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A Study of the Actuarial Profession

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A Senior Thesis submitted in partial fulfillment
of the requirements for graduation
in the Honors Program
Liberty University
Spring 2007

Acceptance of Senior Honors Thesis

This Senior Honors Thesis is accepted in partial fulfillment of the requirements for graduation from the Honors Program of Liberty University.

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Abstract

The aim of this thesis is to educate the reader about the career of an actuary and will put into practice some of the rigorous mathematical applications that are used. There will also be an emphasis on the financial and business applications of the previous information, which encompasses the well roundedness of the profession. The actuarial profession is ranked one of the best in the country and rightfully so because of the skill and time it takes to perform it well. This thesis will guide the reader in understanding more clearly the high rank of the actuarial profession and the details of the statistical mathematics that are used.

A Study of the Actuarial Profession

Due to the uncertainty of future events, life necessarily involves a good deal of risk. In order to cope with this prevalent risk, people often turn to insurance. For one person to take on the entire cost of insuring his or her own future is typically impossible. Therefore, people frequently turn to insurance companies who provide insurance which allows customers to spread the cost of risk among many individuals. In order to keep these companies running smoothly, these risks must be managed. In order to help them manage these risks, these companies employ actuaries.

The actuarial profession is currently regarded as one of the best jobs in the country. An actuarial position is usually held by a college graduate who has focused on business and mathematics (“What is an actuary”, 2006). An actuary must understand statistical mathematics and probability as well as the current economic and financial issues of the world.

Job Responsibilities

Most actuaries work in the life insurance industry, but they are also employed in the government sector and in other insurance industries. This thesis will focus mainly on the responsibilities of the life insurance actuary, but it will also mention those of the property and casualty actuary. The two main responsibilities of the life insurance actuary are performing valuations and pricing products. Although these are the main responsibilities, actuaries are now being used in many areas of product development and risk management. They are even being placed into executive positions (Black & Skipper, 1994): “Actuaries are integrated into the entire operation of the company – not just valuation and pricing, but underwriting, network contracting, medical management, and

product design” (Rosenblatt, 2004, p.103). The value this profession holds as well as its awareness to the general public continues to grow.

Valuations

One of an actuary’s main responsibilities is making valuations. Valuation actuaries did not always exist. In 1985, a group from the American Academy of Actuaries and the Society of Actuaries recommended the creation of valuation actuaries to each life insurance company. In general, this person would perform a periodic assessment of the company’s assets and liabilities (Tulis, 1992).

Importance of Valuations

Generally, 85% of the liabilities of an insurance company are found in life, health, or annuity reserves (Mehr & Osler, 1956). Any small unintentional change to the value of these reserves could significantly affect the earnings of a company. Because of this, it is important for a valuation actuary to periodically assess the company’s assets and liabilities (Tulis, 1992). He or she is concerned with the solvency of the company. Because of the constant cash flow and changing market rates, this position has grown to be indispensable (Black & Skipper, 1994). Without the valuation actuary, a company will not be able to produce accurate financial statements (Tulis, 1992).

Defining Valuations and the Four Major Types

In general, when a valuation of contingent liabilities takes place, the actuary is placing a present value on future events which are unknown. Any time an insurance policy is in force, the company will be collecting premiums on the policy. The actuary is responsible for determining the probability that a claim will take place. During this time, the company must hold the money needed to fulfill the claim, also known as reserves,

should it take place (Tulis, 1992). When reserves are calculated, assumptions are made about future mortality rates and interest rates (Black & Skipper, 1994).

There are four main types of valuations done by an actuary. These include Statutory valuations (STAT), Generally Accepted Accounting Principles (GAAP) valuations, Gross Premium valuations, and Tax Reserve valuations (Tulis, 1992). Each of these has different rules and obligations, yet each is important to the financial well being of the company.

Statutory valuations are the most conservative of all valuations. These valuations help a company determine its financial stability. The rules and methods used to perform statutory valuations are all derived from state law (Tulis, 1992). Many companies will hold even larger reserves than necessary for additional financial stability: “As a matter of fact, the states only prescribe the basis on which minimum reserves are to be calculated, the companies being permitted to use any other basis which will yield reserves equal to or larger than those produced by the statutory method” (McGill, 1967, p. 241).

Choosing Statutory Reserve levels has many effects on an insurance company. Gross premiums are affected in that a company will want to avoid maintaining deficiency reserves. This occurs when the gross premium level selected for a certain underwriting class of a policy is less than the net level premium reserves. STAT reserves also affect product development. As new features are added to products, new requirements must be met for STAT reserves, making it more difficult to design desirable products. Lastly, companies are compared based upon statutory measures. Companies use this knowledge when selecting a statutory reserve basis so that the strength of their company is clearly defined (Tulis, 1992). In conclusion, Statutory valuations are the most conservative

because they require the company to withhold the highest level of reserves and can prepare the company for a worst case scenario.

The second type of valuations are GAAP valuations. These call for reserves to be calculated only if the company is publicly traded. One of the goals of GAAP accounting is that income is recognized in the period that it is earned. Because of this, GAAP considers the present value of future claims to be a liability. Each period, the liability is decreased and a valuation takes place (Black & Skipper, 1994). Even though GAAP Reserves are less conservative than STAT Reserves, they are still considered to be “reasonable and conservative” (Tulis, 1992, p. 4). It is interesting to note that even though some companies are not required to prepare GAAP financial statements, they may prepare similar ones because of how accurately it describes a company’s financial standings (Tulis, 1992). GAAP Reserves differ from STAT reserves because they also include any assumptions made about future policy lapses (Black & Skipper, 1994). Although they are considered less conservative, GAAP Reserves are equally important.

Even less conservative are the requirements of the Gross Premium valuations. These reserves are valued with the purpose of giving a “best estimate of the liabilities of the company” (Tulis, 1992, p. 5). One may wonder why there would be a need for an estimate rather than an accurate number. These numbers are used for the purpose of acquisitions and mergers. A time frame may not allow for STAT or GAAP Reserves to be calculated. Consider that net premiums cover the present value of a future claim while gross premiums include the additional policy owner dividends. Gross premium valuations “offset the present value of future benefits and expenses payable with the present value of future gross, rather than net, premiums” (Tulis, 1992, p. 5). This is done

because it has been found that most future benefits or losses are found in the gross premium balance sheet.

Finally there are Tax Reserve valuations. Any time a company earns income, it must also pay income tax. From 1958 to 1984, Tax Reserves were calculated to be the same as Statutory Reserves. Historically, deficiency reserves were not included as a taxable item, however. During the 1970s, when interest rates rose, the Congress found this method of calculating Tax Reserves to cause inequality. They soon in 1984 gave companies specific new requirements in calculating Tax Reserves which allowed for any two identical policies to be taxed the same in one tax year (Tulis, 1992).

All companies doing life insurance business in the United States are required to perform valuations (Tulis, 1992). Overall, statutory valuations are found to be the most commonly used by companies because it is not required that a company be public to hold statutory reserves. An actuary is responsible for understanding each of these reserves and knowing the methodologies used to calculate these reserves.

Statutory Valuations

Because statutory valuations are of the most importance, its components and requirements will be discussed here. It is important to note that there are three major requirements to any statutory valuation. These are the mathematical calculations, the verification of results, and the actuarial opinion. Each of these is equally important and is discussed in the following.

Mathematical calculations. Actuaries must have a sound background in mathematics in order to fulfill their job responsibilities. Concepts from the theory of compound interest and probability theory are used on a regular basis (Gerber, 1997). One

of the major reserve methods used to follow through with the mathematical calculations mentioned above is the net level premium reserve method. When this method is used, policy owners are paying a constant premium price from year to year. In the early years, he or she is paying premiums that are higher than the cost required to pay claims during those years. The excess money accumulates year after year and is called the net level premium reserve. The net level premium reserve is found using net premiums rather than gross premiums. This fund and all other reserves for any policies in force at one point in time represent the total liability a certain life insurance company may have (Harper & Workman, 1970). Equation 1 shows that the net premium is set equal to a constant percentage of the gross premiums, which have been previously defined. Here,

$$NP = K * GP \quad (1)$$

where NP = net premium, K = constant percentage, GP = gross premium. Figure 1 shows the difference in the company's premiums and claims paid for each year.

Year	Premiums Paid at Beginning of Year	Total Fund at Beginning of Year (Col. 6 previous year plus Col. 2)	Fund Accumulated for One Year (Col. 3 x 1.03)	Claims paid at End of Year (Number of deaths x \$1,000)	Balance in Fund at End of Year after Payment of Claims (Col.4 - Col. 5)
1	\$ 18,385,221.12	\$ 18,385,221.12	\$ 18,936,777.75	\$ 18,481,000.00	\$ 455,777.75
2	\$ 18,349,737.60	\$ 18,805,515.35	\$ 19,369,680.81	\$ 18,732,000.00	\$ 637,680.81
3	\$ 18,313,772.16	\$ 18,951,452.97	\$ 19,519,996.59	\$ 18,981,000.00	\$ 538,996.56
4	\$ 18,227,328.64	\$ 18,816,325.20	\$ 19,380,814.96	\$ 19,324,000.00	\$ 56,814.96

Figure 1. Net level premium reserves by year.

(Harper & Workman, 1970, p. 217)

By year four, this company had over \$56,000.00 in net level premium reserves. These funds are calculated at the end of each year and are known as terminal reserves. There are other methods of calculating terminal reserves in which the previous year's information is not always needed (Harper & Workman, 1970). Overall, this method allows for the highest reserves (Tulis, 1992).

Because the expenses of life insurance companies are significantly higher the first year a policy is in force as compared to the following years, an expense allowance is used. New reserves are created, called modified reserves, and are equal to the previous net level premium reserve less the unamortized expense. Equation 2 represents the expense allowance.

$$EA = B^{MOD} - A^{MOD} \quad (2)$$

Here, EA = expense allowance, B^{MOD} = renewal net premium and A^{MOD} is the first year net premium. From this, the value of the modified reserves is calculated in equation 3.

$$V^{MOD} = V^{NL} - EA * \left(\frac{\ddot{a}_{x+t}}{\ddot{a}_x} \right) \quad (3)$$

Here, V^{MOD} = the value of the modified reserve, V^{NL} = the value of the net level premium reserve, \ddot{a} = an annuity. It is important to know that the expense allowance is found using formulas and not found by the actual expenses of the first year of a policy (Tulis, 1992).

The second major method used to calculate reserves is the Commissioners Reserve Valuation Method (CRVM). While the previous method produced the largest reserves, this method produces the smallest. Policies in which the full preliminary term net premiums are greater than the full preliminary term modified net premiums for a 20-year-life policy are calculated using this method. This is yet another example of a modified reserve, as mentioned above. These policies will create a different modified net premium during the first year than the modified net premiums used in the remaining years of the policy. This method in itself actually uses another method to determine its modified net premiums, the Full Preliminary Term method (Tulis, 1992).

In this situation, the first year net premium is set so that the terminal reserves equal zero. The following remaining years' net premiums are equivalent and are found as if it was a policy issued at one year older for the duration of one less year. For example, a 20-year policy issued at age 30 has a modified first year net premium of \$2.07 while the following 19 years have premiums of \$22.51. The difference of \$20.44 represents the first year's net premium subtracted from any subsequent year's net premium. From this information, equation 4 is used by the Commissioner's method to find the modified net premium.

$$MNP = NLP + \frac{((NLP^1) - (Y))}{PV} \quad (4)$$

Here, MNP = modified net premium, NLP = net level premium, NLP^1 = net level premium for 19-payment life, one age older, Y = 1-year term premium, and PV = present value of life annuity due for one premium period (Harper & Workman, 1970). The

CRVM method and the net level premium methods are the most popular, although other methods continue to be in use (Tulis, 1992).

Verification of results. Once the actuary has completed the mathematical calculations for the reserves, there must be a verification of the results. Part of the actuary's job is to make sure the reserves meet the requirements laid out by the federal and state governments. There are questions that arise as to the accuracy of the assumptions of the interest rates used. The actuary must not only meet the requirements of the law, but must assure others that the reserves will meet the liability levels of the company. The actuary must take into account the future cash flows of the company, known as cash flow testing, the probability of a future epidemic which could affect claims, as well as the fall of a certain product which decreases mortality rates in that area (Tulis, 1992). All of these are extremely important because of the impact they have on the amount of reserves that are to be held. Without proper reserves, an insurance company faces many financial health issues.

Actuarial opinion. The third requirement of a valuation is the actuarial opinion. Mark Tulis (1992) stated:

In 1991 the National Association of Insurance Commissioners adopted a revised model Standard Valuation Law (SVL) which requires an Actuarial Opinion of Reserves to be filed annually. According to the 1991 SVL, "Every life insurance company doing business in this state shall annually submit the opinion of a qualified actuary as to whether the reserves and related actuarial items held in support of the policies and contracts...are computed appropriately, are based on assumptions which satisfy contractual provisions, are consistent with prior

reported amounts, and comply with applicable laws of this state. The commissioner by regulation shall define the specifics of this opinion and add any other items deemed to be necessary to its scope.” (p. 10)

This opinion is made by the appointed actuary and encompasses each of the areas mentioned in the verification of results section (Black & Skipper, 1994). It should list the actuary’s position in relation to the company, the type of work He or she does, and an overview of the items to be discussed (Tulis, 1992). The main focus of the opinion is to describe the financial health of the company (Black & Skipper, 1994).

In summary, valuations are very important to the health of any insurance company. There are many types of valuations, yet statutory valuations are the most commonly used. There are many methods to calculate the required reserves, although the CRVM and net level premium methods are used most often. After calculations are completed, the results must be verified, and an annual statement of actuarial opinion must be submitted. This part of an actuary’s job is vital to the stability of any insurance company.

Pricing

A second major area of interest to the actuary is product development. Although insurance companies frequently have a product development team, actuaries may work with many different departments to help make the product successful. Some of the steps in this process are creation of the product, choosing the underwriting classes, drafting, filing policies with the state, performing an example, and projecting prices (J. Parker, personal communication, August 1, 2006).

As products are created and developed, and given a marketing plan, an equally important part of this development is the pricing process. Premium rates vary from product to product based on the characteristics of the products (Black & Skipper, 1994):

Rate making is one of the most important functions of insurers. If rates are too low, the insurers will sustain underwriting losses and possibly become insolvent.

If rates are too high, the insurer may lose business to competitors with lower rates.

(Webb, Launie, Rokes & Baglini, 1984, p. 1)

There are many factors that must be taken into consideration when setting premium rates. The actuary not only has to take into account mortality rates, persistency, and expenses, but any additional requirements of the managers, owners, and customers. Also, throughout the process the actuary must keep in mind the financial health of the company (Black & Skipper, 1994).

Since the actuary is involved in the underwriting process, he or she must be knowledgeable of all aspects of the company, including the company's investments. Bernard Webb (1983), a professor of Actuarial Science and Insurance, concluded that "Underwriters, who are the final arbiters of pricing, seldom are intimately acquainted with their company's investment results and sometimes seem almost completely unaware of investment income" (p. 1) It is this wealth of knowledge that makes the actuary so valuable.

Actuaries are considered the "managers of the pricing process" (Black & Skipper, 1994, p. 966). They make many assumptions which are relied on heavily by others in the pricing process. This process has four major steps: "defining a pricing plan, establishing

actuarial assumptions, determining products and prices, and operating and managing results” (Black & Skipper, 1994, p. 966).

When actuaries begin to define a pricing plan, they are defining the factors that will affect future pricing decisions. They must also be aware that defining premiums is a circular process. Information learned for a previous round of product development is used to help determine premiums during the current round (Black & Skipper, 1994).

As plans develop to create new products, other departments rely heavily on assumptions given by the actuaries. Rates are assigned based on these assumptions while adding additional costs for profit. These assumptions become principles for the company to which future performance can be compared. Without these actuarial assumptions, there is no starting point for further development (Black & Skipper, 1994).

Pricing Principles

The third step, product pricing, is encompassed by three main principles: “premium rates should be adequate, they should be equitable, and they should not be excessive” (Black & Skipper, 1994, p. 20). If each of these standards is met, the actuary will develop a fair and profitable price for the company and the customer.

Rate adequacy implies that the rates charged for equal policies now plus the reserves must be sufficient to cover future claims and expenses. Equal policies are defined as policies that were sold under the same product name and have the same rates and benefits. Because one does not know for certain the future claims that will be made, the company continues to rely on the assumptions of the actuary. The actuary is responsible during this step of the process to provide as accurate a probability as possible. Mortality rates are used to help provide these numbers (Black & Skipper, 1994).

The second standard is rate equity. This implies that for each classification of customers equal rates are given based on these classifications. If many different groups are created, the expenses may increase, but the customer may also feel that he or she is being treated more fairly. The process of deciding which classification, if any, a customer belongs to is called underwriting. The actuary is responsible for finding the rates that belong to each underwriting class (Black & Skipper, 1994). He or she must take into account factors such as height, weight, sex, age, smoking, etc (J. Parker, personal communication, August 1, 2006). Through this information the actuary will then create different underwriting classes, each with different rates.

The third pricing principle is the standard that rates do not become excessive. Just like any company, sales will decrease if the prices become too high for the product being offered. Since competition is high in the insurance industry, companies must be careful not to ignore this important aspect of product pricing (Black & Skipper, 1994).

Elements of Pricing

There are five major areas that an actuary needs to have an understanding of with relation to product pricing. These areas are the probability of the claims, the benefits of the product, profits, expenses, and the time value of money (Black & Skipper, 1994). To fully understand each of these ideas, an actuary should have sufficient knowledge of a typical life insurance company balance statement. Figure 2 shows a typical balance statement of a life insurance company.

PERIOD ENDING	31-Dec-06	31-Dec-05	31-Dec-04
Assets			
Current Assets			
Cash And Cash Equivalents	7,866,000	4,843,000	4,051,000
Short Term Investments	2,709,000	3,306,000	2,663,000
Net Receivables	14,490,000	12,186,000	6,696,000
Inventory	-	-	-
Other Current Assets	-	-	-
Total Current Assets	-	-	-
Long Term Investments	324,561,000	300,614,000	234,660,000
Property Plant and Equipment	7,000	-	379,000
Goodwill	4,897,000	4,797,000	-
Intangible Assets	-	-	-
Accumulated Amortization	-	-	-
Other Assets	152,334,000	136,258,000	94,023,000
Deferred Long Term Asset Charges	20,851,000	19,641,000	14,336,000
Total Assets	527,715,000	481,645,000	356,808,000
Liabilities			
Current Liabilities			
Accounts Payable	48,271,000	35,501,000	30,170,000
Short/Current Long Term Debt	1,449,000	1,414,000	1,445,000
Total Current Liabilities	-	-	-
Long Term Debt	13,759,000	12,022,000	7,412,000
Other Liabilities	427,882,000	401,623,000	292,206,000
Deferred Long Term Liability Charges	2,278,000	1,706,000	2,473,000
Total Liabilities	493,639,000	452,266,000	333,706,000
Stockholders' Equity			
Redeemable Preferred Stock	278,000	278,000	278,000
Preferred Stock	1,000	1,000	-
Common Stock	8,000	8,000	8,000
Retained Earnings	16,574,000	10,865,000	6,608,000
Treasury Stock	(1,357,000)	(959,000)	(1,785,000)
Capital Surplus	17,454,000	17,274,000	15,037,000
Other Stockholder Equity	1,118,000	1,912,000	2,956,000
Total Stockholder Equity	33,798,000	29,101,000	22,824,000
Net Tangible Assets	\$28,901,000	\$24,304,000	\$22,824,000

Figure 2. Sample annual balance sheet for MetLife Inc.

(“MET”, 2006)

Of these important elements, the probability of the claims and the benefit of the product were discussed in the valuation section, while profits and expenses would be understood from an income statement. The balance statement helps the actuary to understand the concepts of liabilities with relation to the company’s equity.

Mortality Tables

Another important element of concern in pricing is the understanding of mortality tables: “Statistical tables showing probabilities of death are called ‘mortality’ tables” (Mehr & Hedges, 1956, p. 14). These are the basis for determining appropriate premium rates. Utilizing these tables helps to apply the principles mentioned before of rate adequacy, equity, and non-excessiveness (Mehr & Hedges, 1956). Sometimes however, the actuary is not always aware of how these rate tables were created. This is an important characteristic to attain to and helps the actuary understand which of the many tables that exist should be used.

Mortality table construction. There are a few foundational mathematical concepts that the actuary would be familiar with when understanding the construction of a mortality table. A few assumptions used in determining mortality are the uniform distribution of deaths, the Balducci hypothesis, and the constant force of mortality. In each of these, according to Robert Batten in 1978, the following variables are assigned to be:

d_x = the number of persons who are expected to die in a one-year interval between ages x and $x + 1$

q_x = the probability that a life aged x will not survive to age $x + 1$

u_x = the annualized force of mortality operative at age x

While a mortality rate can represent a “relationship of the frequency of deaths of individual members of a group to the entire group membership over a particular time period” (Barons, 2000, p. 318), the force of mortality represents the same relationship yet represented at an instance. To annualize this data is to represent it on an annual basis. (Barons, 2000)

The first assumption is the uniform distribution of deaths, represented by the function $f(t) = {}_tq_x$ being linear over the interval $0 \leq t \leq 1$. With this assumption,

$${}_tq_x = a + bt \quad \text{so,} \quad (5)$$

$${}_1q_x = a + b = q_x \quad \text{and,} \quad (6)$$

$${}_0q_x = a = 0 \quad \text{solving} \quad (7)$$

$$a = 0 \quad \text{and} \quad b = q_x \quad \text{implies} \quad (8)$$

$${}_tq_x = t * q_x \quad (9)$$

As these formulas imply, the survivorship curve, which is being represented here, is a “series of straight lines, each with its own slope” (Batten, 1978, p. 3). The actuary would then be able to assume the following from the previous observations:

$$u_{x+t} = \frac{q_x}{1 - (t * q_x)} \quad (10)$$

$${}_t p_x * u_{x+t} = q_x \quad (11)$$

What is important to note from this assumption is that u_{x+t} is an increasing function of t (Batten, 1978). Since u represents the force of mortality, this observation makes sense because one knows the probability of death is higher as age increases. The basis of all actions performed to price life insurance uses this assumption and, therefore, is the most important observation to be found within this first mortality assumption.

The second mortality assumption that is often used is the Balducci hypothesis. Gaetano Balducci was an Italian actuary who in 1920 assumed that $f(t) = {}_{1-t}q_{x+t}$ was linear over the interval $0 \leq t \leq 1$. Following this assumption, it can be deduced that:

$${}_{1-t}q_{x+t} = a + bt \quad \text{so,} \quad (12)$$

$${}_1q_x = a = q_x \quad \text{and,} \quad (13)$$

$${}_0q_{x+1} = a + b = 0 \quad \text{solving} \quad (14)$$

$$a = q_x \quad \text{and} \quad b = -q_x \quad \text{implies} \quad (15)$$

$${}_{1-t}q_{x+t} = (1-t) * q_x \quad (16)$$

The actuary would be able to assume the following from equations 12-16:

$$I_{x+t} = \frac{I_x * I_{x+1}}{I_{x+1} + (t * d_x)} \quad (17)$$

where I_x represents the survivorship curve mentioned above. It can be seen here that the survivorship curve is a reciprocal of a straight line, also called a hyperbola. Making assumptions based on these results is much more difficult than under the previous assumption. One should also note that u_{x+t} , the force of mortality, is decreasing. This is true, however, only for a certain “single unit age interval” (Batten, 1978, p. 6). Since u_{x+t} is discontinuous at its endpoints, this decreasing factor is applicable (Batten, 1978).

The third mortality assumption is the constant force of mortality. Under this assumption it is obvious that $u_{x+t} = u$ for $0 \leq t \leq 1$. So, in summary the force of mortality is increasing with the uniform distribution hypothesis, decreasing with the Balducci hypothesis, and constant under the constant force of mortality assumption (Batten, 1978). Each of these assumptions is important to help understand how a mortality table is constructed. To fully understand the use of the mortality tables available, the actuary must understand these foundations to the construction process.

Probability calculations using mortality tables. After the actuary understands the construction process, he or she must know how to use these tables to calculate important probabilities. Some of the most common probabilities calculated from these tables are the probability of living and the probability of dying. Many variations of these simple probabilities can be calculated as well (Harper and Workman, 1970).

Probabilities of living and dying will be calculated below using selections from the 1958 Commissioners Standard Ordinary Table for males as seen in figure 3. Four variables are used in this table: x , I_x , d_x , q_x . These tables are used to represent a group of people at the same age, x . I_x represents the number of people from this group who are still living at age x . The variable d_x represents the number of people who will die when they are at age x . Lastly, q_x represents the probability that a person age x will die within one year (Harper and Workman, 1970).

Age (x)	I_x	d_x	q_x	Age (x)	I_x	d_x	q_x
5	9868375	13322	.00135	43	9135122	42382	.00453
10	9805780	11865	.00121	44	9093740	44741	.00492
15	9743175	14225	.00146	45	9048999	48412	.00535
20	9664994	17300	.00179	50	8762306	72902	.00832
25	9575636	18481	.00193	51	8689404	79160	.00832
30	9480358	20193	.00213	80	2626372	288848	.10998
35	9373807	23528	.00251	85	1311348	211311	.16114
40	9241359	32622	.00353	90	468174	106809	.22814
41	9208737	35362	.00384	95	97165	34128	.35124
42	9173375	38253	.00417	99	6415	6415	1.00000

Figure 3. 1958 C.S.O. Table (male)

(Harper and Workman, 1970, p. 375)

To calculate a simple probability of living, written as p_x , one must use a mortality table and be familiar with the symbols involved. The probability that a person age x will live to reach the age $x + 1$ can be written as follows:

$$p_x = \frac{I_{x+1}}{I_x} \quad (18)$$

Calculating the probability that a person age 50 will live at least one whole year would result in the following:

$$p_{50} = \frac{I_{50+1}}{I_{50}} \quad (19)$$

$$= \frac{I_{51}}{I_{50}} \quad (20)$$

$$= \frac{8,689,404}{8,762,306}$$

$$= .991680$$

The result yields a 99.168% chance that a male age 50 will live to age 51. One would expect from this result that the person holds a .932% chance of dying within the next year. This holds true from the fact that

$$p_x + q_x = 1.0 \quad (21)$$

Since only one of these probabilities can be true for a person, their sum equals 1.0.

Knowing either the probability of living or the probability of dying, one could easily find the other probability by subtracting the first from 1.0 (Harper and Workman, 1970).

Suppose one would like to find the probability that a person at age x would either live or die within more than one year. To calculate the probability that a person age x would live for n years, one would use the following formula.

$${}_n P_x = \frac{I_{x+n}}{I_x} \quad (22)$$

As an example, the following calculations represent the probability that a man age 80 will live at least five more years.

$${}_5 P_{80} = \frac{I_{80+5}}{I_{80}} \quad (23)$$

$${}_5 P_{80} = \frac{I_{85}}{I_{80}} \quad (24)$$

$$= \frac{1,311,348}{2,626,372}$$

$$= .49930$$

From this solution, the probability that a man age 80 will live at least five more years is 49.930%. Again, because people can either only live or die within a certain number of years, the following equation holds to be true (Harper and Workman, 1970):

$${}_n P_x + {}_n q_x = 1.0 \quad (25)$$

Suppose one wanted to find the probability of dying without first having to find the probability of living. Equation 26 represents the probability that a male age x will die within n years.

$${}_nq_x = \frac{I_x - I_{x+n}}{I_x} \quad (26)$$

For example, to calculate the probability that a man age 20 will die within the next 30 years, start with equation 27:

$${}_{30}q_{20} = \frac{I_{20} - I_{20+30}}{I_{20}} \quad (27)$$

$${}_{30}q_{20} = \frac{I_{20} - I_{50}}{I_{20}} \quad (28)$$

$${}_{30}q_{20} = \frac{9,664,994 - 8,762,306}{9,664,994} \quad (29)$$

$$= \frac{902,688}{9,664,994}$$

$$= .093398$$

From this result, the probability that a man age 20 will die within the next 30 years is 9.3398%. This number is expected since 50 is still considered a young age to die (Harper and Workman, 1970). Understanding these simple probabilities and having the knowledge to read mortality tables is crucial to the actuary. These probabilities play a large role in the pricing process as well.

Net single premiums using mortality tables. Not only can the actuary calculate probabilities of living and dying from mortality tables, but he or she can also calculate net single premiums, an important aspect of the pricing process. The net single premium is also known as “the present value of the benefits offered by a particular insurance policy” (Harper and Workman, 1970, p. 188). With the use of the mortality table and designated interest rates, a premium can be calculated. The actuary must also increase the premium thereafter to account for expenses and profits of the company.

A few concepts must first be explained in order to understand the calculation of a net single premium. To recall, I_x represents the number of people of a group still living at age x . The variable d_x represents the number of those people who are expected to die during that same year. When calculating a premium, the premium is paid by I_x number of people, while the benefits at the end of the year are paid to d_x number of people. To calculate the premium, the amount each pays in, you must take into account both of these numbers, as well as the interest rate on the money paid in. These concepts result in equation 30 assuming \$1000 of life insurance for a one year period:

$$P * I_x = 1000 * d_x * v \quad (30)$$

Where P is the net single premium, $v = \frac{1}{1+i}$, and i is the interest rate. To change the amount of coverage, one would simply change 1,000 to the appropriate number. Since it is a one year period, this equation could also be considered to find the net single premium for term insurance for one year. If the term were to cover more than one year, $(d_x * v)$ would be replaced with

$$(d_x * v + d_{x+1} * v^2 + d_{x+2} * v^3) \dots \quad (31)$$

and so on until the number of terms equals the number of years insured. This formula allows for interest to be accumulated and the changing number of expected deaths to be accounted for.

As an example, using the 1958 C.S.O. Table and 3% interest, a net single premium at age 40 for \$3,000.00 of life insurance lasting for five years will be calculated. The following line diagram will help to explain this process even further.

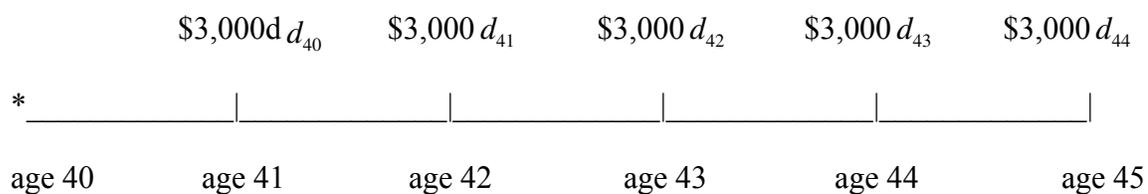


Figure 5. \$3,000 5 Year Term Insurance Age 40 Line Diagram

(Harper and Workman, 1970)

In this example,

$$v = \frac{1}{1 + .03} = .978074 \quad (32)$$

$$P * I_{40} = 3,000(d_{40} * v + d_{40+1} * v^2 + d_{40+2} * v^3 + d_{40+3} * v^4 + d_{40+4} * v^5) \quad (33)$$

$$P * (9,241,359) = 3,000(32,622 * .97807 + 35,362 * .94259 + 38,253 * .91513 \\ + 41,382 * .88847 + 44,741 * .86259) \quad (34)$$

$$P * (9,241,359) = 3,000(31,672,33,332,38,252 + 36,767 + 38,593) \quad (35)$$

$$P * (9,241,359) = 3,000 * 178,616 \quad (36)$$

$$P * (9,241,359) = 535,848,000 \quad (37)$$

$$P = \$57.98 \quad (38)$$

As a result of these formulas, the insured at age 40 would pay a net single premium of \$57.98 for five years of term insurance at a 3% interest. The number of years insured, interest rate, and age of insured can be interchanged within this formula to find any net level premium for term life insurance. In the case of whole life insurance, the same formula may be used, but the number of years insured extends until the end of the mortality table (Harper and Workman, 1970). Once the actuary has utilized the mortality table to calculate premiums, he or she would increase this amount to cover any expenses

and profits of the company. These steps would conclude the pricing process for the actuary.

Overall, the actuary must have sufficient knowledge of many areas to understand the pricing process. These areas include current population trends, mortality tables and their formation, the financial statements of the company, the probability of the claims, and any additional requirements to pricing made by the managers and owners of the company. This is a well rounded base for the understanding of the price development process. This profession requires a great deal of knowledge and the ability to filter and use from what is considered necessary.

The Property and Casualty Actuary

Until this point, only the life insurance actuary has been discussed. This approach was taken because the majority of working actuaries belong to the life insurance industry. Actuaries can also work with pensions, the government, or other financial institutions. Because of the many investment options available today, actuaries can also work in financial institutions to help customers make the best investment decisions (“Careers”, 2005).

Aside from life insurance, the next most popular position for the actuary is in the property and casualty insurance industry. The actuary working in the property insurance field must become aware of contracts made concerning areas such as fire, theft, dishonesty, surety, boiler and machinery, glass, inland marine, and ocean marine insurance (Williams & Heins, 1985). These are only a few of the many areas that property insurance can cover.

Development of Property and Casualty Actuaries

The development of the property and casualty insurance industry has been much slower than the life insurance industry. In the 1970s, the property and casualty actuary began to underwrite policy holders with new classifications of age, sex, and marital status. When equality was becoming an issue of concern in the United States, this idea of classification was disliked, slowing the progress of the underwriting process (Bartlett & Walter, 2000).

Prior to this time, these actuaries were mostly involved in analyzing and preparing rate levels and filing these findings. Some claims had been incurred but not yet filed with the company, and the actuary was responsible for establishing reserves for such claims. During the 1970s, the state governments decided to closely supervise more property and casualty insurance companies. Because of this supervision, the actuary began to take on many more responsibilities. Actuaries were overall more responsible for reporting a company's status to the government in various actuarial opinions and statements. Part of this opinion included a statement that the company was holding enough reserves to cover the contingent liabilities of the company (Bartlett & Walter, 2000).

Although life insurance actuaries were already required to report an actuarial opinion, it was just at this time that property and casualty actuaries were required to do so. Now that the actuary had another major responsibility, the industry felt there was a lack of qualified professionals to do the job (Bartlett & Walter, 2000). As standards for the actuary rose, there need for accountability also rose. The Casualty Actuary Society (CAS) formed and provided a way for businesses to know whether their actuarial employees were meeting government standards. The following information from the CAS website explains the purpose of the organization:

The purposes of the Casualty Actuarial Society are to advance the body of knowledge of actuarial science applied to property, casualty and similar risk exposures, to establish and maintain standards of qualification for membership, to promote and maintain high standards of conduct and competence for the members, and to increase the awareness of actuarial science. (“About CAS”, 2006, p. 1)

The Casualty Actuary Society had fewer than 600 members in the 1970s. The CAS now has more than 4,000 members. Casualty and property actuaries continue to be held accountable for their qualifications (“About CAS”, 2006). As exposure to the profession increases, the number of qualified individuals increases, yet there is still a great demand for actuaries.

The Differences Between the Life, Property, and Casualty Actuaries

Many of the concepts are the same for the actuary in the property and casualty insurance industries, yet there is one main difference. In life insurance, the event being insured is certain to happen. The timing of the event is in question. In property and casualty insurance, the event being insured may never happen. Because of this, certain procedures may differ for the actuaries working in these areas.

The largest difference for a property or casualty actuary is in calculating reserves. There are fewer statutory requirements for the casualty actuary (Bartlett & Walter, 2000). This is quite logical because there are fewer claims of property and casualty policies than life policies. The two main areas of liability reserves that must be met for the property actuary are loss reserves and unearned premium reserves (Huebner, Black, & Cline, 1976). Loss reserves refer to funds set aside for claims that have not yet been reported.

Unearned premium reserves refers to any sum of money paid by policyholders in advance for which they are yet to receive insurance. Because premiums are usually paid each month, the company can only claim those premiums to be earned at the conclusion of each month. If they are paid for a year, yet a month has passed, the additional eleven months of premiums are kept separate as a liability to the customer. A new month's premium can only be considered to belong to the company as that month concludes (Huebner, Black, & Cline, 1976). These two reserves are the main concern of any property insurance actuary.

Profession Requirements

Because the actuarial profession requires so much knowledge and insight, actuaries are constantly tested on this knowledge. Each actuary must meet certain requirements to obtain and keep a job. Most actuaries come from a background of only bachelor's degrees in the areas of actuarial science, mathematics, economics, finance, or accounting. Although a degree in actuarial science is preferred, not all schools offer such a program. Aside from which initial degree is chosen, the potential actuary is encouraged to take additional classes in each of the following areas: mathematics, economics, finance, accounting, and business ("What is an actuary", 2006). Most actuaries are well rounded and knowledgeable of many of these subject matters.

There is also a series of exams required of anyone seeking to become an actuary. There are a total of eight exams, each with its own subject matter. While still considered an actuarial student, actuaries who take these exams are usually working full time in their industry. Many companies will give students a portion of their work day as study time. Many will also give a pay increase for each exam passed ("What is an actuary", 2006).

The actuary must constantly take these exams to move forward in his/her career.

Although given time during work, these exams require a large amount of studying outside of the office. Many students study for hundreds of hours for each test. They may also be required to sit for an exam each session, which occurs twice a year. The first 6-10 years of the actuarial student's career will include taking exams on a consistent basis, yet he or she will also have advanced significantly in their career.

As mentioned before, each actuarial exam has its own subject matter. Those actuaries interested in working in life insurance versus property and casualty will follow an exam schedule given to them by the Society of Actuaries (SOA). Using this as a model, Exam P/1 covers basic probability. It will cover material learned in a typical first and second course of a probability class during college. Exam FM focuses on the theory of interest and life insurance mathematics. Most companies are looking for entry level actuaries who have completed at least Exam P/1, but hopefully Exam FM as well (J. Parker, personal communication, August 1, 2006).

Exam MLC focuses more on life contingent actuarial models. Exam MFE also has a specialized financial economics segment. The construction and evaluation of actuarial models are covered in Exam C. After an actuary has completed the first five exams and met other requirements by the SOA, he or she is considered to be an Associate of the Society of Actuaries (ASA). The actuary may also choose from a variety of topics for the remaining two exams. The sixth and seventh exam topics include advanced finance (AFE), portfolio management (APMV), retirement benefits (CSP-R), individual life and annuities (CSP-I), and group health (CSP-GH) ("Online Exam", 2006). After completing Exam VII, and any other requirements by the SOA, the actuary may become

a Fellow of the Society of Actuaries (FSA) (“What is an actuary”, 2006). Both the ASA and FSA titles are highly regarded.

In addition to the actuarial exams, there are a number of educational courses the actuary must take. If approved, these courses may be taken during college for credit later in the profession. Otherwise, these courses must be taken once one becomes an actuarial student. These credits are called Validation by Educational Experience (VEE) credits (“What is an actuary”, 2006).

Growth and Opportunities of the Actuarial Profession

The actuarial profession in general is growing and continues to be in high demand. Because of the numerous examinations, qualifications may be hard to meet. Since so much work is done to pass each of these exams and meet other requirements, the position is respected and can open the doors to many other areas in life. For those that do qualify and have a desire for the job, finding a well-paid position will be quite simple. Entry level actuaries in 2005 paid more than \$52,000 (“Actuaries”, 2006). “Median annual earnings of actuaries were \$76,340 in May 2004” (“Actuaries”, 2006).

Figure 5 presents a recent survey of an actuarial student’s salaries:

Life & Health October 2006 (in thousands)	0-0.5 yrs excluding sign-on bonus	0.5- 2.5 yrs	2.5- 4.5 yrs	4.5- 6.5 yrs	6.5-9.5 yrs	9.5- 14.5 yrs	14.5- 19.5 yrs	19.5+ yrs
1 course	42-58	47-60	51-64					
2 courses	45-59	50-63	55-69	57-72				
3 courses	48-61	54-66	60-73	62-79				
4 courses	50-64	56-73	62-78	66-83	70-94			
5 courses		59-75	65-82	71-90	77-103			
ASA		64-83	73-93	81- 100	87-115	94-129	101- 212	104- 226+
7 courses				84- 106	89-121	98-134		
8 courses				89- 120	93-131	106- 148		
FSA			89- 124	96- 137	105- 155	118- 181	133- 280	141- 304+

Figure 5. Actuary Salary Survey

(“Actuary Salary Survey”, 2006)

Six out of ten actuaries hired will work in the insurance industry. Many firms are also hiring consulting actuaries to work with risk within the company (“Actuaries”, 2006). The demand is high, yet for those who meet the qualifications and have the desire to work in these fields, the job can be significantly rewarding and very desirable in many ways. In fact, the actuarial profession has been rated as one of the top five jobs in the United States (“What is an actuary”, 2006).

Criticisms of the Actuarial Profession

Despite the demand for the position and its high rank in the United States, some countries do not find the actuarial profession to be quite as highly regarded. In England, a recent study was performed to analyze the actuarial profession. Sir Derek Morris

published his finding on December 17, 2004. From this analysis, Morris found that the profession was unable to adapt quickly enough to the changing circumstances within the country. He also found that the general public lacked the knowledge of the professional level of their actuaries. When actuarial opinions were given, they were not challenged or tested enough. Overall, the standards given to actuaries were low and limited. To claim an actuaries' opinion may not be providing the truth about the solvency of a company could create many potential problems. This type of potential behavior could cause many legal concerns. Because this study was taken over the entire country of England and provided the previous results, this study seemed pertinent to this paper.

Conclusion

The role of the actuary in today's insurance world is crucial. As the responsibility and accountability of the profession has grown, awareness of the profession has increased as well. The actuary primarily works in the life insurance field, but can also work for the government or other insurance industries. With a focus on business and mathematics, the actuary only needs a bachelor's degree to begin his/her career. While working with the Society of Actuaries, the Casualty Actuary Society, or other organizations, the actuary moves through his/her career by completing courses and exams in his/her field ("What is an actuary", 2006).

As mentioned, most actuaries work with life insurance. They are responsible mainly for the valuation of the company's liabilities and pricing their products. Government regulations must be met when valuing the liabilities (Black & Skipper, 1994). The actuary must have knowledge of the four main types of reserves. These include Statutory Reserves, GAAP Reserves, Gross Premium Reserves, and Tax

Reserves. There are multiple methods used to calculate these reserves, although the most common are the net level premium reserve method, and the Commissioners Reserve Valuation Method (Tulis, 1992). The property and casualty actuaries also need to understand these processes, yet these actuaries will calculate fewer reserves since fewer claims occur within this industry (Bartlett & Mavis, 2000). In either industry, the actuary must have complete knowledge of the company's health and reserve methods used.

The main responsibilities within pricing include assuring the fact that "premium rates should be adequate, ... equitable, and ... not ... excessive" (Black & Skipper, 1994, p. 20). Actuaries work with many departments within the company to assure that these principles are true. The actuary should also be familiar with each of the aspects of the company's balance sheet (Black & Skipper, 1994).

Overall, the actuary is a well-rounded employee whose many responsibilities maintain the solvency of a company. He or she is given many duties within the company and must therefore be aware of the many facets of a company's financial health. Because of the amount of responsibility given, the actuary is held accountable for his/her opinions and must be able to comply with the many government and company-based requirements. Actuaries continue to be looked upon highly and the public's awareness of the profession continues to grow. Actuarial science would be a rewarding career for any math, finance, economics, or accounting major.

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Appendix A

Acknowledgments

A special recognition is offered to Joshua Parker of Genworth Financial, Inc. for his kindness and valuable conversations that helped this thesis to become truly insightful. Dr. Xavier Retnam, Dr. Glyn Wooldridge, and Professor Rodney Chrisman are also recognized for their assistance in improving and revising the contents of this thesis.

Appendix B

Insurance Terminology

The following definitions will help the reader in understanding some of the topics discussed in this paper. Each was found and is quoted in full by Rubin in 2000:

Claim: request by an insured for indemnification by an insurance company for loss incurred from an insured peril (p. 85)

Deficiency Reserves: addition to reserves of a life insurance company required by various states because the valuation premium is greater than the gross premium. Without a deficiency reserve, the normal reserve by itself would be less than the actual reserve required (p. 128)

Insurance: mechanism for contractually shifting burdens of a number of pure risks by pooling them (p. 244)

Lapse: 1. in property and casualty insurance, termination of a policy because of failure to pay a renewal premium. 2. in life insurance, termination of a policy because of failure to pay a premium and lack of sufficient cash value to make a premium loan. (p. 269)

Life Insurance: protection against the death of an individual in the form of payment to a beneficiary—usually a family member, business, or institution. In exchange for a series of premium payments or a single premium payment, upon the death of an insured, the face value (and any additional coverage attached to a policy), minus outstanding policy loans and interest, is paid to the beneficiary. (p. 280)

Mortality Rate: relationship of the frequency of deaths of individual members of a group to the entire group membership over a particular time period. (p. 318)

Mortality Table: chart showing rate of death at each age in terms of number of deaths per thousand (p. 319)

Premium: rate that an insured is charged, reflecting his or her expectation of loss or risk. The insurance company will assume the risks of the insured (length of life, state of health, property damage or destruction, or liability exposure) in exchange for a premium payment. Premiums are calculated by combining expectation of loss and expense and profit loadings. Usually, the periodic cost of insurance is computed by multiplying the premium rate per unit of insurance by the number of units purchased. The rate class in which the insured is placed includes large numbers of individuals with like characteristics who pose the same risk. Every individual in a given class will not incur the same loss; rather each has approximately the same *expectation* of loss (p. 392)

Premium Mode: frequency of premium payment, monthly, quarterly, or annually (p. 394)

Underwriting: process of examining, accepting, or rejecting insurance risks, and classifying those selected, in order to charge the proper premium for each. The purpose of underwriting is to spread the risk among a pool of insureds in a manner that is equitable for the insureds and profitable for the insurer (p. 536)