to two different outcomes. Studies may eventually converge on standard definitions and methodologies or they would diverge in such a fashion that application and generalization of research would be impossible. Divergence would prevent a standardization of screening definitions (M. M. Murphy, M. l. M. M. Mazzocco, L. B. Hanich, & M. C. Early, 2007a).

Post Secondary Interventions for MLD

In discussing the appropriateness of MLD classifications and interventions at the post secondary level McGlaughlin, Knoop, & Holliday (2005) state that "the results of this study suggest that students with mathematics disabilities at the college level tend to mirror research findings for students identified with mathematics disabilities at the elementary and secondary levels" (p. 229). While many colleges offer general mathematics support students who demonstrate a possible mathematics disability should receive a comprehensive array of assistance (McGlaughlin, et al., 2005).

Wadlington & Wadlington (2008) pointed out that, "specific mathematical difficulties are diverse; therefore, addressing each individual's problems can be a challenge for students and their teachers" (p. 2). Mazzocco & Thompson (2005) stressed that "It is important to identify risk for MLD, because—like poor reading achievement—poor math achievement is a risk factor for negative outcomes in both childhood and adulthood" (p. 142).

If it becomes apparent that these students have deficits in one or more aspects of working memory, specific interventions could be coordinated with participation in developmental math. Kroesbergen & Van Luit (2003) state that "An intervention is judged *effective* when the students acquire the knowledge and skills being taught and thus appear to adequately apply this information at, for example, posttest" (p. 99). Interventions need to be developmentally appropriate as well. Secondary, and post secondary, math education involves the acquisition of problem solving skills. These skills are directly related to solving word problems and applying knowledge in new situations (Kroesbergen & Van Luit, 2003).

One area of disabilities research that is gaining ground is response to intervention (RTI). A response to intervention (RTI) model generally uses three levels of intervention. First is the general education level, second is research based tutoring, and third is special education. The earlier this identification and intervention can occur the better are the chances that students will be more competent (Fuchs, et al., 2007). Some of the important goals of intervention should be (a) increased confidence and precision with arithmetic combinations, (b) use of developmentally appropriate counting strategies, and (c) an ability to compare the magnitude of numbers (Gersten, et al., 2005).

In reflecting on the predominance of studies in basic skills in a meta-analysis of MLD research Kroesbergen & Van Luit (2003) state that "The interventions in the domain of basic skills nevertheless showed the highest effect sizes" (p. 110). Two other significant factors played a role in the effectiveness of interventions (a) the length of time involved, and (b) the method of instruction. The length of time was negatively correlated with the effect of interventions, suggesting that short and specific interventions were most effective. The method of direct instruction, whether classroom or computer based, provided the most effective intervention in the basic skills domain (Kroesbergen & Van Luit, 2003).

Based on the similarities of MLD between elementary and post-secondary students and the effectiveness of direct focused interventions in basic skills, two specific issues present themselves for further research (a) can the use of directed study in numerical combinations provide an effective intervention, and (b) can this intervention be effective at the post secondary level? In other words, does an intervention in combination mastery significantly improve learning outcomes in developmental math at the college level?

It is important that educational researchers expand the understanding of mathematics disability and seek out effective interventions at all developmental stages. The students affected by MLD cannot afford continued neglect.

Comorbid Reading and Math Learning Disabilities

Two foundational areas of learning are math and reading. It is difficult for students who struggle with either of these subjects. However, when students have significant difficulties in both subject areas it presents a serious challenge to learning. Research into reading disabilities is well established. Research into math disabilities is still developing (Gersten, et al., 2007; Wise, et al., 2008). One of the recent areas of focus in understanding math disabilities is the occurrence of a comorbid relationship between math and reading disabilities.

Learning disabilities often co-occur frequently. The question becomes one of increased severity in both learning domains or a qualitatively different type disability (Cirino, Fletcher, Ewing-Cobbs, Barnes, & Fuchs, 2007). Dirks, Spyer, and de Sonneville (2008) stated that in the studies recently conducted children facing both reading and math disability “not only have more generalized verbal and nonverbal problems but also in most studies appear to be the most impaired in comparison to reading-only or arithmetic-only disability groups” (p. 460).

Dirks et al. (2008) studied the prevalence of combined reading and math disabilities and found that the occurrence exceeded expectations. The expected rate of comorbidity was 4.9% and the actual rate proved to be 7.6%. This strong prevalence has spurred the interest of math disabilities researchers.

The development of literature and research surrounding the comorbidity of math and reading disabilities has it s roots in one of the seminal articles in the study of math disabilities. Geary (1993) included the relationships between reading disabilities (RD) and math disabilities (MD) in his discussion of the convergence of developmental psychology and neuropsychology in the study of math disabilities. Geary pointed out that there is often an occurrence of RD in children experiencing MD. Future research could look into the possible presence of Comorbid deficits in post secondary students.

What is clear, even in light of the current study, is that a significant portion of the post secondary student population struggles to successfully navigate college level mathematics. If our goal is to make post secondary education accessible to all we need to address the issue of mathematic learning disability in the post secondary student population.

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Appendix A Demographic Data

Participants by Gender

	Frequency	Percent
Female	73	59.3
Male	50	40.7
Total	123	100.0

Participants by Academic Level

	Frequency	Percent
Freshmen	72	58.5
Sophomore	37	30.1
Junior	13	10.6
Senior	1	0.8
Total	123	100.0

Participants by Prior Participation in Developmental Math

	Frequency	Percent
Yes	28	22.8
No	95	77.2
Total	123	100.0

Appendix B Cover Sheet

Number Line Research

Thank you for your willingness to participate in this research project. The survey and answers are completely anonymous. In order to provide for the maximum effectiveness of the research please complete the survey items below.

Are you: ☐ Female ☐ Male

Are you a: ☐ Freshman ☐ Sophomore ☐ Junior ☐ Senior

Have you taken MATH 100 before: ☐ Yes ☐ No

The following pages each have a number at the top of the page and a number line from 0 to 100. The goal is to mark on the number line the approximate location of the number printed at the top of the page. This is not a test so you may answer quickly.

57

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0 100

Please mark the approximate location of the number on the number line