

TRANSACTIONAL ALGORITHM FOR SUBTRACTING FRACTIONS:

GO SHOPPING

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Transactional Algorithm for Subtraction of Fractions: Go Shopping

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Abstract

James Pinckard. TRANSACTIONAL ALGORITHM FOR SUBTRACTION OF FRACTIONS: GO SHOPPING. (Under the direction of Dr. John Pantana) School of Education, November, 2009.

The purpose of this quasi-experimental research study was to examine the effects of an alternative or transactional algorithm for subtracting mixed numbers within the middle school setting. Initial data were gathered from the student achievement of four mathematics teachers at three different school sites. The results indicated students who utilized the transactional algorithm demonstrated greater comprehension, retention, and computational accuracy than those who utilized traditional algorithms. The difference between the scores of the two groups was statistically significant. The follow-up investigation employed a quasi-experimental nonrandomized Test 1-Test2 control research with two teachers. An analysis of variance (ANOVA) was used to analyze the data obtained from 7th graders at one middle school setting. The null hypothesis was that there would be no difference in the achievement levels of students regardless of which algorithm was used to solve subtraction of mixed numbers. The null hypothesis was rejected as the difference between the two groups was statistically significant. Overall, the use of the transactional algorithm for subtracting fractions improved student performance in both the short term and long term. The implication of this study was that multiple strategies, especially ones that provided connections with real life experiences of the child, increased student achievement within the classroom.

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Chapter One: Introduction

In one of the most defining moments of his career, a certain young teacher was challenged to confront his preconceptions regarding the processes by which students learn and understand mathematical algorithms. One day, during a typical review of the standard “renaming” procedure for subtracting mixed numbers with seventh grade students, one girl suddenly threw her pencil on her desk and folded her arms across her chest. Unable to contain her frustration any longer, the girl burst out and cried, “I don’t get this, and I’m not learning anymore about fractions until you can make it real to me.” Shaken by her uncharacteristic attitude, the instructor struggled to regain his confidence. Throughout the day, he was haunted by the realization if he couldn’t help this student, he would be solely responsible for her stagnation in mathematics.

Knowing that he only had until the next day to find a suitable analogy for the subtraction of mixed numbers to the life of a twelve-year-old, the teacher desperately began brainstorming various examples as he drove home. As he thought, the teacher realized that this girl was not merely complaining but that she had discovered a serious flaw in the teaching of mathematics. Most of the examples and word problems in the mathematics textbooks dealt with either measuring distance or cutting fabric; neither situation applied to her or the other thirty-three seventh grade students in the class “These math textbooks and procedures *are* irrelevant,” the teacher thought. “Teaching these tired methods to the students just because this is the way I was taught makes as much sense as giving them a slide-rule instead of a calculator.”

Years later, the teacher would discover that many renowned experts in the field of mathematics had expressed similar misgivings about the bequeathing of these reliable, yet inapplicable algorithms from one generation to another. These methods can be identified as “relatively standardized techniques which are not especially interesting from an intellectual point of view” (Browder, 1976, p. 251). These algorithms are enshrined in the halls of textbooks, never to be challenged either by the instructor or student.

Without a direct correlation to the reality each student experiences, these sacred procedures cannot be established by means of reason and therefore must be committed to memory. However, years of educational philosophy have reported that the mere memorization of algorithms runs contrary to the true interactive nature of learning. In order for students to truly internalize a particular concept and not be superficial learners, the instruction must emphasize intellectual engagement rather than the regurgitation of information (Burke, 1995; O’Brien, 1999).

Amid his thoughts about the frustrated student, the teacher suddenly remembered that his wife had asked him to pick up a gallon of milk on his way home. Spotting a convenience store immediately to his right, he pulled into the parking lot. A few minutes later, the teacher put his milk on the counter and began to pull a few bills out of his wallet. As he handed the money to the cashier, the memories of the mixed number problem flooded into his mind. He realized that by using cash in the transaction, he had just found the difference of two mixed numbers without going through the painful process of renaming.

The teacher spent the evening rehearsing the transaction and solving problem

after problem. By the time his wife came to check on him, he had developed an entirely new method for subtracting mixed numbers independent of the tedious “renaming” procedure. With new hope, the teacher tidied his desk and turned off the light, excited to share his discovery with the girl.

The next morning, standing in front of the white board as the students filed into their seats, the teacher noticed the frustrated student staring at him from the middle row. Her arms were crossed again, and the look on her face reminded the teacher of a judge at a track meet. A little nervous, he cleared his throat and quieted the class.

Addressing the girl, the teacher began by saying that he might have something to help her with the fraction problems. Although the only change in her expression was a slightly raised eyebrow, he continued, asking her if she liked shopping. Of course she did; she was a twelve-year-old with a big appetite for good music. As he explained how monetary transactions were very similar to the mixed number problems, her body posture relaxed. After reviewing the shopping procedure a few times, she appeared intrigued with what was presented to her and volunteered to try a few problems on the white board. She faltered slightly over certain steps, but whenever the teacher gave her simple hints about shopping and monetary transactions, she nodded and continued working. After she completed several more problems correctly, she beamed at the teacher and sat down, pencil in hand and eagerness in her eyes.

With a grin, the teacher explained the procedure to the entire class and called student after student to the board. He spent the rest of the lesson laughing whenever a

suspicious student asked, “That’s it?” and reminding the hesitant ones to “Go shopping!” Although some students appeared cautious about this new technique, the teacher was sure that by the end of the week, everyone would at least understand the concept of subtracting mixed numbers better. When the bell rang and all the seventh graders left his room smiling and waving, the teacher breathed a sigh of relief. He was amazed that connecting such a common activity to the personal experience of his students helped them internalize the concept so much better than the textbook alone. Already, he could not wait to continue his lesson the next day.

Mayer (2002) considered *retention* and *transfer* to be two of the most important goals of the educational system. The concept of *retention* was closely associated with the ability to remember the material or knowledge in the same setting as was presented to the learner. *Transfer* went beyond mere memorization of the knowledge; it involved the ability to use the knowledge in new and appropriate settings.

Many studies have declared the importance of fractional understanding and the difficulties students have with fractions (Ben-Chaim, Fey, Fitzgerald, Benedetto, & Miller, 1998; Bottge & Hasselbring, 1993; Brown & Quinn, 2006; Carnine & Jitendra, 1997). Researchers have agreed that the change from numerical symbolism of whole and decimal numbers to fractional or rational number representations often complicated the conceptual understanding of fractions for students (English & Halford, 1995; Hasemann, 1981; Mack, 1990). Hasemann (1981) stated that the rules or procedures students learned for the natural numbers are quite different than those required to solve fraction problems. Students who experienced difficulties performing

operations on whole numbers, especially subtraction requiring borrowing or regrouping, were bound to experience complications when they attempted to solve subtraction of mixed numbers. The remedy for the inability of students to solve problems correctly has been an overemphasis of instruction based on memorization or mechanized procedures. Though these algorithms may have led many students to correct solutions to specific mathematical concepts such as subtraction of fractions, students were unable to relate the procedures to any other event or activity outside the classroom setting.

Problem Statement

Brownell (1954) declared that premature emphasis on memorization acted as a barrier to sound learning by students. He asserted that meaningful learning consisted of instruction deemed significant to the child in terms of transfer and application. The transition or accentuation of intelligent mastery skills over mechanical skills marked positive movement towards meaningful mathematics (Brownell, 1954). This overemphasis on the teaching of algorithms rather than development of understanding through connections has led students to believe that learning in mathematics means abandonment of thinking skills and dependence upon memorization or “doing without understanding” (Brown & Quinn, 2007).

In addressing the term *meaningful*, Freudenthal (1981) deemed it as what is “meaningful to the learner” and suggested that instruction must view placement of “the real world”, or context, ahead of “mathematising” (p. 8). The focus of mathematical instruction forged on the combination of meaningful learning and

connection to real world context for the student increases the probability of proper application of skills within the proper contextual setting.

Whitney (1987) pointed out that most people use different methods of calculations in their everyday lives than those which were taught in classroom settings. The same author noted that only gradual progress was being made through “increasing efforts in the U.S.A., as in much of the world, to make ‘math’ more relevant to schoolchildren” (p. 229). This meant multiple generations of students had received the same irrelevant algorithms to solve mathematical problems such as subtraction of mixed numbers.

This research examined the effect of an alternative or *transactional algorithm* for the subtraction of fractions on student achievement in the short and long term. The transactional algorithm paralleled a familiar event in the life of a student. As students recalled their experiences in their role as consumers they were better able to connect the procedures of the transactional algorithm with subtraction of mixed numbers. Students solved subtraction of mixed numbers problems through the verbalization of their recent shopping trip. They simply began the self talk with, “I have this much money including change. The cashier said that total cost for the purchase was this particular amount.” This was followed up with, “I will just give them the next nearest whole dollar amount and get some change back.” The arithmetic that followed was to simply subtract the dollar amount given to the clerk from the original amount the student began with. The result represented the excess amount of money the student did not need to access for this particular transaction. The change received from the

cashier is combined with the original change, or fractional portion, to determine the new amount of funds the student has.

Traditional algorithms for subtraction of mixed numbers taught to students for generations lack a direct connection to the activities students experience in their everyday lives. The delivery of instruction through the use of an analogy, such as shopping, offered procedures that students were able to associate with and recite later. Rittle-Johnson & Koedinger (2005) thought the use of stories encouraged students to consider alternative or informal strategies to problem solving which in turn increased their comprehension levels. The focal point of this study was the used of an algorithm that exhibited connections to monetary transactions. Data has been collected and analyzed to quantify the effect on student achievement and retention when students utilize a transactional algorithm.

Purpose

The continued use of the same archaic algorithm resulted in the same low levels of achievement from generations of learners in the area of subtraction of mixed numbers requiring renaming or regrouping. Reflection on this repeated use of a computational procedure based on memorization led this researcher to investigate the effectiveness of an alternative algorithm that increased student understanding and achievement.

The primary purpose of this study was to determine and describe the impact of transactional or alternative mathematical algorithms for subtracting fractions on achievement levels of middle school students. The secondary area of inquiry of this research was to compare the long-term achievements of students who utilize the

transactional algorithm against those of students who employ the traditional algorithm.

Focus and Intent

The focus of this research project was the measurement of the effect of an alternative, reality-based algorithm for subtraction of mixed numbers by seventh grade students. The expected outcome was that use of the transactional algorithm would reaffirm the positive results achieved in previous research studies connecting mathematical instruction with real-life situations. The algorithm introduced to the participating students related the common experience of monetary transactions to the mathematical procedure of subtracting mixed numbers. This strategy or method differed significantly from the conventional or traditional procedures that relied on the memorization of strict, impassive rules that magically led students through a series of mathematical steps to the correct solution. Conversely, the transactional algorithm connected itself to an activity that middle school students have consistently experienced in their life - going shopping.

The learning of mathematics by students is not simply confined to the area of numeric or symbolic manipulation. The learning cannot be considered complete if the student simply duplicated the mathematical steps or procedures provided by a textbook or teacher and applied that learning exclusively within the school environment. The transfer of knowledge from the classroom setting to the learner must be connected to the experiences and language of the learner. Said in another fashion, the formalized education received within the classroom must be related closely to the language and experiences of the learner.

The anticipated conclusion of this research is that a notable impact will be observed in students' comprehension of subtraction of fractions due to the connectivity of the transactional algorithm to real life experiences. The expectation of this study is that an increase in achievement levels will also be detected in subtraction of mixed numbers as students verbally articulate the transactional algorithm on a regular basis.

Situation to Self

By fortuitous events, this teacher became involved in the San Diego Math Project and California Mathematics Renaissance Project in the early 1990s. The primary goal of the first project was and continues to be the improvement of mathematics education in schools servicing San Diego and Imperial Counties through development of leadership among the teaching ranks (San Diego Math Project, n.d.). The secondary and lofty goal of this project was to transform middle school mathematics programs so that "all students - especially those from groups whose mathematics achievement has historically lagged - become empowered mathematically" (Acquarelli & Mumme, 1996, p. 478). Many of the activities promoted through these reform projects were designed to engage both the students and teachers in the meaningful process of understanding mathematics. These two projects, along with the experience of working in a group research project for the teaching of rational numbers through the National Center for Research in Mathematical and Science Education, transformed the way this teacher viewed student learning and effective instruction. These programs challenged the participants to set aside their roles as mathematics educators and model the attitude of reflective,

inquiring learners.

The most common struggle with fraction operations became evident after just a few years of teaching at the middle school level. Students' efforts to memorize and reproduce a series of ordered steps or procedures had little effect in increasing their computational accuracy or retention. In reflection, this writer has been changing his instructional methodology away from the traditional mechanical style to incorporating contextual connections to the everyday life events of the student. This change has meant the focus of classroom activities must allow for students to generate their own algorithms or be guided towards some nontraditional algorithms in solving mathematical problems such as subtraction of mixed numbers.

Guiding Questions

The following questions guided the investigator in this research project:

Research Question 1: Are there statistically significant differences in the achievement levels (as measured by the researcher-developed assessment instrument) between students who only received instruction with traditional algorithms for subtraction of mixed numbers as compared with students who received instruction in both traditional and alternative algorithms for subtraction of mixed numbers?

Null Hypothesis 1-H₀: There will be no significant difference in mathematics achievement between students who only received instruction with traditional algorithms for subtraction of mixed numbers and students who received instruction with both traditional and alternative algorithms for subtraction of mixed numbers.

Research Question 2: Are there differences in computational achievement between students using the traditional algorithm and those using the transactional algorithm over a course of 16 weeks?

Null Hypothesis 2-H₀: There will be no significant difference in mathematics achievement between students using the traditional algorithm and those using the transactional algorithm over a course of 16 weeks.

Significance of the Study

Algebra has been identified as the pivotal mathematics course for generations of learners. Included in the educational transition from basic arithmetic to the rigors of algebra, students are obliged to journey through the realm of fractions. Some considered the understanding of fractions as the bridge from the concrete concepts of arithmetic to the abstraction of algebra and remarked that there is no avoidance of fractions within algebra (Brown & Quinn, 2006; Wu, 2001).

The struggle to master this minuscule part of mathematics has continued to plague students of all ages (Anderson, Anderson, & Wenzel, 2000; Clarke, Roche, & Mitchell, 2008; Lamon, 1999; Saenz-Ludlow, 1994). The traditional algorithms presented to students on operations of fractions offer very little connections to real life events. The investigation into the use the transactional algorithm presented in this study can prove to counteract the low achievement levels for subtraction of mixed numbers.

Definition of Terms

Algorithm is defined as a step-by-step procedure to use to correctly solve a mathematical problem.

Alternative algorithm is defined as a type of substitute algorithm that, when utilized, will produce an answer to a mathematical problem. This type of algorithm may or may not be efficient based upon the number of steps required to arrive at a solution.

Borrowing or regrouping is defined as the mathematical process needed to increase the fractional element of the minuend by the value of one.

Fraction is defined as numerical representation of $\frac{a}{b}$ where a and b are integers and b cannot equal 0.

Improper fraction is defined as a fraction in which the numerator has a greater value than the denominator.

Lowest terms are defined as a proper fraction in which the common factors of both numerators and denominators have been removed through simplification.

Mixed number is defined as a number represented by the combination of a counting number and a fraction.

Rubric scoring is defined as a rating scale that assigned a numeric value for performance based on an established criterion.

Traditional or standard algorithm is defined as a type of algorithm that has been taught within the school system and is more than likely presented by publishers in their textbooks as the algorithm that should be used by students to solve mathematical problems.

Transactional algorithm is defined as a type of substitute algorithm utilized with subtraction of mixed numbers based on the analogy of monetary transactions that typically occurred within store settings.

Chapter Two: Review of the Literature

A review of related literature was conducted to discover previous research studies and information available relative to the comparative analysis of the effectiveness of different algorithms for subtraction of fractions. This initial search led this researcher to a prior investigation conducted by Carney (1973). His study determined the effectiveness of two methods used to teach addition and subtraction concepts of fractions or rational numbers. Carney's study did not specifically compare the effectiveness of the traditional algorithm for subtracting fractions with an alternative algorithm. However, the nature of the study was a comparison of an experimental approach to the established or standard approach of teaching addition and subtraction of fractions. This limited search of relevant information led the researcher to investigate other factors with noteworthy impact on academic achievement and retention by students in the mathematics classroom. Some of these factors included content knowledge, the role of algorithms, real life connections, cognition, and effective teaching.

An examination of student work on fraction tests led Brownell (1933) to a nonstandard method of adding fractions for that particular era. The study discovered that students from two of the seven schools had utilized an algorithm that he labeled as the "crutch" (p. 5). Brownell's study was concerned not only with the psychological principles of learning, but also with the effectiveness of the use of this algorithm on student achievement. Brownell thought that the students' use of this nontraditional method clearly pointed to their performances under the influence of a

form of *deliberate instruction*. He judged that this deliberate instruction would have received condemnation from researchers of educational psychology and teachers of arithmetic due to its stress on mechanized learning. Brownell (1933) proceeded to research the method employed by students in the two schools to determine whether its use would “advance or retard sound learning” (p. 7). The study concluded that the learning of fractions was not impeded by the use of the “crutch” method.

Pedagogical Content Knowledge

Though volumes of work exist on the components of effective teaching and learning, the simplest statement heard by this writer about this topic through an unknown source was “It’s not what we teach but how we teach that will make a lasting impression.” Shulman (1986) characterized pedagogical content knowledge as the teachers’ capabilities to make subject matter understandable for their students. Some of the tools Shulman suggested that teachers employ to accomplish this lofty goal included “powerful analogies, illustrations, examples, explanations, and demonstrations” (1986, p. 9).

Prior to the formalization of Shulman’s pedagogical content knowledge, Hativa (1983) listed four strategies that made mathematics lessons more comprehensible: embedding, sequencing, rationalizing steps, and sensitivity to students. She defined embedding as “identifying pre-existing information” and integrating it in a “relevant” manner into new information or concepts (p. 402).

Sequencing, as described by Hativa (1983), dealt with the purposeful arrangement of lesson components from known to unknown, simple to complex. Students may have learned or memorized particular approaches to solve different

types of fraction problems; however, they often employed the wrong procedure. The use of incorrect process was determined to be the second most common error exhibited by students when solving subtraction of fractions problems (Brueckner, 1928).

Hativa designated *rationalizing* as the detailed verbalization of the teacher's methods beyond just the transfer of knowledge. Teachers not only explained what they did, but also provided the rationale for the use of the method. The strategy of rationalizing meant that teachers displayed diligence in their clarification of a particular technique over another, thus aiding in comprehension and application of the method (p. 404).

Hativa added "sensitivity to students" to her list as another factor leading to an improved understanding of mathematical concepts (p. 404). Within this factor Hativa provided several descriptors, including being concerned, discernment of boredom or confusion, encouragement of questions, and relating concepts to "students' lives and experiences" as attempts by which teachers could advance mathematical understanding through the affective domain (p. 404).

Algorithm

The source of the word *algorithm* has been traced to the ninth-century Muslim mathematician al-Khwarizmi (Anthes, 2008). Originally, *algorithm* meant "the art of calculating" (Brumbaugh, Ashe, Rock, & Ashe, 1997, p. 265). The term *art*, in this case, would be associated with Webster's definition of a "skill acquired by experience, study, or observation." As generations have passed, mathematical education as an art

has been transformed into a collection of procedures to be bequeathed to the next generation as unquestionable rules to follow.

The 1478 Treviso manuscript marked the controversial transition from the use of Roman numerals to the Hindu-Arabic numerical system and algorithms (Swetz, 1987). This arithmetic primer was used to train students for the profession of “reckoning” or what is considered in modern era as accounting. The intended use of the book was not for the privileged few but for anyone who wished to learn computation skills. The book did not contain multiple strategies for mathematical problem solving. The opinion of Swetz (1987) was that the Treviso manuscript contained what its author deemed as the necessary techniques for students to learn and practice. Swetz also believed that the book’s author avoided a discussion of arithmetic operation of fractions for two reasons. First, the topic may have been judged as being too confusing for students. Second, the topic itself was unimportant to the lives of everyday citizens or merchants.

The approach of mathematical instruction based on handing down mathematical algorithms from one generation focused on the transmission of facts, rules, and procedures (O'Brien, Shapiro, & Reali, 1971). Mathematical instruction of this type ran contrary to the principles of Piaget’s strategies of building children’s mental operations. This method of mathematics instruction by teachers has remained relatively unchanged since the 19th century (Saracho & Spodek, 2009). In this present age, the study of mathematics continues to be associated with the memorization and application of near-sacred procedures or algorithms rather than with highly-developed problem solving skills. The escalated dependence on a “prescribed step-by-step set of

instructions” and the lack of comprehensive connections to real life experiences of the students often led to the misapplication of these procedures by students (Aksu, 1997; Brown & Quinn, 2007, p. 24).

Slesnick (1982) advised that the successful use of algorithms does not necessarily indicate conceptual understanding. In differentiating skill performance and conceptual learning, Slesnick declared mathematical understanding is often associated with the capacity of a student “to manipulate an arithmetic algorithm with or without understanding” (p. 143). In other words, the gauge of the students’ proficiency in mathematics has been based on their ability to recall a proper sequence of steps and provide the correct solution to a problem without questioning their understanding of the process.

As mathematical concepts increased in difficulty, students relied heavily on rote memorization of textbook algorithms or teacher-developed shortcuts to survive mathematics courses (Simon, 1986). Mathematical operations on rational numbers continues to be taught through a series of specialized rules that must be “memorized like lines of a poem, in the belief that they later direct the processes involved in computation” (Carragher & Schliemann, 1985, p. 37). One of the characteristics listed by Streefland (1982) on the instruction of fractions was the immediate presentation of and dependence on algorithms. Students were expected to perform mathematical operations on fractions without coherent understanding of the process. Teachers and students sometimes relied on non-mathematical ways, such as creating mnemonics to memorize algorithms. An example of this would be the saying of unknown origin

associated with dividing fractions, “Ours is not to reason why; just invert and multiply.”

The amount of time spent on mechanical operations or “stodgy routine work” (Godfrey, 1910, p. 232) has drawn students away from the development of conceptual understanding. This obligation of learning methods, which are often misunderstood, led to many of the mathematical difficulties students have in school (Wheatley, 1992). This lack of development of conceptual understanding and overemphasis on procedural strategies in mathematical education was summarized by Carver (1937):

Even with our best students mechanical manipulation may be greatly overemphasized, while with the poorest of them the teacher is always tempted to be satisfied with memorized processes even though he knows that the student has no understanding of what he is doing. (p. 360)

Numerous studies have been published on the difficulty students have with computations of mathematical problems involving fractions. Researchers identified incorrect application of procedure as one of the most common errors of students make in approaching mathematical operations on fractions (Aksu, 1997; Brueckner, 1928; Morton 1924). In his investigation, Brownell (1933), found similar evidence of students initiating faulty procedures in their attempts to solve addition of fractions problems. Of the four basic operations of addition, subtraction, multiplication, and division of fractions, Brueckner (1928) reported that subtraction of fraction with renaming challenged students the most.

Vinner, Hershkowitz, and Bruckheimer (1981) published a report that analyzed errors in addition of fractions. Within this report they submitted several

factors for students' miscalculations. These factors included *wrong reconstruction of details*, *misidentification*, *wrong analogy*, *wrong interpretation of symbols*, and *compartmentation*. Although this study dealt with addition of fractions, some of the factors also apply to errors that students exhibit in their attempts to subtract fractions. The researchers aligned the term, *wrong reconstruction of details*, with the perception of student forgetfulness (Vinner et al., 1981, p. 72). It implied that students displayed only partial recollection of an algorithm which led to an erroneous solution. In subtraction of fractions some students can recall the need for transformation to common denominators yet often fail to regroup or borrow from the whole correctly. Another factor leading to error, *wrong analogy*, was described as the application of logical reasoning in an incorrect manner. This error was illustrated as the use of addition throughout the problem, e.g. the student adds the denominators together along with the addition of numerators. In a problem involving subtraction of fractions, this oversight is illustrated by the subtraction of the denominators by the student. The factor labeled as *misidentification* was defined as the process in which the student used an algorithm correctly except that it was inappropriate for the problem setting, for example using the multiplication algorithm on a division problem. Students simply applied the most complete algorithm they had memorized regardless of the appropriateness of the situation.

Rittle-Johnson and Koedinger (2005) proclaimed that the strong integration of contextual, conceptual, and procedural knowledge allows people to solve problems and to recall more information than the memorization of facts or procedures. Contextual knowledge was drawn out of real-world situations as opposed to

manipulation of problems represented in a purely symbolic manner. Conceptual knowledge was defined as the ability of students to correctly apply knowledge acquired to solve “novel problems” (Rittle-Johnson & Koedinger, 2005, p. 317). Procedural knowledge was associated with the step-by-step process that when specifically followed, led students to a solution without a guarantee of their acquisition of some form of conceptual understanding.

Connectivity

Baranes, Perry, & Stigler, (1989) suggested the fundamental problem in education was the process in which children connected classroom knowledge with the outside world. The lack of connectivity between rational numbers (fractions) and the tangible world of middle school students often encumbered the conceptual understanding of this branch of mathematics. Mastery of mathematical concepts was often based on the student’s ability to acquire knowledge and apply it to life independently and appropriately. Eventually, students must extend their newly gained knowledge from the classroom setting to the “real world.” In his speech pleading for reform in mathematical education, Whitehead (1916) labeled those concepts taught to students which were of no benefit or use as “inert ideas” (p. 192). Berliner (1992) expressed the thought in a similar fashion: “If what we learn is out of context - like so much of mathematics and language as learned in school - it becomes inert” (p. 155).

Merrill (2001) wrote of four distinct phases of effective learning. One of the phases, “activation of prior knowledge,” dealt with the incorporation of new knowledge into the students’ existing knowledge (p. 462). Merrill believed that too much emphasis has been placed on use of intangible representations. He stated that

students had an inadequate foundation to understand these abstract representations. In his other phase called “integration of these skills into real-world activities,” Merrill stated that unless the knowledge and skill were integrated into the life of the student they would soon be forgotten. Zohar (2006) regarded the connection between everyday experiences and concepts studied within the school environment as critical for understanding to take place. In 1901, Langley, Godfrey, and Siddons raised the question, “How are we to fit the new teaching onto the knowledge which the boy brings to his public school?” (p. 106).

Dewey (1920) wrote:

From the standpoint of the child, the great waste in the school comes from his inability to utilize the experiences he gets outside the school in any complete and free way within the school itself; while, on the other hand, he is unable to apply in daily life what he is learning at school. That is the isolation of the school—its isolation from life. When the child gets into the schoolroom he has to put out of his mind a large part of the ideas, interests, and activities that predominate in his home and neighborhood. So the school, being unable to utilize this everyday experience, sets painfully to work, on another tack and by a variety of means, to arouse in the child an interest in school studies. (p. 67)

Whitehead (1916) wrote that continued instruction of disconnected curriculum would result in the demise of the liveliness of school curriculum and natural inquisitiveness of the student. He generalized that students would rather study something of worth than assume the roles of “intellectual minuets” (p. 196).

The process by which people arrived at mathematical solutions in non-school settings was far different than the formalized procedure expected in the classroom setting. How important was it for educators to incorporate these informal procedures in non-school settings? This question directed educators to consider the importance of the knowledge a student brings into the classroom rather than manufacture and dictate a disconnected series of *symbolic manipulations* (Baroody, 2006). Baroody added that this disconnection portrayed schools as isolated institutions which lack application to real life. One of three key *inhibitors* which thwart mathematical understanding was identified by Pogrow (2004) as the students' perception that math is "a series of arbitrary, unintelligible rules imposed by adults" (p. 298). These studies suggested that significant negative impact occurred in learning due to the artificial learning environment and lack of connectivity with students' daily lives. Forno (1929) proclaimed the need for teachers to build up a "foundation of the subject on actual experience, so that students may be able to see the reason for certain procedures" (p. 18). She continued the thought by expressing that "reasoning" rather than a reliance "on mere memorization of certain rules and formulas" may serve as preventive measures in future mathematical errors (p. 18). Lovell (1972) suggested that teachers needed to provide some structure in their instruction but stressed the importance of that structure having relevance to real life, especially for the "weakest school-educable pupils" (p. 177). He placed a great deal of emphasis on transitioning slowly from concrete operational to formal operational stages of Piaget's cognitive development theory. Lovell remarked that teachers should not forcibly attempt to

instill mathematical understanding in their students. He equated meaningful learning with the ability of the students to transform “one reality state into another” (p. 178).

Some considered the use of mathematical word problems as a means to bridge the gap between everyday mathematics and formal mathematics (Arcavi, 2002, Baranes et al., 1989, Bonotto, 2005). Nevertheless, Arcavi regarded these connections as “merely artificial disguises or excuses for applying a certain mathematical technique” (2002, p. 21). These word problems were described as “symbolic puzzles” that students considered as detached from the real-world (Baranes et al., 1989, p. 288). They also noted that even with problems presented in real-world contexts, students still had difficulty in correctly applying the mathematics needed to solve these word problems. These researchers concluded that students increased their problem solving effectiveness by their exploitation of strategies such as “mapping problems onto a quantitative system such as money” (p. 316).

Boaler (1993b) also suggested that real-world situations should be related to the mathematical operation before the introduction of any formal algorithm to the student. Many have agreed that the development of conceptual understanding prior to teaching standard procedures will produce a higher rate of understanding by students (Steffe, 1983; Baroody & Ginsburg, 1990; Davis, Maher, & Noddings, 1990; Sowder, 1998). In essence, the suggestion here was that effective understanding can be supported through the connection of students’ prior experiences and knowledge with the new knowledge such as in the case of subtracting fractions.

The lack of connections students had between school subjects and their daily lives called for a curricular integrated project titled Mathematics, Art, Research,

Collaboration, and Storytelling (Reilly & Pagnucci, 2007). The design of the project was to assist students in contextualization of their learning at a middle school in Pennsylvania. This project integrated a writing component to problem solving strategies. Students wrote stories that showed they understood the mathematical concepts presented to them within their class. Student projects demonstrated how mathematical knowledge was related to real life situations. No quantitative results were presented by the authors of the article. However the article stated that as students reviewed for their state tests, their ability to recall these stories directed their attention to the mathematical concepts associated with the stories.

Teaching and Learning

Schools were portrayed by O'Brien (1976) as factories where students were "passive receptors of information, rules, facts and recipes" (p. 93). He offered a description of mathematics education as the "transmission" of facts, rules, procedure, and nomenclature designed to arrive at solutions as quickly and accurately as possible (p. 93). Whitehead (1916) proclaimed the following:

Education is the acquisition of the art of the utilization of knowledge. This is an art very difficult to impart. Whenever a textbook is written of real educational worth, you may be quite certain that some reviewer will say that it will be difficult to teach from it. Of course, it will be difficult to teach from it. If it were easy, the book ought to be burned; for it cannot be educational. In education, as elsewhere, the broad primrose path leads to a nasty place. This evil path is represented by a book or a set of lectures which will practically

enable the student to learn by heart all the questions likely to be asked at the next external examination. (p. 195)

Worse yet was the description of mathematical learning for the student as “the subsequent capacity to regurgitate a copy of what was taught” (Kieran, 1994, p. 598). What was taught has been unfavorably described by Boaler (1993a) as “a cold, detached, remote body of knowledge” (p. 13). Barnett (1934) quipped, “As a matter of fact, for most students mathematics and common sense are not to be mixed” (p. 77). In the mathematics classroom, the learner placed a great deal of significance on a teacher’s input. At some point, the vast majority of the mathematical content being taught is intangible to the daily lives of students. Conversely, when the students were able to connect problem-solving methods to their own life experiences, they were empowered by the relevance and reality of their knowledge (Brown & Quinn, 2007).

Education has been thought of as a complex set of circumstances dealing with not only the transfer of knowledge, but also the utilization of knowledge. Educators and institutions have high expectations of the quantity of knowledge students will be required to acquire throughout their elementary and secondary years. However, a struggle exists within students when the required knowledge is not significant to them. For decades, many have called for mathematics to reflect on this need for meaning and purpose. In order to accomplish this reform, Snead (1998) recommended a movement away from teaching in the traditional fashion and towards a constructivist teacher viewpoint. The role of the constructivist teacher became one of a facilitator, assisting students in their quest to construct “meaningful mathematical knowledge for themselves” (p. 295).

Relevant arithmetic, as defined by Brownell (1947), is the “instruction which is deliberately planned to teach arithmetic meanings and to make arithmetic sensible to children through its mathematical relationships” (p. 257). He considered “meaningless arithmetic” as the opposite, possessing little or no specific purpose for students (p. 257). Students in this situation are deemed to have acquired mathematical knowledge through incidental paths. In his closing comments, Brownell (1947) declared that a “high positive correlation between meaningfulness and rate of learning holds under a very wide range of conditions” (p. 265). Simply stated, students attain a clearer understanding and retain more of the concepts being taught when they are allowed to form a personal relationship to the context of the problem presented to them.

Traditional mathematics education for many generations has been under the same instructional form where knowledge is transferred from teacher to student through an “explain-practice” method of instruction (Wheatley, 1992). Much has been said about the techniques mathematics teachers are likely to use in their classrooms. The culture of the math classroom has not changed in the past hundred years because teachers often teach in a similar manner as they were taught a generation or more ago (Bennett, 1991). Teachers typically reviewed the previous day’s homework, presented new concepts, monitored seatwork, and then assigned a new set of homework for students. This type of traditional mathematics instruction is usually described as *basic skills focused*, *procedure-oriented*, and *drill and practice* instruction. In this educational environment, the teacher is described as someone who has taken on the role of “giver” of knowledge and student as the “receiver” (Wilkins,

2000). This description is aligned with Nabhan & Trimble's (1994) portrayal of education as "crime of deception;" what is taught holds more weight than the actual experiences one brings into an educational setting (pp. 106-7).

Research that compared school cultures of 1968 and 1998 (Seaman, Szydlik, Szydlik & Beam, 2005) found that even though the school culture moved "toward a more constructivist philosophy," the teaching of mathematics remained focused on the memorization of rules, formulas, and procedures. Since the last decade, the dominant method of instruction continues to be described as the "development of rote procedural skills" (Alsup & Sprigler, 2003). Gardner (1991) noted that formal schooling tended to ignore the "organized beliefs" that children utilized in their attempts to understand the world around them. Though the school system may downplay the role of a person's intuitive understanding, Gardner concluded that it is likely to return "once the person leaves a scholastic milieu" (p. 86).

Peters (1970) defined *discovery learning* as the rules or principles that children developed for themselves in an attempt to increase their own understanding of concepts. The outcome of this type of learning was that students were more able to transfer the use of the concept to new situations. Discovery learning was subdivided into five types by Biggs (1994). She labeled these as impromptu discovery, free exploratory discovery, guided discovery, directed discovery, and programmed learning. The categories were established based on the amount of freedom or control in the learning process. On one end of the spectrum, Biggs considered impromptu discovery as the only student-initiated form of learning. Within the remaining four categories, the learning process originated from either the teachers or work cards

provided by the publishers. In these cases, the teachers had a preconceived idea of what they wanted their students to learn or discover. Programmed learning, a type of scripted instructional strategy, is positioned on the other end of the spectrum. Biggs noted that this type of discovery learning may have students completing tasks, yet it may not provide the teacher with evidence that any learning actually took place.

An analogy was constructed generations ago by Carver (1937) in which he stated that memorization and correct pronunciation of a Russian phrase without understanding is simply “making Russian noises” (p. 359). Similarly, with mathematics, the ability to recall formulas for areas of a circle, trapezoid, rectangle, or triangle does not indicate a learned fluency of geometry. The student must be able to apply that knowledge in the appropriate situation.

The range of mathematical understanding for students can be varied from having absolutely no understanding to having the ability to proficiently apply and communicate mathematical concepts in a variety of situations. Zohar (2006) outlined an outstanding characterization of “learning for understanding” (p. 1579). Zohar hypothesized that “the learner forms multiple, intricate connections among the concepts” in both academic and non-academic settings (p. 1579). Later in her work, Zohar categorized two types of learning as “surface or rote” and “deep or meaningful” learning (2006, p. 1584). This surface or rote learning only allowed the student to replicate the information that was acquired. The deep or meaningful learning described by Zohar (2006) was the evidence that the student was able to “think and act flexibly with what one knows” (p. 1585).

A student unable draw upon either previous knowledge or instruction to begin solving a problem would be considered as having no mathematical understanding in relation to the problem. The student who approached the problem incorrectly does possess some mathematical intuition. His or her strategy in solving the problem may be complete or partial, the initiative more than likely stemming from an association with another learned procedure. An example of this would be the common error students had in multiplying fractions. Students often utilized the procedure for dividing fractions, inverting and multiplying, rather than simply multiplying numerators and denominators to find their solution. In contrast, the student who demonstrated mathematical understanding would be able to lift the knowledge learned within one environment and utilize it in another setting. For example, the process of determining the amount of paint needed to paint a bedroom. The solution to this problem required an integrated knowledge and computation of surface area, proportional reasoning, and application of budgetary constraints.

Relational understanding was considered the ability of the learner to both correctly utilize and understand the rational application of a procedure for a particular situation (Mellin-Olsen, 1981). Instrumental understanding was expressed as the knowledge of a particular rule or set of rules to follow. Aksu (1997) equated instrumental understanding with the ability to perform mathematical calculation, computation “without connections to concepts or conceptual rationale” (p. 375). Relational understanding defined by Skemp (1978) was the combination of a student’s knowledge as to what to do and why. Relational and instrumental understandings were distinguished by their respective inclusion and exclusion of

connective “conceptual rationale[s]” (Aksu, 1997). Although instrumental learning ignores the vital relationship between intellectual reasoning and new material, it is not useless; Skemp suggested that instrumental instruction restores a student’s self-confidence more successfully than relational instruction (p. 8).

The differences between these two types of understandings may be demonstrated in teaching the area of circles. A student may be provided with the simple formula, “area of a circle is equal to pi times radius squared.” Suppose the student solved 20 problems perfectly when diagrams show circles with the radii appropriately labeled; however, when problems were presented displaying a circle’s diameter rather than a radius, the student was confused and did not know how to approach it on his or her own. Unless the teacher continued with an instrumental teaching of another formula for finding the area of a circle, one that deals with the use of the diameter, students must rely on their relational understanding of circles that a diameter is twice the length of a radius to build their own procedure to find the area of a circle.

Aksu (1997) represented the terms *procedural* and *conceptual knowledge* in the same manner that Skemp used *instrumental* and *relational understanding*. Procedural knowledge remains centered on the rules for calculating solutions to particular situations. The difference between Aksu’s *procedural* and *conceptual knowledge* was the transferability of conceptual knowledge to other mathematical concepts and ideas. Aksu believed that the goals of mathematics education should be the development of both procedural and conceptual knowledge. A study that evaluated the effectiveness of a cognitive tool, a prototype computer program, on the

comprehension of procedural knowledge within the concept of addition and subtraction of fractions with unlike denominators suggested conceptual understanding of equivalent fractions precedes the use of the cognitive tool for procedural knowledge (Kong, 2008).

In addressing the corrosion of conceptual understanding in students, some educational researchers suggested a movement of guiding students towards the building or inventing of their own algorithms (Huinker, 1998; Streefland, 1982). The major advantages for students listed by Huinker were increased interest in problem solving, utilization of multiple strategies, improved proficiency in dealing with different type of mathematical representations, and efficiency in communication skills through the students' thinking and reasoning processes (p. 181). The consideration for utilizing the knowledge of the child goes further. Streefland (1982) supported the proposal of instruction on fractions to be a gradual learning of algorithms. This aligned with the principle of "progressive schematization," which called for a smoother rather than an abrupt approach to the use of abstract algorithms by students (Freudenthal, 1981).

Beatley (1950) concluded his article with the use of an analogy. He portrayed the current education of a child as a tidy house. In an educational environment that was considered "neat and orderly," Beatley (1950) advocated a need for students to display some initiative in their learning, to clean an "untidy room." The analogy used called for students to depend less on the "guide" or textbook and use of a trial-and-error method of problem solving. Beatley believed that through participation, students would develop ownership of their education.

Informal knowledge

Students have not entered classrooms across this nation void of any knowledge. The development of some of their knowledge has been outside of the formal school setting. The formal or school mathematics is regarded as the transitional bridge that carries the students from their informal knowledge to the symbolic and abstract concepts of mathematics (Ginsburg & Amit, 2008).

Resnick (1987) correlated *informally acquired knowledge* with that knowledge people obtain through engagement within social contexts outside of the school environment. She contrasted school learning as mostly “symbol-based” which disconnected people from objects and events outside the school classrooms (p. 14). The acquisition of socially based knowledge was also studied by Saxe (1988). He found that unschooled urban children selling candy in Brazil performed more adequately in transactional subtraction problems than urban nonsellers. Saxe concluded that these street vendors created their own problem-solving procedures through adaption to their everyday experiences. Carraher and Schliemann (1985) provided the label “natural routines” for the procedures children use instead of school-taught procedures, as in circumstances comparable to the Brazilian street vendors (p. 43).

Leinhardt (1988) regarded *intuitive knowledge* as an application-based “circumstantial knowledge” built by the students through their real life experiences and applied to problem scenarios that appeared to be familiar to them. This type of knowledge was described as being somewhat disorganized and inefficient. Leinhardt

believed instruction based on intuitive knowledge would be difficult due to the highly contextualized nature of the experiences.

Situated cognition (Brown, Collins, & Duguid, 1989) and other designations have been given to nontraditional or alternative procedures students employ in their attempts to solve mathematical problems. Mack (1990) consolidated and characterized different types of knowledge that related to real-life situations as *informal knowledge*. She defined this type of knowledge as knowledge built by students through their experiences. Students applied this knowledge either correctly or incorrectly in problem situations that appeared familiar to them.

Cognitively guided instruction (Chambers & Lacampagne, 1994) was presented as an instructional strategy that centered on allowing students to approach problem solving through formal or informal means. Students were “encouraged” to employ a variety of strategies and provide an explanation of their problem solving process to an audience of their teachers and peers. The results of this study indicated that students of teachers who trained under cognitively guided instruction were able to recall number facts more effectively than the control group of students. Cobb & Steffe (1983) wrote that constructivist teachers continuously examine instructional exchange from the students’ points of view. This scrutiny of verbal and nonverbal exchanges is meant to increase the probability that students will interpret correctly the intentions of the teacher.

Summary

Schaaf (1945) warned his readers of overemphasizing proficiency in computational skills and intellectual capacity for abstract mathematical principles. He

stated that the “ultimate outcome of instruction” should be focused on the integration of mathematics, students, and “the realities of life” (p. 407). Although the topic of this study was narrowly focused on the subject of subtraction of mixed numbers, the majority of the literature review dealt with strategic approaches to improve student achievement. These articles and studies proclaimed the complexity of teaching and learning. This particular study was conducted along the same line as Brownell’s (1933) study on a mathematical “crutch” method used by students on the addition of fractions. It provided a description of the effect the use of a nonstandard algorithm for subtracting mixed numbers had on student understanding and retention.

Chapter Three: Research Process and Methodology

Research Design

This study employed a quasi-experimental nonrandomized pretest-posttest method. This approach was chosen because it provided the format that did not disrupt the typical school situation (Ary, Jacobs, Razavieh, & Sorensen, 2006). School and class schedules were not rearranged to create a randomized control and treatment group. The composition of each class remained under the control of the site administrator.

In an informal manner, this inquiry began years ago when one student expressed her frustration with the disconnection between the concept being taught and her personal life. For nearly two decades, this researcher has incorporated the instruction of the transactional algorithm alongside the traditional algorithm in teaching subtraction of fractions. The effect of this intuitive instruction led this researcher to formalize the experience by collecting, analyzing, and reporting on the findings from various school sites and classrooms. The goal of this research was to evaluate any differences in student achievement through the use of the transactional or alternative algorithm for subtraction of mixed numbers.

Selection of Sites

The mathematics classes for this research were chosen from three middle schools in the southwest Riverside County region of California. These three schools were part of two different school districts. One of the sites, School A, was where the researcher was employed as a mathematics teacher. The second school site, School B,

was the oldest middle school in the same district as School A. The selection of this particular site was based on the professional relationship the researcher had with the site administrator and the participating teacher who volunteered to take part in this study. The third middle school site, School C, used in this study was located within another school district. Once again the selection of this school site was based on the professional relationship developed between the site administrator, participating teachers and this researcher.

The general demographics of each school are displayed in Table 1. The following descriptive data for each of the schools were from the 2007 – 2008 school year. School A had a student population of 992 in sixth through eighth grades; of these students, 41% were Caucasian, 5% were African American, 26% were Hispanic, and 28% were divided among other ethnic groups. Approximately 8% of students attending School A were considered English Learners. An estimated 18% of the students qualified for free or reduced meals. Finally, there was a 23 to 1 student-to-teacher ratio in sixth through eighth grades for School A.

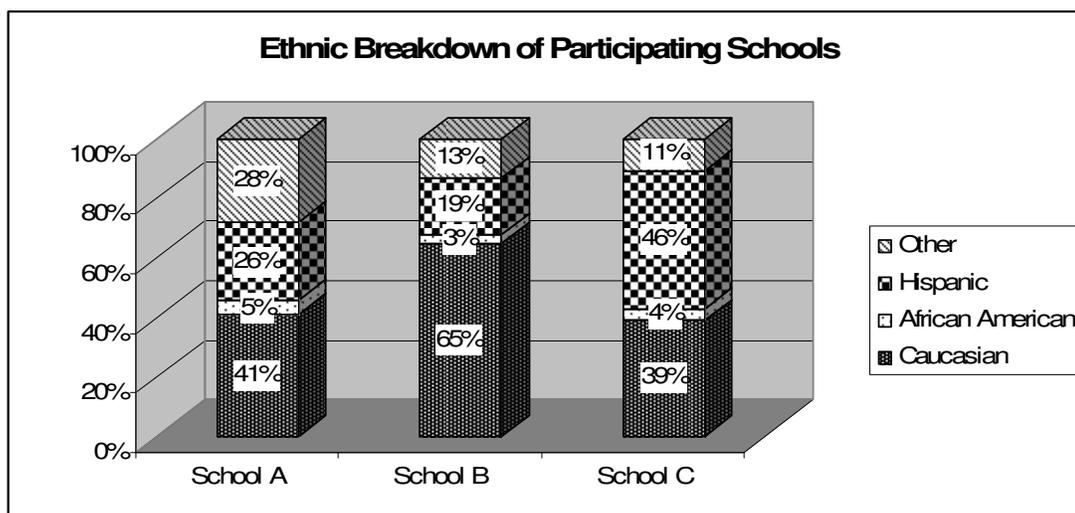
School B had a student population of 1370 in sixth through eighth grades; of these students, 65% were Caucasian, 3% were African American, 19% were Hispanic, and 13% were divided among other racial groups. Approximately 5% of the students were considered English Learners. An estimated 11% of the students qualified for free or reduced meals. There was a 21 to 1 student-to-teacher ratio in sixth through eighth grades.

School C had a student population of 1505 in sixth through eighth grades; of these students, 39% were Caucasian, 4% were African American, 46% were Hispanic,

and 11% were divided among other ethnic groups. Approximately 16% of the students were considered English Learners. An estimated 35% of the students qualified for free or reduced meals. There was a 24 to 1 student-to-teacher ratio in sixth through eighth grades.

Figure 1

Ethnic Breakdown of Participating Schools



Teacher Participants

Initially six teachers expressed interest in participating in this study. One of the teachers withdrew from the investigation stating that she was overwhelmed by her job-related tasks. Classes that participated in this study came from teachers and students representing three different school sites and two different school districts. Only the researcher taught the transactional algorithm to his classes. The other teachers utilized the traditional algorithm for subtraction of fractions.

Two teachers who participated in this study came from the same school, School A. The teachers included the researcher (Teacher A) and the other teacher

(Teacher B). Both teachers were assigned to teach mathematics to seventh graders. The third teacher (Teacher C) was assigned to another school within the same district as Teacher A and Teacher B. This teacher was also assigned to teach mathematics at the seventh grade level. The fourth and fifth teachers were employed within a different school district. They were assigned to teach mathematics at the same school site. The fourth teacher (Teacher D) instructed both seventh and eighth grade mathematics including an algebra course. Teacher E taught sixth and seventh grade mathematics.

Student data from the fifth teacher's classes were eliminated from this study. Inspection of the data received from this teacher's classes revealed many of the students did not provide work or evidence that supported their solutions. Additionally, there were indications that students cheated on the quizzes by copying solutions from one another or received assistance from an outside source. An investigation revealed that a substitute teacher had been in charge of the classes for that particular day of testing.

The Teacher Background Questionnaire (Appendix A) was collected from the participating teachers along with other math teachers at the four school sites. Data were also collected from anonymous mathematics teachers attending a conference in San Diego, CA sponsored by the Greater San Diego Area Math Council. Additional surveys were sent to three other school sites within the southwest Riverside County region.

Information from these questionnaires indicated that each of the four teachers who participated in the initial investigation had, on the average, over 20 years

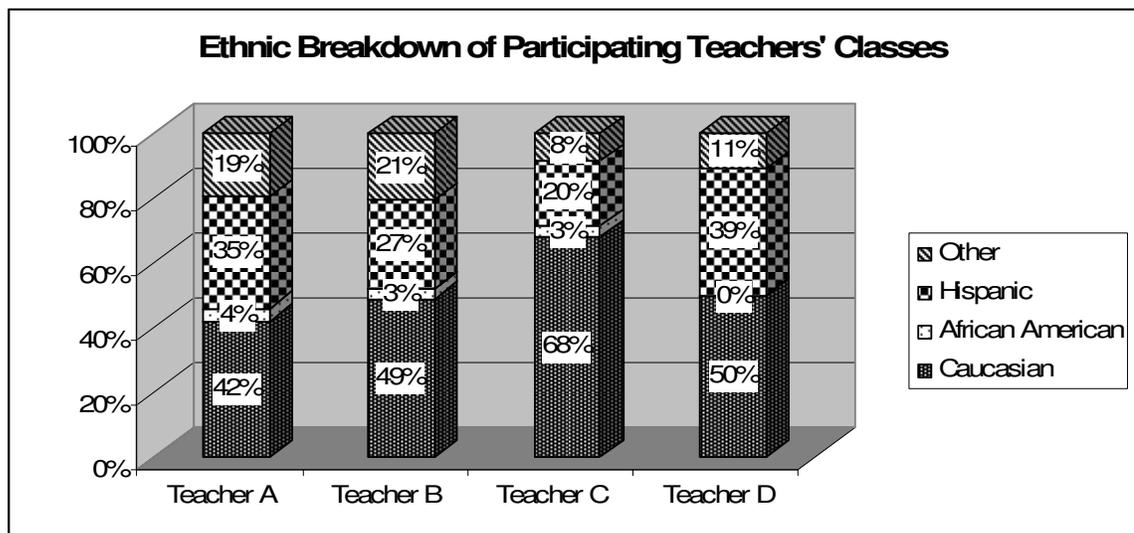
teaching experience. These teachers were also considered to be “highly qualified” under the guidelines of the national No Child Left Behind Act of 2001.

Student Participants

The primary function of this investigation was to compare the effectiveness of various algorithms for subtracting fractions as seen from the perspective of middle school students. The general demographics of each teacher’s classes are displayed in Table 2. These students were solicited from three middle schools in two different school districts within the Riverside County region of southern California.

Figure 2

Ethnic Breakdown of Participating Teachers’ Classes



A one-way ANOVA was used to test for differences in mathematical aptitude of students based on the Performance Levels assigned to students from the 2008 California Standards Test for mathematics. This test found student scores from Teacher B’s classes to be statistically significant, $F(3, 526) = 13.44$, $p = 0.0000000183$. Tukey post-hoc comparisons of the four groups, in Table 1, indicated

that students from Teacher D's classes were collectively at a lower level of mathematical aptitude than all of the other students who participated in this study.

Table 1

Comparison of Mathematical Aptitude

Post hoc analysis

Tukey simultaneous comparison t-values (d.f. = 526)

		Teacher D	Teacher A	Teacher C	Teacher B
		3.2	3.7	3.8	3.9
Teacher D	3.2				
Teacher A	3.7	4.22			
Teacher C	3.8	5.16	0.89		
Teacher B	3.9	5.90	1.92	1.12	

critical values for experiment wise error rate:

0.05	2.60
0.01	3.18

The analysis also revealed no statistically significant differences among students from classes under Teacher A, Teacher B, and Teacher C.

Instruments

The development of the instruments used in this study was a collaborated project with Teacher D. The instrument began as an attempt to indentify students for a school wide mathematics intervention class. The initial instrument used by mathematics teachers was an online product from Renaissance Learning™ called STAR Math™. This online assessment tool used 25 problems to progressively measure student performance. These assessments revealed that most students struggled in the area of fractions. The multiple choice format of this assessment tool proved to be a hindrance in identification-specific rationale for students incorrectly

solving problems. Teachers could not categorize the common types of errors students made in their computations.

Teacher D and the researcher recognized the need to develop a free response form of assessment to analyze student work in the area of fractions. The revised instrument became a 20 problem quiz that covered addition, subtraction, multiplication, and division of fractions. Field testing of this instrument resulted in almost complete disaster. Most of the problems were not even attempted by the students. The field test revealed that students were unable to complete all of the problems in one class period. Results of the field tests were ambiguous from the limited amount of data, whether the students had partial or complete understanding of any of the operations on fractions.

Additional discussion between Teacher D and the researcher about the assessment tools resulted in an agreement to focus specifically on the area of subtraction of mixed numbers. The rationale for honing in on this one particular mathematical operation was based on discussion of the computational skills needed to process this problem type. The working knowledge required by students to correctly solve these problems included understanding of common denominators, equivalent fractions, recognition of the need to borrow or regroup, and solving addition of fractions.

The final instrument of four problems, equally presented in horizontal and vertical format, was agreed upon by Teacher D and the researcher. The decision also included the use of the rubric scoring system rather than a simplified correct or incorrect indication for each of the problems attempted. The rationale for the rubric

scoring system was that the differentiated scores provided a better representation of the level of understanding students had in solving subtraction of mixed numbers.

The assessment tool was previewed and endorsed by three middle school mathematics instructors and one retired mathematics specialist. The instrument was field tested in the same format as the researcher's classes in the prior school year. Test 1 was included in Student Quiz & Survey (Appendix B).

Data Collection Process and Methodology

Research Question #1. Are there statistically significant differences in the achievement levels (as measured by the researcher-developed assessment instrument) between students who only received instruction with traditional algorithms for subtraction of mixed numbers and students who received instruction with both traditional and alternative algorithms for subtraction of mixed numbers?

Null Hypothesis 1-H₀: There will be no significant difference in mathematics achievement between students who only received instruction with traditional algorithms for subtraction of mixed numbers and students who received instruction with both traditional and alternative algorithms for subtraction of mixed numbers.

In addition to the literature review on the topic of mathematical operations on fractions, the researcher analyzed student performance on fraction problems from the Student Quiz & Survey (Appendix B). These four problems on the student surveys were known as Test 1 throughout this research. Data were also collected from the Subtraction of Fractions Test 2 (Appendix C) administered to seventh grade students from Teacher A and B. In addition to tabulating the percentage of correct answers in Test 1 and Test 2, the researcher utilized an eight-point rubric scoring system from

the Rubric for Evaluation of Fractions Quiz (Appendix D) to measure the level of student competence in the problem solving process.

The student assessment consisted of four subtraction problems on the Student Quiz & Survey (Appendix B). These fraction problems were presented in both vertical and horizontal layouts. The directions explicitly told the students to show their work in the space allocated on the assessment. A review of the work provided by the student separated the data into two categories of algorithms, either traditional or transactional. The data from students who showed evidence of attempting to solve problems using both algorithms were classified within the transactional algorithm category.

Rather than simply categorizing student solutions as correct or incorrect, this researcher was concerned with the process students used in their problem solving techniques. An eight-point rubric from the Rubric for Evaluation of Fraction Quiz (Appendix D) was used to numerically qualify the process that students used to arrive at their correct or incorrect solutions. This scoring system was designed to distinguish the difference between an incorrect answer caused by a simple miscalculation and one caused by a faulty understanding of the appropriate strategies to solve the problem. Students were given a score of eight points if their procedure was logically utilized and the solution was correct, regardless of which algorithm was used by the student.

A score of six points indicated a simple error was committed by students. The assignment of six points was based on the assumption that students needed just one additional step to arrive at a correct solution. The most common example of solutions awarded six points was the failure to simplify fractions to the lowest terms. Another

rationale for awarding six points was simple arithmetic errors performed by students. In both of these cases, students were considered to have reasonable knowledge of the procedure necessary to solve the problem correctly.

A score of four points indicated that multiple, yet minor, errors were present. The majority of the students who received a score of four points both made calculation errors and failed to simplify their fractions. Another error type within this score included the failure of students to arrive at the correct common denominators.

A score of two points indicated that students had made major errors in their attempt. The initial procedure utilized by students in this case clearly indicated a confused knowledge of procedures necessary to approach the problem. Students who received a score of two points typically knew that the problem involved the transformation of the fractions to equivalent fractions with common denominators. The error exhibited by majority of the students was a computational mistake. Students either subtracted the smaller valued numerator from the larger valued numerator or used the base-ten form of regrouping, incorrectly placing a one in front of the numerator.

Also, students who received a score of zero exhibited little or no evidence of conceptual understanding of operations on fractions. These students tended not to transform fractions into equivalent fractions with common denominators. They also tended to subtract numerators from numerators and denominators from denominators without converting the fractions into equivalent fractions with common denominators. Students who wrote an answer without any evidence of process received zero points.

Research Question #2. Are there differences in computational achievement between students using the traditional algorithm and those using the transactional algorithm over a course of 16 weeks?

Null Hypothesis 2-H₀: There will be no significant difference in mathematics achievement between students using the traditional algorithm and those using the transactional algorithm over a course of 16 weeks.

The researcher conducted a long-term study (16 weeks) on the effectiveness and retention of algorithms used by his students through the weekly assessments. These weekly assessments contained 15 problems; one of these problems required students to subtract fractions in a format that necessitated students to borrow, regroup, or rename the fractions. The results from this assessment were categorized by the types of algorithm and whether the student succeeded or failed to arrive at the correct solution. Only data that clearly showed the student's attempt to solve the problem were included in this part of the research. If no attempt was made, the data were discarded. As the quizzes or assessments were graded, data were recorded into the weekly grade sheets. The data were separated into four categories: traditional algorithm with correct solution, traditional algorithm with incorrect solution, transactional algorithm with correct solution, and transactional algorithm with incorrect solution. According to standard classroom procedure, the weekly quizzes were graded, the data were collected, and papers were returned to the students. The timeframe of data collection lasted for 16 weeks, including one break of one week and another break of two weeks.

The student assessments, (Test 1 and Test 2), and teacher questionnaires were administered at the school sites within a designated number of instructional days from the beginning of the school year. The number of instructional days for all school sites ranged from 65 to 70. This range of instructional days also applied for the two teachers working under the year-round school schedule. This particular range of days was based on when the school sites were expected to provide instruction to their seventh grade students on the concept of subtraction of fractions.

Distribution and Collection of Data

The researcher was in charge of the distribution and collection of test materials at School A. Since there was only one participating teacher, Teacher C, at School B the researcher made arrangements to distribute and collect test materials with Teacher C. Teacher D was assigned as the lead teacher for School C. Test materials were delivered to Teacher D who in turn distributed the materials to the other participating teacher at the site. All test materials were picked up by the researcher from the three school sites.

Once the surveys and tests were collected from the participating teachers, all items were transcribed into digital format. To protect student privacy, names were replaced with numeric identification codes. Three working copies of this digital information were utilized during the study. The copies were distributed among a variety of electronic storage devices: a computer, a flash memory drive, and rewriteable compact disc. The original paper documents of data and resources have been carefully safeguarded and will remain so until six months after the successful defense of this dissertation. Likewise, the digital data will be stored on a designated

memory drive during this period. After a full year has elapsed from the successful defense of this dissertation, the data stored on the flash drive will be deleted.

However, the information stored on rewriteable compact disc will be carefully preserved for an additional two years.

Coding, Evaluation, and Interpretation

This quasi-experimental research collected data on the use of algorithms for subtraction of fractions from multiple sources. This research utilized Test 1 and Test 2 for data collection. A comparative guide, Rubric for Evaluation of Fractions Quiz (Appendix D), was used by the researcher to score student solutions. For each problem, the student received a score ranging from zero to eight points reflecting the competency level for completing the task of subtracting mixed numbers which required borrowing or renaming. A composite score of 32 points indicated that the student exhibited complete understanding of the procedures necessary to successfully arrive at the correct solution for all four problems. A composite score of zero indicated that a student's effort or approach was clearly illogical in solving problems on subtraction of fractions. Scores between these two extremes revealed the relative competence or understanding of the essential procedures needed to arrive at the correct solution of the problems.

The longitudinal study of 16 weeks was conducted within Teacher A's classes. The researcher evaluated these problems differently than Test 1 and Test 2. Problems were either marked correct or incorrect. Records were maintained on a weekly basis to document the success or failure to solve a problem on subtraction of mixed numbers requiring borrowing or renaming. For each student the data was transcribed

into four categories: traditional algorithm with correct solution, traditional algorithm with incorrect solution, transactional algorithm with correct solution, and transactional algorithm with incorrect solution. At the end of 16 weeks, data from each student were consolidated to determine their success rate based on the use of algorithm type. The definition of success rate in this portion of the study meant the student was successful at correctly solving the problems at least 50% of the time they use a particular algorithm on the weekly assessments on the subtraction of mixed numbers.

Justification of Analysis Methodology

No suitable instrument was found in the literature that directly corresponded to the needs of this study. The researcher developed a four-problem assessment based on a pilot study conducted the previous school year. These four subtraction problems on mixed numbers required students to borrow or rename in order to arrive at a correct solution. The presentation of the problems was equally arranged in both vertical and horizontal format. This researcher-developed assessment was previewed and endorsed by three middle school mathematics instructors and one retired mathematics specialist. The researcher had also field tested the same format with his classes a year prior to data gathering for this study.

Statistical Analysis Procedures

The data collected from this quasi-experimental study necessitated several methods of statistical analyses. This study sought to measure the effectiveness of algorithms students used on four fraction problems. The researcher utilized an eight-point rubric scoring system from the Rubric for Evaluation of Fraction Quiz

(Appendix D) to measure the level of student competence in problem solving process. The analysis of variance (ANOVA) was used to compare the means of accuracy of students utilizing two types of algorithms on subtraction of fractions from the three different school sites. The alpha level was set at 0.05.

The nominal scale best describes the data collected for the longitudinal study of the effectiveness of students' use of a particular type of algorithm. This study did not isolate a particular group as the control or experimental group. Students were allowed to utilize the algorithm of their choice from one week to the next. Sixteen weeks of data were obtained on the effectiveness of the type algorithm used by each student. The data was categorized by the choice of algorithm used to correctly solve the fraction problems. The comparisons of individual successes were best described with the use of percentiles as opposed to the comparison of the means.

Threats to Internal Validity

History. The history effect or "disruptive factors" (Ary et al., 2006, p. 292) are considered as those events occurring during the experiment that may have influenced the outcomes of the study. In this study, teacher participants were asked to administer Test 1 within the same two week period. The scheduling of the assessment was based on the scope and sequence of the seventh grade mathematics curriculum for the researcher's school district. Specifically, the date for administration of Test 1 was based on the completion of mathematics lessons on subtraction of fractions. The conjecture of this researcher was that events within each classroom and school were very similar in nature. No significant incident, other than the issue of the substitute teacher being in charge of Teacher E's classes, was reported to the researcher. As

previously mentioned, data from the fifth teacher was removed from this study due to several assessments reflecting blatant duplication of either correct or incorrect methods. There was also evidence that students received outside assistance from either the substitute teacher or teacher's aide in solving the fraction problems.

The presentation of the lesson on subtraction of mixed number differed for the researcher's classes. The researcher presented separate lessons on the use of traditional and transactional algorithms for subtraction of mixed numbers. After these lessons were presented the students within the researcher's classes were allowed to choose for themselves which algorithm to use in solving fraction problems that required regrouping. The threat of history had no effect on the portion of the study that investigated the longevity effectiveness of algorithms. The probe into the longitudinal effectiveness of the algorithms on student achievement occurred over a period of sixteen weeks for Teacher A's classes.

Maturation. This type of threat dealt with biological or psychological changes as a function of the passage of time (Ary, et al., 2006). The scheduling of Test 1 coincided with the sequence of district expectancies for seventh grade mathematics for the three participating teachers from the same school district. The scheduling of Test 1 for Teacher D and Teacher E was affected by the year-round calendar utilized within their school district. Since the number of days of instruction ranged from 65 to 70 days at all of the school sites involved in this study, the threat of maturation was considered minimal.

Testing. Repetitive use of the same form of test may affect the results; this was considered the *testing effect* (Ary et al., 2006). This study utilized equivalent

forms of the assessment for the Test 1 and Test 2. This decision lessened the threat of testing effect. All of the participating teachers used Test 1 in December 2008. The large interval of time, five months, also lessened the testing effect. Test 2 was administered by the two seventh grade mathematics teachers at the researcher's school site. The longevity study was only conducted with Teacher A's classes. The use of equivalent forms of the weekly fraction problem also minimized this testing threat during the longevity study.

Instrumentation. To maintain reliability of the instruments, the research relied on the same free response type of assessments rather than blending different types of assessments throughout the study. These assessment instruments were designed to capture the depth of students' knowledge and application of knowledge in their attempts to solve subtraction of mixed numbers. Measurement of the different levels of understanding required the use of a rubric scoring system. By using the same forms throughout this study, the threat of instrumentation was minimized.

The threat of instrumentation was also minimized for the longevity study on the effectiveness of traditional and alternative algorithms. The researcher utilized the same horizontal format in presenting the subtraction of mixed numbers throughout the sixteen week period.

Statistical regression. The description of this threat was the tendency of participants to achieve scores which regressed towards the mean on a second measure (Ary et al., 2006). The heterogeneous assignment of students into Teacher A's and Teacher B's classes, based on scholastic achievement, minimized this type of threat.

An analysis of the mathematical aptitude levels showed that students from Teacher A's classes and Teacher B's classes were not statistically different.

Students who participated in the longitudinal effectiveness study were allowed to choose the type of algorithm from week to week. The effect of this threat may have impacted the longitudinal study if a considerable portion of the students with high mathematical ability utilized the transactional algorithm throughout the sixteen week. However, an application of a one-way ANOVA (see Table 2) confirmed no significant difference existed in the mathematical aptitude between students who consistently used the transactional algorithm and students who used the traditional algorithm during the longitudinal study.

Table 2

Mathematical Aptitude Comparison – Longitudinal Study

	<i>Mean</i>	<i>n</i>	<i>Std. Dev</i>	
	360.2	70	63.38	Traditional Algorithm Users
	378.9	66	55.29	Transactional Algorithm Users
	369.3	136	60.11	Total

ANOVA table					
<i>Source</i>	<i>SS</i>	<i>df</i>	<i>MS</i>	<i>F</i>	<i>p-value</i>
Treatment	8,840.67	1	8,840.668	2.61	.1085
Error	453,706.68	134	3,385.871		
Total	462,547.35	135			

Selection. As noted in the demographics, students from Teacher D's classes were collectively at a lower level of mathematical aptitude level than all of the other students who participated in this study. This may have affected the results from Test 1 of this study.

Experimental mortality. For the comparative study of the four different teachers' classes, the threat was minimized due to the single administration of the test. For the longitudinal study, the displacement of two students during the course of the study had little impact on the reliability of data collected. The addition of three new students during the study may have posed a minimal threat to the study since they have not received a formal instruction in utilizing the transactional algorithm for subtraction of mixed numbers.

Selection-maturation interaction. There was no threat of this category for the students from the same school district. A minimal threat may have been produced from students from the neighboring district which utilized the year-round calendar for its schools. The students under the year-around calendar received twenty to thirty additional days of instruction compared to the non year-round school systems.

Experimenter effect. This unintentional effect of the researcher's bias would have the greatest impact on this study. The assumption of this study was that no significant differences existed with respect to instructional strategies among the participating teachers. The researcher was aware that his own bias toward what he considered as a better algorithm may have influenced the rubric scoring of the tests. To address this threat, the same score was assigned or given for correct solutions to problems regardless of the type of algorithm used. In the researcher's own classes, no student was forced to use or penalized for utilization of one algorithm over or another. For students of other teachers, the decision of which algorithm a student would use was of no consequence since none of the teachers taught the alternative or transactional algorithm to their students during this school year.

Subject effects. The Hawthorne effect would have occurred within all of the participating teachers' classrooms where students would put forth a concerted effort to please their teachers by performing at a higher achievement level. The researcher detected an irregularity in the data collected from Teacher E's classes. Several tests were submitted with the same exact incorrect solutions. These tests required students to "show their work" on the paper provided to them. The tests in questions had the same exact work written in support of their solutions. Additionally, several more tests has evidence that more than one person had attempted to solve the problems. There were indications that students put forth less than their normal effort in completing the assessment. The assumption for this researcher was that students cheated by collaborating together on the assessments. An inquiry into the administration procedure of the assessment revealed that a substitute teacher had taken over for the normal teacher on that particular day. For the integrity of this study, the data from Teacher E's classes were eliminated from this study.

Ary et al (2006) stated that the John Henry effect is likely to occur in a study comparing a conventional method to an innovative method of instruction. The John Henry effect which is also known as the compensatory rivalry is associated with increased performance efforts on the part of the control group. This researcher avoided making announcements about the purpose of the tests or what other students were participating in this study.

For the longitudinal study both the Hawthorne and John Henry effect may have posed a threat. Students who utilized the standard or traditional algorithm may put forth an unusual amount of effort to compete against those students who used the

transactional algorithm. Students who used the transactional algorithm may have viewed the importance of “doing well” as a means to please their teacher.

Diffusion. This threat was minimized by the fact that the students involved in this study came from different teachers and schools which removed opportunities to share instructional strategies prior to the administration of the assessments.

Ethical Issues

All participants of this research were treated in a fair and ethical manner. No student was either punished or rewarded for utilizing a particular algorithm on any of the assessments.

Summary

This quasi-experimental study investigated the differences of student achievement in subtraction of mixed numbers based on their use of either the traditional or the transactional algorithm. The study involved seventh grade students at three different middle schools in southern California. The initial assessment of student performance was analyzed soon after the instruction of the concept utilizing both the traditional and the transactional algorithms. A second assessment was administered five months later to collect data on the retention of the procedures by the students at the same school site. Data from the researcher-developed instruments were analyzed through the use of analysis of variance (ANOVA). Details of the results from this study were presented in Chapter Four.

Chapter Four: Results

Results

Teacher questionnaires and student assessments were the instruments used to determine the most common algorithm used to solve subtraction of fractions problems. Information retrieved from the Teacher Background Questionnaire (Appendix A) provided the researcher with insight into the teachers' own mathematical education. These questionnaires also provided information about teachers' opinions on instructional practices as they were applied to subtraction of fractions. Data from these questionnaires were consolidated in the Results of Teacher Survey (Appendix E). Likewise, student quizzes and surveys identified the students' preferred methods for subtracting fractions, measured competency in the procedure, and analyzed the students' confidence in their abilities. Finally, the long-term study on the effectiveness of the transactional algorithm was analyzed through the students' performances on weekly quizzes in Teacher A's mathematics classes over 16 weeks.

The teacher questionnaire solicited information about the teachers' educational background such as undergraduate major and number of years as a teacher. Teachers were asked to review four examples of strategies for subtracting fractions and identify which one was deemed to be the most common algorithm used in classroom instruction. Teachers were also asked to determine which algorithm best matched the method they used in their roles as students to solve subtraction of fractions. Teachers were also solicited for their opinion as to which algorithm had a stronger possibility for arithmetic errors by students.

In the Teacher Background Questionnaire, (Appendix A), teachers were asked to identify the common algorithm for subtracting fractions. By an overwhelming margin, the teachers recognized the Traditional Algorithm - Decomposition (Appendix F) as the most common algorithm used to subtract fractions.

The first step of the decomposition algorithm converted the fractional elements of the problem into equivalent fractions with common denominators. Assuming that “borrowing” was required, the next step of the decomposition procedure renamed the minuend as a whole number and a mixed number. Then, the mixed number must be converted into an improper fraction. With common denominators established, the next procedure required the subtraction of numerators to produce the fractional solution. This fractional element was simplified to the lowest terms if necessary and added to the difference of the whole numbers to arrive at the final answer.

The method which converted both mixed numbers into improper fractions was regarded as the second most popular strategy for solving subtraction of fractions problems. This method is presented in Traditional Algorithm - Conversion to Improper Fractions (Appendix G). After the initial step of conversion of mixed numbers into improper fractions, the next process was the transformation of these improper fractions into equivalent fractions through the conversion to common denominators. The difference between the numerators was placed over the common denominator. The next step was to simplify the improper fraction into a mixed number. Once again, if needed the final step required that the fractional element was expressed in the lowest terms.

The four examples of subtracting fractions algorithms were presented to the teachers. The reverse side of the questionnaire used a Likert-scaled response format asking teachers to rate statements about these algorithms along with their general views of mathematics learning, teaching, and resources. The Method One algorithm represented the algorithm that transformed the mixed numbers into improper fractions as demonstrated in Traditional Algorithm - Conversion to Improper Fractions (Appendix G). The next process in this algorithm was to convert the improper fractions into equivalent fractions with common denominators. The example proceeded into displaying subtraction of the numerators. The final step in this process was the simplification the answer into a mixed number.

Teachers were surveyed to discern which of the traditional algorithms were, in their opinion, the most popular. Data from the questionnaires were consolidated into Table 3. Results indicated that 13% of the teachers agreed strongly that the most popular algorithm for subtraction of fractions was the method that converted mixed numbers into improper fractions. The same question also yielded an outcome that 17% of the teachers disagreed strongly that this algorithm was the most popular.

The Method Two algorithm resembled the regrouping or renaming strategy of whole number subtraction. Fractional elements were converted into equivalent fractions with common denominators. The next process in this algorithm was to determine whether or not to “borrow” from the ones unit. Since these problems required borrowing, the example showed extraction of one from the minuend. The value of one was converted into a fraction with common denominator. Addition of the numerators produced an improper fraction. The subtrahend is then subtracted from

the improper fraction. If needed the fractional element must be simplified to the lowest terms. The final step with this algorithm was to combine whole number and fractional element.

Table 3

Most Popular Algorithm from Teacher Questionnaires

	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree
Method One, traditional – decomposition algorithm	17%	22%	9%	39%	13%
Method Two, traditional – conversion to improper fractions algorithm	0%	22%	17%	35%	26%
Method Three & Four, transactional algorithms are new to me.	4%	17%	9%	48%	22%

For Method Two, the regrouping or renaming strategy, 26% of the teachers regarded the algorithm as the most popular with no one disagreeing strongly. Thus conclusion was drawn that the regrouping or renaming strategy was more than likely to be the algorithm most teachers were accustomed to utilizing in subtracting mixed numbers.

Methods Three and Four represented the transactional algorithm. Method Three was configured in a horizontal format and Method Four into a vertical format. The process showed subtracting the subtrahend mixed number from the next higher whole number resulting in a fractional solution. Next, the fractional solution was then added to the fractional element of the minuend using a common denominator. If needed, this fraction was simplified into a fraction with lowest terms.

The transactional algorithm as presented in Transactional Algorithm (Appendix H) is also known as Method Three and Method Four in the teacher questionnaires. The results from the questionnaires determined that 70% of teachers either agreed or strongly agreed that this algorithm was considered “new” to them. In contrast, only 4% of teachers indicated that they strongly disagreed with the statement. This signified that a small portion of the teachers had some knowledge of this strategy for subtracting fractions.

When teachers were asked about the algorithm they used for solving subtraction of fractions, 56% stated that they use the same algorithm they learned in junior high or middle school. This opinion was affirmed with 63% of teachers agreeing or strongly agreeing that textbook algorithms for subtraction of mixed numbers have remained unchanged over the years.

Many previous studies on fractions have analyzed the errors common to particular operations on fractions. Several of these studies indicated students had extreme difficulty in solving fraction problems that required borrowing or renaming (Bottge & Hasselbring, 1993; Brueckner, 1928; Guiler, 1946; Ramharter & Johnson, 1949; Scott, 1962). In their research, Ramharter & Johnson (1949) sought to find any differences in “methods of attack used by ‘good’ and ‘poor’ achievers” in solving and correcting errors involving subtraction of fraction problems (p. 586). The investigators reported that problems of subtraction of mixed numbers with borrowing were the most difficult regardless whether the student was categorized as “good” or “poor” (p. 590). This study substantiated that “poor” students continued to incorrectly solve these fractions problems even after repeated attempts. The authors provided no

opinion for continued failure of “poor” students in subtraction of fractions.

Table 4 below summarized the results of the both Test 1 and Test 2 from Teacher A’s and Teacher B’s classes. The outcome revealed that slightly less than 60% of the students were able to solve three or four problems correctly on both the Test 1 and Test 2. The results also indicated that passage of time did not significantly improve student achievement on subtraction of fraction.

Table 4

Summary of Test 1 and Test 2 Results

Correct Solutions	Test 1	Test 2
3 problems	34.4%	35.1%
4 problems	24.2%	24.2%
3 or 4 problems	58.6%	59.3%

Additionally, 61% of the teachers either agreed or strongly agreed that the number of standards or concepts they must teach does not allow them to present students with innovative methods or algorithms.

Analysis of the student quizzes or Test 1 in the research found that the majority of the students favored the use of the traditional algorithm for subtraction associated with Method Two. This method or strategy was previously depicted as the algorithm which finds the common denominator of the fraction elements then proceeds to regroup from the whole number as presented in Traditional Algorithm - Decomposition (Appendix F). Table 5 displays the breakdown of the type algorithm students used within each participating teachers’ classes. It should be noted that all these teachers considered Method Two as the most popular algorithm used in

classrooms for the subtraction of fractions. Only Teacher A presented Methods Two, Three, and Four to his classes.

Table 5

Common Use of Algorithm Type by Students

Algorithm Type	Teacher A	Teacher B	Teacher C	Teacher D
Traditional (Regrouping)	54%	91%	95%	98%
Traditional (Improper)	3%	9%	5%	2%
Transactional	43%	0%	0%	0%

Research question #1. Are there statistically significant differences in the achievement levels (as measured by the researcher-developed assessment instrument) between students who only received instruction with traditional algorithms for subtraction of mixed numbers as compared with students who received instruction in both traditional and alternative algorithms for subtraction of mixed numbers? This study was designed to test the effectiveness of the transactional algorithm against the traditional algorithms for subtracting fractions. Four teachers were instructed to administer the four-problem assessment to their students after their classes had received instruction on subtraction of fractions. Students from Teacher A's classes were taught both the traditional and transactional algorithm for subtracting fractions. Students from Teacher A's classes were allowed to use the algorithm of their choice on Test 1. Analysis of the data, illustrated in Table 6, indicated that students from Teacher A outperformed students from the other teachers regardless of the type of

algorithm used on Test 1. Analysis of the mathematical aptitude showed that students from Teacher D's classes were significantly lower than the other teachers' students. However, the analysis of Test 1 revealed that they performed about the same as the students from Teacher B and Teacher C. Overall, the impression through this analysis was that student use of the transactional algorithm was more effective than the traditional algorithm.

Table 6

Success Rate on Four Fraction Problems (Test 1)

Correct	Transactional	Traditional				Total	TOTAL
	A	A	B	C	D	Traditional	
3 problems	20.0%	16.0%	13.7%	19.0%	20.4%	17.4%	17.8%
4 problems	32.9%	36.0%	22.9%	18.5%	21.3%	23.0%	24.3%
3 or 4 problem	52.9%	52.0%	36.6%	37.5%	41.7%	40.4%	42.1%

A one-way ANOVA was used to test for effectiveness of algorithms used on Test 1 for subtracting fractions among students from the five groups. The use of the transactional algorithm differed significantly, $F(4, 548) = 3.88$, $p = .0040$. The Null Hypothesis $1-H_0$ was rejected due to the statistically significant difference in achievement levels between seventh grade students using the traditional algorithm for subtraction of fractions and students using the transactional algorithm for subtracting fractions.

Tukey post-hoc comparisons of the five groups, shown in Table 7, indicated that the achievement levels of Teacher A's students A ($M = 26.0$), who used the transactional algorithm were significantly higher than Teacher D's students ($M = 21.5$), $p = 3.66$. Achievement comparisons between students who utilized the

transactional algorithm from Teacher A and students who used the traditional algorithm from the other remaining groups were not statistically significant at $p < .05$.

Table 7

Post-hoc Analysis of Test 1

Tukey simultaneous comparison t-values (d.f. = 548)					
	D	B	C	A-Traditional	A-Transactional
	21.5	23.1	23.4	24.7	26.0
D	21.5				
B	23.1	1.59			
C	23.4	1.92	0.26		
A-Traditional	24.7	2.67	1.34	1.18	
A-Transactional	26.0	3.66	2.40	2.28	0.97
critical values for experiment wise error rate:					
	0.05	2.77			
	0.01	3.33			

As previously noted, an analysis of student data indicated a significant difference in mathematical aptitude between students from the different school districts. Application of a one-way ANOVA comparing students from the same district confirmed once again a significant difference in the effective use of the transactional algorithm on the Test 1, $F(3, 440) = 2.71$, $p = .0450$.

A one-way ANOVA was used to test for the effectiveness of algorithms used on Test 1. The achievement level of students who used the transactional algorithm differed significantly from students who used the traditional algorithm at the same school site, $F(2, 273) = 3.34$, $p = .0369$. Tukey post-hoc comparisons of the three groups indicate that use of the transactional algorithm ($M = 26.0$) gave significantly higher achievement ratings than Teacher B's students ($M = 23.1$), $p = 2.53$.

Comparisons between the use of the transactional algorithm and traditional algorithm

within Teacher A's students were not statistically significant at $p < .05$.

Ex post facto research was conducted on the Test 1 data from students who were not exposed to the transactional algorithm. A one-way ANOVA was used to test for differences between students from the three teachers who only taught the traditional algorithm to their students. The achievement scores for the use of traditional algorithm within the groups were not statistically significant at the $p < .05$.

The administration of Test 2 was limited to the students of the same school site. An analysis of Test 2 on four subtraction problems, summarized in Table 8, resulted in slightly less than 16% of the total population correctly solving three of four problems. Once again only 24.2% of the students correctly solved all four problems. For students who employed the traditional algorithms, the percentages for correctly solving three and four problems were respectively 16.1% and 20.0%. Comparatively, students utilizing the transactional algorithm were more successful 14.7% and 35.3%.

Table 8

Success Rate on Four Fraction Problems (Test 2)

Correct Solutions	Algorithm Type		
	Transactional	Traditional	Both
Three	14.7%	16.1%	15.7%
Four	35.3%	20.0%	24.2%
Three or four	50.0%	36.1%	39.9%

A one-way ANOVA was used to test for the effectiveness of the algorithms used on Test 2 for subtracting fractions among students from Teacher A and Teacher

B. Results indicated the use of the transactional algorithm differed significantly compared to the use of the traditional algorithm, $F(2, 245) = 10.48, p = .0000431$. Tukey post-hoc comparisons of the three groups indicated that use of the transactional algorithm ($M = 26.6$) gave significantly higher achievement ratings than students from Teacher B group ($M = 21.9$), $p = 4.40$. The Tukey post-hoc results indicated that students from Teacher A using traditional algorithm ($M = 24.9$) achieved slightly higher ratings than students from Teacher B ($M = 21.9$), $p = 2.89$. Comparisons between the use of the transactional algorithm and traditional algorithm within Teacher A's classes were not statistically significant at $p < .05$. One-way ANOVA analysis indicated a significant difference between the students' use of traditional algorithm from Teacher A's and Teacher B's classes $F(1, 178) = 7.73, p = .0060$.

Comparisons between Test 1 and Test 2 scores from students who used the traditional algorithm in the Teacher A's classes were not statistically significant at $p < .05$. The comparison between Test 1 and Test 2 scores from students who used the traditional algorithm in Teacher B's classes also resulted in a declaration of being not statistically significant at $p < .05$. As for the students within the Teacher A's classes who utilized the transactional algorithm, results indicated no significance between student scores on Test 1 and Test 2 at $p < .05$.

The simple comparison of the use of a particular type of algorithm from Test 1 to Test 2 is displayed in Table 9. The data for students who solved all four problems correctly revealed a greater increase by those students who utilized the transactional algorithm as opposed to a slight increase by those students who utilized the traditional algorithm.

Table 9

Comparative Results for Test 1 and Test 2

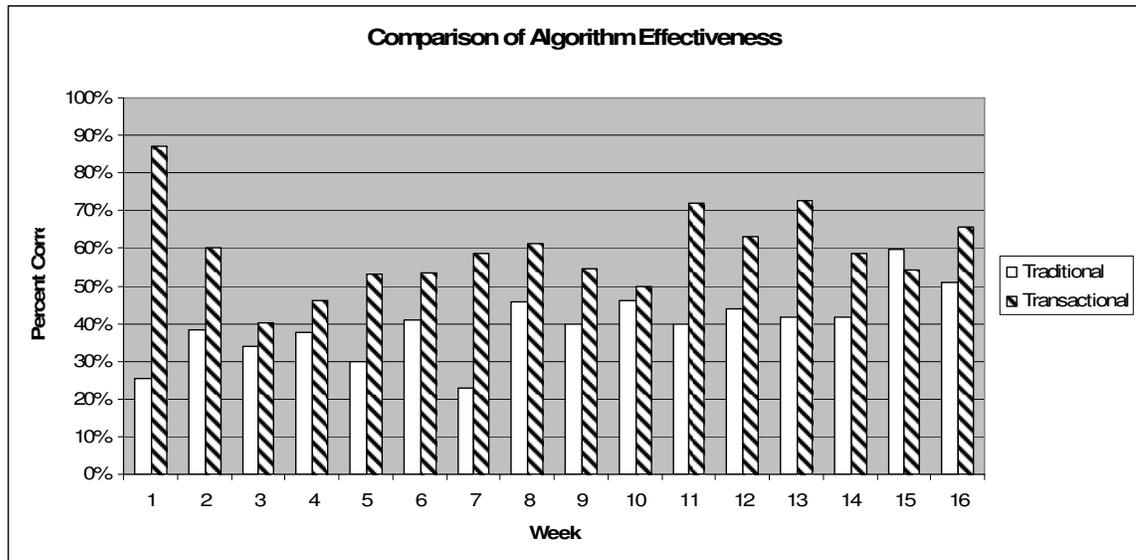
Correct Solutions	Algorithm Type			
	Traditional		Transactional	
	Test 1	Test 2	Test 1	Test 2
Three	15.8%	16.1%	20.0%	14.7%
Four	19.7%	20.0%	32.9%	35.3%
Three or four	35.5%	36.1%	52.9%	50.0%

Research question #2. Are there differences in computational achievement between students using the traditional algorithm and those using the transactional algorithm over a course of 16 weeks? A parallel study was conducted to compare the long term effectiveness of the transactional algorithm for students in the Teacher A's classes. The students were allowed to use their choice of algorithm from week to week. This portion of the research analyzed solving one problem on subtraction of mixed numbers on a weekly basis for 16 weeks. Data collected from the longevity study was evaluated differently than the data obtained from Test 1 and Test 2. Solutions to the problems were categorized as either correct or incorrect. The weekly comparison of the achievement rates based on either the traditional or transactional algorithm has been illustrated in Figure 3.

The initial week showed a high percentage (87%) of correct solutions by students using the transactional algorithm. The lowest percentage (40%) of correct solutions occurred on the third week of this study. The use of the transactional algorithm consistently outperformed the use of the traditional algorithm throughout the study except at the 15th week.

Figure 3

Comparison of Algorithm Effectiveness



Detailed examination showed that only 32% of the students attempted to use the transactional algorithm during the first week (see Figure 4). The largest percentage, 53% of students, who attempted to utilize the transactional algorithm during the 16-week study, came at the 15th week. By the end of the sixteen weeks, the trend displayed in the bar graph was that students were equally divided in their choice of what algorithm to use for subtraction of fractions.

The Longitudinal Study of the Transactional Algorithm (Appendix I) illustrated the data for the 16-week study on the comparative effectiveness in the use of the traditional and transactional algorithms. Data presented in Table 10 displays an overall rate of 39.1% correct for students who attempted to use the traditional algorithm throughout the 16 weeks. Comparatively, the success rate of 59.2% for students who attempted the transactional algorithm indicated, to this researcher, that this alternative algorithm had a clear advantage over the traditional algorithm.

Figure 4

16-Week Study: Algorithm Choice

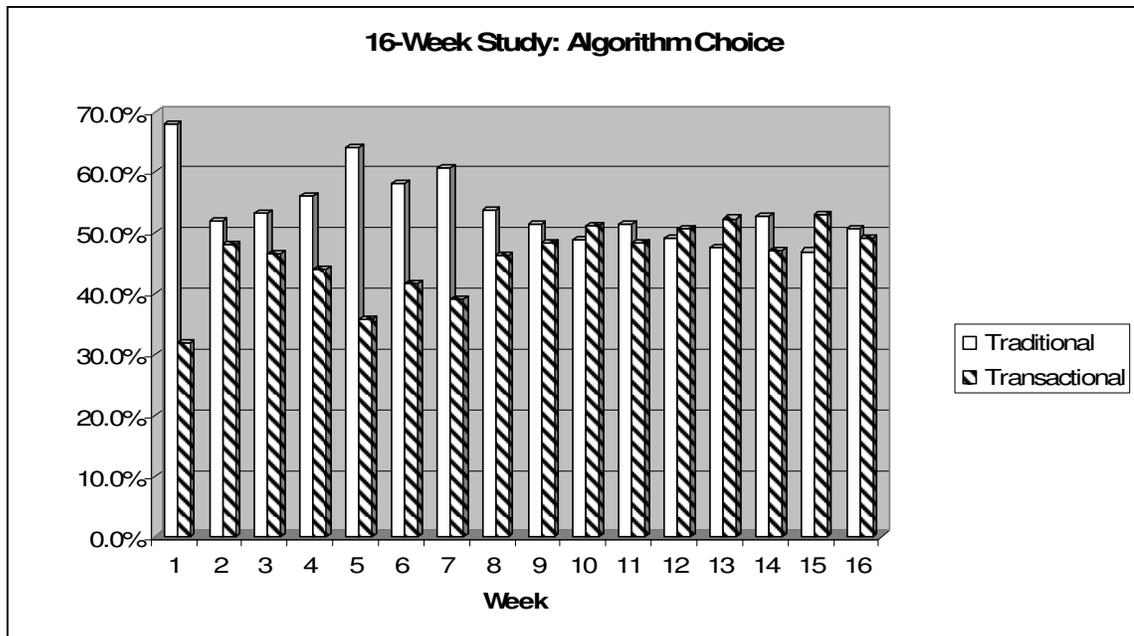


Table 10

Longitudinal Study of Transactional Algorithm (16 weeks)

	Traditional Algorithm			Transactional Algorithm		
	Attempted	Correct	Percent	Attempted	Correct	Percent
Totals	1114	436	39.1	950	562	59.2

A one-way ANOVA was used to investigate the long term effectiveness of algorithms used by students from the Teacher A's classes for 16 weeks. The results indicated the effectiveness of the transactional algorithm used by students differed significantly compared to the use of traditional algorithm, $F(1, 30) = 29.45$, $p = .00000699$. Therefore, this study calls for the rejection of the Null Hypothesis H_0 stating that no significant difference exists in mathematical achievement of seventh grade students in the long term (16 weeks) between students using traditional algorithm for subtraction of fractions when compared to students using transactional

algorithm for subtracting fractions.

Chapter Five: Summary and Discussion

Overview

This final chapter summarizes the research presented in the previous chapters and discusses the results. This chapter is divided into the following sections summarizing: (a) purpose of the study; (b) review of the research methodology; (c) discussion of the results; (d) implications; (e) limitations; and, (f) suggested recommendations for further research.

Purpose of the Study

The study of fractions has been problematic for students for generations (Brown & Quinn, 2007; Carnine & Jitendra, 1997; Sleight, 1943). The primary purpose of this investigation was to determine the effectiveness of an alternative algorithm for subtraction of fractions. This alternative or transactional algorithm was based on the everyday event of consumerism - going shopping. The secondary purpose was to study the long term effectiveness of student use of this transactional algorithm compared to the use of the traditional algorithm over 16 weeks.

Although the traditional algorithm for subtraction of fractions was considered efficient, it did little to relate itself with the experiences of a typical middle school student. This lack of connectivity, of the algorithm to life experiences, inhibited students' conceptual understanding as well as the retention of problem-solving procedures. For this teacher, conceptual knowledge was considered the direct opposite of inert or useless knowledge that Whitehead described in 1916. The data from this study upheld statements made by previous researchers that subtraction of fractions continues to be a major obstacle for middle school students.

This researcher has witnessed firsthand the difficulty students have with mathematical operations involving fractions especially subtraction of mixed numbers requiring regrouping or renaming. Though students may have mastered subtraction of whole or natural numbers, they are overwhelmed by the number of steps required to solve subtraction of fractions. Traditionally, students approached subtraction of mixed numbers by converting the fractional elements to equivalent fractions. The next step compared the fractional elements to decide whether borrowing or regrouping was necessary. Students were then required to apply the procedure of borrowing a value of one from the whole number of the subtrahend. The algorithm's following step involved a transformation of the one "borrowed" into a fraction to be added to the initial fraction of the subtrahend. These beginning steps were followed by the mathematical operations with specialized rules that apply to fractions. Only the numerators of the fractions were subtracted, while the denominators remained the same. For this teacher, no matter how well the lesson was presented, the look of confusion and frustration still appeared throughout the classroom.

For a few years, this teacher relied heavily on the traditional teaching method of emphasizing textbook algorithms. These lessons were quickly followed up with plenty of drill and practice for students. The challenge placed by one student to make the process of subtracting fractions relevant to her life changed that dependency. The challenge to make math, specially the procedure for solving subtraction of fractions, relevant placed this teacher on a long journey that led to this quasi-experimental research.

Review of the Research Methodology

The focus of this study was pursued through the quasi-experimental method. The use of this particular research method compensated for the lack of a true randomization of teachers and students into treatment and control groups. Teachers and student were placed into their classes by the onsite administrator prior to this research. The reorganization of the subjects to accommodate this study was considered too inconvenient for each school community.

For several years, this investigator reflected and revised his classroom instruction on subtraction of fractions through the use of the transactional algorithm. This investigation was about the formalization of these revisions and quantification of effective teaching strategies. Results from this study confirmed the same opinion as previous studies, that students have difficulties in subtracting fractions (Bottge & Hasselbring, 1993; Brueckner, 1928; Guiler, 1946; Ramharter, 1949; Scott, 1962), especially subtraction of mixed numbers that required borrowing. This research once again validated what writers penned ages ago about students and effective learning: students are more able to comprehend mathematical procedures that have some similarity to events of their lives.

Discussion of the Results

Algorithms. The overwhelming preference for teachers and students in subtracting fractions was the use of the traditional decomposition algorithm. Nearly every student asked was lost for words when they were solicited to relate the traditional algorithm to a real-life event. Some of the students tried to recall the word problems associated with the concept of subtracting fractions. However, careful

examinations of these word problems revealed the lack of strong connections to real-life events of a typical middle school student. For example, a word problem designed for subtracting fractions centered on a doll artist cutting fabric in mixed number lengths. Students were tasked to find out what length of lace was left over after a portion of the lace was cut from a known length. In another example, students were presented with a time problem. Students were tasked to find the difference between two specified times. Although students may have associated themselves with the scenario of working on homework, the problem contained fractional elements of an hour rather than simply the units of minutes. Beyond having solved the problem correctly through the use of an algorithm, students were not able to transfer the process to another situation.

When asked to solve a problem, students complained that they did not know how to start. With the transactional algorithm, a simple suggestion of, “Go shopping,” jump started students’ efforts to solve the problem. This connection of the transactional algorithm with an everyday event such as shopping becomes even more effective because students were able to make the visual and verbal connections through the storytelling process. In contrast, this researcher found that it took more than a simple hint when assisting students who were utilizing the traditional algorithm to solve their problems.

Test 1. An analysis of Test 1 of four subtraction problems on fractions resulted in slightly more than 17% of the total population correctly solving three of four problems. Only 24.2% of the students correctly solved all four problems. For students who employed traditional algorithms to solve their problems, the success

rates were 17.4% and 23.0%. Comparatively, students who utilized the transactional algorithm were successful at a rate of 20.0% and 32.9% respectively. These figures not only pointed to the continued difficulty of subtracting of fractions by seventh graders, but also the effectiveness of the transactional algorithm on Test 1.

Test 2. As previously reported, one-way ANOVA results for Test 2 indicated a significant difference between student scores based on which algorithm was used, $F(2, 245) = 10.48, p = 0.0000431$. The Tukey post-hoc results indicated that students from Teacher A using the traditional algorithm ($M = 24.9$) achieved slightly higher ratings than Teacher B's students ($M = 21.9$), $p = 2.89$.

Comparisons between the use of the transactional algorithm and the traditional algorithm within Teacher A's classes were not statistically significant at $p < .05$.

When instruction of the algorithms took place in Teacher A's classes, a conscious effort was made to present the lessons without giving one algorithm more credibility over another. One-way ANOVA analysis indicated a significant difference in Teacher A's students using the traditional algorithm when compared to students from Teacher B, $F(1, 178) = 7.73, p = .0060$. The weekly exposure of Teacher A's students to a problem involving subtraction of fractions may have attributed to this difference.

All of the students in Teacher B's classes used the traditional algorithm for subtracting fractions. A one-way ANOVA analysis of the Test 1 and Test 2 scores resulted in no significant difference from students of Teacher B. However, a detailed review of the Test 1 and Test 2 data from Teacher B's students indicated an overall decrease in student achievement. This decrease in student achievement may be attributed to diminished practice in subtracting fractions by Teacher B's students

between the administration of Test 1 and Test 2. As shown in Table 11, students in Teacher B's classes averaged a decrease of approximately two points from their achievement on Test 1 to their scores on Test 2.

Table 11

Test 1/Test 2 Differences - Teacher B

Teacher B's Classes			
Algorithm Used		Total Difference in Scores	Average Effect
Test 1	Test 2		
Traditional	Traditional	-192	-1.8

Students in Teacher A's classes were not mandated in the use of a particular algorithm on both the Test 1 and Test 2. They were given the choice of using whichever algorithm they felt comfortable with. The information illustrated in Table 12 summarized how many of Teacher A's students either utilized the same algorithm or switched the algorithm type in their attempts to solve problems on Test 1 and Test 2. The data indicated nearly an equal proportion of students used the same algorithm for both tests. The data also indicated that an equal portion of students switched algorithm types used from the first test to the second.

Table 12

Changes in Algorithm Used – Teacher A's Students

Teacher A's Classes		
Test 1	Test 2	Students
Traditional	Traditional	55
Traditional	Transactional	16
Transactional	Transactional	52
Transactional	Traditional	16

A one-way ANOVA analysis of the Test 1 and Test 2 scores from Teacher

A's students resulted in no significant difference in achievement levels based on the algorithm used on the problems. Further inspection of Test 1 and Test 2 scores revealed that Teacher A's students who used the traditional algorithm on both tests underwent a slight increase in student achievement. Those students who switched from the traditional to the transactional algorithm also experienced a slight increase in their Test 2 scores. The illustrated data, as shown in Table 13, were noteworthy in that students who switch from the transactional to traditional algorithm exhibited an average decrease of 4.3 points between their Test 1 and Test 2 scores. A one-way ANOVA analysis of students who used the traditional algorithm on both Test 1, and Test 2 resulted in no significant difference in achievement levels.

Table 13

Test 1/Test 2 Differences – Teacher A

Teacher A's Classes			
Algorithm Used		Total Difference in Scores	Average Effect
Test 1	Test 2		
Traditional	Traditional	10	0.2
Traditional	Transactional	8	0.5
Transactional	Transactional	76	1.5
Transactional	Traditional	-68	-4.3

Implications

Many reports pointed to the struggles students had with fractions. These struggles went beyond the classroom and into their lives as adults (Johanning, 2008; Ross & Bruce, 2009). Both studies commented on the need for “context-laden” and “contextual mathematical settings” as ways to develop situational understanding in the use of fractions. Year after year, the traditional classroom instruction has depended on the efficient transmission of algorithms. This need for efficiency led

educators to develop algorithms that have very little to do with real-life situations as the traditional or standard algorithm for subtracting mixed numbers. When teachers were questioned about why a particular algorithm was taught, most said that it was the method they were taught years ago, or they stated that these steps would guide students in producing the correct solution every time. When asked about the rationale for not teaching more than one method, most teachers stated that students would more than likely become confused having to choose which algorithm to use. The results of this study suggested that student achievement and retention improved for subtracting fractions when they were exposed to multiple problem solving strategies.

Discussion

For this particular study, lessons were presented to students that attached the multiple step process of solving subtraction of mixed numbers with narrative explanation of monetary transactions. This research found significant differences in achievement levels between the standard or traditional algorithm for subtracting mixed numbers and an alternative or transactional algorithm. The increase in mathematical achievement was observed in both the short and long timeframes. Brenner (1998) emphasized the need of making mathematics “more meaningful to children” through the connection of the child’s life with the mathematical instruction within the classroom (p. 123). A child’s education is much more than listening to a teacher and perusing through their textbook in a certain number of days. For learning to be meaningful, the learner must place significance not only on the teacher’s input but also on the learning process. The student must have a “reconciliation of personal

experience” with new knowledge for effective understanding to take place (Brown, 1996, p. 64).

Though not as meticulous as Brownell’s study, this research was confronted with the same analytical questions: Does the use of the algorithm infringe certain psychological principles of education? To what degree if any does the algorithm advance or impede sound learning?

Delimitations and Limitations

Delimitations. The study was limited to seventh grade students at three middle schools in Riverside County of the state of California. The mathematical content standards that covered operations on fractions were assigned to seventh grade level by the state of California (California Department of Education, 1997). This study utilized the researcher as one of the participating teachers. Test 1 data were collected from classes of four teachers. Data collected and analyzed on Test 2 came from two mathematics teachers at the same school site.

Limitations. Several factors affected the accurateness of this study. The administration of Test 2 was limited to just two seventh grade mathematics teachers. The scheduling of Test 2 created a conflict with the other two participating teachers. Both of these teachers voiced a concern about losing a day or two of review for the statewide assessment at the time Test 2 was scheduled to be administered to their students. The teachers also believed students would not put forth an honest effort on Test 2 if they administered the test after the statewide assessment.

Recommendations for Further Research

This research provided evidence of positive achievement by students who

utilized the transactional algorithm for subtraction of mixed numbers. The typical progression of mathematical study began with simple number recognition and progressed to the four basic operations of arithmetic. From these four basic operations students moved into the realm of fractions. Studies have recorded the difficulties, which continue to this day, that students have with fractions (Aksu, 1997; Carnine & Jitendra, 1997). An extensive study on the difficulties students had with operations on fractions found that subtraction of fractions had the largest number of errors as a group (Breucker, 1928). The knowledge required to solve mathematical problems involving the subtraction of mixed numbers demanded students demonstrate more than the mastery of the four basic arithmetic operations. Students were required to have command of fractional understanding such as equivalent fractions and common denominators.

Mathematical algorithms provided learners with an ordered set of procedures that led them to correct solutions without regard to whether the students understood the mathematical concepts. In the name of efficiency, teachers relied heavily on their own educational experiences and utilized the same algorithms they learned years ago or utilized the algorithms contained in the textbooks as the central focus of mathematical lessons. The conventional methods for subtracting fractions involved strict memorization of steps that were outside the realm of learners' experiences. The transactional algorithm used by students in this study for subtracting mixed numbers provided students with an alternative method or algorithm. The transactional algorithm was developed through a real-world experience, monetary transaction, which dealt with shopping. Most students have experienced some type of

transactional activity prior to receiving instruction on subtracting mixed numbers. This connection between a real-life event and classroom instruction provided them with a tool which not only aids in recall but also can be utilized outside of the school setting.

This investigator makes the following recommendations:

- (1) Conduct a replication study in a true pretest/posttest format to support or refute the findings that the students' use of the transactional algorithm increases achievement in subtraction of mixed numbers.
- (2) Conduct a similar study with several school sites, removing the researcher as one of the participating teachers.
- (3) Design a study to measure the differences in the affective domain of students. Does the use of transactional algorithm affect student attitude towards learning mathematics? How does the use of transactional algorithm affect student attitudes towards mathematics? This research should be designed to compare the attitudes of students towards mathematics education who are instructed strictly with the traditional algorithm compared to students taught the transactional algorithm for subtraction of mixed numbers.
- (4) Study the effectiveness of the use of transactional algorithm by students of "special needs." How does the use of transactional algorithm by "special needs" students effect their achievement in subtraction of mixed numbers?
- (5) Conduct a study comparing student achievement based on the conversion to improper fractions method for subtraction of fractions. The use of this method resulted in more incorrect solution

- (6) Analyze any achievement level differences in the use of transactional algorithm based on gender. Is there any difference in mathematical achievement for subtraction of mixed based on the gender of the student?
- (7) Modify the transactional algorithm to be employed with the subtraction of whole numbers and decimals. The design of this study would be to measure any difference in achievement level by students based on the use of an alternative algorithm for subtraction of whole numbers and decimals.

This research showed that a significant difference on achievement levels occurred due to a reality-connected algorithm. The presentation of this algorithm reaffirmed what some considered commonsense, that people learn best with concepts that have stronger connections to their activities and life-experiences.

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Teacher Background Questionnaire (page 2)

Please circle the letter that best matches with your choice to the following statements.		Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree
1.	Method (1) is the most popular algorithm for solving subtraction fractions.	A	B	C	D	E
2.	Method (2) is the most popular algorithm for solving subtraction fractions.	A	B	C	D	E
3.	The algorithm I use for subtracting fractions is the same as the one I learned in junior high/middle school.	A	B	C	D	E
4.	Method (3) & (4) are algorithms that are new to me.	A	B	C	D	E
5.	There appears to be more chances that students will make arithmetic errors with methods (1) & (2) than with methods (3) & (4).	A	B	C	D	E
6.	Students should learn more than a single method to solve math problems.	A	B	C	D	E
7.	Math answers are either right or wrong.	A	B	C	D	E
8.	In general, the teaching of math has remained the same as when I was in junior/senior high school.	A	B	C	D	E
9.	In general, mathematical algorithms in textbooks have remained unchanged.	A	B	C	D	E
10.	The number of math standards/concepts that needs to be taught does not allow for instruction in innovative methods or algorithms.	A	B	C	D	E
11.	The state test correctly assesses the mathematical ability of students.	A	B	C	D	E
12.	I believe I use multiple strategies to help students of different levels learn math in my classroom.	A	B	C	D	E
13.	Textbooks provide relevant examples for students to their life experiences.	A	B	C	D	E
14.	Computational skills are needed to assure student success in math.	A	B	C	D	E

Appendix B: Student Quiz & Survey

Show your work in the space provided.

<p>(1) Subtract and simplify if possible.</p> $\begin{array}{r} 22\frac{2}{5} \\ - 7\frac{3}{4} \\ \hline \end{array}$	<p>(2) Subtract and simplify if possible.</p> $61\frac{3}{8} - 13\frac{3}{5}$
<p>(3) Subtract and simplify if possible.</p> $\begin{array}{r} 45\frac{3}{8} \\ - 11\frac{5}{6} \\ \hline \end{array}$	<p>(4) Subtract and simplify if possible.</p> $52\frac{2}{7} - 36\frac{2}{3}$

Please complete the survey on the back of this page.

Student Quiz & Survey (page 2)

Please circle the letter that best matches with your choice to the following statements.		Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree
1.	I am sure that I can learn math.	A	B	C	D	E
2.	Last year learning math was easy for me.	A	B	C	D	E
3.	Math will not be important to me in my life's work.	A	B	C	D	E
4.	Last year learning about fractions was really hard.	A	B	C	D	E
5.	I'll need mathematics for my future work.	A	B	C	D	E
6.	I am sure of myself when I do math.	A	B	C	D	E
7.	Last year I got good grades in math.	A	B	C	D	E
8.	Math is a worthwhile, necessary subject.	A	B	C	D	E
9.	Last year I felt successful when I'm in my math class.	A	B	C	D	E
10.	Math has been my worst subject.	A	B	C	D	E
11.	This year learning math was easy for me.	A	B	C	D	E
12.	I will use mathematics in many ways as an adult.	A	B	C	D	E
13.	This year I'm getting good grades in math	A	B	C	D	E
14.	I see mathematics as something I won't use very often when I get out of high school.	A	B	C	D	E
15.	This year I feel successful when I'm in my math class.	A	B	C	D	E

Appendix C: Subtraction of Fractions Test 2

Show your work in the space provided.

<p>(1) Subtract and simplify if possible.</p> $\begin{array}{r} 31\frac{1}{4} \\ - 5\frac{3}{5} \\ \hline \end{array}$	<p>(2) Subtract and simplify if possible.</p> $49\frac{2}{3} - 12\frac{3}{4}$
<p>(3) Subtract and simplify if possible.</p> $\begin{array}{r} 58\frac{3}{8} \\ - 16\frac{5}{6} \\ \hline \end{array}$	<p>(4) Subtract and simplify if possible.</p> $25\frac{3}{4} - 9\frac{6}{7}$

Appendix D: Rubric for Evaluation of Fractions Quiz

Points	General Description	Algorithm & Solution
8	Work displays clear evidence of understanding the use of the algorithm. No mistakes were made in solving the problem.	Knowledge of algorithm is very clear. Common denominators and correct equivalent fractions are used. Exact solution calculated.
6	Work displays clear evidence of understanding the use of the algorithm. Minor mistakes were made in solving the problem. Student was within one step of correctly solving the problem.	Knowledge of algorithm is very clear. Minor errors in basic math, finding common denominators and equivalent fractions.
4	Work displays some evidence of understanding the use of the algorithm. Multiple minor mistakes were made in solving the problem. Student was within two steps of correctly solving the problem.	Knowledge of algorithm is not clear. Multiple errors in basic math, finding common denominators and equivalent fractions.
2	Work displays lack of understanding the correct use of the algorithm. Major mistakes were made in solving the problem. Student would need more than two steps of correctly solving the problem.	Knowledge of algorithm is not clear. Major multiple errors in basic math, finding common denominators and equivalent fractions.
0	Work displays complete lack of understanding an algorithm to use in this situation. Major mistakes were made in solving the problem. Student would need to start from the beginning to solve the problem.	Evidence shows complete lack of mathematical understanding in solving fraction problems.

Appendix E: Results of Teacher Survey (percentages)

	Strong Disagree	Disagree	Neutral	Agree	Strongly Agree
1. Method (1) is the most popular algorithm for solving subtraction fractions.	17	22	9	39	52
2. Method (2) is the most popular algorithm for solving subtraction fractions.	0	22	17	35	26
3. The algorithm I use for subtracting fractions is the same as the one I learned in junior high/middle school.	4	22	17	43	13
4. Method (3) & (4) are algorithms that are new to me.	4	17	9	48	22
5. There appears to be more chances that students will make arithmetic errors with methods (1) & (2) than with methods (3) & (4).	9	17	35	26	13
6. Students should learn more than a single method to solve math problems.	14	5	18	27	36
7. Math answers are either right or wrong.	9	27	18	32	14
8. In general, the teaching of math has remained the same as when I was in junior/senior high school.	0	45	23	27	5
9. In general, mathematical algorithms in textbooks have remained unchanged.	4	13	17	57	9
10. The number of math standards/concepts that needs to be taught does not allow for instruction in innovative methods or algorithms.	4	26	9	39	22
11. The state test correctly assesses the mathematical ability of students.	22	26	30	17	4
12. I believe I use multiple strategies to help students of different levels learn math in my classroom.	4	4	9	52	30
13. Textbooks provide relevant examples for students to their life experiences.	13	35	30	17	4
14. Computational skills are needed to assure student success in math.	0	0	4	57	39

Appendix F: Traditional Algorithm - Decomposition

$$\begin{array}{r}
 42\frac{2}{5} = \frac{16}{40} = 41 + 1\frac{16}{40} = 41 + \frac{56}{40} \\
 -25\frac{5}{8} = \frac{25}{40} \qquad \qquad -25 + \frac{25}{40} \\
 \hline
 16 + \frac{31}{40} \\
 = 16\frac{31}{40}
 \end{array}$$

Decomposition Algorithm – Shorten Version

$$\begin{array}{r}
 42\frac{2}{5} = 41 + \frac{7}{5} = \frac{56}{40} \\
 -25\frac{5}{8} = -25 + \frac{5}{8} = \frac{25}{40} \\
 \hline
 16 + \frac{31}{40} \\
 = 16\frac{31}{40}
 \end{array}$$

Appendix G: Traditional Algorithm - Conversion to Improper Fractions

$$42\frac{2}{5} = \frac{(42 \times 5) + 2}{5} = \frac{212}{5} \left(\frac{8}{8}\right) = \frac{1696}{40}$$

$$- 25\frac{5}{8} = \frac{(25 \times 8) + 5}{8} = \frac{205}{8} \left(\frac{5}{5}\right) = \frac{1025}{40}$$

$$\frac{671}{40}$$

$$40 \overline{) 671}$$

$$\underline{40}$$

$$271$$

$$\underline{240}$$

$$31$$

$$= 16\frac{31}{40}$$

Appendix H: Transactional Algorithm

Horizontal Format

$$\begin{array}{r}
 42\frac{2}{5} - 26 = 16\frac{2}{5} = \frac{16}{40} \\
 26 - 25\frac{5}{8} = +\frac{3}{8} = \frac{15}{40} \\
 \hline
 16\frac{31}{40}
 \end{array}$$

Vertical Format

$$\begin{array}{r}
 26 \\
 42\frac{2}{5} - 25\frac{5}{8} \\
 26 \\
 \hline
 16\frac{2}{5} + \frac{3}{8} \\
 16\frac{16}{40} + \frac{15}{40} = 16\frac{31}{40}
 \end{array}$$

Appendix I: Longitudinal Study of the Transactional Algorithm

Week	Standard or Traditional Algorithm			Transactional Algorithm		
	Attempted	Correct	Percent	Attempted	Correct	Percent
1	83	21	25.3	39	34	87.2
2	68	26	38.2	63	38	60.3
3	71	24	33.8	62	25	40.3
4	69	26	37.7	54	25	46.3
5	84	25	29.8	47	25	53.2
6	78	32	41.0	56	30	53.6
7	79	18	22.8	51	30	58.8
8	72	33	45.8	62	38	61.3
9	68	27	39.7	64	35	54.7
10	63	29	46.0	66	33	50.0
11	68	27	39.7	64	46	71.9
12	66	29	43.9	68	43	63.2
13	60	25	41.7	66	48	72.7
14	60	25	41.7	57	34	59.6
15	62	37	59.7	70	38	54.3
16	63	32	50.8	61	40	65.6
Totals	1114	436	39.1	950	562	59.2