**Proposal**

**Title –** Calculus on a Real Associative Algebra

**Program of Study –** Mathematics

**Presentation Type – Choose one of the following:** PowerPoint

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**Category:** Experimental (Theoretical)

**Abstract**: In this paper, we give an introductory study of various topics regarding calculus on a real associative algebra. Much like the study of complex variables generalizes real analysis, calculus on an algebra further broadens this study. The concepts learned in first and second semester calculus classes generalize to these algebras. We first describe the structure and different representations of a general algebra and of its binary operations, as well as functions mapping from an algebra into itself. We define the derivative of a function on an algebra as well as the conditions required for a function to be differentiable, namely the generalized Cauchy-Riemann equations of complex analysis. We also study antidifferentiation and integration on an algebra and their relation to the conditions set by the algebra. We cover sequences and series over an algebra and the general theorems of convergence that generalize, including many limit laws and the Cauchy criterion. With these tools, we consider power series in an algebra, which allow us to study special functions on it, such as the exponential, sine, cosine, and hyperbolic sine and cosine. In differential equations, we can interchange ordinary differential equations in an algebra and real partial differential equations. Power series also help us to define functions that are especially significant in a specific algebra. Indeed, as the sine and cosine functions can be defined from the exponential in the complex numbers, we can derive corresponding functions

specific to each algebra. These new "trigonometric" functions in an algebra with certain properties generalize the Pythagorean Identity by parameterizing solutions to equations obtained from the determinant of the matrix representation. These fascinating results give us a broader view of calculus and special functions on a general algebra.

**Christian worldview integration**: My Christian worldview inspires me to study God's creation and to seek to understand it better. God reveals himself through nature, as Paul says in Romans 1:20: "For since the creation of the world God's invisible qualities—his eternal power and divine nature—have been clearly seen, being understood from what has been made, so that people are without excuse" (NIV). I believe that researching and furthering our understanding of the truth in mathematics and science bring glory to God, even though they do not seem to directly relate to Him. Even when a study does not directly pertain to other people or does not seem to have any application to them, it is still quite valuable. For example, it could become applicable in the future in ways that we do not see while we are researching. However, I think that the greatest part of research's value is in the fact that it is God's truth and that it helps to expand our knowledge of the world. Thus my research is guided by a desire to organize and expand on ideas I've found in other papers that were published on the subject I study. Generalization is the key facet of this study, and I believe that the fact that it broadens our understanding of calculus makes this research important. When it comes to communicating the results of my research with others, I wish to emphasize the value of its abstraction and simply of its truth. However, I think it is also important to communicate its potential to impact society through its applications.